Magnetoelastic theory of type-II superconductors in the mixed state

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(Received 18 October 1993; revised manuscript received 21 March 1994)

This paper presents a macroscopic theory for analyzing the magnetoelastic response of elastic type-II superconductors in the mixed state. The theory includes not only the vortex-dynamic effect, the normal-current effect, and the flux-flow Hall effect, but also the effect of the London moment induced by the local motion of the deformable superconductor in the mixed state. The theory is considered to be valid within the framework of the generalized Galilean relativity at the magnetoquasistatic approximation. By this theory, a set of linearized coupled wave equations is derived to study some problems concerning magnetoelastic wave propagation in type-II superconductors in the mixed state. Attenuation and dispersion behaviors of the magnetoelastic wave are then analyzed. It is shown that the nonlocal effect on the length scale of the London penetration depth may be of importance in analyzing the propagation behavior of the magnetoelastic wave at frequencies higher than the depinning frequency. It is also shown that the effect of the London moment induced by dynamic deformation can be of significance as compared with the effect of the normal Lorentz current on the magnetoelastic coupling behavior of elastic type-II superconductors in the mixed state. Furthermore, it is found that there is a phase change between the transverse elastic wave and the magnetic wave induced in the type-II superconductor in the mixed state. This effect is found to be closely related to the vortex-dynamic properties of the superconductor.

I. INTRODUCTION

Recently, there has been growing interest in research on electromagnetomechanical interactions in materials due to recent progress in sophisticated device technology and their potential applications.[1-3] During the past three decades, many theories have been proposed and subsequently developed to study the phenomena of electromagnetomechanical interactions in various types of materials. A summary of these theories may be found in the recent work of, for instance, Eringen and Maugin.[4] Although much understanding of the behavior of deformable normal conductors has been gained and some effort has been made to study the electrodynamic and mechanical behavior of superconductors,[5-12] the theoretical basis of describing the electromagnetomechanical behavior of deformable type-II superconductors in mixed states has not yet been fully established.

During the past, much effort[13-19] has been made, both theoretically and experimentally, to study the dynamic behavior of the vortex structures in type-II superconductors in the mixed state due to its importance in understanding the macroscopic electrodynamic properties of the type-II superconductors. Recently, a series of very interesting theoretical work has been done by Coffey and Clem,[20-25] in which they have developed a nice theoretical model, using a self-consistent approach to vortex dynamics, to describe electrodynamic responses of the type-II superconductors in the mixed state. In their model, a continuum approximation of the London equation with a vortex term is used and the effect of quasiparticle excitations is included by a normal-current density contribution. Their model is considered to be applicable to a wide range of electrodynamic phenomena involving vortex dynamics and over a wide range of frequencies, including radio and microwave frequencies (below superconducting-gap frequencies). However, the Coffey-Clem model cannot be simply used to analyze electromagnetomechanical behavior of deformable type-II superconductors in the mixed state, where the effect of the motion of the superconductor has to be properly taken into account.

In this paper, we shall, therefore, introduce a theoretical model to analyze the magnetoelastic response of elastic type-II superconductors in the mixed state with the aid of the Coffey-Clem self-consistent approach.[20-25] In this model, the effect of the London moment induced by the local motion of the elastic type-II superconductor in the mixed state and the effect of the Lorentz force on the magnetoelastic behavior of the elastic type-II superconductor will be taken into account. The theoretical model will also include the flux-flow Hall effect, which can be of both theoretical and practical interest.[14,17,26-29]

We have noticed that the London moment was first predicted theoretically by London[30] for rigid-body rotating superconductors and, later, was observed experimentally first by Hildebrandt.[31] Recently, the London moment has also been observed by Verheijen *et al.*[32] for some rotating high-temperature oxide superconductors. In practice, the effect of the London moment has been used to construct, for instance, gyroscopes for some space flight scientific experiments, among others. The generalization of the London phenomenological theory to deformable superconductors in the Meissner state has been made recently by Zhou[11,12] to include the effect of the London moment induced by the local motion (dynamic deformation) of the elastic superconductor rather than the global rotation of the material body within the frame-

0163-1829/94/50(1)/354(8)/$06.00

work of the generalized Galilean relativity at the magnetoquasistatic approximation. In this paper we shall further study the effect of the London moment on the behavior of elastic type-II superconductors in the mixed state. We shall assume that the elastic type-II superconductor considered is isotropic and nonmagnetic for the sake of simplicity. It is, however, possible to extend the work to anisotropic superconductors. Some treatments on deformable magnetic superconductors may also be found in the recent work of Zhou and Miya.[8] For the type-II superconductor in the mixed state, we shall assume that the vortex displacements are small in comparison with the intervortex spacing so that a linear elastic approximation of the vortex lattice continuum can be adopted. We also assume that the superconductor is in an isothermal state so that thermal forces due to temperature gradient can be ignored. As a consequence, magnetic history, critical state effect, flux creep effect, and thermoelectric effect in the type-II superconductor will not be treated here. In addition, we shall assume that the elastic deformation of the superconductor is small so that a linear elasticity theory can be applied for the superconductive material medium.

With the aid of this theoretical model, some magnetoelastic wave phenomena in the elastic type-II superconductor in the mixed state will be studied in this paper. We shall see how the effects of the London moment, the vortex dynamics, and the normal conduction fluid may influence the dynamic response of the elastic type-II superconductor in the mixed state.

II. BASIC EQUATIONS FOR ELASTIC TYPE-II SUPERCONDUCTORS IN THE MIXED STATE

It is known that, at the microscopic level, a flux line consists of superelectrons moving with a certain density and velocity distribution around the center of the flux line. This means that forces on a flux line will actually be experienced by the electrons and can, therefore, be caused by electric and magnetic fields. At the macroscopic level, we are dealing with phenomena that can be described in terms of average macroscopic fields and currents. Phenomenologically, in a reference frame $S'(\mathbf{x},t')$ attached to the material medium, we may write the following equation of motion for the flux-line lattice continuum:

$$m_f \frac{\partial^2 \mathbf{w}}{\partial t'^2} = -\eta \frac{\partial w}{\partial t'} - K\mathbf{w} + q_{\text{eff}} \mathbf{E}'_v + \mathbf{J}' \times \mathbf{B}'_v \ , \tag{1}$$

where m_f denotes the effective-mass density associated with the flux-line lattice. So far, little is known about the effect of acceleration as a cause for macroscopic forces on flux lines. Some studies have shown that the inertial force is usually negligibly small compared to pinning forces in most hard superconductors. However, inertial forces could be sufficient to trigger flux movements in samples with very low pinning strengths.[19] A recent study of Coffey and Clem[21] suggested that the inertial effect may be significant in the vortex dynamics of high-temperature superconductors. The vector $\mathbf{w}$ denotes the vortex displacement vector, measured from an equilibrium pinning site in the medium in S'. η is the flux-flow viscosity for the isotopic superconductor. The current density vector $\mathbf{J}'$, the vortex magnetic field $\mathbf{B}'_v$, and the vortex electric field $\mathbf{E}'_v$ are all averaged quantities over microscopic fields and current distributions in the superconductor. In Eq. (1) we have adopted a simplified model[16,20] in which, for the case of local vibration of flux lines without global flux-flow motion, the effect of the pinning force and the deformation of the flux lattice is modeled by a simple restoring force of the form $f^{\text{rest}} = -K\mathbf{w}$ for the isotropic type-II superconductor, where K is the spring coefficient.

It can be seen that the third and fourth terms on the right-hand side of Eq. (1) are, respectively, the electric and magnetic driving forces on the flux lines. As we shall show later, the electric force, caused by the vortex electric field,

$$\mathbf{E}'_v = -(\partial \mathbf{w}/\partial t') \times \mathbf{B}'_v \ ,$$

induced by the motion of the flux lines, may cause the flux-flow Hall effect. We have noticed that although the flux-flow Hall effect of the type-II superconductors in the mixed state was observed early in the 1960's (Refs. 34–36) and studied by a number of researchers,[14,17,37,38] its microscopic mechanism seems not yet to have been fully understood, especially for the recently discovered high-temperature oxide superconductors.[26–28] Here, we shall use simply a phenomenological treatment by introducing an effective charge density q_{eff} for the flux-line lattice continuum. It is possible that Eq. (1) may also include a random (or Langevin) force to model the effect of flux creep.[24] For some simplicity, however, we shall ignore here the Langevin force term by limiting our study to cases where the effect of flux creep is not significant and, in addition, we shall assume that the superconductor is in an isothermal state so that thermal forces due to temperature gradient can be ignored.

When the flux lines move, they must also obey a continuity equation[33]

$$\nabla' \times \mathbf{E}'_v = -\frac{\partial \mathbf{B}'_v}{\partial t'} \ . \tag{2}$$

In many cases, electrodynamic phenomena of superconductors can be analyzed with the aid of knowledge gained from studying the behavior of the superconductor in time-harmonic fields ($\sim e^{i\omega t}$). In the time-harmonic fields, we may get from Eq. (1) the following relation:

$$w_k = \frac{1}{i\omega B_v'^2} R_{kl} e_{lpq} J'_p B'_{vq} \ , \tag{3}$$

in which ω is the radian frequency, e_{ijk} is the permutation symbol, and R_{kl} is defined by

$$R_{kl} = \left[\sigma_v \delta_{kl} + \frac{q_{\text{eff}}}{B'_v} e_{klp} n'_p \right]^{-1} , \tag{4}$$

where $\mathbf{n}'$ is the unit direction vector of the vortex magnetic field ($\mathbf{n}'=\mathbf{B}_v'/B_v'$) and B_v' is the magnitude of the vortex magnetic field. $\sigma_V=\sigma_{V1}-i\sigma_{V2}$ is the complex flux-flow conductivity, defined by

$$\sigma_{V1}=\frac{\eta}{B_v'^2} \quad \text{and} \quad \sigma_{V2}=\frac{K-m_f\omega^2}{\omega B_v'^2} \ . \tag{5}$$

If the direction of the vortex magnetic field $\mathbf{B}_v'$ is supposed to be along the z' axis, we may find

$$[R_{kl}]=\begin{bmatrix} \dfrac{\sigma_v^2}{\sigma_v(\sigma_v^2+q_{\text{eff}}/B_v'^2)} & \dfrac{-q_{\text{eff}}}{B_v'(\sigma_v^2+q_{\text{eff}}/B_v'^2)} & 0 \\ \dfrac{q_{\text{eff}}}{B_v'(\sigma_v^2+q_{ef}/B_v'^2)} & \dfrac{\sigma_v^2}{\sigma_v(\sigma_v^2+q_{\text{eff}}/B_v'^2)} & 0 \\ 0 & 0 & \dfrac{1}{\sigma_v} \end{bmatrix} . \tag{6}$$

We may also derive the following expressions for the electric current density components:

$$J_{x'}'=\sigma_V E_{vx'}'+\frac{q_{\text{eff}}}{B_v'}E_{vy'}' \ , \tag{7}$$

$$J_{y'}'=\sigma_V E_{vy'}'-\frac{q_{\text{eff}}}{B_v'}E_{vx'}' \ , \tag{8}$$

which shows that the coefficient q_{eff}/B_v' characterizes effectively the flux-flow Hall effect. Introducing the Hall angle α_H,[39] we may write $q_{\text{eff}}=\eta(\tan\alpha_H)/B_v'$. In general, η and α_H may be determined experimentally. There are, however, some microscopic models that may be used to analyze these parameters.[14,17] The derived current-vortex displacement relation, given by Eq. (3), can be used self-consistently in electrodynamic field equations in a similar way, proposed first by Coffey and Clem.[20–25]

Let us now discuss the electrodynamic and mechanical field equations for the study of elastic type-II superconductors in the mixed state in the reference frame $S(\mathbf{x},t)$ of the laboratory, which is supposed to be fixed in space. To model the electrodynamics of the type-II superconductor in the mixed state, we shall use the following Maxwell equations at the magnetoquasistatic approximation:

$$\nabla\times\mathbf{H}=\mathbf{J} \quad \text{and} \quad \nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t} \ , \tag{9}$$

and the modified second London equation,

$$\mu_0\lambda_L^2\nabla\times\mathbf{J}_s-\frac{m^*}{e^*}\nabla\times\mathbf{V}=-(\mathbf{B}-\mathbf{B}_v) \ , \tag{10}$$

where $\mathbf{J}_s$ is the supercurrent density, $\mathbf{V}$ is the local velocity of the superconductive medium ($\mathbf{V}=\partial\mathbf{U}/\partial t$ with $\mathbf{U}$ being the elastic displacement vector of the medium), $\mathbf{B}$ is the magnetic induction field, and $\mathbf{B}_v$ is the local vortex magnetic field which is, in general, not equal to the magnetic induction field $\mathbf{B}$ due to the nonlocal effect on the length scale of λ_L and to the presence of normal conducting fluid.[20] μ_0 is the permeability of free space and λ_L is the London penetration depth. m^* and e^* are, respectively, the mass and electric charge of the Cooper pair superconducting electrons. It can be seen from Eq. (10) that the Coffey and Clem formulation[20–25] has been modified here by the introduction of a new term, the second term on the left-hand side of Eq. (10). The inclusion of this new term is due to the effect of the London moment induced by the local motion (dynamic deformation) of the elastic superconductor.[11,12] The concept of the London moment originated from London's work[30] on rotating superconductors, which predicated that a uniform magnetic field may be generated within the body of any superconductor which is set into rotation. Such a magnetic field was first observed by Hildebrandt[31] and is now often called the London moment. Essentially, the London moment is believed to be due to the effect of the acceleration of the motion of the superconductor. Here, the London approximation has been used to model electrodynamic phenomena in type-II superconductors in the mixed state. Thus, one may expect that the model is appropriate for the field region $B_{c1}<B\ll B_{c2}$. Such an intermediate field region is of interest especially for recently discovered high-temperature oxide superconductors due to their small lower critical field and large upper critical field.

In addition, to account for the effect of normal currents, we shall use the two-fluid model, which may be expressed by

$$\mathbf{J}=\mathbf{J}_s+\mathbf{J}_n \ , \tag{11}$$

where $\mathbf{J}_n$ denotes the normal-current density, given by

$$\mathbf{J}_n=\sigma_n(\mathbf{E}+\mathbf{V}\times\mathbf{B}) \ , \tag{12}$$

with σ_n being the local electric conductivity of the normal fluid. Here, we shall ignore the Hall effect due to the normal conduction fluid in the presence of the magnetic field for simplicity.

Considering now the equation of motion for the elastic type-II superconductor in the presence of a magnetic field, we may write

$$\rho\frac{\partial^2\mathbf{U}}{\partial t^2}=\nabla\cdot\mathbf{t}+\frac{1}{\mu_0}[(\mathbf{B}\cdot\nabla)\mathbf{B}-\tfrac{1}{2}\nabla(\mathbf{B}\cdot\mathbf{B})] \ , \tag{13}$$

where ρ is the mass density of the superconductor, $\mathbf{U}$ is the elastic displacement vector, and $\mathbf{t}$ is the Cauchy stress tensor, which, at the linear approximation, may be expressed by Hooke's law $t_{ij}=C_{ijkl}U_{k,l}$, with C_{ijkl} being the elastic modulus tensor. For the isotropic elastic superconductor, we have

$$C_{ijkl}=(\Xi-\tfrac{2}{3}G)\delta_{ij}\delta_{kl}+G(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}) \ , \tag{14}$$

where Ξ is the elastic bulk modulus and G is the elastic shear modulus of the superconductor. One may note that for the type-II superconductor in the mixed state, the elastic moduli C_{ijkl} may differ from their values in the normal state due to possible anomalies arising from the superconducting phase transition and from the effect of magnetic flux structures in the superconductor in the mixed state, details of which are referred to in, for instance, the work of Zhou and Miya.[9]

It is shown that the given set of self-consistent-field equations for analyzing magnetoelastic behavior of the elastic type-II superconductors in the mixed state are, in general, nonlinear and coupled. These field equations should be supplemented with appropriate boundary conditions and initial conditions in order to solve some mixed boundary-value, evolution problems. A general discussion about the electromagnetic and mechanical initial and boundary (interface) conditions may be found elsewhere.[4,10] In what follows, two special examples will be given to study some problems of wave propagation in elastic type-II superconductors in the mixed state.

III. MAGNETOELASTIC PLANE WAVE IN ELASTIC TYPE-II SUPERCONDUCTORS IN THE MIXED STATE

In this section we shall study the problem of magnetoelastic wave propagating in the elastic type-II superconductors in the mixed state. Here, we shall be particularly interested in the wave problem in which the superconductor is initially in a static magnetic field $\mathbf{B}_0$ and it may have the static elastic displacement $\mathbf{U}_0$, which can be caused by some static mechanical forces and/or the magnetic force. When the type-II superconductor is in the mixed state, magnetoelastic waves may be induced by a small perturbed time-varying loading, which can be of electromagnetic and/or mechanical origin. In such a case, we may write the total magnetic induction field by $\mathbf{B}=\mathbf{B}_0+\mathbf{b}$ and the total elastic displacement by $\mathbf{U}=\mathbf{U}_0+\mathbf{u}$ in the superconductor. Here, $\mathbf{b}$ and $\mathbf{u}$ are, respectively, the small perturbed time-varying magnetic field and elastic displacement field to be determined. As the first-order approximation, in what follows, we shall assume that all material properties concerned are independent of the perturbed fields $\mathbf{b}$ and $\mathbf{u}$, and are determined only in those static fields at a constant and uniform temperature T. Furthermore, we shall ignore the flux-flow Hall effect for simplicity. After these considerations, we may now derive and solve the following linearized coupled magnetoelastic wave problem.

From Eqs. (9)–(12) we may get

$$\nabla^2\mathbf{B}=\mu_0\sigma_n\frac{\partial\mathbf{B}}{\partial t}+\frac{1}{\lambda_L^2}(\mathbf{B}-\mathbf{B}_v)-\frac{m^*}{e^*\lambda_L^2}\nabla\times\mathbf{V}-\mu_0\sigma_n\nabla\times(\mathbf{V}\times\mathbf{B}) , \quad (15)$$

from which we may see that the third and fourth terms on the right-hand side of this equation are caused by the local motion of the elastic superconductor, and they represent, respectively, the effect of the London moment induced by dynamic deformation and the effect of the normal Lorentz current. A rough estimation of the relative importance of these two terms can be made by choosing some typical values of $\sigma_n=5\times10^6/\Omega\,\text{m}$, $\lambda_L=1$ μm, and $B=1$ T, which shows that the effect of the London moment is, at least, of the same order of importance as compared with the effect of the normal Lorentz current. If the London penetration depth λ_L is smaller, for instance, $\lambda_L=0.1$ μm for some high-T_c oxide superconductors, and/or σ_n is smaller at lower temperature $T/T_c\ll1$, the effect of the London moment may become even more important.

Now, let us study further Eq. (15). By integrating the vortex continuity equation (2) with respect to time, we may find

$$\mathbf{B}_v=\mathbf{B}_0+\mathbf{b}_v=\mathbf{B}_0-\nabla\times[\mathbf{B}_0\times(\mathbf{w}+\mathbf{u})] , \quad (16)$$

at the linear and magnetoquasistatic approximation. Thus, by Eq. (3), where $\mathbf{B}'_v$ is replaced by $\mathbf{B}_0$ and $\mathbf{J}'$ is replaced by $\mathbf{J}$ at the linear and magnetoquasistatic approximation, we may derive, after some manipulations, the following equation in time-harmonic fields:

$$\tilde{\lambda}^2\nabla^2\mathbf{b}=\mathbf{b}-\nabla\times(\mathbf{u}\times\mathbf{B}_0)+\frac{(i/2)\delta_{VC}^2\nabla\times[(\nabla\times\mathbf{b})\cdot\mathbf{n}]\mathbf{n}-(i\omega m^*/e^*)\nabla\times\mathbf{u}}{1+2i\lambda_L^2/\delta_N^2}, \quad (17)$$

where $\tilde{\lambda}$ is the complex penetration depth defined by

$$\tilde{\lambda}^2(\omega,B_0,T)\equiv\frac{\lambda_L^2-i\delta_{VC}^2/2}{1+2i\lambda_L^2/\delta_N^2} \quad (18)$$

and $\delta_N=(2/\mu_0\omega\sigma_n)^{1/2}$ is the normal skin depth, and $\delta_{VC}=(2/\mu_0\omega\sigma_V)^{1/2}$ is a complex skin depth. We have noticed that the complex penetration depth $\tilde{\lambda}$ was first introduced by Coffey and Clem.[20] In Eq. (17), $\mathbf{n}$ is the unit direction vector of $\mathbf{B}_0$, i.e., $\mathbf{n}=\mathbf{B}_0/B_0$. Here, $\mathbf{B}_0$ is assumed to be a constant and uniform magnetic field. It is shown that the appearance of the second and third terms on the right-hand side of Eq. (17) is due partly to the effect of current flow parallel to the magnetic field $\mathbf{B}_0$ and partly to the effect of the local mechanical motion (dynamic deformation) of the elastic type-II superconductor in the mixed state.

Furthermore, from Eq. (13), we may get

$$-\rho\omega^2\mathbf{u}=(\Xi+G/3)\nabla(\nabla\cdot\mathbf{u})+G\nabla^2\mathbf{u}+\frac{1}{\mu_0}[\mathbf{B}_0\cdot\nabla\mathbf{b}-\nabla(\mathbf{B}_0\cdot\mathbf{b})] . \quad (19)$$

Equations (17) and (19) constitute a set of coupled linearized field equations for the determination of the perturbed magnetic field $\mathbf{b}(\mathbf{x},t)$ and the perturbed elastic displacement field $\mathbf{u}(\mathbf{x},t)$ in the elastic type-II superconductor in the mixed state.

We shall now look at the following type of magnetoelastic plane wave:

$$u_x=u_0\exp[i(\omega t-kx)] , \quad (20)$$

$$b_z=b_0\exp[i(\omega t-kx)] , \quad (21)$$

where u_0 and b_0 are constants relevant to the wave amplitudes of the elastic displacement and the magnetic field, respectively. Other field components are $u_y=u_z=0$ and $b_x=b_y=0$. ω is the radian frequency of the wave and k is the propagation constant which is, in general, a complex quantity $k=k_0-i\alpha$, with α being the attenuation coefficient. Here, we have chosen the z axis along the direction of the magnetic field $\mathbf{B}_0$. Equations (20) and (21) describe a type of magnetoelastic wave that is longitudinal in the elastic displacement $\mathbf{u}$ but transverse in the magnetic field $\mathbf{b}$. This wave propagates along the x-axis direction, being perpendicular to the direction of the magnetic field $\mathbf{B}_0$.

Substitution of Eqs. (20) and (21) into Eqs. (17) and (19) gives the following equations:

$$(\omega^2-k^2c_L^2)u_0+i\frac{kB_0}{\mu_0\rho}b_0=0 \ , \tag{22}$$

$$ikB_0u_0-(1+k^2\tilde{\lambda}^2)b_0=0 \ , \tag{23}$$

for the determination of u_0 and b_0. Here,

$$c_L=[(\Xi+4G/3)/\rho]^{1/2}$$

is the longitudinal sound velocity. The condition for the existence of a nontrivial solution of Eqs. (22) and (23) gives the following dispersion equation:

$$(\omega^2-k^2c_L^2)(1+k^2\tilde{\lambda}^2)=k^2c_A^2 \ , \tag{24}$$

where $c_A=B_0/(\rho\mu_0)^{1/2}$ denotes the Alfvén speed, which is a measure of the relative importance of magnetic effects in comparison with mechanical ones insofar as wave propagation is concerned. It is noted that the wavelength of the magnetoelastic wave considered is usually much longer than 1 μm in most practical cases where the wave frequency is less than 1 GHz. The continuum model presented here may work well. For higher frequency applications, the model may work in some cases if, however, the effect of microstructures of the material can be taken into account effectively. Two special cases may be of interested. First, if one lets $\lambda_L\to\infty$ mathematically, by noting Eq. (18), one may find that Eq. (24) recovers its classical form for a normal elastic conductor. Second, if one lets $\lambda_L\to 0$, one may find that Eq. (24) recovers the result obtained by Shapira and Neuringer[5] as far as the longitudinal elastic wave is concerned. In the general case, we may introduce an effective complex conductivity $\sigma_{\text{eff}}=\sigma_{e1}-i\sigma_{e2}$, defined by

$$\sigma_{\text{eff}}\equiv\frac{1}{i\mu_0\omega\tilde{\lambda}^2} \ , \tag{25}$$

from which, by noting Eq. (18), we may find, after some manipulations,

$$\sigma_{e1}=\frac{(\sigma_{V1}+\mu_0\omega\sigma_n\sigma_{V2}\lambda_L^2)(1+\mu_0\omega\sigma_{V2}\lambda_L^2)-\mu_0\omega\sigma_{V1}\lambda_L^2(\sigma_{V2}-\mu_0\omega\sigma_n\sigma_{V1}\lambda_L^2)}{(1+\mu_0\omega\sigma_{V2}\lambda_L^2)^2+(\mu_0\omega\sigma_{V1}\lambda_L^2)^2} \ , \tag{26}$$

$$\sigma_{e2}=\frac{(\sigma_{V2}-\mu_0\omega\sigma_n\sigma_{V1}\lambda_L^2)(1+\mu_0\omega\sigma_{V2}\lambda_L^2)+\mu_0\omega\sigma_{V1}\lambda_L^2(\sigma_{V1}+\mu_0\omega\sigma_n\sigma_{V2}\lambda_L^2)}{(1+\mu_0\omega\sigma_{V2}\lambda_L^2)^2+(\mu_0\omega\sigma_{V1}\lambda_L^2)^2} \tag{27}$$

Mathematically, for $\lambda_L\to 0$, one has $\sigma_{e1}=\sigma_{V1}$ and $\sigma_{e2}=\sigma_{V2}$, and for $\lambda_L\to\infty$, one has $\sigma_{e1}=\sigma_n$ and $\sigma_{e2}=0$, as expected. If the effect of the normal conduction fluid is negligible in some cases, we may set $\sigma_n=0$ mathematically, which results in the following expressions:

$$\sigma_{e1}=\frac{\sigma_{V1}}{(1+\mu_0\omega\sigma_{V2}\lambda_L^2)^2+(\mu_0\omega\sigma_{V1}\lambda_L^2)^2} \ , \tag{28}$$

$$\sigma_{e2}=\frac{\sigma_{V2}+\mu_0\omega(\sigma_{V1}^2+\sigma_{V2}^2)\lambda_L^2}{(1+\mu_0\omega\sigma_{V2}\lambda_L^2)^2+(\mu_0\omega\sigma_{V1}\lambda_L^2)^2} \ , \tag{29}$$

which shows that the nonlocal effect on the length scale of λ_L leads effectively to the reduction of the flux-flow conductivity of the superconductor in the mixed state.

To solve the dispersion equation (24), for simplicity, we make the approximation by replacing the term k^2 which appears in the second parentheses on the left-hand side of Eq. (24) by its zero-field value $(\omega/c_L)^2$, which may be justified by noting the fact that the effect of magnetic fields modifies only slightly the phase velocity of the elastic wave. With this approximation, we may now obtain the attenuation coefficient α by

$$\alpha=\frac{\omega}{c_L}\left\{\frac{a}{2}[\sqrt{1+(b/a)^2}-1]\right\}^{1/2} \tag{30}$$

and the phase velocity c_{me} of the magnetoelastic wave by

$$c_{me}=\frac{\omega}{k_0}=c_L\left\{\frac{a}{2}[\sqrt{1+(b/a)^2}+1]\right\}^{-1/2} \ , \tag{31}$$

where the parameters a and b are given by

$$a=\frac{1+(1+c_A^2/c_L^2)(\sigma_{e1}^2/\sigma_0^2+\sigma_{e2}^2/\sigma_0^2)+(\sigma_{e2}/\sigma_0)(2+c_A^2/c_L^2)}{1+(1+c_A^2/c_L^2)^2(\sigma_{e1}^2/\sigma_0^2+\sigma_{e2}^2/\sigma_0^2)+(2\sigma_{e2}/\sigma_0)(1+c_A^2/c_L^2)} \tag{32}$$

and

$$b=\frac{(c_A^2/c_L^2)(\sigma_{e1}/\sigma_0)}{1+(1+c_A^2/c_L^2)^2(\sigma_{e1}^2/\sigma_0^2+\sigma_{e2}^2/\sigma_0^2)+(2\sigma_{e2}/\sigma_0)(1+c_A^2/c_L^2)} \ , \tag{33}$$

where σ_0 is defined by $\sigma_0 = \omega/(\mu_0 c_L^2)$.

It is shown that Shapira and Neuringer's result[5] has been modified here by accounting for the nonlocal effect on the length scale of λ_L and the normal-current effect, which may be seen from the complex conductivity given by Eqs. (26) and (27). In this example, the effect of the London moment does not show up since the wave is longitudinal in the elastic displacement. Some numerical calculations are shown in Figs. 1 and 2, where material parameters are chosen to be illustrative rather than to describe any particular material sample.

Shown in Fig. 1 is the variation of attenuation coefficient α with respect to the normalized frequency ω/ω_0, calculated from Eq. (30). Here, ω_0 is called the depinning frequency, defined by the relation $\sigma_{V2}/\sigma_{V1} = \omega_0/\omega$. If the Bardeen-Stephen model[14] and the Gittleman and Rosenblum model[16] are used, ω_0 can be expressed by

$$\omega_0(T,B_0) = \frac{2\pi J_c(T,B_0)\sqrt{B_0}}{\sigma_N(T)B_{c2}(T)\sqrt{\Phi_0}} , \tag{34}$$

where Φ_0 is the flux quantum ($\Phi_0 = 2.07\times10^{-15}$ Wb). Numerically, if we take some values of $B_{c2}=25$ T, $J_c = 5\times10^4$ A/cm^2, $\sigma_N = 2\times10^7/\Omega$ m, and $B_0 = 4$ T, we get $\omega_0 \approx 2.76\times10^8$ rad/s. Intuitively, we might argue that ω_0 should not increase with an increase in the magnetic field B because the surface resistance should not decrease for increasing magnetic field B. If this argument, which has yet to be verified experimentally, is correct, we may expect that the field dependence of the dc critical current density J_c should be of the form $J_c \propto B^{-\gamma}$ with $\gamma \geq \frac{1}{2}$ if the model is proper. Indeed, Kim, Hempstead, and Strand[40] have proposed a model in which one has $J_c \propto (B+B_1)^{-1}$, where B_1 is a small constant. The Bean model[41] in which J_c is supposed to be independent of the field B (i.e., $\gamma=0$) seems not to obey the criterion of $\gamma \geq \frac{1}{2}$. However, if the operating frequencies of concrete problems are far from (for instance, much lower than) the depinning frequency, the Bean model may still work because the result will not be sensitive to the exact value of the depinning frequency. Other forms of the field dependence of the dc critical current density were also proposed in the past, such as, for instance, $J_c \propto B^{-1/2}$ (Yasukochi *et al.*[42]) and $J_c \propto B^{-1/2}(\mu_0 H_{c2} - B)$ (Campbell, Evetts, and Dew-Hughes[43]).

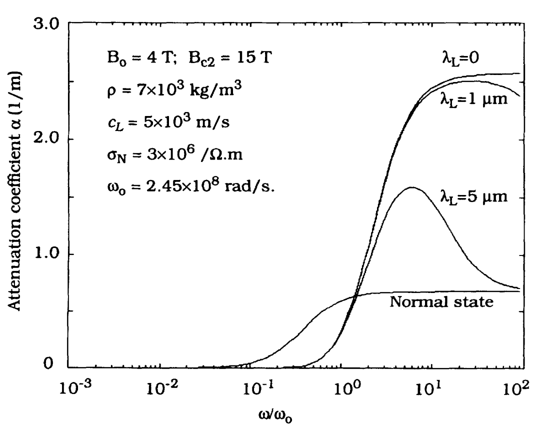

FIG. 1. Variation of the attenuation coefficient α with the frequency ω.

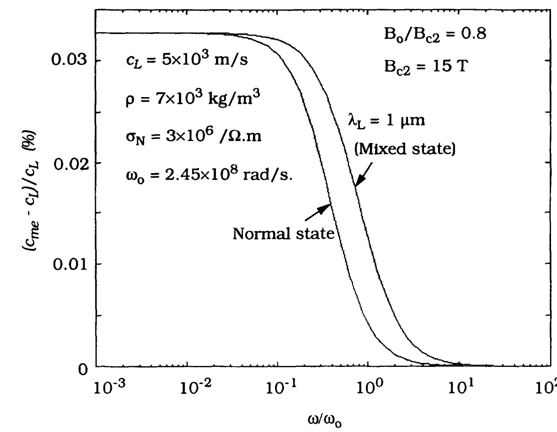

FIG. 2. Dispersion behavior of the magnetoelastic wave.

It can be seen from Fig. 1 that the attenuation of the magnetoelastic wave in the type-II superconductor in the mixed state is smaller than that in the normal state when the wave frequency ω is less than the depinning frequency ω_0. However, for wave frequencies larger than the depinning frequency, the attenuation of the magnetoelastic wave in the type-II superconductor in the mixed state can be larger than that in the normal state. In particular, Fig. 1 shows the effect of the London penetration depth λ_L. The Shapira-Neuringer model correspond to the case of $\lambda_L = 0$. It can be seen that when the effect of the London penetration depth is taken into account, the result predicted by Eq. (30) coincides with the result from the Shapira-Neuringer model at frequencies near or less than the depinning frequency ω_0, but the results show their difference at higher frequencies. At very high frequencies much larger than the depinning frequency, Eq. (30) shows that the attenuation of the magnetoelastic wave approaches its normal-state value for nonzero London penetration depth, which is physically reasonable. It would be valuable if one could verify experimentally the high-frequency behavior of attenuation of the magnetoelastic wave in the type-II superconductor in the mixed state. So far, it seems that no such experimental results have been proven, probably because of the fact that high-frequency (GHz) ultrasonic devices are not easy to obtain. Recent progress in research on hypersound (or quantum acoustics[44]) may help to improve the situation.

Shown in Fig. 2 is the dispersion behavior of the magnetoelastic wave. It can be seen that the strong dispersion of the magnetoelastic wave in the elastic superconductor in the mixed state appears at frequencies near the depinning frequency. When the superconductor is in the normal state, the wave becomes strongly dispersive at frequencies lower than those in the mixed state.

We may note that, in the above example, the effect of the London moment does not show up. This is because the wave considered is longitudinal in the elastic displacement. In Sec. IV we shall study the effect of the London moment by considering transverse waves in the type-II superconductor in the mixed state.

IV. PHASE CHANGE IN SOME WAVES PROPAGATING IN ELASTIC TYPE-II SUPERCONDUCTORS IN THE MIXED STATE

In this section we shall study the effect of the London moment on the magnetoelastic behavior of the elastic type-II superconductor in the mixed state. We consider the following problem of a magnetic plane wave induced by a transverse elastic plane wave in the elastic type-II superconductor in the mixed state. We shall assume that the elastic type-II superconductor is in a static and uniform magnetic field $\mathbf{B}_0$ and at a temperature $T \ll T_c$ so that the effect of the normal current due to thermal excitation is negligible. We shall also ignore the small effect of the magnetic force on the behavior of the elastic wave in the special case. Furthermore, we assume that the magnetic field $\mathbf{B}_0$ is along the z-axis direction and that the elastic type-II superconductor is in the mixed state. The superconductor is then subjected to an elastic disturbance, described by a transverse elastic plane wave of the following form:

$$u_y = u_0 \exp\left[i\omega \left(t - \frac{x}{c_T} \right) \right] , \tag{35}$$

with $u_x = u_z = 0$. Here, u_0 (>0) denotes the amplitude of the elastic plane wave, and $c_T = (G/\rho)^{1/2}$ is the speed of the transverse elastic plane wave.

To study the electrodynamic response of the superconductor by such a given mechanical disturbance, we look for the following type of magnetic wave:

$$b_z = b_0 \exp\left[i\omega \left(t - \frac{x}{c_T} \right) \right] , \tag{36}$$

with $b_x = b_y = 0$. By substituting Eqs. (35) and (36) into Eq. (17) in the absence of normal currents, we may obtain, after some manipulations, the following result:

$$b_0 = |b_0| e^{i\gamma} , \tag{37}$$

where the amplitude of the magnetic wave, $|b_0|$, is expressed by

$$|b_0| \frac{m^* \omega^2 u_0}{e^* c_T} \sqrt{a_1^2 + a_2^2} \tag{38}$$

and the phase factor γ by

$$\gamma = \tan^{-1}\left[\frac{a_2}{a_1} \right] . \tag{39}$$

Here, the two parameters a_1 and a_2 are given, respectively, by

$$a_1 = \frac{1 + \left[\frac{\omega}{c_T} \right]^2 \left[\lambda_L^2 + \frac{\sigma_{V2}}{\mu_0 \omega (\sigma_{V1}^2 + \sigma_{V2}^2)} \right]}{\left[1 + \left[\frac{\omega}{c_T} \right]^2 \left[\lambda_L^2 + \frac{\sigma_{V2}}{\mu_0 \omega (\sigma_{V1}^2 + \sigma_{V2}^2)} \right] \right]^2 + \frac{\omega^2 \sigma_{V1}^2}{c_T^4 \mu_0^2 (\sigma_{V1}^2 + \sigma_{V2}^2)^2}} , \tag{40}$$

$$a_2 = \frac{\omega \sigma_{V1} / [c_T^2 \mu_0 (\sigma_{V1}^2 + \sigma_{V2}^2)]}{\left[1 + \left[\frac{\omega}{c_T} \right]^2 \left[\lambda_L^2 + \frac{\sigma_{V2}}{\mu_0 \omega (\sigma_{V1}^2 + \sigma_{V2}^2)} \right] \right]^2 + \frac{\omega^2 \sigma_{V1}^2}{c_T^4 \mu_0^2 (\sigma_{V1}^2 + \sigma_{V2}^2)^2}} . \tag{41}$$

The result shows that the transverse elastic plane wave propagating in the elastic type-II superconductor in the mixed state may induce a transverse magnetic plane wave with the same phase velocity propagating in the superconductor due to the effect of the London moment induced by the local motion of the elastic type-II superconductor. Furthermore, the result shows that there is a phase change between the elastic plane wave and the induced magnetic plane wave, characterized by the phase factor γ. This phase factor γ is found to be dependent on the material properties of the elastic type-II superconductor in the mixed state. It is also frequency dependent. Mathematically, if we let $\sigma_{V1}, \sigma_{V2} \to \infty$ ($B_0 \to 0$), which corresponds to the Meissner state, we may find that γ becomes zero, and Eq. (38) recovers the result derived earlier by Zhou[11] for elastic superconductors in the Meissner state, where that is no such phase change between the transverse elastic plane wave and the induced transverse magnetic plane wave. This indicates that the effect of the phase change is closely related to the vortex-dynamic properties of the elastic type-II superconductor in the mixed state. So far, it seems that no experiments have been reported about this effect. Because of possible theoretical and practical interest, it would be worthwhile to investigate this effect experimentally.

V. CONCLUSIONS

A macroscopic theory has been introduced in this paper to describe phenomenologically the magnetoelastic behavior of elastic type-II superconductors in the mixed state. In this theory we have self-consistently included the vortex-dynamic effect, the normal-current effect, the flux-flow Hall effect, the London moment effect, and the deformation effect in the elastic type-II superconductors in the mixed state. The theory is considered to be applicable to the analysis of some electromagnetic and mechanical phenomena in elastic type-II superconductors in the mixed state, where effects of magnetic history and flux creep can be ignored. The theory is valid within the framework of the generalized Galilean relativity at the magnetoquasistatic approximation. By using this theory, we have studied both analytically and numerically some problems concerning magnetoelastic wave propagating in elastic type-II superconductors in the mixed state at the linear approximation. It is shown that the attenuation of the magnetoelastic wave in the type-II superconductor in the mixed state is smaller than that in the normal state when the wave frequency ω is less than the depinning frequency ω_0. However, for wave frequencies larger than the depinning frequency, the attenuation of the magnetoelastic wave can be larger than that in the normal state. It is also shown that the magnetoelastic wave in the superconductor shows strong dispersion at frequencies near the depinning frequency. Furthermore, we have studied the effect of the London moment on some wave propagating in elastic type-II superconductors in the mixed state. It is shown that there is a phase change between the transverse elastic plane wave and the induced transverse magnetic plane wave propagating in the elastic type-II superconductor in the mixed state. This effect is shown to be closely related to the vortex-dynamic properties of the elastic type-II superconductor in the mixed state.

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