Critical behavior of crystals with long-range correlations caused by point defects with degenerate internal degrees of freedom

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We analyze the possibility of the appearance of long-range correlations of states of quenched defect systems in disordered solids. For randomly distributed defects with a finite number of degenerate internal degrees of freedom such correlations are shown to appear near the defect percolation threshold if they obey certain nearest-neighbor correlation rules. The corresponding effective Hamiltonian can be viewed as that of a generalized m-component long-range correlated Weinrib-Halperin model. Our renormalization-group investigation shows, however, that it asymptotically decomposes into a set of m noninteracting one-component Weinrib-Halperin models. The coordinates of the stable fixed point of this model are determined to $O(\epsilon^{1/2})$

I. INTRODUCTION

In the last two decades wide investigations have been focused on the properties of disordered solids. In particular, of special interest was the critical behavior of crystals containing quenched defects. In the major part of the literature the considerations have been restricted to the case of point defects with small concentrations so that the corresponding random fields have been assumed to be Gaussian and δ correlated in the frame of the continuous approximation.

An important achievement in this field was the formulation of the Harris criterion.[1] In accordance with this criterion subsequent renormalization group (RG) treatments confirmed that defects of scalar type (random critical temperature defects) are relevant only for the Ising model, and, moreover, only a slight modification of the critical exponents takes place in three dimensions[2] while in two dimensions only logarithmic corrections arise.[3–7] Because the crossover to the impurity-modified critical behavior is governed by the specific heat exponent α which is very small in three dimensions and zero in two dimensions, the modifications due to the impurities are not likely to be observable in real disordered solids.

The latter conclusion is, of course, not true in general. For example, even a small concentration of defects of random field type induces drastic changes in the critical behavior of solids. Unfortunately, in spite of great efforts, the behavior of even the simplest random field impurity Ising model is not known very well.[8,9] This is also true for some other models of disordered solids.[10]

Therefore it is worthwile to investigate defect models which are of physical significance and can be solved analytically and are also open to experimental observations (including numerical simulations).

For this purpose we consider a crystal containing randomly distributed quenched defects with a finite number of degenerated internal degrees of freedom. Such defects can be nonspherical molecules, defect pairs of impurity-impurity or impurity-defect type, defect centers in dumbbell configurations, and other defect complexes having several equivalent energy minima.

It will be shown that near the defect percolation threshold long-range (LR) correlations of states of the defects appear in a rather natural way if simple rules arising from the strong short-range (SR) interaction between the defects are assumed to hold. The presence of LR correlations induces strong changes of the critical behavior of the disordered solid in a rather wide temperature range near a second-order phase transition point.

The paper is organized as follows: In Sec. II we describe the structure of the defects and introduce the rules of their interactions. Then we derive formulas for the pair correlation functions which describe the random defect configurations. In Sec. III we derive the effective Hamiltonian of the model which is found to be an extension of the Hamiltonian considered by Weinrib and Halperin.[11] Section IV is devoted to the analysis of the RG equations which are derived by simultaneous analytical and dimensional regularization of the effective Hamiltonian. It will be shown that due to the existence of a special stable fixed (SF) point in a case which corresponds to diagonal defect matrices all m components of the fluctuating order parameter are decoupled asymptotically in the impurity critical region. Correspondingly, the effective Hamiltonian decomposes into a sum of m Hamiltonians each of which is equivalent to the one-component Weinrib-Halperin (WH) Hamiltonian. Therefore the values of the critical exponents are those of the one-component WH model.

Since WH did not find all coordinates of the SF point we study this partial problem anew. We have been able to determine the position of the SF point and are able to show that its accessibility (a positive value of the SF coordinate for the LR invariant charge) is governed by

the extended Harris criterion.[11]

Section V is devoted to the analysis of the possibility to observe such a critical behavior. Finally we make some concluding remarks including the possibility to extend the considered effect to other systems.

II. MODEL HAMILTONIAN AND DEFECT CORRELATION FUNCTIONS

We start with a most simple model Hamiltonian corresponding to a disordered crystal in the vicinity of a second-order phase transition containing defects which have only two internal (for example orientational) degrees of freedom:

$$\mathcal{H} = \mathcal{H}_c + \mathcal{H}_{\rm imp}, \tag{1}$$

with

$$\mathcal{H}_{\rm imp} = \int d^d\mathbf{r} \sum_{i,k,t} \tilde{V}_{ik}^{(t)}(\mathbf{r})\varphi_i(\mathbf{r})\varphi_k(\mathbf{r}) \tag{2}$$

and the host crystal Hamiltonian which is given in a two-component Landau-Ginzburg-Wilson form

$$\begin{aligned}\mathcal{H}_c = \frac{1}{2}\int d^d\mathbf{r} \{&\tau_0 \left[\varphi_1(\mathbf{r})^2 + \varphi_2(\mathbf{r})^2\right] \\ &+[\nabla\varphi_1(\mathbf{r})]^2 + [\nabla\varphi_2(\mathbf{r})]^2 \\ &+\lambda_0\left[\varphi_1(\mathbf{r})^4 + \varphi_2(\mathbf{r})^4\right] + 2g_0\varphi_1(\mathbf{r})^2\varphi_2(\mathbf{r})^2\},\end{aligned} \tag{3}$$

where $\tau_0 = (T - T_c^{(0)})/T_c^{(0)} < 0$ is the bare mass parameter. The fluctuating quenched random fields $\tilde{V}_{ik}^{(t)}(\mathbf{r})$ are assumed to have the form

$$\tilde{V}_{ik}^{(t)}(\mathbf{r}) = \Theta(\mathbf{r})V_{ik}^{(t)}. \tag{4}$$

Here $\Theta(\mathbf{r})$ is the indicator function with $\Theta = 1$ at the position of the defects (distributed randomly in space with concentration x) and $\Theta = 0$ otherwise. The index t indicates the different internal states of the identical defects (for example at different but crystallographic equivalent positions inside a unit cell). These states are assumed to be distributed with equal weight so that the symmetry of the crystal on the average is conserved.

Let us in the following consider the case when the random field matrices $V^{(t)}$ are diagonal ones:

$$V^{(1)} = \begin{pmatrix} V_1 & 0 \\ 0 & V_2 \end{pmatrix}, \quad V^{(2)} = \begin{pmatrix} V_2 & 0 \\ 0 & V_1 \end{pmatrix}.$$

This means that we have tensor-type defects the symmetry of which is relatively high (random anisotropy axis type). The general case will be commented on at the end of the paper.

The simplest assumption concerning the SR interaction between the defects is to postulate that nearest-neighbor defects have just the same internal states t.

Let us now calculate the pair correlation functions $\mathcal{K}_{ik}(\mathbf{r}-\mathbf{r}')$ of the random matrix elements $\tilde{V}_{ik}^{(t)}$:

$$\begin{aligned}\mathcal{K}_{ik}(\mathbf{r}-\mathbf{r}') = &\left\langle\left\langle \tilde{V}_{ii}^{(t)}(\mathbf{r})\tilde{V}_{kk}^{(t')}(\mathbf{r}')\right\rangle\right\rangle \\ &-\left\langle\left\langle \tilde{V}_{ii}^{(t)}\right\rangle\right\rangle\left\langle\left\langle \tilde{V}_{kk}^{(t')}\right\rangle\right\rangle,\end{aligned} \tag{5}$$

where the double brackets denote a consecutive average over the internal degrees of freedom t and over the spatial defect configurations. In particular

$$\left\langle\left\langle \tilde{V}_{ii}^{(t)}\right\rangle\right\rangle = \langle\theta(\mathbf{r})\rangle\,\overline{V_{ii}^{(t)}} = \frac{x}{2}(V_1 + V_2). \tag{6}$$

Here and henceforth we use the notation $\langle\cdots\rangle$ for an average over the spatial configurations only, and a bar stands for an average over the internal states for a fixed spatial configuration. Let $g(\mathbf{r},\mathbf{r}')$ denote the probability function with the following significance: $g(\mathbf{r},\mathbf{r}') = 1$ if both points $\mathbf{r},\mathbf{r}'$ are occupied by defects of the same cluster[12] and $g(\mathbf{r},\mathbf{r}') = 0$ otherwise. Then an element of the pair correlation function (5), for example, $\mathcal{K}_{11}(\mathbf{r}-\mathbf{r}')$, can be written as follows:

$$\begin{aligned}\mathcal{K}_{11}(\mathbf{r}-\mathbf{r}') = &\left\langle[1-g(\mathbf{r},\mathbf{r}')]\overline{V_{11}^{(t)}(\mathbf{r})}\,\overline{V_{11}^{(t')}(\mathbf{r}')}\right\rangle \\ &+\left\langle g(\mathbf{r},\mathbf{r}')\overline{V_{11}^{(t)}(\mathbf{r})V_{11}^{(t)}(\mathbf{r}')}\right\rangle - \left\langle\overline{V_{11}^{(t)}}\right\rangle^2,\end{aligned} \tag{7}$$

where in the second term we set $t = t'$ according to our SR defect interaction rule. The first term describes the contribution of the defects which belong to different clusters, and so the indices t and t' are independent.

Performing the consecutive average in (7) we obtain

$$\begin{aligned}\mathcal{K}_{11}(\mathbf{r}-\mathbf{r}') &= x[1-\langle g(\mathbf{r},\mathbf{r}')\rangle]\left(\overline{V_{11}^{(t)}}\right)^2 \\ &\quad+\langle g(\mathbf{r},\mathbf{r}')\rangle\,\overline{(V_{11}^{(t)})^2} - x\left(\overline{V_{11}^{(t)}}\right)^2 \\ &= \frac{x}{4}(V_1-V_2)^2\langle g(\mathbf{r},\mathbf{r}')\rangle = \mathcal{K}_{22}(\mathbf{r}-\mathbf{r}')\end{aligned} \tag{8}$$

and, similarly,

$$\begin{aligned}\mathcal{K}_{12}(\mathbf{r}-\mathbf{r}') &= -\frac{x}{4}(V_1-V_2)^2\langle g(\mathbf{r},\mathbf{r}')\rangle \\ &= -\mathcal{K}_{11}(\mathbf{r}-\mathbf{r}').\end{aligned} \tag{9}$$

It is easy to see that the quantity $G(\mathbf{r}-\mathbf{r}') = \langle g(\mathbf{r},\mathbf{r}')\rangle$ is nothing else but the Green's function for the percolation problem. Its Fourier transform $G(q)$ has the form

$$G(q) \propto \begin{cases} (q^2 + R_c^{-2})^{-1}R_c^{-\eta}, & q \ll R_c^{-1}, \\ q^{-2+\eta}, & q \gg R_c^{-1}, \end{cases} \tag{10}$$

with $R_c(x) = R_0\,|(x-x_c)/x_c|^{-\nu}$ if the defect concentration x is near the percolation threshold $x_c \approx \tilde{x}_c(a_0/R_0)^d$. Here a_0 is of order of the geometric size of a single defect, R_0 is the radius of interaction between the defects, and $\tilde{x}_c$ is the threshold of the site percolation problem. η is Fisher's exponent for the d-dimensional percolation problem and ν is the corresponding critical exponent of the correlation length. When $x \to x_c$ the asymptotics of $G(r)$ becomes a pure power law.

Thus we arrive at the important conclusion that near the defect percolation threshold x_c LR correlations arise which are absent in the case of scalar defects without

internal degrees of freedom (for which $V_1 = V_2$).

It is clear that this conclusion is valid also for tensor-type defects which are described by symmetric matrices $V^{(t)}$ of arbitrary rank m. For example, if we consider defects with $m!$ different internal states (i.e., $t = 1, ..., m!$) which are described by diagonal matrices

$$V_{ik}^{(1)} = V_i \delta_{ik}, \quad i = 1, ..., m, \tag{11}$$

and the other matrices are given by all permutations of $V_1, ..., V_m$ we obtain

$$\mathcal{K}_{ii}(\mathbf{r} - \mathbf{r}') = G(\mathbf{r} - \mathbf{r}') \frac{x}{m} \sum_{i=1}^{m} V_i \left(V_i - \frac{1}{m} \sum_{j=1}^{m} V_j \right) \tag{12}$$

and for $i \neq k$

$$\mathcal{K}_{ik}(\mathbf{r} - \mathbf{r}') = -\frac{1}{(m-1)} \mathcal{K}_{ii}(\mathbf{r} - \mathbf{r}'). \tag{13}$$

In the remaining part of this paper we shall study mainly this particular case.

III. EFFECTIVE HAMILTONIAN

We consider a system in which the pure crystal has a second-order phase transition with an m-component order parameter. Let us assume that in the presence of frozen defects the additional Hamiltonian is given by (2) with diagonal matrices $V^{(t)}$ according to (11).[13] We may use (10), (12), and (13) for the description of the LR asymptotics of the pair correlators of the random matrices $\tilde{V}^{(t)}(\mathbf{r})$ at the phase transition temperature $T_c \neq 0$ if the characteristic energy J of the SR defect interaction is much greater than the host interaction energy $J_{\text{host}} \approx T_c$. Indeed, in such a case thermal fluctuations destroy the arrangement of the internal states inside the defect clusters at $x \approx x_c$ only for exponentially large distances $r \approx R_0 \exp\{J/T_c\}$,[14] and our postulated rule works well.[15]

We shall investigate in particular the critical behavior of a disordered crystal which arises at $x \approx x_c$ due to the presence of the LR defect correlations. It should be pointed out that x_c can be much smaller than $\tilde{x}_c$; i.e., the dilution of the crystal by the defects can be rather weak. Moreover, the disturbance of the crystal by an isolated defect can be weak, too, so that $xV_{ik} \ll \lambda_0, g_0$. Therefore, in the absence of the LR correlations due to the defects one may assume that the system is described by the Khmelnitsky Hamiltonian[16] and in the presence of the correlations it is sufficient to take into account only the lowest-order defect correlation functions (12), (13).

Keeping these remarks in mind and using the standard replica method for the m-component version of (3) we can write down the effective Hamiltonian of our problem in the form

$$\mathcal{H}_{\text{eff}} = \int d^d\mathbf{r} \left\{ \sum_{i=1}^{m} \left[\frac{1}{2} \left(\tau \Phi_i^2 + |\nabla \Phi_i|^2 \right) + \sum_{\alpha=1}^{n} \left(\lambda_0 \varphi_{i\alpha}^4 + g_0 \varphi_{i\alpha}^2 \sum_{k \neq i} \varphi_{k\alpha}^2 \right) \right] \right. \tag{14}$$

$$\left. + \int d^d\mathbf{r}' \left[[u_0 \delta(\tilde{\mathbf{r}}) + u_1^{(0)}(\tilde{\mathbf{r}})] \sum_{i=1}^{m} (\Phi_i^2)^2 + 2 \left[v_0 \delta(\tilde{\mathbf{r}}) + v_1^{(0)}(\tilde{\mathbf{r}}) \right] \sum_{i<k}^{m} \Phi_i^2 \Phi_k^2 \right] \right\},$$

where $\tilde{\mathbf{r}} = \mathbf{r} - \mathbf{r}'$, $\tau = \tau_0 + x(\sum_{i=1}^{m} V_i)$, $\Phi_i = \{\varphi_{i1}, \ldots, \varphi_{in}\}$, and $\Phi_i^2 = \sum_{\alpha=1}^{n} \varphi_{i\alpha}^2$. The replica number $n \to 0$. In (14) the functions u_1 and v_1 are proportional to the LR asymptotics of (12), (13), respectively, and the vertices u_0 and v_0 describe the SR scattering of the fluctuations.

We note that the effective Hamiltonian (14) reduces to the one of the WH m-component model[11] when $\lambda_0 = g_0$, $u_0 = v_0$, and $u_1 = v_1$.

For renormalizing (14) we employ the scheme of analytical and dimensional regularization, which was used for similar problems in Refs. 17, 18. The Fourier-transformed defect interaction with the tensor structure $(\varphi_{i\alpha})^2(\varphi_{i\beta})^2$ takes the form $u^{(0)} + u_1^{(0)} q^{-2\xi}$ while that with the tensor structure $(\varphi_{i\alpha})^2(\varphi_{j\beta})^2$ may be written as $v^{(0)} + v_1^{(0)} q^{-2\xi}$, where $u^{(0)}, u_1^{(0)}, v^{(0)}$ are positive and, in accordance with (13), $u_1^{(0)} = -(m-1)v_1^{(0)}$. Carrying out the renormalization we work in the limits $4 - d = \epsilon \to 0$ and $\xi \to 0$. At the end of the calculations one should set $\epsilon = 1$, $\xi = 1 - \eta/2$.

IV. RENORMALIZATION GROUP EQUATIONS

At first glance it seems that we have six invariant charges $\lambda, g, u, u_1, v, v_1$ (which correspond to $\lambda_0, g_0, u_0, u_1^{(0)}, v_0, v_1^{(0)}$, respectively) and, correspondingly, six independent RG equations for these quantities. Nevertheless the coupling constants u_1 and v_1 obey the following exact relation:

$$u_1 = -(m-1)v_1, \tag{15}$$

from which follows that in fact only five charges are independent.

This can be readily seen from the general diagrammatic representation of the RG equations. In Fig. 1 we have displayed the structure of the contributions to the Gell-Mann–Low functions β_{u_1} and β_{v_1}. The corresponding analytical RG equations can be written explicitly as

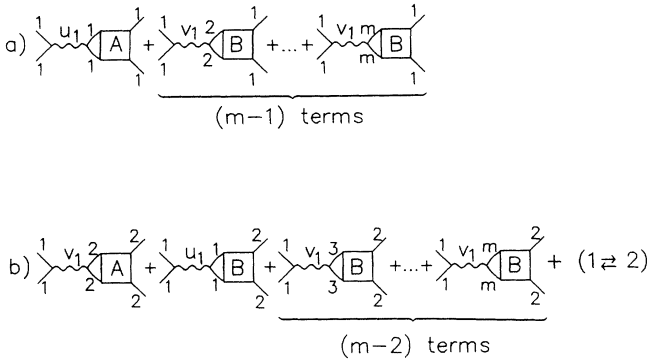

FIG. 1. Structure of the contributions to the Gell-Mann–Low functions (a) β_{u_1} and (b) β_{v_1}.

$$\frac{du_1}{dt} = Au_1 + (m-1)Bv_1,$$
$$\frac{dv_1}{dt} = Av_1 + Bu_1 + (m-2)Bv_1. \tag{16}$$

Here $t = \ln\{R_c/R_0\}$, where $R_c(T)$ is the correlation length. It is obvious that if the relation (15) has been true for the initial values of u_1 and v_1 the same condition must hold for any value of t. Relation (15) is, therefore, a direct consequence of the symmetry relation $u_1^{(0)} = -(m-1)v_1^{(0)}$ obtained from (13).

The critical behavior of the disordered crystal described by the effective Hamiltonian (14) can be analyzed by studying the fixed points of the RG equations. We are interested only in those SF points which can be reached from the region of the initial parameters of (14) and shall not investigate the rest.

There is no necessity for writing down the rather lengthy system of the RG equations for all five invariant charges because the existence and the stability of a unique SF point can be proved in a general manner.

Let us use, as before, the notation β_i for the Gell-Mann–Low functions of an invariant charge i. It is easy to see that if $g = 0$ then β_i for $i = \lambda, u, u_1$ do not depend on v and v_1 Therefore these functions at $g = 0$ coincide with the β functions obtained by WH in the case of the $(m = 1)$-component order parameter model. WH established that in the subspace spanned by the invariant charges λ, u, u_1 a SF point exists. Thus we need to consider the fixed point with the following coordinates:

$$\lambda^* = \lambda^*_{\mathrm{WH}}, \quad u^* = u^*_{\mathrm{WH}},$$
$$u_1^* = u^*_{1(\mathrm{WH})}, \quad g^* = 0, \tag{17}$$

where λ^*_{WH}, u^*_{WH}, $u^*_{1(\mathrm{WH})}$ are the coordinates of the SP point of the one-component WH model.[19] The value of the coordinate of the new charge v^* is unknown. Therefore let us consider the RG equation for v:

$$\frac{dv}{dt} = v\left[\frac{\epsilon}{2} - 24\lambda + 16(u+v) - 16u_1\right] + 16u_1^2 + \cdots . \tag{18}$$

Here we have written down all the second-order terms and have used the exact relation (15). It is easy to see that there exists a fixed point v^* of order $\epsilon^{3/2}$ with a coordinate

$$v^* = \frac{4(u_1^*)^2}{3\lambda^*}\left[1 + O(\epsilon^{1/2})\right]. \tag{19}$$

Writing down (19) we took into account that WH have obtained for the $m = 1$ case the following values of coordinates for their "long-range" SP point:

$$\lambda^* = \frac{\xi}{12} + O(\epsilon), \quad u^* = \frac{\xi}{16} + O(\epsilon),$$
$$u_1^* = O(\epsilon). \tag{20}$$

Moreover, WH pointed out that the fixed point of the short-range impurity Ising (SRII) model (with $u_1^* = 0$) loses its stability if

$$\xi > \xi_c, \quad \xi_c = \xi_c^{(0)} + O(\epsilon), \quad \xi_c^{(0)} = \left(\frac{6\epsilon}{53}\right)^{1/2}. \tag{21}$$

Therefore the value of the regularizator ξ must be of the order $O(\epsilon^{1/2})$ at the LR fixed point (20) which eventually becomes stable instead of the SRII fixed point when (21) is satisfied.

In their investigation WH did not determine the value of the coordinate u_1^* for the LR point (20); neither did they check whether this coordinate is positive. The latter is a necessary condition for the accessibility of point (20) from the region of the initial parameters of (14).

However, the value of the coordinate u_1^* can be easily found if one notes that the coordinates of the SRII and LR fixed points coincide at $m = 1, \xi = \xi_c^{(0)}$. Since the former fixed point is stable for $\xi < \xi_c^{(0)}$ and becomes unstable at $\xi > \xi_c^{(0)}$ it is natural to suggest that a new "long-range" SF point arises just at $\xi = \xi_c^{(0)}$ and, consequently, its coordinate obeys a relation

$$u_1^* \propto \left(\xi - \xi_c^{(0)}\right)^{\delta}, \tag{22}$$

with $\delta > 0$.

Indeed, the RG equation for the charges λ and u for small values of u_1^* have solutions

$$\lambda^* = \frac{1}{12}\left(\frac{6\bar{\epsilon}}{53}\right)^{1/2} + O(\epsilon),$$
$$u^* = \frac{1}{16}\left(\frac{6\bar{\epsilon}}{53}\right)^{1/2} + O(\epsilon), \tag{23}$$

where $\bar{\epsilon} = \epsilon + 96u_1^*$. Substituting (23) into the RG equation for u_1 we obtain

$$\xi = \left[\frac{6}{53}(\epsilon + 96u_1^*)\right]^{1/2} \tag{24}$$

and the coordinate of the LR fixed point u_1^* is given by

$$u_1^* = \frac{53}{576}\left(\xi^2 - \xi_c^2\right), \quad \xi_c = \xi_c^{(0)} + O(\epsilon^{3/2}). \tag{25}$$

Thus we have proved the existence of the LR fixed point of the one-component WH model. It has a positive coordinate u_1^* for $\xi > \xi_c$ (in the range where the SRII fixed point loses its stability) and $\delta = 1$. Simultaneously we have established that there is a fixed point with coordinates (17), (19), (23), and (25) for our generalized model (14).

Before we analyze the stability of this fixed point we shall consider the extended Harris criterion for the $m = 1$ WH model. According to this criterion the presence of LR-correlated defects affects the critical behavior of a disordered crystal if and only if

$$\xi > \xi_c = -\frac{\alpha_I}{2\nu_I} > 0, \tag{26}$$

where α_I and ν_I are the exponents of the specific heat and the correlation length, respectively, of the SRII model. Substituting the $\epsilon^{1/2}$ expansion for these exponents one obtains again expression (25) for ξ_c. Since after its formulation the validity of the extended Harris criterion has been confirmed to all orders of renormalized perturbation theory[17] we can be sure that u_1^* in (25) is a positive number, because this is guaranteed by this criterion. Therefore we have proved that, starting from the initial values of the parameters of Hamiltonian (14), one can reach the fixed point given by Eqs. (17), (19), (23), and (25).

The stability of this fixed point can be established by analyzing the general structure of the matrix $\partial\beta_{x_i}/\partial x_j$ $(x_i, x_j = \lambda, g, u, u_1, v, v_1)$. It is easily seen that (a) $\beta_g \propto g$ and correspondingly $\partial\beta_g/\partial x_i|_{g=g_*=0} = 0$ at $x_i = \lambda, u, u_1, v$; (b) the charge v appears in the functions β_i $(i = \lambda, u, u_1, g)$ always in the combination $v^a g^b$ $(a, b \geq 1)$, from which follows $\partial\beta_i/\partial v|_{v_*=g_*=0} = 0$; (c) the charge v_1 has the same properties as v, and so there are no additional terms of the type $(\partial v_1/\partial u_1)(\partial\beta_i/\partial v_1)$ with $i = \lambda, u$.

Therefore we have the following eigenvalues of the stability matrix: Three of them coincide with those of the one-component WH model and the other two are simply

$$\frac{\partial\beta_g}{\partial g} = \alpha/2\nu < 0,$$
$$\frac{\partial\beta_v}{\partial v} = -12\lambda^*\left[1 + O(\epsilon^{1/2})\right] < 0, \tag{27}$$

where α and ν are the specific heat and correlation length exponents of our full model. As the additional eigenvalues (27) are negative the LR fixed point (17), (19), (23), and (25) is stable.

Because we have $g_* = 0$ all the critical exponents for the model (14) are independent of v^*, v_1^*. Hence the critical behavior of this model for any value of m is equivalent to the behavior of a set of m independent one-component WH models.

In particular, the correlation length exponent ν and, correspondingly, the exponent of the specific heat α are known exactly and are given by[11,17]

$$\nu = \frac{1}{d - 2 + \eta},$$
$$\alpha = \frac{2(\eta - 2)}{d - 2 + \eta}, \tag{28}$$

where η is the Fisher exponent for the continuum percolation problem.

V. DISCUSSION

In this section we would like to compare the Weinrib-Halperin (WH) model with ours (14) and add some general remarks.

WH (Ref. 11) have introduced long-range (LR) correlations into their model in an abstract mathematical manner. The authors emphasize that they considered the LR correlation function only as an *ad hoc* quantity without any realization by means of a concrete physical mechanism. Therefore, in particular, WH restricted their considerations only to the isotropic version of the model, although in solids the anisotropic case is quite usual.

The LR correlation functions (12), (13) in our model arise from the presence of a strong short-range (SR) interaction between defects with internal degrees of freedom. As indicated before such cases may arise quite often in physical systems.

The local-rule condition employed by us may, of course, be generalized.[20] One may consider the conditions of the so-called bootstrap percolation problem[22,23] in which sites must have at least some minimum number of nearest neighbors to be a "bootstrap site." It is known that the nontrivial clusters of these sites are fractal objects so that the corresponding correlators are of LR type.

More nontrivial types of the SR local rule arise if one considers the antiferromagnetic version of Pott's model (AP). For example for this model with three and four states on a triangular and fcc lattice, respectively, it has been demonstrated that LR order disappears already below a critical concentration $x_c^{\mathrm{AP}} < x_c$ (at $T = 0$).[24] This is due to the fact that as for the bootstrap percolation problem mere connectivity is not sufficient for the penetration of the local order paramater across the diluted lattice.[21] It is interesting to note that there exists also another mechanism for a shift of the percolation threshold for disordered systems with a frustrated ground state.[25]

Our SR defect interaction rule is simple enough for application in a computer simulation. We think it is worthwhile to check our theoretical prediction (28) for two- and three-dimensional disordered lattices. Indeed, unlike for the short-range impurity Ising (SRII) model the crossover exponents for the LR fixed point are expected to be of the order of unity. Moreover, the critical exponents (28) differ appreciably from those for a pure crystal. So the defect critical regime must be rather wide.

On the other hand numerical investigations of the crit-

ical behavior of models with a bootstrap type of SR interactions may turn out to be useful for understanding the interplay of clusters of different topological structures. This interplay, apparently, leads to novel crossover effects which have been observed for the SRII model.[26,27]

The attention of the authors of Refs. 22–25 has been concentrated mainly on the fact of the shift of x_c for some models in comparison with the usual percolation problem. We would like to emphasize a novel feature of such disordered solids with more complicated local interaction rules: The appearance of LR correlations due to the presence of degenerated internal degrees of freedom of defects may lead to the same consequences as the existence of a mass fractal structure in real space.

In particular, this aspect might also affect the scattering properties of crystals containing impurities with internal degrees of freedom[28] and therefore be important for interpreting scattering data of light or phonons from low-lying excitations (as well as low-temperature thermal anomalies) of the so-called orientational glasses.[29,30] Of course the problem of a full description of these excitations (including phonon-fracton crossover effects) for LR-correlated disorder remains a subject for further investigations.

In conclusion we would like to point out the main results of our paper: The presence of defects with degenerated internal degrees of freedom may induce long-range correlations near different types of *defect* percolation thresholds leading to a drastic change in the critical behavior. The critical behavior of our model (14) is the same as that of the one-component Weinrib-Halperin model.[11] We have shown that near the defect percolation threshold x_c the corresponding effective Hamiltonian (involving an m-component order parameter that couples to defects with $m!$ internal states) asymptotically decomposes into a set of m noninteracting one-component WH models.[33]

We should mention also that in our investigation we have considered only the case of relatively symmetrical point defects which are described by diagonal matrices. In the case of defects with arbitrary low symmetry the inclusion of off-diagonal disorder (i.e., $V_{ik}^{(t)} \neq 0$ for $i \neq k$) renders the LR fixed point unstable. One may conclude that in this case the trajectories of the RG equations run away to infinity as in the low-concentration limit of the model (14) without the LR vertices.

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