Weak-localization and Maki-Thompson superconducting fluctuation effects in crystalline disordered Ti-Al-(Sn,Co) alloys at $T > T_c$

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We have measured the magnetoresistivities of crystalline disordered $Ti_{1-x}Al_{x-y}(Sn,Co)_y$ ($x=0.072$ and $y \leq 0.03$) alloys between 1.7 and 15 K in magnetic fields up to 3 T. With a total doping level of 7.2 at. % for Ti, our samples are disordered enough to manifest quantum corrections due to weak-localization and electron-electron interaction effects. As the magnetic field B is increased, magnetoresistivities $\Delta\rho(B)/\rho(0) = [\rho(B) - \rho(0)]/\rho(0)$ of the order of 10^{-4}–10^{-3} are observed. Both the functional forms and magnitudes of the observed $\triangle\rho(B)$ are interpreted in terms of weak-localization and Maki-Thompson superconducting fluctuation effects. Our sample compositions (i.e., additions of $y=0.000$, 0.030 of Sn, and 0.0005 of Co, respectively, for Al) have been chosen so as to tailor the various aspects of both effects. We find that spin-orbit scattering is greatly increased in the alloy containing 3.0 at. % of Sn. As a result, $\triangle\rho(B)$ increases monotonically with increasing B in all our measuring magnetic fields and at all measuring temperatures. In the alloy containing 0.05 at. % of Co, superconducting fluctuation effects are greatly enhanced at our measuring temperatures, and a diverging inelastic scattering rate is observed in our lowest temperature range of 1.7–1.9 K. From comparison of our experimental results with theory, the inelastic scattering times have been inferred and ascribed to electron-phonon interaction in the presence of strong impurity scattering. Our results suggest that, in the crystalline disordered metals like our Ti-Al-(Sn,Co) alloys, the electron-phonon scattering rate varies with the square of the temperature, $\tau_{ph}^{-1} \sim T^2$, in the temperature range 3–15 K. Particularly, the absolute magnitudes of the measured τ_{ph} are consistent within $\sim 40\%$ with the theoretical prediction.

I. INTRODUCTION

Over the past decade or so, it has been realized that weak-localization (WL) effects result in noticeable magnetoresistivities in disordered metals at low temperatures and in low magnetic fields.[1–6] Analysis of those "anomalous" magnetoresistivities has proved to provide quite quantitative information of the various phase-relaxation mechanisms of the electron wave functions, including inelastic, spin-orbit, and magnetic spin-spin scattering processes. In particular, the electron phase-breaking times have been studied and become quite well understood in one and two dimensions (i.e., small-diameter wires and thin films, respectively).[4–6] It is now widely accepted that the dominating phase-breaking mechanism in reduced dimensions is electron-electron scattering. The temperature T dependence of the electron-electron scattering has also been determined, and found to be different from that in the pure case. In disordered systems, electron-electron scattering possesses much weaker dependences on T than that in pure systems.[4] In three dimensions (3D), on the other hand, the situation is less conclusive. Frequently, experiments find that electron-phonon scattering is the dominating inelastic scattering.[7–10] However, the T dependence of the electron-phonon scattering times, $\tau_{ph}(T)$, observed in various experiments are not always in agreement with one another. Various theoretical calculations[11–13] predict values of p ranging from 2 to 4, where p is the exponent determining the T dependence of τ_{ph}, i.e., $\tau_{ph} \sim T^{-p}$. The usual materials used for studies in this direction are thick granular films,[14] thick quench-condensed metal films,[15] doped semiconductors,[16] and metallic glasses,[7–10,17] but not crystalline disordered metallic systems. Obviously, the lack of experiments on crystalline disordered metals is due to the fact that it is extremely difficult to fabricate metal samples microscopically homogeneous in the bulk and having high resistivities.

In this work we have carried out systematic measurements of the magnetoresistivities of $Ti_{1-x}Al_{x-y}(Sn,Co)_y$ ($x=0.072$ and $y \leq 0.03$) alloys between 1.7 and 15 K and in magnetic fields up to 3 T. As has been reported previously,[18] these alloys are crystalline disordered metals, and are ideal for studies of WL and electron-electron interaction effects. The total doping level of 7.2 at. % for Ti in each sample has been chosen such that every sample has a residual resistivity $\rho(10\ \text{K}) \sim 100\ \mu\Omega\,\text{cm}$. Experimentally, the magnetoresistivities, $\triangle\rho(B) = \rho(B) - \rho(0)$, in samples with impurity resistivities of this order can be accurately measured. Our samples are as follows. (i) We make a $Ti_{0.928}Al_{0.072}$ alloy (hereafter referred to as Ti96) as the parent sample. Here aluminum atoms are introduced into a titanium host mainly to produce desirable impurity scattering. (ii) We make a $Ti_{0.923}Ag_{0.005}Al_{0.072}$ alloy (hereafter referred to as Ag1).

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Since the addition (0.5 at.%) of Ag for Ti is minor, we expect that the electronic properties of Ag1 are not appreciably affected from that of Ti96, except for a slight increase in spin-orbit scattering rate. Thus, a quantitative comparison of the $\Delta\rho(B)$ of this sample with that of Ti96 will provide a stringent consistency check of our experimental method (e.g., signal detection and data analyses) and the credibility of the theory. (iii) We make a $Ti_{0.928}Al_{0.042}Sn_{0.030}$ alloy (hereafter referred to as Sn8). In this alloy, the addition (3.0 at.%) of Sn for Al is comparatively high, e.g., relative to the addition of Ag in Ag1. Therefore, the spin-orbit scattering rate is significantly increased from that of Ti96. (iv) We make a $Ti_{0.928}Al_{0.0715}Co_{0.0005}$ alloy (hereafter referred to as Co1). Previously, it has been found [19] that the addition of a small amount of Co for Ti results in a remarkable increase in the superconducting transition temperature, T_c, from that of pure Ti. Therefore, fluctuational superconductivity will be enhanced in this sample at our measuring temperatures, compared with that in Ti96, Ag1, or Sn8. Experimentally, this enhancement is very clearly demonstrated in the relatively large low-B magnetoresistivities. Our results have been analyzed in terms of WL and Maki-Thompson[20–23] superconducting fluctuation effects. The electron phase-breaking times and their T dependence are reliably inferred, and the dominating phase-relaxation process is ascribed to electron-phonon interaction in the presence of strong impurity scattering. In particular, we obtain close agreement between experiment and theory.

This paper is organized as follows. In Sec. II, we discuss the pertinent theories required to interpret our measured magnetoresistivities and inferred inelastic scattering times. In Sec. III, we describe our experimental method for sample fabrication and magnetoresistivity measurements. In Sec. IV our experimental results are presented and compared with the theories discussed in Sec. II. Section V contains our conclusion.

II. THEORY

A. Magnetoresistivities

Physically, weak-localization effects in a disordered metal result from coherent back-scattering of two complementary electron waves traveling a closed path in opposite directions. The phases of the two waves remain coherent when they return to the origin unless there are any (scattering) mechanisms which break the time-reversal symmetry between the two waves.[6] Typical time-reversal-symmetry breakers are spin-flip scatterings and external magnetic fields.[4] In fact, even a *weak* magnetic field B can cause a noticeable phase difference between the two complementary waves, because the traveled closed path could be long, e.g., several thousands Å, at liquid-helium temperatures. A weak magnetic field can thus suppress the coherent backscattering or WL effects. Here a weak magnetic field means a field in which the classical magnetoresistivity due to Lorentz force is negligibly small. (On the other hand, it is understood that we are dealing with external magnetic fields $B > B_\phi$, the electron phase-breaking field, such that the WL effects are suppressed.) In 3D normal metals, the magnetoresistivities $\Delta\rho(B)$ due to WL effects have been calculated by Fukuyama and Hoshino,[24] and others.[22,25] In the calculations of Fukuyama and Hoshino, in addition to the inelastic scattering, spin-orbit scattering, and Zeeman splitting of spin subbands have been taken into consideration. For superconducting materials at temperatures above T_c, $\Delta\rho(B)$ has also been calculated.[21–23] In this latter case, effects resulting from fluctuational superconductivity need to be concerned. More precisely, one has to consider the Maki-Thompson superconducting fluctuation effects, if one considers a temperature region with T close to T_c but still far from the immediate vicinity of T_c, i.e., $2\pi k_B(T-T_c) \gg \hbar/\tau_\phi$ (which applies to all of our measurements), where τ_ϕ is the electron phase-breaking time. [On the other hand, in the immediate vicinity of T_c, i.e., $2\pi k_B(T-T_c) \ll \hbar/\tau_\phi$, one has to include a contribution from the Aslamazov-Larkin term.[26] However, in this work we will concentrate on the region $2\pi k_B(T-T_c) \gg \hbar/\tau_\phi$ and ignore the Aslamazov-Larkin contribution to the magnetoresistivity.] Noteworthily, Larkin has shown that the superconducting fluctuation contribution to $\Delta\rho(B)$ corresponding to the Maki-Thompson diagram has the same B dependence as the WL contribution in the absence of spin-orbit scattering, but with opposite sign and with a coefficient called $\beta(T)$ which diverges at T_c.[21,22] (β is the Larkin's electron-electron attraction strength.) The work of Larkin therefore connects nicely to seemingly unrelated areas of research, i.e., WL and fluctuational superconductivity.

The total magnetoresistivity including both the WL effects and Maki-Thompson term is given by[21,22,24]

$$\frac{\Delta\rho(B)}{\rho^2(0)} = \frac{e^2}{2\pi^2\hbar}\sqrt{\frac{eB}{\hbar}}\left(\frac{1}{2\sqrt{1-\gamma}}\left[f_3\left(\frac{B}{B_-}\right)-f_3\left(\frac{B}{B_+}\right)\right]-f_3\left(\frac{B}{B_2}\right)+\beta(T)\,f_3\left(\frac{B}{B_\phi}\right)\right.$$
$$\left.-\sqrt{\frac{4B_{so}}{3B}}\left[\frac{1}{\sqrt{1-\gamma}}\left(\sqrt{t_+}-\sqrt{t_-}\right)+\sqrt{t}-\sqrt{t+1}\right]\right), \quad (1)$$

where

$$t = \frac{3B_\phi}{4(B_{so}-B_s)},$$

$$\gamma = \left[\frac{3g^*\mu_B B}{8eD(B_{so}-B_s)}\right]^2,$$

$$B_\phi = B_i + 2B_s,$$

$$B_2 = B_i + \frac{2}{3}B_s + \frac{4}{3}B_{so} ,$$

$$B_\pm = B_\phi + \frac{2}{3}(B_{so} - B_s)(1 \pm \sqrt{1-\gamma}) ,$$

and

$$t_\pm = t + \frac{1}{2}(1 \pm \sqrt{1-\gamma}) .$$

Here g^* is the electron Lande-g factor (we take $g^* = 2.0$ in this work), μ_B is the Bohr magneton, and D is the diffusion constant. The characteristic fields B_j's are defined by $B_j = \hbar/4eD\tau_j$, where $j = i$, so, and s refer to the inelastic, spin-orbit, and magnetic spin-spin scattering times (fields). The exact expression of the function f_3 in Eq. (1) is an infinite series which has been calculated by Kawabata.[25] In analyzing our experimental results in this work, instead, we use an approximate expression for f_3 given by Baxter *et al.*,[27] which has been shown to be accurate to be better than 0.1% for all argument z. The approximate expression derived by Baxter *et al.* is[27]

$$f_3(z) \approx 2\left[\sqrt{2+\frac{1}{z}} - \sqrt{\frac{1}{z}}\right] - \left[\left(\frac{1}{2}+\frac{1}{z}\right)^{-1/2} + \left(\frac{3}{2}+\frac{1}{z}\right)^{-1/2}\right] + \frac{1}{48}\left(2.03+\frac{1}{z}\right)^{-3/2} .$$

Clearly, the term $\beta(T)f_3(B/B_\phi)$ in Eq. (1) is the Maki-Thompson superconducting fluctuation contribution. This term is calculated in the weak magnetic field limit $4eDB < 2\pi k_B(T - T_c)$ (which applies to all of our measurements). For larger magnetic fields, this term becomes a constant.[23] The values of the coefficient β depend on temperature, but they are not affected by spin-orbit scattering since this term is concerned with the singlet part of electron-electron interaction in the Cooper channel.[21,22] This contribution will be suppressed in exactly the same manner as the WL effects would be in the presence of spin-spin scattering. Because β has a logarithmic T dependence, this contribution can be significant even at T well above T_c. Since fluctuational superconductivity is progressively suppressed by an increasing magnetic field, the Maki-Thompson contribution to $\Delta\rho(B)$ is always positive, in contrast to the contribution from the WL effects [Eq. (1) except the $\beta f_3(B/B_\phi)$ term], which can be either positive or negative, depending on the ratio τ_i/τ_{so} and the strength of B.

B. Inelastic scattering times

The phase-breaking times τ_ϕ of the electron wave functions in disordered systems have attracted much theoretical and experimental attention recently. In principle, τ_ϕ is not necessarily identical with the inelastic time τ_i. In disordered metals impurity scattering causes the electron-phonon (e-ph) scattering to change. Many authors have studied e-ph interaction in the presence of strong impurity scattering, frequently in connection with the study of dirty superconductors.[11–13] In a series of papers, Schmid and co-workers[13] have demonstrated that, in the destruction of phase coherence in WL, the phase-breaking rate due to e-ph interaction is identical to the inelastic collision rate. By means of a heating measurement and of a proximity-effect measurement in normal metals and superconductors, respectively, Bergmann[28] has also emphasized the effects of e-ph interaction on the phase coherence. For electrons at the Fermi level, the inelastic e-ph scattering rate is given by[13]

$$\frac{1}{\tau_{ph}} = 4\pi \int_0^\infty d\omega \, \frac{\alpha^2 F(\omega)}{\sinh(\hbar\omega/k_B T)} , \tag{2}$$

where $\alpha^2 F(\omega)$ is the Eliashberg function composed of the e-ph coupling constant α and phonon density of states $F(\omega)$. For crystalline metals, $\alpha^2 F(\omega)$ is Debye-like, varying as ω^2, so from Eq. (2) we obtain $\tau_{ph}^{-1} \sim T^3$. For disordered metals, it has been found, both theoretically and experimentally,[11,29] that $\alpha^2 F(\omega)$ varies as ω in the low-ω regime. From Eq. (2) we see that it yields $\tau_{ph}^{-1} \sim T^2$. In the present case of our three-dimensional Ti-Al-(Sn,Co) alloys, we have e-ph scattering as the dominating inelastic scattering, and $\tau_\phi \approx \tau_i \approx \tau_{ph}$.

Theoretically, Bergmann has explicitely calculated the electron-phonon interaction in a dirty metal.[11] He points out that the impurities participate in the lattice oscillations (i.e., the phonons) and change the e-ph interaction. He finds additional e-ph processes which do not conserve the lattice momentum. These additional e-ph processes contribute particularly to processes with low-energy transfer, so that $\alpha^2 F(\omega)$ depends linearly on the energy in the low-energy region, as mentioned. Takayama has also studied this problem.[12] The results given by Bergmann and Takayama agree almost exactly. The impurity contribution to the inelastic e-ph scattering rate according to Takayama is [12]

$$\frac{1}{\tau_{ph}} = \frac{2\pi^2\lambda}{k_F \ell}\frac{(k_B T)^2}{\hbar^2 \omega_D} = \frac{2\pi^2\lambda}{k_F \ell}\frac{k_B T^2}{\hbar \theta_D} , \tag{3}$$

where k_F is the Fermi wave number, ℓ is the elastic mean free path, ω_D (θ_D) is the Debye frequency (temperature), and λ is a constant given by $\lambda = nmv_F^2 q_D^2/(6n_i M v_s^2 k_F^2)$, where n (n_i) is the electron (ion) density, m (M) the electron (ion) mass, v_F (v_s) the Fermi (sound) velocity, $q_D = \omega_D/v_s$, and Z the valence. Using $q_D = (1/Z)^{1/3}k_F$ and the Bohm-Staver relation for sound velocity[30] $v_s^2 = Zmv_F^2/(3M)$, we rewrite λ in the following form:

$$\lambda = \left(\frac{2}{Z}\right)^{2/3}\frac{k_F^3}{6\pi^2 Z n_i} . \tag{4}$$

Takayama has pointed out that Eq. (3) is of great importance in the frequency range $(\omega/v_s)\ell < 1$ (Ref. 12). Recently, the inelastic scattering rate τ_i^{-1} has been realized to be directly related to the pair-breaking parameter δ of Maki-Thompson theory,[31] $\delta = \pi\hbar/8k_B T\tau_i$.

TABLE I. Values of relevant parameters for $Ti_{1-x}Al_{x-y}(Sn,Co)_y$ alloys. The compositions x and y are *nominal*. Values of $k_F\ell = 3\pi^2\hbar/(\rho k_F e^2)$ are calculated using $k_F = 1.46\times10^{10}$ m^{-1} (the Fermi wave number for Ti, Ref. 34), and the measured values of ρ(10 K). ρ is in $\mu\Omega$ cm, D in m^2/s, and B_{so} and B_s in T.

Sample	Composition	ρ (300 K)	ρ (10 K)	$k_F\ell$	D ($\times10^{-5}$)	B_{so}	B_s
Ti96	$Ti_{0.928}Al_{0.072}$	129	90.7	9.2	8.6	0.25	0.0
Ag1	$Ti_{0.923}Ag_{0.005}Al_{0.072}$	146	101	8.3	7.8	0.32	0.0
Sn8	$Ti_{0.928}Al_{0.042}Sn_{0.030}$	152	114	7.3	6.9	0.54	0.0
Co1	$Ti_{0.928}Al_{0.0715}Co_{0.0005}$	141	93.9	8.9	8.3	0.25	0.0

III. EXPERIMENTAL METHOD

$Ti_{1-x}Al_{x-y}(Sn,Co)_y$ alloys were prepared by a standard arc-melting method reported previously.[32] The *nominal* compositions x and y are listed in Table I. Appropriate amounts of Ti (99.995% pure), Al (99.999% pure), and/or Ag (99.995% pure), Sn (99.999% pure), Co (99.9965% pure) were arc-melted several times. Before the alloys were arc-melted, the furnace was thoroughly flushed with high-purity (99.9999%) helium gas to remove any appreciable residual gases. A Zr getter was also used. With such careful precautions, *no* sign of sample oxidation and/or magnetic-impurity contamination was detected from, e.g., magnetoresistivity measurements.

Two differently sized samples were sliced from each alloy ingot. A large sample, typically 1 mm × 1 mm × 10 mm, was used for a calibration of the alloy resistivity. Mainly owing to uncertainties in the sample shapes, the absolute values of ρ reported in this work are accurate to about 4%. A small sample, typically 0.2 mm × 0.2 mm × 10 mm, was obtained by grinding and polishing it with a fine sandpaper. This second sample, having a typical resistance of $\sim$0.2–0.3Ω, was used for magnetoresistivity measurements. Platinum electrodes were spot-welded onto the samples. dc magnetoresistivities were measured by a four-probe technique, using a Keithley 220 current source and a Keithley 181 nanovoltmeter. Great care had been taken to make low-noise electrical contacts and wiring. The measurements were perfomed in a quiet environment and a signal resolution of 10 nV (equivalent to ≈ 1 $\mu\Omega$) was readily achieved, using an applied current of ≈ 10 mA. This corresponds to a resolution of a relative resistivity change $\delta\rho/\rho \approx 4\times10^{-6}$. The magnetic fields were produced by a superconducting magnet immersed in liquid helium, and the temperature was measured with a calibrated carbon-glass thermometer. For each sample, 7–9 magnetoresistivity curves were measured, with each curve obtained by increasing B from zero to 3 T while fixing the temperature at a certain value between 1.7 and 15 K. Care was taken to avoid any appreciable temperature fluctuations during the period of the measurement of a given magnetoresistivity curve.

IV. RESULTS AND DISCUSSION

A. Magnetoresistivities

We first estimate the values of the diffusion constant D for our alloys. To do this, we use independently determined electronic specific heat $\gamma T = 315T$ J/m^{3}K for our alloys[33] to calculate the density of states at the Fermi surface $N(0)$, using the free-electron model $\gamma = \pi^2 k_B^2 N(0)/3$. D is then obtained through the Einstein relation $D = 1/\rho e^2 N(0)$, where ρ is the measured impurity resistivity. Our best values of D thus obtained are listed in Table I. Note that if, instead, the value of $N(0)$ (= 12.0 states/atom Ry, both spins) from the recent band-theory calculations for Ti were used,[34] one would obtain values of D a factor $\sim$60% higher than those listed in Table I. This is in line with the frequent observation in the literature[35,36] that using band-theory parameters usually results in overestimating D. We note that with those values of D presented in Table I, the Zeeman energy of electron spin is comparable to the orbital dephasing energy, i.e., $g^*\mu_B B \sim eDB$. Therefore, inclusion of the effects of Zeeman splitting as considered by Fukuyama and Hoshino in Eq. (1) is essential to account for our experimental results.

Figure 1 shows the representative normalized magnetoresistivities (or negative magnetoconductivities) $\triangle\rho(B)/\rho^2(0)$ as a function of the magnetic field B for Ti96 at several temperatures as indicated in the caption to Fig. 1. The symbols are the experimental results, and the curves are the predictions of the theory (1). Clearly, the $\triangle\rho(B)/\rho^2(0)$ behavior with B shown in this figure

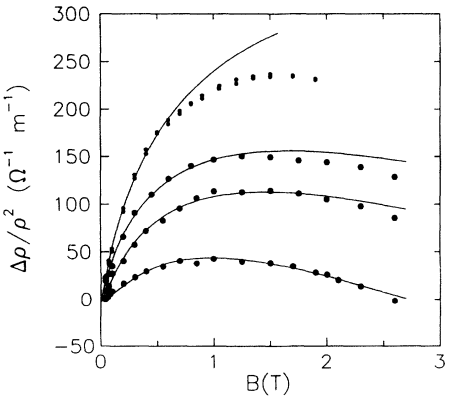

FIG. 1. $\triangle\rho(B)/\rho^2(0)$ as a function of magnetic field B for $Ti_{0.928}Al_{0.072}$ alloy at (from bottom to top) 10.0, 6.00, 3.00, and 1.73 K, respectively. The symbols are the experimental results, and the curves are the predictions of the theory (1) with the values of the relevant parameters given in Table I, the values of β (crosses) shown in Fig. 5, and the values of B_i (closed circles) shown in Fig. 6.

is characteristic of the WL effects with moderate spin-orbit scattering (since both Ti and Al have moderate atomic weights). In low B, the magnetoresistivities are positive at all measuring T. At higher B, $\Delta\rho(B)/\rho^2(0)$ can decrease with increasing B and become negative (not shown) if B and/or T is sufficiently high. The magnitude of $\Delta\rho(B)/\rho(0)$ is of the order of several parts in 10^{-4}. To avoid any complications from electron-electron interaction effects[3–5] and from classical magnetoresistivities due to Lorentz force,[37] we shall only concentrate on the low-B regime in the following analyses. Also, in the low-B regime, the coefficient β in Larkin's theory[21] is negligibly depressed. Our results are compared with the predictions of (1).

In performing least-squares fits of our experimental results to the predictions of (1), we focus on the regime $B/T < 0.5$ T/K.[9,27] (Same as in Figs. 2–4.) Our fitted values of the relevant parameters are listed in Table I, the values of β (crosses) are plotted in Fig. 5, and the values of B_i (closed circles) are shown in Fig. 6. Figure 1 clearly shows that Eq. (1) can well describe our experimental results. (The theoretical curves at high magnetic fields, where deviations between the theory and experiment are noticeable, are only drawn for reference.) We have found that the Maki-Thompson term is required for a quantitative description of our experimental results. If only the WL effects were included, the difference between the theory and experiment would become more and more pronounced as T is lowered, as can be expected since superconducting fluctuations are more important as T approaches T_c. For comparison, we note that the behavior of $\Delta\rho(B)$ with B we observe in Fig. 1 is qualitatively similar to that recently observed in thin Ti films (80–1000 Å thick) by Vangrunderbeek *et al.*[36] Moreover, taking effects of diffusivity into account, our inferred value of the spin-orbit field B_{so} ($=0.25$ T) is essentially the same as theirs. This is encouraging, since spin-orbit scattering is a material property and its strength should be fairly independent of system dimensionality. (Our samples are three dimensional with regard to WL effects, while the films of Vangrunderbeek *et al.* are two dimensional.) A spin-orbit field of 0.25 T for Ti96 corresponds to a $\tau_{so} \approx 7.7\times10^{-12}$ s. Since Al atoms are light, our doping of Al into a Ti host to make Ti96 does not increase the spin-orbit scattering rate from that of pure Ti to within our experimental uncertainties. Further discussion of our determination of the values of B_{so} and spin-spin field B_s is given in the following subsection.

Figure 2 shows the representative normalized magnetoresistivities $\Delta\rho(B)/\rho^2(0)$ as a function of B for Ag1 at several temperatures as indicated in the caption to Fig. 2. The symbols are the experimental results, and the

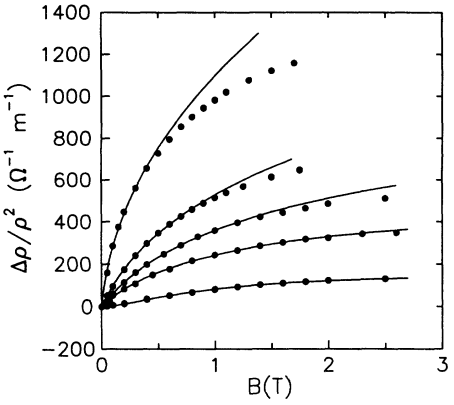

FIG. 3. $\Delta\rho(B)/\rho^2(0)$ as a function of magnetic field B for $Ti_{0.928}Al_{0.042}Sn_{0.030}$ alloy at (from bottom to top) 10.0, 3.50, 2.45, 2.00, and 1.70 K, respectively. The symbols are the experimental results, and the curves are the predictions of the theory (1) with the values of the relevant parameters given in Table I, the values of β (open squares) shown in Fig. 5, and the values of B_i (open squares) shown in Fig. 6.

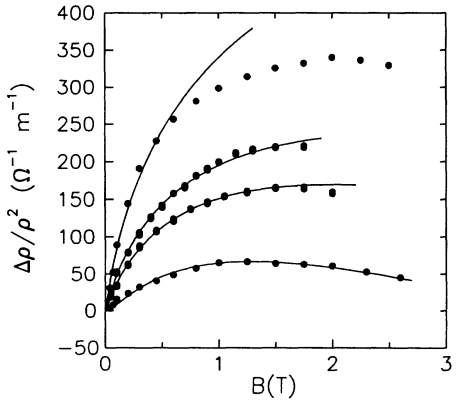

FIG. 2. $\Delta\rho(B)/\rho^2(0)$ as a function of magnetic field B for $Ti_{0.923}Ag_{0.005}Al_{0.072}$ alloy at (from bottom to top) 7.00, 3.00, 2.00, and 1.70 K, respectively. The symbols are the experimental results, and the curves are the predictions of the theory (1) with the values of the relevant parameters given in Table I, the values of β (open triangles) shown in Fig. 5, and the values of B_i (open triangles) shown in Fig. 6.

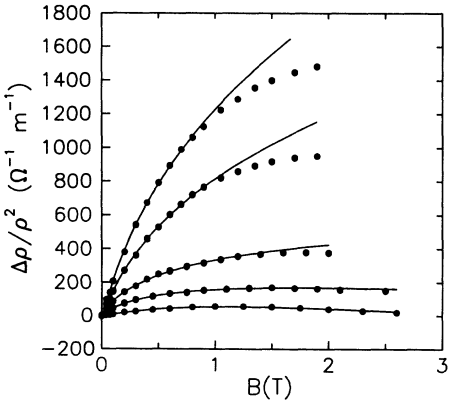

FIG. 4. $\Delta\rho(B)/\rho^2(0)$ as a function of magnetic field B for $Ti_{0.928}Al_{0.0715}Co_{0.0005}$ alloy at (from bottom to top) 8.00, 3.02, 2.06, 1.85, and 1.70 K, respectively. The symbols are the experimental results, and the curves are the predictions of the theory (1) with the values of the relevant parameters given in Table I, the values of β (open circles) shown in Fig. 5, and the values of B_i (closed squares) shown in Fig. 6.

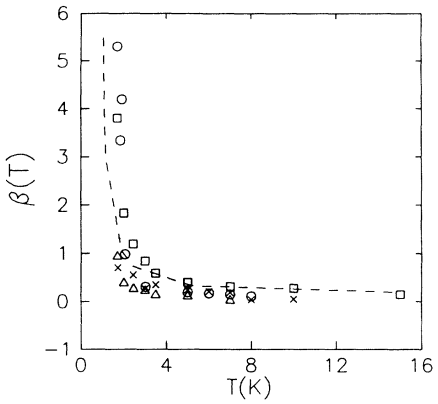

FIG. 5. The coefficient β for the Maki-Thompson contribution to magnetoresistivity as a function of temperature T for $Ti_{0.928}Al_{0.072}$ (crosses), $Ti_{0.923}Ag_{0.005}Al_{0.072}$ (open triangles), $Ti_{0.928}Al_{0.042}Sn_{0.030}$ (open squares), and $Ti_{0.928}Al_{0.0715}Co_{0.0005}$ (open circles) alloys. The dashed curve is the prediction of Larkin's theory (Ref. 21) with a $T_c = 0.7$ K.

curves are the predictions of the theory (1). Inspection of this figure indicates that the $\Delta\rho(B)/\rho^2(0)$ behavior with B for this sample is quite similar to that of Ti96, Fig. 1. Again, the least-squares fitted values of the relevant parameters are listed in Table I, the values of β (open triangles) are plotted in Fig. 5, and the values of B_i (open triangles) are shown in Fig. 6. Our results reveal that, with the addition of a minor amount (0.5 at.%) of Ag for Ti, B_{so} is slightly increased to 0.32 T. This is readily expected, since Ag has a larger atomic weight than that of Ti. Otherwise, the inelastic fields are barely changed, see Fig. 6. Thus, our observed $\Delta\rho(B)/\rho^2(0)$ for Ag1, both B and T dependences and magnitudes, and the inferred inelastic fields, are all in good accord with those observed in Ti96. Note that we expect that the electronic properties (other than the spin-orbit scattering) of Ag1 and Ti96 are essentially similar such that WL effects and superconducting fluctuations in them are essentially identical. Our results on Ag1 and Ti96 therefore provide a convincing consistency check of our experimental method and the credibility of the theory (1).

Figure 3 shows the representative normalized magnetoresistivities $\Delta\rho(B)/\rho^2(0)$ as a function of B for Sn8 at several temperatures as indicated in the caption to Fig. 3. The symbols are the experimental results, and the curves are the predictions of the theory (1). Again, the $\Delta\rho(B)/\rho^2(0)$ behavior with B is characteristic of the WL and Maki-Thompson superconducting fluctuation effects. There are two features which deserve discussion. First, in contrast to those in Ti96 and Ag1, this figure reveals that $\Delta\rho(B)$ are monotonically increasing functions of B in all magnetic fields and at all temperatures used in this work. Obviously, this is due to the increased spin-orbit scattering rate in this alloy. Second, the magnitudes of $\Delta\rho(B)/\rho^2(0)$ are largely increased from those in Ti96, especially at T below about 2 K. At 1.7 K, the $\Delta\rho(B)/\rho^2(0)$ in Sn8 is a factor ~ 3–4 larger than the corresponding magnetoresistivity in Ti96. This latter behavior is partly due to an onset of superconductivity which is observed in this alloy around this temperature (see Fig. 7). [The large magnetoresistivities observed in our measuring magnetic fields result partly from the increased spin-orbit scattering rate in this sample, check Eq. (1).]

Our least-squares fitted values of the relevant parameters for Sn8 are listed in Table I, the values of β (open squares) are plotted in Fig. 5, and the values of B_i (open squares) are shown in Fig. 6. Our results reveal that, with the addition of 3.0 at. % of Sn, the value of B_{so} is increased by a factor ~ 2 (from 0.25 T for Ti96 to 0.54 T for Sn8), as expected since Sn has a large atomic weight relative to that of Ti, and also the doping level is high.

Figure 4 shows the representative normalized magnetoresistivities $\Delta\rho(B)/\rho^2(0)$ as a function of B for Co1 at several temperatures as indicated in the caption to Fig. 4. The symbols are the experimental results, and

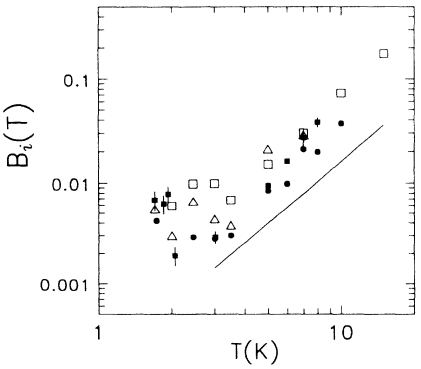

FIG. 6. Inelastic fields B_i as a function of temperature for $Ti_{0.928}Al_{0.072}$ (closed circles), $Ti_{0.923}Ag_{0.005}Al_{0.072}$ (open triangles), $Ti_{0.928}Al_{0.042}Sn_{0.030}$ (open squares), and $Ti_{0.928}Al_{0.0715}Co_{0.0005}$ (closed squares) alloys. The straight line is drawn proportional to T^2, and is a guide to the eye.

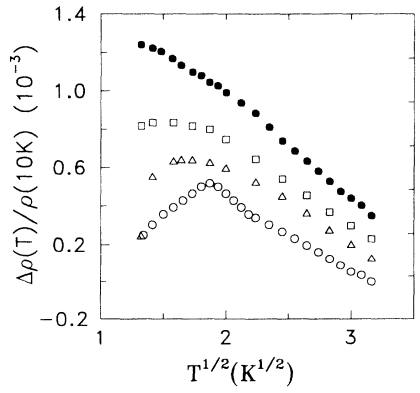

FIG. 7. Variations of the resistivity rises $\Delta\rho(T) = \rho(T) - \rho(10\text{ K})$ normalized to $\rho(10\text{ K})$ with $\sqrt{T}$ for $Ti_{0.928}Al_{0.072}$ (closed circles), $Ti_{0.923}Ag_{0.005}Al_{0.072}$ (open squares), $Ti_{0.928}Al_{0.042}Sn_{0.030}$ (open triangles), and $Ti_{0.928}Al_{0.0715}Co_{0.0005}$ (open circles) alloys. These resistivity rises are measured in zero magnetic field.

the curves are the predictions of the theory (1). Clearly, the $\Delta\rho(B)/\rho^2(0)$ behavior with B is characteristic of the WL and Maki-Thompson superconducting fluctuation effects. Moreover, there are two features which deserve attention. First, in contrast to those in Sn8, this figure reveals that $\Delta\rho(B)$ increases first with increasing B, but can then be reduced with increasing B at higher magnetic fields. This is most clearly demonstrated in the curves at 3.02 and 8.0 K, respectively. This behavior suggests that spin-orbit scattering is not strong in this alloy, as Co is only slightly heavier than Ti, besides the doping level (0.05 at.%) is extremely low. Second, the magnitudes of $\Delta\rho(B)/\rho^2(0)$ are largely increased from those in Ti96, especially at T below about 3 K. At 1.7 K, the $\Delta\rho(B)/\rho^2(0)$ in Co1 is a factor $\sim$6–7 larger than the corresponding magnetoresistivity in Ti96. This latter behavior is due to the enhanced superconductivity in this particular alloy. As mentioned in the Introduction, this is due to the fact that T_c is highly increased in Co-doped Ti. For instance, Matthias *et al.*[19] previously found that T_c is monotonically increased from $\approx$0.4 K for pure Ti to $\approx$2.8 K for Ti-Co with a doping level of 3 at.% of Co. (See further discussion below.) Our least-squares fitted values of the relevant parameters for Co1 are listed in Table I, the values of β (open circles) are plotted in Fig. 5, and the values of B_i (closed squares) are shown in Fig. 6. Indeed, our quantitative analyses reveal that, with the addition of a minor amount of Co, the value of B_{so} (= 0.25 T) remains *unchanged* to within our experimental uncertainties. On the other hand, the low-T values of β and B_i increase largely relative to the corresponding values for Ti96.

The Maki-Thompson superconducting fluctuation contribution to the low-T and low-B magnetoresistivities in a 3D disordered system is governed by the term $\beta(T)f_3(B/B_\phi)$ in Eq. (1). The values of β as a function of T for a given superconductor (with a given T_c) has been calculated and tabulated by Larkin.[21] In Fig. 5, we plot the variations of the fitted values of β with temperature for our Ti96 (crosses), Ag1 (open triangles), Sn8 (open squares), and Co1 (open circles) alloys. Because the T_c's of our alloys have not been directly measured, we plot the variations of β with T, instead of with the reduced temperature T/T_c. The dashed curve shown in this figure is the prediction of Larkin with a $T_c = 0.7$ K. This figure shows that the general behavior of our experimental β is in fair agreement with Larkin's theory. Since, once the value of T_c is given, there are no other adjusting parameters involved in determining this theoretical curve, the agreement between the theory and experiment thus strongly supports Larkin's theory. (Thus, the rather low T_c's of our alloys can be inferred from measurements of the magnetoresistivities at much higher temperatures. However, in the present work we have not attempted to estimate the individual value of T_c for each alloy.) We note that, from about 4 down to 1.7 K, the values of β for Ti96 and Ag1 only show slight increases, while the values for Sn8 and Co1 show rapid increases. This rapidly increasing behavior of β in Sn8 and Co1 results in large magnetoresistivities as has been seen in Figs. 3 and 4, respectively. The value of β at 1.7 K for Co1 is a factor $\sim$1.5 higher than that for Sn8, while it is a factor $\sim$6.5 higher than that for Ti96. At this temperature, $\beta \approx 0.8$, 1.0, 3.8, 5.3 for Ti96, Ag1, Sn8, and Co1, respectively.

B. Inelastic scattering times

It has been well established that measurements of the magnetoresistivities due to WL effects and Maki-Thompson fluctuational superconductivity can provide quantitative information of the various electron relaxation times in disordered systems. Along this line, we have in this work carefully determined the values of the inelastic field B_i, spin-orbit field B_{so}, and magnetic spin-spin field B_s for our alloys by quantitatively comparing our measured magnetoresistivities with the predictions of (1). Such work is of great interest especially because there has been basically no report available on the inelastic scattering times in 3D crystalline disordered metals. Among the three characteristic fields B_i, B_{so}, and B_s, only B_i is T dependent. For a given sample, the other two characteristic fields B_{so} and B_s are T independent and should be the same for the whole set of magnetoresistivity curves measured at different temperatures. First, we consider the values of B_{so}. In performing least-squares fits of our experimental results to the predictions of (1), we first treat B_{so} as a free parameter for each magnetoresistivity curve. Then, for a given sample, we take the average value (exclude 3–4 curves taken at our lowest and highest measuring temperatures) as the fixed value of B_{so} for the whole set of magnetoresistivity curves. The determined value of B_{so} for each alloy is listed in Table I, and the physical implications of its strength have been discussed in the above subsection where appropriate. Second, consider the values of B_s. Since we have used high-purity starting materials to fabricate our alloys (Sec. II), we do not expect our alloys to contain magnetic impurities, i.e., we do not expect spin-spin scattering to play any noticeable role. Indeed, we find that all of the magnetoresistivity curves in our various alloys can be quantitatively interpreted with a zero B_s (Table I).[19] This absence of spin-spin scattering manifests itself in the T dependences of B_i shown in Fig. 6, where *no* sign of a saturation of B_i (or, more precisely, B_ϕ) is observed in any alloy at the lowest measuring temperatures.[38] Also, the remarkable positive contribution from the $\beta(T)f_3(B/B_\phi)$ term in each sample (especially in Sn8 and Co1) at low temperatures, Figs. 1–4, is a good indication of null spin-spin scattering. If there were any appreciable spin-spin scattering in our alloys, the Maki-Thompson contribution would have been greatly diminished.[21,22]

Figure 6 shows the variations of B_i with T for Ti96 (closed circles), Ag1 (open triangles), Sn8 (open squares), and Co1 (closed squares) alloys. The straight line is drawn proportional to T^2, and is a guide to the eye. For clarity, our experimental uncertainties for the values of B_i are only given for Co1, which also serve to represent the typical uncertainties we derive in this work. This figure clearly demonstrates that the values of $B_i(T)$ of these four alloys are basically the same. This is plausible, since one would not expect a minor addition of Ag,

Sn, or Co to cause a substaintial change in the inelastic scattering. (Recall that all of our alloys have similar impurity resistivities. See Table I.) Most significantly, this figure indicates that the $B_i(T)$'s for all four alloys vary with T^2 at above about 3 K. This T^2 dependence is in accordance with the prediction of the theory (3) for electron-phonon scattering in disordered metals.

Quantitatively, Eq. (3) predicts a $\tau_{\rm ph} \approx 7.5\times10^{-11}$ s for Ti96 at 10 K, assuming $\theta_D = 420$ K (the Debye temperature for elemental Ti, Ref. 39), $k_F\ell \approx 9.2$ (Table I), and $\lambda \approx 0.2$. Here the value of λ has been estimated from Eq. (4) using the values for pure Ti: $k_F = 1.46\times10^{10}$ $\rm m^{-1}$ (Ref. 34), $Z = 3$–4, and $n_i = 5.66\times10^{28}/\rm m^3$ (Ref. 39). Experimentally, we find $B_i \approx 0.037$ T for Ti96 at 10 K. With $D = 8.6\times10^{-5}$ $\rm m^2/s$ for this alloy, we obtain $\tau_i \approx 5.2\times10^{-11}$ s. Thus, our experimental results for τ_i is consistent within $\sim 40\%$ with the theoretical prediction of Takayama for the electron-phonon scattering in disordered metals.[12] In three dimensions such *close* agreement between theory and experiment has rarely been reported in the literature.[40] This observation suggests the advantage of the use of crystalline disordered metallic systems over glassy-metal systems for the study of WL and superconducting fluctuation effects. With this value of τ_i, we are well in the temperature regime $2\pi k_B(T - T_c) \gg \hbar/\tau_i$, where only the Maki-Thompson term is important while the Aslamazov-Larkin contribution can be entirely ignored.[21,22,26]

Close inspection of Fig. 6 for the B_i behavior with T for Co1 reveals a clear deviation from the T^2 dependence at T below about 3 K. At 2.0 K, a deviation is already observable. Between about 1.7 and 1.9 K, in constrast to a decrease with reducing T, the values of B_i increase pronouncedly from the values as would be extrapolated from the T^2 dependence at high temperatures. For example, we obtain $B_i(1.8\ \rm K) \approx 0.007$ T, compared to $B_i(2.0\ \rm K) \approx 0.002$ T. This pronounced increase in B_i with reducing T is caused by the intensification of electron-electron interaction in the Cooper channel near the critical temperature. Physically, as T approaches T_c, τ_i^{-1} will diverge because of the increasing probability that an electron will meet another electron of nearly opposite momentum and spin and condense into a superconducting fluctuation.[4,31] Similar diverging behavior of τ_i^{-1} (or, equivalently, B_i) with decreasing T at low temperatures has been first observed in thin Al films by Gordon *et al.*[31] In 3D, to the best of our knowledge, we are not aware of any experimental report of such phenomenon. This observation of a rapid increase in B_i below 2.0 K is consistent with the fact that a minor addition of Co enhances the superconductivity in Ti.

We also discuss briefly the observed leveling-off of the $B_i(T)$ for Ti96 between ≈ 2.5 and 4 K. Taking fluctuational superconductivity into consideration, we believe that this leveling-off behavior is not related in any way to magnetic spin-spin scattering.[38] Rather, this is most likely related to intensification of electron-electron interaction as T approaches T_c. In this case, however, the intensification is less pronounced as that in Co1. And even at our lowest measuring temperature of 1.7 K, the value of B_i is only increased from that at 3 K by a factor ~ 1.3. [In Co1, $B_i(1.7\ \rm K)/B_i(2.0\ \rm K) \approx 3.6$.]

In Fig. 7, we plot the variations of the resistivity rises $\Delta\rho(T) = \rho(T) - \rho(10\ \rm K)$ normalized to $\rho(10\ \rm K)$ with $\sqrt{T}$ for Ti96 (closed circles), Ag1 (open squares), Sn8 (open triangles), and Co1 (open circles) alloys. For clarity, we only show the data for T below 10 K. These resistivities are measured in zero magnetic field. This figure reveals that $\Delta\rho$ scales essentially with $\sqrt{T}$ above about 4 K. This behavior has been quantitatively explained in terms of electron-electron interaction effects.[18] In 3D disordered metals in zero magnetic field, electron-electron interaction effects should dominate over WL effects and cause a resistivity rise proportional to the square root of T (while in low magnetic fields, WL effects dominate).[1–5] At T below about 4 K, a downward deviation from the $\sqrt{T}$ dependence becomes noticeable in every alloy, signifying the importance of superconducting fluctuation effects ("paraconductivity") as the superconducting transition is approached. Notice that the downward deviation is most significant in Co1. This result strongly supports our observation of large values of β (Fig. 5) as well as of a "diverging" behavior of $B_i(T)$ (Fig. 6) at our lowest measuring temperatures in this alloy. By applying a magnetic field of 6 T, we have observed that superconductivity is completely quenched in every alloy, and that the $\sqrt{T}$ dependence is followed down to 1.6 K.[41]

Finally, we mention that the inelastic fields in our alloys cannot be explained by electron-electron scattering. For electron-electron scattering, Schmid has given a general expression for the scattering rate in 3D disordered metals and it can be expressed as[42]

$$\frac{1}{\tau_{ee}} = \frac{\pi}{8}\frac{(k_BT)^2}{\hbar E_F} + \frac{\sqrt{3}}{2}\frac{1}{(k_F\ell)^{3/2}}\frac{(k_BT)^{3/2}}{\hbar\sqrt{E_F}}\ , \qquad (5)$$

where E_F is the Fermi energy. A similar expression has also been derived by Al'tshuler and Aronov.[43] The first term in (5) dominates in the pure case while the last term dominates in the strong disorder limit. Using $E_F = \hbar k_F v_F/2$ and the values of k_F quoted above and $v_F = 0.32\times10^6$ m/s (Ref. 34) for Ti, and $k_F\ell = 9.2$ for Ti96, we obtain $\tau_{ee} \approx (3.4\times10^{-7}T^{-2} + 3.3\times10^{-8}T^{-3/2})$ s. The scattering time due to the first term is ~ 2 orders of magnitude longer than our experimental value, although it possesses a T^{-2} dependence. The second term possesses a $T^{-3/2}$ dependence which is not consistent with our results. Besides, this latter term predicts a scattering time a factor ~ 20 longer than our experimental value. Therefore, our observations in Fig. 6 can by no means be accounted for by electron-electron scattering.

V. CONCLUSION

We have measured the magnetoresistivities of $\rm Ti_{0.928}Al_{0.072-y}(Sn,Co)_y$ alloys with values of $y \le 0.03$. First, Al atoms are introduced into a Ti host to create a disordered system in which weak-localization and Maki-Thompson superconducting fluctuation effects are manifest. This is a crystalline disordered metallic system, in contrast to,

e.g., the glassy-metal systems usually used in the literature for studies of quantum corrections to resistivities and magnetoresistivities in three dimensions. Addition of a minor amount (0.5 at.%) of Ag for Ti has been performed to produce a parallel system for a stringent consistency check of our experimental method and the credibility of the theory. A third sample containing 3.0 at.% of Sn is then fabricated, in which spin-orbit scattering is quite significant. Therefore, the magnetoresistivities in it are monotonically increasing functions of the external magnetic field. Our fourth sample contains 0.05 at.% of Co, in which fluctuational superconductivity is largely enhanced at our measuring temperatures. From our magnetoresistivity measurements, values of the various electron scattering fields have been inferred, including inelastic, spin-orbit, and spin-spin fields. In particular, the temperature dependence of the inelastic field B_i has been determined to be $B_i \sim T^2$ between 3 and 15 K. Our experimental results for $B_i(T)$ agree to within $\sim 40\%$ with the theoretical prediction of electron-phonon scattering in disordered metals by Takayama. In three dimensions such close agreement between theory and experiment has rarely been reported in the literature. In addition, a diverging behavior of $B_i(T)$ at our lowest measuring temperatures (1.7–1.9 K) due to intensification of superconducting fluctuations has been observed in the alloy with Co addition. Our overall results have provided a consistent and quantitative check of the predictions of the weak-localization theory and Maki-Thompson theory.

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