Quantum shot noise in a normal-metal–superconductor point contact

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(Received 18 April 1994)

The distribution function for the current noise in a normal-metal–superconductor quantum point contact is calculated. It is shown that this distribution describes independent processes when a charge $\pm e_0$ or $\pm 2e_0$ passes through the contact. At low temperature and voltage only processes with double-charge transfer are relevant. At zero temperature and low voltage the distribution has a binomial form.

I. INTRODUCTION

Discreteness of charge carriers gives rise to the shot noise, i.e., to time-dependent fluctuations of the current around its mean value I. Recent interest in the shot noise in mesoscopic devices at low temperature was inspired by the pioneering works of Khlus[1] and Lesovik.[2] They derived an expression for the mean square of current fluctuations and found that, due to the quantum correlations imposed by the Pauli exclusion principle, the intensity of the shot noise is suppressed with respect to its classical value. Later, their results were generalized and applied to various cases.[3–5]

It was shown by Lesovik and Levitov (LL) that not only is the second correlator suppressed, but the entire distribution function changes in the quantum regime from Poissonian to binomial.[6,7] LL also showed[8] that when a small ac component with frequency Ω is added to the voltage V, steps occur on the noise-voltage characteristic at voltages $V=\hbar\Omega n/e_0$ with integer n.

The noise of a normal-metal–superconductor (NS) junction was first considered by Khlus,[1] and recently his analysis was extended to a disordered case by de Jong and Beenakker.[9] Our purpose was to find the distribution function for the shot noise in a clean NS point contact. We did this using a slight modification of the LL procedure, and found that the Andreev reflection at the NS boundary leads to several new effects. At low temperature and voltage the distribution remains binomial, but with a double charge. Thus, the LL steps on the noise-voltage characteristic of an ac-biased junction exist at even n only. At arbitrary voltage the distribution function is complex, and its first and second moments are found to be in agreement with the results of Refs. 10 and 1, respectively.

II. NORMAL METAL CASE

To pinpoint the most important elements of the problem, we discuss briefly the shot noise distribution function for an normal-normal (NN) quantum point contact. Let us consider the Landauer resistor, i.e., a one-dimensional channel with a scattering potential, and two bulk electrodes attached to the left (L) and right (R) sides of the channel. The electrodes are assumed to be in equilibrium and biased with respect to each other by voltage V. We also assume that there are no inelastic processes in the channel. The asymptotic forms of the wave functions ψ_L and ψ_R of the electrons incident from left to right, respectively, with energy $E=\hbar^2k^2/2m$ are

$$\begin{bmatrix}\psi_L\\ \psi_R\end{bmatrix}(x)=\begin{cases}\begin{bmatrix}1 & r\\ 0 & t\end{bmatrix}\begin{bmatrix}e^{ikx}\\ e^{-ikx}\end{bmatrix}, & x\to-\infty\\ \begin{bmatrix}t & 0\\ r & 1\end{bmatrix}\begin{bmatrix}e^{ikx}\\ e^{-ikx}\end{bmatrix}, & x\to\infty\end{cases} \tag{1}$$

where t and r are the transmission and reflection amplitudes, respectively. The annihilation operator for a particle in the channel is

$$\hat{\Psi}_E(x)=\psi_L(x)\hat{a}_E+\psi_R(x)\hat{b}_E\ , \tag{2}$$

where $\hat{a}_E$ and $\hat{b}_E$ are annihilation operators in the left and right electrodes, respectively. The current operator $\hat{I}$ can be expressed via the number of particles in the channel, say, on the left-hand side of the barrier propagating right ($\rightarrow$) or left ($\leftarrow$):

$$\begin{aligned}&\hat{I}=e_0(\hat{N}_{\rightarrow}-\hat{N}_{\leftarrow})\ ,\\ &\hat{N}_{\rightarrow}=\hat{a}^+\hat{a},\quad \hat{N}_{\leftarrow}=\hat{\phi}^+\hat{\phi}\ ,\\ &\hat{\phi}=t\hat{b}+r\hat{a}\ ,\end{aligned} \tag{3}$$

where e_0 is the electron charge.

We want to calculate the distribution function $P(Q,t_0)$ of a charge Q transferred through the channel in time t_0. Without inelastic processes, electrons on different energy levels propagate independently. Thus, the characteristic function $\chi(\lambda)=\sum_k P(e_0k,t_0)e^{i\lambda k}$ is a product of contributions from different energies:

$$\ln\chi_Q(\lambda,t)=\frac{t_0}{2\pi\hbar}\int dE\,\ln\chi_E(\lambda)\ . \tag{4}$$

Actually, because of the uncertainty relation, the levels in the energy interval $\Delta E\sim\hbar/t_0$ are mixed. This effect can be neglected only for long measurement times (see Ref. 11 for details), and we restrict ourselves to this situation only.

The single energy characteristic function $\chi_E(\lambda)$ is connected with the probability $P_E(k)$ of transferring k electrons with energy E through the channel. The latter can be expressed in the form

$$P(k)=\sum_{l=0}^{\infty} P(l+k,l) , \tag{5}$$

where $P(m,n)$ is the joint probability of m electrons propagating to the right and n to the left. Now the expression for χ_E acquires the form

$$\chi_E(\lambda)=\sum_{k=0}^{\infty} P(k)e^{i\lambda k}=\sum_{m,n=0}^{\infty} P(m,n)e^{i\lambda m}e^{-i\lambda n}$$
$$=\langle e^{i\lambda \hat{N}_{\rightarrow}} e^{-i\lambda \hat{N}_{\leftarrow}} \rangle , \tag{6}$$

where $\langle\ \rangle$ stands for the statistical average over the Fermi distributions in the electrodes. Note, that the operators $\hat{N}_{\rightarrow}$ and $\hat{N}_{\leftarrow}$ do not commute, so the last identity in (6) is not obvious. It does not matter which of the exponents comes first, because $\hat{N}_{\rightarrow}$ is diagonal in $\hat{a}$ and $\hat{b}$. It is important that there is a product of the exponents in (6), and not the exponent of a sum of $\hat{N}$ operators. The prescription (4) and (6) of writing characteristic functions for transmitted charge is equivalent to that presented in Ref. 6.

It is essential for the calculation of $\chi_E(\lambda)$ that the operators $\hat{N}_{\rightarrow}$ and $\hat{N}_{\leftarrow}$ are projectors, i.e., $\hat{N}^2_{\rightarrow(\leftarrow)}=\hat{N}_{\rightarrow(\leftarrow)}$. This is obvious for $\hat{N}_{\rightarrow}$ and for $\hat{N}_{\leftarrow}$ is a consequence of the anticommutation relation

$$\hat{\phi}^+\hat{\phi}+\hat{\phi}\hat{\phi}^+=|t|^2+|r|^2=1 . \tag{7}$$

The projecting character of $\hat{N}_{\rightarrow(\leftarrow)}$ allows us to simplify the exponent $e^{i\lambda\hat{N}_{\rightarrow(\leftarrow)}}=1+(e^{i\lambda}-1)\hat{N}_{\rightarrow(\leftarrow)}$ and leads to the LL result for $\chi(\lambda)$:

$$\chi_Q(\lambda,t)=\exp\left\{\frac{t_0}{2\pi\hbar}\int dE\,\ln\{1+|t|^2(e^{i\lambda}-1)n_L(E)[1-n_R(E)]+|t|^2(e^{-i\lambda}-1)n_R(E)[1-n_L(E)]\}\right\} , \tag{8}$$

where $n_L(n_R)$ is the occupation number in the left (right) electrode.

Formula (8) has a simple physical meaning: electrons and holes from the left electrode can travel to the right with probabilities $P_e=|t^2|n_L(1-n_R)$ and $P_h=|t^2|n_R(1-n_L)$, respectively. Therefore, the characteristic function is the sum of three contributions: $P_e e^{i\lambda}$ for electrons, $P_h e^{-i\lambda}$ for holes and $(1-P_e-P_h)$ for the processes when no net charge is transferred. This sum coincides with the expression under the logarithm in (8).

Formula (8) gives the binomial distribution in the zero-temperature limit:

$$\chi_Q(\lambda,t)=(|t|^2e^{i\lambda}+1-|t|^2)^{e_0Vt_0/(2\pi\hbar)} , \tag{9}$$

where $e_0Vt_0/(2\pi\hbar)$ plays the role of the number of attempts. From the semiclassical point of view it means that only the electrons in the energy interval e_0V contribute to the noise.

The sketch above is by no means a substitute for rigorous derivation, which can be found in the LL original papers. Our goal was merely to present some physical motivations for Eqs. (4) and (6). These formulas are used in the following NS point contact analysis.

III. NS BOUNDARY SCATTERING AMPLITUDES

In order to modify the above model for an NS point contact, we follow the procedure described in Ref. 10. The NS point contact is characterized by the gap function $\Delta(x)$ and potential $U(x)$. The electron annihilation operator in a superconductor is a linear combination of creation and annihilation operators for the BCS quasiparticles $\hat{b}$ and $\hat{b}^+$:

$$\psi=\hat{b}u-\hat{b}^+v^* , \tag{10}$$

and the x dependence of coefficients u and v is governed by the Bogolyubov–de Gennes equation:[12]

$$\hat{\mathcal{H}}\begin{bmatrix}u(x)\\ v(x)\end{bmatrix}=E\begin{bmatrix}u(x)\\ v(x)\end{bmatrix} , \quad \hat{\mathcal{H}}=\begin{bmatrix}-\dfrac{\hbar^2}{2m}\dfrac{d^2}{dx^2}-\mu+U(x) & \Delta(x)\\ \Delta^*(x) & -\left[-\dfrac{\hbar^2}{2m}\dfrac{d^2}{dx^2}-\mu+U(x)\right]\end{bmatrix} . \tag{11}$$

On the left-hand side of the contact ($x \to -\infty$), in a normal metal, Δ vanishes and Eq. (11) decouples into two ordinary Schrödinger equations with plane-wave solutions:

$$\begin{aligned}
\psi_L^{(1)} &= e^{-i(p_F+E/v_F)x}\begin{bmatrix}1\\0\end{bmatrix},\\
\psi_L^{(2)} &= e^{-i(p_F-E/v_F)x}\begin{bmatrix}0\\1\end{bmatrix},\\
\psi_L^{(3)} &= e^{i(p_F-E/v_F)x}\begin{bmatrix}0\\1\end{bmatrix},\\
\psi_L^{(4)} &= e^{i(p_F+E/v_F)x}\begin{bmatrix}1\\0\end{bmatrix}.
\end{aligned} \tag{12}$$

On the right-hand side of the contact ($x \to \infty$) we have a superconductor with a constant gap function $\Delta(x)=\Delta_0=\text{const}$. There are four plane-wave solutions provided $E > \Delta_0$:

$$\begin{aligned}
\psi_R^{(1)} &= C_0 e^{-i(p_F+\xi/v_F)x}\begin{bmatrix}u_0\\v_0\end{bmatrix},\\
\psi_R^{(2)} &= C_0 e^{-i(p_F-\xi/v_F)x}\begin{bmatrix}v_0\\u_0\end{bmatrix},\\
\psi_R^{(3)} &= C_0 e^{i(p_F-\xi/v_F)x}\begin{bmatrix}v_0\\u_0\end{bmatrix},\\
\psi_R^{(4)} &= C_0 e^{i(p_F+\xi/v_F)x}\begin{bmatrix}u_0\\v_0\end{bmatrix},
\end{aligned} \tag{13}$$

where

$$\xi=\sqrt{E^2-\Delta_0^2}\,, \quad \begin{bmatrix}u_0\\v_0\end{bmatrix}=\frac{1}{\sqrt{2}}\begin{bmatrix}\sqrt{1+\xi/E}\\\sqrt{1-\xi/E}\end{bmatrix}, \tag{14}$$

and $C_0=(u_0^2-v_0^2)^{-1/2}$ is some normalization constant which we discuss later.

The asymptotic states $\psi_{L,R}^{(2),(4)}$ propagate to the right and $\psi_{L,R}^{(1),(3)}$ to the left in accordance with the sign of the group velocity (see Fig. 1).

The Hamiltonian (11) preserves the probability density of finding either an electron or a hole at a given point (see, e.g., Ref. 10). Namely, $P(x,t)=|u(x,t)|^2+|v(x,t)|^2$ obeys the continuity equation

$$\frac{\partial P}{\partial t}+\operatorname{div}J_P=0\,, \tag{15}$$

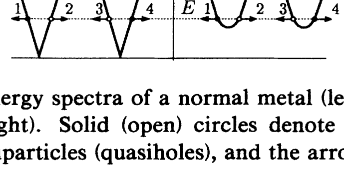

FIG. 1. Energy spectra of a normal metal (left) and a superconductor (right). Solid (open) circles denote the asymptotic states of quasiparticles (quasiholes), and the arrows show direction of their velocities. The numbers 1–4 are related to the asymptotic-state wave-function superscripts in Eqs. (12) and (13).

where $J_P=(\hbar/m)\text{Im}(u^*\nabla u - v^*\nabla v)$ is the probability current. The normalization constant C_0 in (13) is chosen so that the asymptotic states ψ_L and ψ_R carry the same probability current.

To introduce the scattering amplitudes, we consider a solution of Eq. (11), $\psi^{(e)}$, with an asymptotic form containing $\psi_L^{(4)}$ as the only state incident on the barrier:

$$\psi^{(e)}(x)=\begin{cases}\psi_L^{(4)}+r\psi_L^{(1)}+r_A\psi_L^{(3)}, & x\to-\infty\\ t\psi_R^{(4)}+t_A\psi_R^{(2)}, & x\to\infty\,.\end{cases} \tag{16}$$

Equation (16) determines four main scattering amplitudes r, r_A, t, and t_A for the reflection, Andreev reflection, transmission, and Andreev transmission, respectively. The two latter processes are simply conversion of an electron into a BCS electronlike or holelike quasiparticle.

For an arbitrary solution of Eq. (11) with the asymptotic form

$$\psi(x)=\begin{cases}\sum_\alpha B_{L,\alpha}\psi_L^\alpha, & x\to-\infty\\ \sum_\alpha B_{R,\alpha}\psi_R^\alpha, & x\to\infty\end{cases} \tag{17}$$

the coefficients $B_{L,\alpha}, B_{R,\alpha}$ are connected by the scattering matrix, which transforms the incoming waves into outgoing ones:

$$\begin{bmatrix}B_{L,1}\\B_{L,3}\\B_{R,4}\\B_{R,2}\end{bmatrix}=S\begin{bmatrix}B_{L,4}\\B_{L,2}\\B_{R,1}\\B_{R,3}\end{bmatrix}. \tag{18}$$

There are many relations between the scattering matrix elements. First, the probability conservation (15) leads to S-matrix unitarity. Secondly, for a bulk superconductor we can neglect the influence of the supercurrent through the contact and $\Delta(x)$ can be chosen real, provided there is no external magnetic field. Thus, Eq. (11) becomes real and complex conjugation gives another symmetry. To summarize, the symmetries of the S matrix are

$$\begin{aligned}
&SS^+=1 \text{ (unitarity)}\,,\\
&SS^*=1 \text{ (complex conjugation)}\,.
\end{aligned} \tag{19}$$

Thus, S is a symmetric unitary matrix:

$$S=\begin{bmatrix} r & r_A & t & t_A\\ r_A & r' & t'_A & t'\\ t & t'_A & r_< & r_{A<}\\ t_A & t' & r_{A<} & r'_< \end{bmatrix}. \tag{20}$$

The particular values of the scattering amplitudes and their energy dependence are determined by the NS point contact geometry. The case with potentials $U(x)=2Zp_F\delta(x)$ and $\Delta(x)=\Delta_0\theta(x)$ is discussed in more detail in Appendix B. There are no states in the superconductor with energy $E<\Delta_0$. Under this condition the S matrix describes the reflection and Andreev reflection of electrons and holes coming from the left:

$$S=\begin{bmatrix} r & r_A \\ r_A & r' \end{bmatrix} . \tag{21}$$

IV. NS POINT CONTACT CHARACTERISTIC FUNCTION

The calculation of the characteristic function χ_Q for an NS point contact will be performed in the same way as for an NN junction. In order to avoid uncertainty with the BCS quasiparticles charge, we calculate the current on the normal-metal side of the point contact. We introduce the operators $\hat{\phi}_e$ and $\hat{\phi}_h$, which annihilate either a particle or a hole propagating in the normal metal to the left. These operators play the same role as the operator $\hat{\phi}$ in (3), and are linear combinations of the creation and annihilation operators $\hat{a}$, $\hat{a}^+$, $\hat{b}$, and $\hat{b}^+$ for electrons in the normal metal and BCS quasiparticles in the superconductor. It is convenient to introduce another notation for the electron annihilation operators in order to present the results in a compact form:

$$\hat{a}_4=\hat{c}_1,\quad \hat{a}_2^+=\hat{c}_2,\quad \hat{b}_1=\hat{c}_3,\quad \hat{b}_3^+=\hat{c}_4 . \tag{22}$$

The coefficients in the expression for $\hat{\phi}_e$ and $\hat{\phi}_h$ are the first two rows of the scattering matrix (20):

$$\hat{\phi}_e=\sum_i S_{1i}\hat{c}_i,\quad \hat{\phi}_h=\sum_i S_{2i}\hat{c}_i . \tag{23}$$

The unitarity of S matrix guarantees the anticommutation relations

$$\phi^+_{e,(h)}\phi_{e,(h)}+\phi_{e,(h)}\phi^+_{e,(h)}=1,\quad \phi_e^+\phi_h+\phi_h\phi_e^+=0 . \tag{24}$$

The generalization of Eq. (6) for the characteristic function has now the form

$$\chi_E(\lambda)=\langle e^{i\lambda(\hat{N}^{(e)}_{\rightarrow}-\hat{N}^{(h)}_{\rightarrow})}e^{-i\lambda(\hat{N}^{(e)}_{\leftarrow}-\hat{N}^{(h)}_{\leftarrow})}\rangle , \tag{25}$$

where we introduced operators for the numbers of electrons and holes propagating left and right:

$$\begin{aligned} &\hat{N}^{(e)}_{\rightarrow}=\hat{c}_1^+\hat{c}_1,\quad \hat{N}^{(h)}_{\rightarrow}=\hat{c}_2^+\hat{c}_2 ,\\ &\hat{N}^{(e)}_{\leftarrow}=\hat{\phi}_e^+\hat{\phi}_e,\quad \hat{N}^{(h)}_{\leftarrow}=\hat{\phi}_h^+\hat{\phi}_h . \end{aligned} \tag{26}$$

Note, that the electrons and holes carry opposite charges and the operators $\hat{N}^{(e)}_{\leftarrow}$ and $\hat{N}^{(h)}_{\leftarrow}$ commute. The $\hat{N}$ operators involved are projectors because of the identity (24). Therefore, the characteristic function $\chi_E(\lambda)$ may be expressed in the form

$$\begin{aligned} \chi_E(\lambda)=\langle &\{1+(e^{i\lambda}-1)\hat{N}^{(e)}_{\rightarrow}\}\{1+(e^{-i\lambda}-1)\hat{N}^{(h)}_{\rightarrow}\}\\ &\times\{1+(e^{-i\lambda}-1)\hat{N}^{(e)}_{\leftarrow}\}\{1+(e^{i\lambda}-1)\hat{N}^{(h)}_{\leftarrow}\}\rangle . \end{aligned} \tag{27}$$

In order to obtain the characteristic function for the total transmitted charge as a function of the applied voltage and temperature, the averaging in Eq. (27) must be performed in accordance with the rules

$$\begin{aligned} &\langle \hat{c}_1^+\hat{c}_1\rangle\equiv n_1=n(E-e_0V) ,\\ &\langle \hat{c}_2^+\hat{c}_2\rangle\equiv n_2=1-n(-E-e_0V) ,\\ &\langle \hat{c}_3^+\hat{c}_3\rangle\equiv n_3=n(E-\delta\mu) ,\\ &\langle \hat{c}_4^+\hat{c}_4\rangle\equiv n_4=1-n(-E-\delta\mu) . \end{aligned} \tag{28}$$

It should be mentioned, that the energy E is measured with respect to the Cooper pairs chemical potential. In Eq. (28) $n(E)$ denotes the Fermi distribution, V applied voltage, and $\delta\mu$ the difference between the chemical potentials of the quasiparticles in the superconductor and that of the Cooper pairs. In an equilibrium superconductor $\delta\mu$ always equals zero, while the case with $\delta\mu\neq 0$ is commonly referred to as branch imbalance.[13]

The result of averaging (27) can be presented in a compact form introducing the density matrix $\rho_{ij}=n_i\delta_{ij}$ and rewriting the $\hat{N}$ operators from Eq. (26) in the form

$$\hat{N}^{(h,e)}_{\rightarrow(\leftarrow)}=\sum_{i,j}\{P^{(h,e)}_{\rightarrow(\leftarrow)}\}_{ij}\hat{c}_i^+\hat{c}_j , \tag{29}$$

where matrices P are projectors on the appropriate states:

$$\begin{aligned} &\{P^{(e)}_{\leftarrow}\}_{ij}=\delta_{ij}\delta_{i1},\quad \{P^{(h)}_{\leftarrow}\}_{ij}=\delta_{ij}\delta_{i2} ,\\ &\{P^{(e)}_{\rightarrow}\}_{ij}=S_{i1}S_{1j},\quad \{P^{(h)}_{\rightarrow}\}_{ij}=S_{i2}S_{2j} . \end{aligned} \tag{30}$$

The characteristic function for the NS point contact can be expressed in the form (see Appendix A)

$$\begin{aligned} \chi(\lambda)=\int\frac{t_0dE}{2\pi\hbar}\ln\det[1-\rho+\rho e^{i\lambda(P^{(e)}_{\rightarrow}-P^{(h)}_{\rightarrow})}\\ \times e^{-i\lambda(P^{(e)}_{\leftarrow}-P^{(h)}_{\leftarrow})}] . \end{aligned} \tag{31}$$

This is our main result, analogous to that of LL for the NN point contact. One can see that our formula corresponds to the special four-channel case of the general many-channel formula presented in Ref. 7.

Expression (31) can be rewritten in the form

$$\chi(\lambda)=\int\frac{t_0dE}{2\pi\hbar}\ln\left[1+\sum_{n=-2}^{2}A_n(e^{i\lambda n}-1)\right] , \tag{32}$$

which clearly shows that the elementary shot noise processes are associated with either $\pm e_0$ or $\pm 2e_0$ charge transfer. General expressions for the coefficients A_n through the scattering amplitudes and occupation numbers are given in Appendix B. In the limiting case of low temperature $T\ll\Delta_0$ and no branch imbalance the coefficients A_n have the form

$$\begin{aligned} &A_1=n_1(1-|r|^2-|r_A|^2) ,\\ &A_{-1}=n_2(1-|r|^2-|r_A|^2) ,\\ &A_2=n_1(1-n_2)|r_A|^2 ,\\ &A_{-2}=(1-n_1)n_2|r_A|^2 . \end{aligned} \tag{33}$$

In the limit $T\ll e_0V\ll\Delta_0$ the energy dependence of scattering amplitudes can be neglected and integration in (32) gives the binomial distribution

$$\chi_Q=\{(1-|r_A|^2)+|r_A|^2\exp(2i\lambda\,\mathrm{sgn}V)\}^{e|V|t_0/(2\pi\hbar)} . \tag{34}$$

It is clearly seen from Eq. (34) that the elementary act corresponds to a double-charge transfer into the superconductor (either two electrons at $V>0$ or two holes at $V<0$). This very fact leads to the changes in the noise-voltage characteristic of the NS junction with a small ac component of frequency Ω in applied voltage. The original calculations of LL (Ref. 8) can be repeated with the characteristic function (34) as a starting point and show that steps occur now at voltages $V=\hbar\Omega n/e_0$ with even n only. At higher voltages $V>\Delta_0$ single-electron transport also exists, and steps at odd n arise.

In the other limit, $e_0V \ll T \ll \Delta_0$, the calculation analogous to that in Ref. 7 gives a characteristic function:

$$\chi_Q(\lambda)=\exp\frac{-2\lambda_*^2 t_0 T}{\pi\hbar}, \quad \sin\lambda_*=|r_A|\sin\lambda , \tag{35}$$

which leads to the Nyquist formula for the mean square of the current:

$$\langle\langle I^2\rangle\rangle=\frac{2e_0^2T}{\pi\hbar}|r_A|^2 . \tag{36}$$

For arbitrary temperature and voltages the second moment of the distribution (31) can be calculated. The results concur with those obtained by Khlus using another method.[1]

ACKNOWLEDGMENTS

This work was supported by a SERC research grant. The authors are thankful to L. Levitov and G. Lesovik for numerous discussions and M. J. M. de Jong and C. W. J. Beenakker for an opportunity to read their paper prior to publication.

APPENDIX A

We show here how to perform averaging of exponents products, such as

$$\left\langle \exp\left[i\lambda\sum_{i,j}C_{ij}\hat{a}_i^+\hat{a}_j\right]\exp\left[-i\lambda\sum_{i,j}D_{ij}\hat{a}_i^+\hat{a}_j\right]\right\rangle , \tag{A1}$$

where $\hat{a}_i$ are the electrons annihilation operators, C,D are Hermitian projectors $C^2=C$, $D^2=D$, and the density matrix ρ is diagonal in electronic subscripts $\rho_{ij}=\langle\hat{a}_i^+\hat{a}_j\rangle=n_i\delta_{ij}$.

By means of the Bogolyubov transformation the exponent can be presented in the form

$$\exp\left[i\lambda\sum_{i,j}C_{ij}\hat{a}_i^+\hat{a}_j\right]=:\exp\left[\sum_{i,j}(e^{i\lambda C_{ij}}-1)\hat{a}_i^+\hat{a}_j\right]: , \tag{A2}$$

where : : denotes normal ordering. Because of normal ordering, the mean value of the right-hand side in (A2) can be evaluated by means of zero-temperature Green-function technique.[14] For a general expression of the form

$$U(x)=\left\langle :\exp\left[x\sum_{i,j}(e^{i\lambda C_{ij}}-1)\hat{a}_i^+\hat{a}_j\right]:\right\rangle \tag{A3}$$

its logarithmic derivative can be written as

$$\frac{d\ln U}{dx}=\mathrm{Tr}\{(e^{i\lambda C}-1)G\} , \tag{A4}$$

where the Green function G_{ij} is

$$G_{ij}=\frac{\langle :\hat{a}_i^+\hat{a}_j\exp\{x(e^{i\lambda C}-1)\hat{a}^+\hat{a}\}:\rangle}{U} . \tag{A5}$$

After solving the Dyson equation we get

$$\begin{aligned} &G=\{1+x\rho(e^{i\lambda C}-1)\}^{-1}\rho , \\ &U(1)=\det(1-\rho+\rho e^{i\lambda C}) . \end{aligned} \tag{A6}$$

Analogously, the mean value of two exponents

$$\begin{aligned}\chi&=\langle e^{i\lambda\sum Ca^+a}e^{-i\lambda\sum Da^+a}\rangle \\ &=\langle :\exp[(e^{i\lambda C}-1)a^+a]::\exp[(e^{-i\lambda D}-1)a^+a]:\rangle\end{aligned}$$

can be calculated by the above method, using operators ordered along the Keldysh contour. All operators thus become matrices in the Keldysh space and in the same way we arrive at the expression

$$\chi=\det\left\{\begin{bmatrix}e^{i\lambda C}-1 & 0\\ 0 & e^{-i\lambda D}-1\end{bmatrix}\begin{bmatrix}\rho & \rho\\ \rho-1 & \rho\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right\} . \tag{A7}$$

In our particular case $C=P^{(e)}_{\rightarrow}-P^{(h)}_{\rightarrow}$, and $D=P^{(e)}_{\leftarrow}-P^{(h)}_{\leftarrow}$ [see Eq. (25)]. Under these conditions, formula (A7) can be further simplified, taking into account that both C and ρ are diagonal:

$$\chi=\det(1-\rho+\rho e^{i\lambda C}e^{-i\lambda D}) . \tag{A8}$$

This is the result presented in the text in formula (31).

APPENDIX B

We present the general formulas for the coefficients A_n in Eq. (32) in the case of a special geometry of the NS boundary.

Following Ref. 10, we restrict ourselves to a model NS boundary with the δ-function potential $U(x)=2Zp_F\delta(x)$ and steplike gap function $\Delta(x)=\Delta_0\theta(x)$. Under these conditions, the scattering matrix elements for $E>\Delta_0$ are

$$\begin{aligned} &r=r_<=\frac{(v_0^2-u_0^2)(Z^2+iZ)}{\gamma}, \quad r'=r'_<=\frac{iZ+1}{iZ-1}r , \\ &r_A=-r_{A<}=\frac{u_0v_0}{\gamma} , \\ &t=\frac{u_0(1-iZ)}{\gamma}\sqrt{u_0^2-v_0^2}, \quad t'=\frac{1+iZ}{1-iZ}t , \\ &t_A=\frac{iv_0Z}{\gamma}\sqrt{u_0^2-v_0^2}, \quad t'_A=-t_A , \\ &\gamma=u_0^2+Z^2(u_0^2-v_0^2) , \end{aligned} \tag{B1}$$

where u_0 and v_0 are from Eq. (14). For $E<\Delta_0$ there are no states in the superconductor and the amplitudes t, t', t_A, and t'_A vanish, while the formulas (B1) for r, r', and

r_A are still valid. It should be mentioned that the characteristic length in the scattering problem is of the order $L_s \sim \Delta E/(\hbar v_F)$, where $\Delta E \sim \max(T_0, e_0 V, \Delta_0)$. It means that the formulas (B1) for the scattering amplitudes remain valid for the more general case, when both $U(x)$ and $d\Delta(x)/dx$ are located in the region much smaller than L_s.

Note that electrons and holes are scattered essentially in the same way, namely, the following identities are valid for $E > \Delta_0$:

$$r' = r^*, \quad t' = t^*, \quad t'_A = t^*_A .$$

From S-matrix unitarity it also follows that $tt_A + rr_A = 0$.

These relations between the scattering amplitudes enable us to present the coefficients A_n in the expression for the characteristic function (32) in the form

$$\begin{aligned} A_1 &= T[n_1(1-n_3)+(1-n_2)n_4] + T_A[n_1(1-n_4)+(1-n_2)n_3] \\ &\quad -(T-T_A)^2[n_1 n_2(1-n_3)(1-n_4) + 2n_1(1-n_2)(1-n_3)n_4 + (1-n_1)(1-n_2)n_3 n_4] \\ &\quad +2(T-T_A)(T_A+R_A)n_1(1-n_2)(n_3-n_4) , \\ A_2 &= R_A n_1(1-n_2) + (T-T_A)^2 n_1(1-n_2)(1-n_3)n_4 - (T-T_A)(T_A+R_A)n_1(1-n_2)(n_3-n_4), \quad \text{for } E > \Delta_0 \\ A_1 &= 0, \quad A_2 = R_A n_1(1-n_2), \quad \text{for } E < \Delta_0 , \end{aligned} \tag{B2}$$

where n_1, n_2, n_3, and n_4 are the occupation numbers introduced in (28), and T, T_A, and R_A are probabilities of transmission, Andreev transmission, and Andreev reflection, respectively:

$$T = |t|^2, \quad T_A = |t_A|^2, \quad R_A = |r_A|^2 .$$

The formulas for A_{-1}, A_{-2} can be obtained from (B2) with substitution $n_1 \to n_2$, $n_3 \to n_4$ and vice versa.

The coefficients in (B2) contain only the probabilities of different scattering processes, not their amplitudes. Nevertheless, each coefficient in the characteristic function (32) cannot be reduced to a sum of contributions of independent processes of charge transfer contrary to the NN point contact case. The origin of this difference is quantum interference of the transmission and the Andreev transmission processes. It is only due to the S-matrix symmetry that the result can be expressed through the probabilities.

The coefficients A_1 and A_2 contain the contributions due to branch imbalance, which are proportional to $n_3 - n_4$ and disappear for the NN junction, when both T_A and R_A are equal to zero. In the latter case the characteristic function (32) coincides with that for the NN point contact [formula (8)]. Note, that the limits of energy integration in Eqs. (8) and (32) are different. In (8) the integration is performed in the region $-\infty < E < \infty$, while in (32) in the region $0 < E < \infty$.

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