Singlet pairing in the double-chain t-J model

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Applying the bosonization procedure to constrained fermions in the framework of the one-dimensional t-J model, we discuss a scenario of singlet superconductivity in a lightly doped double chain where all spin excitations remain gapful.

In past years the problem of non-Landau Fermi liquid behavior in quasi-one-dimensional systems again attracted a strong interest. This time it was stimulated by Anderson's idea about an effectively one-dimensional (1D) dynamics of excitations in the normal state of layered high-T_c cuprates.[1]

The basic problem here is a complete description of a "dimensional crossover" which may occur as a result of varying coupling between 1D systems (chains) forming a two- or three-dimensional array. In particular, one of the main issues is whether a coherent transport between chains establishes at arbitrary small interchain coupling or whether there is a finite threshold resulting from the "confinement" phenomenon.[1]

Various weak coupling studies of the infinite array problem do not seem to confirm Anderson's picture, although one might think that the situation becomes different at strong coupling. On the other hand systems of a finite number of chains provide interesting examples of a peculiar behavior in an "intermediate dimension." In analogy with a purely 1D case one might expect that these also allow a consistent strong coupling treatment.

Moreover these models can also describe properties of such real materials as $(VO)_2P_2O_7$ or $Sr_2Cu_4O_6$ which contain weakly coupled metal-oxide-metal double-chain ladders. It was also pointed out in Ref. 2 that higher stoichiometric compounds $Sr_{n-1}Cu_{n+1}O_{2n}$ provide a physical realization of weakly coupled n-chain ladders.

A proper Hamiltonian of a strongly correlated double chain is that of the t-J model with antiferromagnetic spin exchanges:[2]

$$H = -\sum_{i,\sigma}\left(t\sum_{f} c^{\dagger}_{i,\sigma,f}c_{i+1,\sigma,f} + t_{\perp}c^{\dagger}_{i,\sigma,u}c_{i,\sigma,d} + \text{H.c.}\right) + \sum_{i}\left[J\left(\mathbf{S}_{i,f}\mathbf{S}_{i+1,f} - \frac{1}{4}n_{i,f}n_{i+1,f}\right) + J_{\perp}\left(\mathbf{S}_{i,u}\mathbf{S}_{i,d} - \frac{1}{4}n_{i,u}n_{i,d}\right)\right], \quad (1)$$

where $f = u, d$ is the chain index. The Hamiltonian (1) has to be complemented by the no-double occupancy constraint

$$n_{i,f} = \sum_{\sigma} c^{\dagger}_{i,\sigma,f}c_{i,\sigma,f} \leq 1. \quad (2)$$

Accordingly, a weak coupling regime can be studied in the framework of the Hubbard model.

At half filling the Hamiltonian (1) describes $S = 1/2$ double-chain Heisenberg model. Available numerical results[3,4] as well as a mean field analysis using the Gutzwiller projection[5] indicate that in contrast to the case of a single chain there are no gapless spin excitations in a double chain. Moreover the spin gap appears to be robust against doping and survives in some range around half filling. These observations are in agreement with the conjecture[2] that a lightly doped double-chain system becomes a singlet superconductor of a modified d-wave type.

Recent weak coupling renormalization group (RG) studies of the double-chain Hubbard model did reveal some spin gapful fixed points characterized by an enhanced singlet pairing in both cases $U < t \sim t_{\perp}$ (Ref. 6) and $t_{\perp} < U < t$.[7] Some enhancement of power-law decaying pairing correlations was also shown numerically.[4]

To clarify the essence of the double-chain physics it is worthwhile to review properties of the single chain t-J model. In the region of ratios $J/t \lesssim 1$ the model can be only found in the so-called Tomonaga-Luttinger (TL) regime which corresponds to both gapless spin and charge excitations.[8,9] It is customary to describe the TL behavior in terms of spin and charge correlation exponents K_s and K_c.

The spin exponent K_s equals to unity everywhere in the TL regime while K_c gradually increases from the value 1/2 which it reaches at $J = 0$ and any density ρ as well as at $\rho \to 1$ and arbitrary J/t as J/t increases or ρ gets smaller. The TL regime persists up to $J/t \approx 2.5$ where a spin gap with strong pairing correlations occurs at small enough fermion density ($\rho \lesssim 1/3$). In fact, one can understand the occurrence of the region of attraction at small ρ as resulting from the existence of a two-particle bound state at zero density. A finite threshold in the attraction strength follows from vanishing of the bound state wave function at zero separation due to the no-double-occupancy constraint. On the other hand in the regime of strong correlation at ρ close to unity one can argue that gapless spin fluctuations drive couplings of the charge sector to the repulsive region and to get an effective attraction ($K_c > 1$) one has to exceed some threshold value of J/t.

However if both ρ and the critical value of J/t are large then the attraction of charges actually leads to the phase separation rather than to the real superconducting pairing. Even the inclusion of the short-range repulsion which is supposed to postpone the onset of phase separation to higher J/t does not extend the region of singlet pairing.[10]

If, on the contrary, spin fluctuations are gapful then these may not renormalize charge couplings significantly. Therefore the charge correlation exponent may not receive its basic contribution leading to $K_c < 1$ at small J/t and the mechanism of attraction may work without any threshold in J/t. Various possibilities to get such a behavior by means of the frustrating spin exchange interactions in a single chain were considered in Refs. 11 and 12.

Coupling between Luttinger chains provides an alternative way to produce a spin gap favoring attraction between charges.

Preceding analytical studies of the problem in the framework of the bosonized "g-ology" Refs. 6, 13, and 7 were restricted on the case of weak coupling.

In the present paper we find additional arguments in favor of the above scenario by using a bosonic representation of the t-J model which is an adequate tool to study a strong coupling behavior.

Although the method of bosonization is conventionally applied to 1D weakly interacting fermions it might be possible to formulate a consistent procedure for the opposite limit when the interaction is extremely strong, that is for the case of constrained fermions. Various versions of the bosonization procedure in the framework of the t-J model were discussed in the literature.[14,15] Recently a modification of the approach proposed in Ref. 15 was shown to give correct exponents for the one- and two-particle correlation functions as well as a good approximation for the energy spectrum of the single-chain t-J model.[16]

According to the method of Ref. 16 the constrained fermion operator $c_{i,\sigma}$ can be represented as a product of a spinless fermion Ψ_i and a spin-one-half operator (hard-core boson) $S_i^{\pm}$:

$$c_{i\uparrow} = P_i \Psi_i S_i^- P_i^\dagger, \qquad c_{i\downarrow} = P_i \Psi_i S_i^+ P_i^\dagger, \tag{3}$$

where the projection operator P_i reduces the space of four on-site states ($|\text{hole}\rangle \otimes |\text{spin}\rangle = |1,\uparrow\rangle, |1,\downarrow\rangle, |0,\uparrow\rangle, |0,\downarrow\rangle$) to the physical Hilbert space formed by the set $|0\rangle$, $|\uparrow\rangle, |\downarrow\rangle$.

In turn, the spin-one-half operator $S_i^{\pm}$ can be expressed in terms of a spinless Jordan-Wigner fermion

$$\begin{aligned} S_i^+ &= \chi_i^\dagger \exp\left(i\pi \sum_{j<i} \chi_j^\dagger \chi_j\right), \\ S_i^- &= \chi_i \exp\left(-i\pi \sum_{j<i} \chi_j^\dagger \chi_j\right), \\ S_i^z &= \chi_i^\dagger \chi_i - \frac{1}{2}. \end{aligned} \tag{4}$$

The authors of Ref. 16 also argued that the local constraint (2) and the sum rule for the constrained fermions $\sum_\sigma \int_0^\infty \frac{d\omega}{2\pi} \text{Im}\langle\{c_{i,\sigma}^\dagger(\omega), c_{i,\sigma}(-\omega)\}\rangle = 1 + \delta$ (where δ is doping) are obeyed even in the approximation which discards the projector P_i. Therefore this approximation which reportedly yields correct exponents of correlation functions and momentum distribution was supposed to provide a better account of fermion correlations than other approaches which do not treat the hard-core condition properly.

With the neglect of projectors P_i representation (3) allows one to rewrite (1) in the form

$$\begin{aligned} H = &-\sum_{i,\sigma}\left(t\sum_f \Psi_{i,f}^\dagger \Psi_{i+1,f}(S_{i,f}^+ S_{i+1,f}^- + S_{i,f}^- S_{i+1,f}^+) + t_\perp \Psi_{i,u}^\dagger \Psi_{i,d}(S_{i,u}^+ S_{i,d}^- + S_{i,u}^- S_{i,d}^+) + \text{H.c.}\right) \\ &+ \sum_i (J n_{i,f} \mathbf{S}_{i,f} \mathbf{S}_{i+1,f} n_{i+1,f} + J_\perp n_{i,u} \mathbf{S}_{i,u} \mathbf{S}_{i,d} n_{i,d}) - \mu \sum_{i,f} \Psi_{i,f}^\dagger \Psi_{i,f} \end{aligned} \tag{5}$$

with a local charge density defined as $n_{i,f} = \Psi_{i,f}^\dagger \Psi_{i,f}$ and a local spin $\mathbf{S}_{i,f}$ given in terms of $\chi_{i,f}$ according to (4).

To get a bosonic form of the Hamiltonian (5) one can use the representation ($\theta_{c,s}^f$ are dual to $\phi_{c,s}^f$)

$$\Psi_{i,f} \sim \sum_{\mu=R,L} e^{i\mu\pi(1-\delta)x + i\mu\phi_c^f(x) + i\theta_c^f(x)}, \quad \chi_{i,f} \sim \sum_{\mu=R,L} e^{i\mu\pi x + i\mu\phi_s^f(x) + \theta_s^f(x)}. \tag{6}$$

Keeping the most relevant operators in the continuous limit we obtain the bosonized Hamiltonian density

$$\begin{aligned} H_B = \frac{1}{2}\sum_{\pm}\Bigg[& v_c^{\pm}[(\partial\theta_c^{\pm})^2 + (\partial\phi_c^{\pm})^2 + \partial\phi_c^{\pm}\partial\phi_s^{\pm} - \partial\theta_c^{\pm}\partial\theta_s^{\pm}] \\ &+ v_s[(\partial\theta_s^{\pm})^2 + (\partial\phi_s^{\pm})^2] + J\left(1 - \delta + \frac{1}{\sqrt{2}\pi}(\partial\phi_c^+ \pm \partial\phi_c^-)\right)^2 \cos 2(\phi_s^- \pm \phi_s^+)\Bigg] \\ &+ t_\perp[\cos\phi_c^- + \cos(\phi_c^+ + 2\delta x)\cos\phi_s^+]\cos\theta_c^-\cos\theta_s^- \\ &+ J_\perp\left(1 - \delta - \frac{1}{\sqrt{2}\pi}(\partial\phi_c^+ + \partial\phi_c^-)\right)\left(1 - \delta - \frac{1}{\sqrt{2}\pi}(\partial\phi_c^+ - \partial\phi_c^-)\right)(\cos 2\phi_s^+ + \cos 2\phi_s^- + \cos\theta_s^-), \end{aligned} \tag{7}$$

where $\phi^{\pm}_{c,s} = \frac{1}{\sqrt{2}}(\phi^u_{c,s} \pm \phi^d_{c,s})$ and $\theta^{\pm}_{c,s} = \frac{1}{\sqrt{2}}(\theta^u_{c,s} \pm \theta^d_{c,s})$. As usual, the values of correlation exponents $K^{\pm}_{c,s}$ can be affected by short-wavelength renormalizations. The bare velocities of charge and spin excitations are given by the formulas $v_c = 2t \sin \pi\delta$ and $v_s = 2J[(1-\delta)^2 - (\frac{\sin \pi\delta}{\pi})^2] + 4\pi t \sin \pi\delta$, although these can be altered too.

We note that expression (7) can be also obtained by using the CP^1 coherent state representation first introduced in Ref. 17 and applying a somewhat different bosonization scheme discussed in Ref. 14. We believe that it verifies a neglect of the projecting operators when deriving a relevant part of the continuous bosonic Hamiltonian.

The first three terms in (7) can be recognized as the Hamiltonian of the charge carrying spinless fermion coupled to the Abelian gauge field $A_\mu = \epsilon_{\mu\nu}\partial_\nu \phi_s = (\partial\phi_s, \partial\theta_s)$ which describes a surrounding spin background. Notice that once the constraint was explicitly resolved one obtains only two independent fields instead of three (spinon, holon, and a gauge field) appearing in mean field studies of the t-J model.

In the case of a single chain the basic TL phase of the t-J model corresponds to both ϕ_c and ϕ_s being gapless. Scaling dimensions of operators of the form $\cos\beta\phi \cos\beta'\theta$ can be estimated by means of the formula

$$\Delta = \frac{1}{4}\left(K\beta^2 + \frac{\beta'^2}{K}\right). \tag{8}$$

At $\delta = 0$ (7) becomes equivalent to the bosonized Heisenberg double chain due to the effective freezing of the charge degrees of freedom. The resulting expression essentially coincides with the relevant part of the one obtained in Ref. 18 where a more general XXZ symmetrical case was considered.

It was argued in Ref. 18 that at least one of the operators $\cos 2\phi^-_s$ and $\cos\theta^-_s$ appears to be relevant and drives the "$-$" spin sector toward a strong coupling regime where either ϕ^-_s or θ^-_s gets locked and a corresponding cosine acquires a nonzero expectation value.

Additionally, at $K^+_s < 1$ the "+" sector gets to a strong coupling regime where ϕ^+_s is locked. Therefore in general there are four possible phases with finite v_s which can be identified as follows ($\eta = 2/K^+_s$): (1) both ϕ^-_s, ϕ^+_s are locked—the antiferromagnetically ordered state; (2) both θ^-_s and ϕ^+_s are locked—singlet state (all spin excitations are gapful); (3) θ^-_s is locked—the XY-type phase characterized by correlations:

$$\langle S^z_f(x) S^z_f(0)\rangle \sim \frac{1}{x^2} + (-1)^x e^{-x},$$
$$\langle S^+_f(x) S^-_f(0)\rangle \sim (-1)^x \frac{1}{x^\eta} + e^{-x}; \tag{9}$$

(4) ϕ^-_s is locked—another gapless phase having different spin correlations:

$$\langle S^z_f(x) S^z_f(0)\rangle \sim \frac{1}{x^2} + (-1)^x \frac{1}{x^{1/\eta}}$$
$$\langle S^+_f(x) S^-_f(0)\rangle \sim e^{-x}. \tag{10}$$

All these states besides the last one were argued in Ref. 18 to appear on an extended phase diagram of the double chain XXZ symmetrical (Heisenberg-Ising) spin model. The analysis carried out in Ref. 18 leads to the conclusion that at $J_\perp > -\frac{1}{4}J^z$ the gapless line $J_\perp = 0$ becomes unstable against arbitrary small $J_\perp$ of any sign.

It has to be noticed that at $J_\perp < 0$ and $J_\perp > 0$ the nature of the singlet ground state is quite different. In the former case every pair of spins on one rung of the ladder tends to form an $S = 1$ state and the system effectively behaves similar to $S = 1$ Heisenberg chain while in the latter case spin pairs couple preferably into singlets which then form a "dimer liquid."

According to the phase diagram proposed in Ref. 18 the spin isotropic point at $J_\perp > 0$ is located deeply inside the gapful "dimer liquid" phase with both θ^-_s, ϕ^+_s being locked. It agrees with numerical[3,4] and mean field[5] results.

We also note that in a general case of N-chain ladders one may expect that a spin gap is present at even N only. To see that one can apply arguments due to Haldane.[19] In Ref. 19 a topological term governing a long-wavelength dynamics of the 2D lattice Heisenberg model was found in the form $\sum_y (-1)^y Q_y(x,t)$, where $Q_y(x,t)$ is a topological θ term appearing in a purely 1D case and distinguishing between integer and half-odd integer spins.[20] In the 2D case with periodic boundary conditions this sum is equal to zero which means the absence of a 2D counterpart of the 1D θ term. However applying this formula to the finite width strip one can see that for odd N the above sum does not vanish and therefore an effectively 1D long-wavelength dynamics remains gapless. In contrast, for even N the gap survives and then scales as $\Delta(N) \sim \exp(-N)$.

On the basis of the results obtained in Refs. 3–5 (and recently confirmed in Ref. 21) we assume that at small doping the hopping terms in (7) can be treated as perturbations which do not destroy the gapful spin state. Then we get the effective Hamiltonian describing a charge dynamics in the spin gap state

$$H_c = \frac{1}{2}\sum_{\pm} v^{\pm}_c \left(K^{\pm}_c (\partial\theta^{\pm}_c)^2 + \frac{1}{K^{\pm}_c}(\partial\phi^{\pm}_c)^2\right)$$
$$+ \bar{t}_\perp \cos\phi^-_c \cos\theta^-_c, \tag{11}$$

where $\bar{t}_\perp = t_\perp \langle \cos\theta^-_s \rangle$. Charge correlation exponents which can be easily read off from (5) are given by the formula

$$K^{\pm}_c = \left(1 + \frac{J}{\pi t}\langle \mathbf{S}_f \mathbf{S}_f \rangle \pm \frac{J_\perp}{\pi t}\langle \mathbf{S}_u \mathbf{S}_d \rangle\right)^{-1/2}. \tag{12}$$

Due to the short-range antiferromagnetic order we encounter the case of $K^+_c > 1$ while $K^-_c - 1$ can be, in principal, of both signs.

The physical origin of attraction between charges in the paramagnetic spin gap state with a short-range antiferromagnetic order can be understood on very general grounds. A straightforward manifestation of this phenomenon in the framework of the t-J model is a negative sign in front of the product of charge densities $n_f n_{f'}$ staying in (5) if $\langle \mathbf{S}_f \mathbf{S}_{f'} \rangle < 0$.

A progressive understanding of the spinless double chain problem[22–24] shows that despite the possible vanishing of the single-particle hopping the "−" charge sector of the system always evolves to the strong coupling regime due to the development of either coherent particle-particle or particle-hole pair hopping. This phenomenon was previously discussed by many authors in the context of quasi-one-dimensional conductors.[25]

For the account of these processes triggered by the single-particle hopping the charge Hamiltonian (11) has to be supplemented by the extra terms

$$\delta H_c = g_{ph} \cos 2\phi_c^- + g_{pp} \cos 2\theta_c^- \qquad (13)$$

generated in the course of renormalization. At small $\bar{t}_\perp/t$ a conventional RG procedure applied to the extended Hamiltonian (11) and (13) leads to the system of equations describing a renormalization flow in the "−" charge sector[24,7] (ξ is a scaling variable):

$$\frac{dg_{ph}}{d\xi} = 2(1 - K_c^-)g_{ph} + \bar{t}_\perp^2 \left(K_c^- - \frac{1}{K_c^-} \right),$$
$$\frac{dg_{pp}}{d\xi} = 2\left(1 - \frac{1}{K_c^-}\right) g_{pp} + \bar{t}_\perp^2 \left(\frac{1}{K_c^-} - K_c^- \right),$$
$$\frac{d\ln K_c^-}{d\xi} = \frac{1}{2}\left(-K_c^- g_{ph}^2 + \frac{1}{K_c^-} g_{pp}^2 \right). \qquad (14)$$

The analysis of the solutions of (14) first performed in Ref. 24 shows that depending on the sign of $g_- = g_{ph} - g_{pp}$ either $\cos 2\phi_c^-$ (at $g_- > 0$) or $\cos 2\theta_c^-$ (at $g_- < 0$) acquires a nonzero expectation value. The asymptotic behaviors of the correlation exponent in the two cases are $K_c^-(\xi) \to 0$ and $K_c^-(\xi) \to \infty$, respectively. By considering four possible order parameters for the spinless case

$$CDW_+ \sim \cos(\phi_c^+ + \phi_c^-)\ ,\ CDW_- \sim \cos(\phi_c^+ + \theta_c^-),$$
$$SS_+ \sim \cos(\theta_c^+ + \theta_c^-)\ ,\ SS_- \sim \cos(\theta_c^+ + \phi_c^-), \qquad (15)$$

one concludes that at $g_- > 0$ the competing types of ordering are (intrachain) CDW_+ and (interchain) SS_- while at $g_- < 0$ the relevant orderings are CDW_- and SS_+. In turn, the result of the competition between them depends on the sign of $g_+ = g_{ph} + g_{pp}$.

By mapping the Hamiltonian (11) and (13) onto the spin $S = 1/2$ chain in an external magnetic field the authors of Ref. 24 also argued that the above statements hold at strong coupling too.

Considering different order parameters relevant for fermions with spin

$$CDW_+ = \sum c_{\mu\sigma}^{f\dagger} c_{-\mu,\sigma}^{f} \sim \cos(\phi_c^+ + \phi_c^-)\cos(\phi_s^+ + \phi_s^-),$$
$$CDW_- = \sum c_{\mu\sigma}^{f\dagger} c_{-\mu,\sigma}^{-f} \sim \cos(\phi_c^+ + \theta_c^-)\cos(\phi_s^+ + \theta_s^-),$$
$$SDW_+ = \sum c_{\mu\sigma}^{f\dagger} c_{-\mu,-\sigma}^{f} \sim \cos(\phi_c^+ + \phi_c^-)\cos(\theta_s^+ + \theta_s^-),$$
$$SDW_- = \sum c_{\mu\sigma}^{f\dagger} c_{-\mu,-\sigma}^{-f} \sim \cos(\phi_c^+ + \theta_c^-)\cos(\theta_s^+ + \phi_s^-),$$
$$SS_+ = \sum \sigma c_{\mu\sigma}^{f} c_{-\mu,-\sigma}^{f} \sim \cos(\theta_c^+ + \theta_c^-)\sin(\phi_s^+ + \phi_s^-),$$
$$SS_- = \sum \sigma c_{\mu\sigma}^{f} c_{-\mu,-\sigma}^{-f} \sim \cos(\theta_c^+ + \phi_c^-)\sin(\phi_s^+ + \theta_s^-),$$
$$TS_+ = \sum \sigma c_{\mu\sigma}^{f} c_{-\mu,\sigma}^{f} \sim \cos(\theta_c^+ + \theta_c^-)\sin(\theta_s^+ + \theta_s^-),$$
$$TS_- = \sum \sigma c_{\mu\sigma}^{f} c_{-\mu,\sigma}^{f} \sim \cos(\theta_c^+ + \phi_c^-)\sin(\theta_s^+ + \phi_s^-), \qquad (16)$$

we observe that in contrast to the spinless case, any paramagnetic spin gapful state has only two relevant types of orderings which are SS_- and CDW_- containing fields θ_s^- and ϕ_s^+.[7]

Examining the divergencies of corresponding response functions with the use of (8) and (12) we obtain that at $J_\perp\langle \mathbf{S}_u \mathbf{S}_d\rangle < 0$ the interchain singlet pairing SS_- appears to be the leading instability while in the opposite case $J_\perp\langle \mathbf{S}_u \mathbf{S}_d\rangle > 0$ the ground state is the "flux phase" CDW_-.

The latter state is characterized by the commensurate with density "flux" $\Phi = 2k_F$ defined as a circulation of a phase of the on-rung order parameter $\langle u_i^\dagger d_i + d_i^\dagger u_i\rangle$ through a plaquette formed by two adjacent rungs of the ladder. In the case of spinless fermions this type of ordering called the "orbital antiferromagnet" was first discovered in Ref. 26 as a counterpart of 2D flux states.

It is also instructive to express the above order parameters in terms of the hybridized states corresponding to the mean field "bonding" and "antibonding" bands $B, A = \frac{1}{\sqrt{2}}(u \pm d)$:

$$CDW_- = \sum_\sigma A_{R\sigma}^\dagger A_{L\sigma} - B_{R\sigma}^\dagger B_{L\sigma},$$
$$SS_- = \sum_\sigma A_{R\sigma}^\dagger A_{L,-\sigma}^\dagger - B_{R\sigma}^\dagger B_{L,-\sigma}^\dagger. \qquad (17)$$

Considering the distribution of signs of the order parameter SS_- on the "four-point Fermi surface" $[\mathbf{k} = (k_F, 0), (-k_F, 0), (k_F, \pi), (-k_F, \pi)]$ we observe that it corresponds to the "d-wave" type pairing. One might expect that in a two-dimensional array of weakly coupled double chains with a continuous Fermi surface this type of ordering does transform into an ordinary d-wave pairing.

It follows from the preceding discussion that both instabilities develop without a threshold in J/t or $J_\perp/t$. Note that this statement is in agreement with the results of the weak coupling analysis of the double-chain Hubbard model.[7]

We also add that in the antiferromagnetically ordered state where all spin excitations acquire Ising gaps, the only relevant order parameters could be SS_+ and CDW_+ containing fields ϕ_s^- and ϕ_s^+. However at all $J\langle \mathbf{S}_f \mathbf{S}_f\rangle < J_\perp^2/t$ the intrachain pairing SS_+ is always favored.

In summary, in the present paper we applied the method of bosonization to constrained fermions in the context of the double-chain t-J model. As a result, we found further arguments supporting the recently proposed scenario of singlet superconductivity in the spin gap state of the double-chain problem.[2]

Our analysis was based on the assumption that a small doping does not destroy the spin gap and a spin dynamics remains essentially the same as in the insulating case. This conjecture is supported by the results of numerical studies,[3,4,24] the Gutzwiller projected mean field,[5] and weak coupling "g-ology."[6,7]

In conclusion, we also comment on a recent claim[27] about an existence of a strong coupling fixed point where some spin excitations remain gapless. The authors of Ref. 27 considered the double-chain t-J model without an interchain spin exchange ($J_\perp = 0$). Then on the bare level their Hamiltonian can be assigned to the universality class of the purely forward scattering model considered in Ref. 13. Indeed, in this special case the only field becoming massive is θ_s^-. In principle, it cannot be ruled out that for some specific double-chain models only one of two spin fields becomes gapful. However an investigation of the double-chain Hubbard model[6,7] demonstrates that the presence of the interchain one-particle hopping is already sufficient to generate the antiferromagnetic spin exchange term with $J_\perp \sim \frac{t_\perp^2}{U}$ which will eventually make all spin modes gapful. We believe that it is a general feature of spin isotropic models of strongly correlated fermions on double chains.

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