Superconductor–normal-metal–superconductor junction in a strong magnetic field

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(Received 29 October 1993)

We consider an *SNS* junction, *N* being the two-dimensional electron gas, subjected to a magnetic field exceedingly large for weak superconductivity. We show that under the condition of Landau quantization the proximity effect induces a thin superconducting paramagnetic strip near the boundary of the *N* region. We obtain the expression for the supercurrent that accounts for multiple Andreev scattering of magnetic edge states. We also address Josephson coupling by bulk magnetic states.

I. INTRODUCTION

Maximum dc Josephson current in a superconductor–normal-metal– superconductor junction only weakly depends on the thickness of the normal metal barrier at low temperature.[1] The reason is that Cooper pairs can penetrate far in the normal metal, a phenomenon called the proximity effect. A weak external magnetic field H shifts the phases of superconducting leads, which results in a Fraunhofer-like diffraction dependence of the current on a magnetic flux through the contact. In a strong magnetic field, it is necessary to take into account the influence of the field upon the proximity effect.[2] The maximum current is expected to be exponentially decreasing function of a distance between superconducting leads when distance becomes larger than magnetic length.

In another paper[3] we considered the weak superconductivity of *SNS* junctions in a strong magnetic field, which leads to Landau quantization of electrons in the metal barrier. We considered a two-dimensional (2D) electron gas, where only a few Landau levels are occupied, as the normal barrier. We showed that magnetic edge states (MES's) are important for the dc Josephson current of the junction.

The edge states are responsible for the transport properties of narrow channels;[4,5] they also determine the Aaronov-Bohm type oscillations of physical quantities in quantum dots.[6,7] A comprehensive review of the transport by MES's in mesoscopic structures can be found in Ref. 8. For a recent development in the theory of the edge states of a fractional quantum Hall liquid, see Refs. 9 and 10.

Let us recall the results of Ref. 3. The geometry of *SNS* junctions is shown in Fig. 1. Superconducting leads couple to the MES's of a closed 2D gas. Supercurrent flows in the two-dimensional system along side the surface. Provided a constant drift velocity of electrons along the edge, the maximum supercurrent does not depend on the position of contacts on the surface. The magnitude of the maximum current is determined by the ratio of the length of the perimeter of the two-dimensional system and the coherence length, which is equal to $\ell_T = V/2\pi T$ (V is the electron drift velocity along the surface, and T is the temperature). The current only weakly depends on the perimeter, provided the perimeter is smaller than the coherence length. In the opposite limit, the current becomes exponentially small by enlarging the perimeter. As a function of the external magnetic flux, the current experiences *AB*-type oscillations, similar to those of the thermodynamic and kinetic quantities of the quantum dots.[6,7]

We believe that this kind of *SNS* system might exhibit interesting physics, and so we continue the study of the basic properties of the *SNS* junction in a strong magnetic field. This paper continues the study mostly with the intent of taking into account multiple Andreev scattering. To this purpose, neglecting Zeeman splitting, we derive the system of equations for the Green's function in terms of MES's of the lowest Landau level (Sec. II). In Sec. III we apply these equations for studying the proximity effect. The dc Josephson current and Andreev spectrum of the edge states will be considered in Sec. IV. The resonant transmission of pairs will be considered in detail. The geometry of the junction where superconducting leads couple to MES's of an open 2D system will also be studied in Sec. IV. Josephson coupling by the bulk states will be considered in Sec. V. Finally, we summarize the main results of the paper.

II. DERIVATION OF BASIC EQUATIONS

In this section we derive the Green's function equations that describe the Andreev reflection of MES's. As in Ref. 3, we consider a structure consisting of a superconductor and two-dimensional metal, which are connected by a point contact. This simple model comprises Andreev scattering and allows one to get rid of complications caused by a distribution of the superconducting order parameter along the thick contact. Figure 1 presents a schematic picture of the junction.

The Hamiltonian of the system, in Namby's representation, has the form

$$H = H_{\rm BCS} + H_{\rm 2D} + t\{\psi^\dagger(\mathbf{r}_1)\sigma_Z\varphi(\boldsymbol{\rho}_1) + \varphi^\dagger(\boldsymbol{\rho}_1)\sigma_z\psi(\mathbf{r}_1)\} \ . \tag{1}$$

Here $H_{\rm BCS}$ is the BCS Hamiltonian of the supercon-

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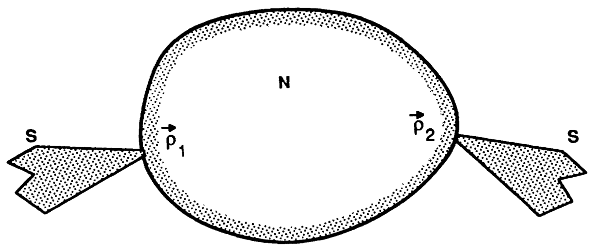

FIG. 1. Schematic picture of a *SNS* junction. Shaded at the left and right sides are the superconducting leads *S*. The point contacts located at ρ_1 and ρ_2 connect them to the 2D system (*N*), the shaded area inside of which denotes the region of the proximity-induced superconductivity. The thickness of this strip is of the order of the magnetic length.

ductor and H_{2D} is the Hamiltonian of the two-dimensional electron gas. The third term in (1) describes coupling of the point $\mathbf{r}_1$ of the superconductor to the point ρ_1 of the two-dimensional system. t is the hopping integral $\psi(\mathbf{r}_1)$ and $\varphi(\rho_1)$ are Namby operators of the superconductor and two-dimensional system, respectively, and σ_i, $i=x,y,z$, are the Pauli matrices.

In the case of more than one point contact, it is necessary to sum over all contacts in (1).

The system of Gorkov equations for the Green's function has the form

$$(i\omega_n - H_{BCS})\hat{G}(\mathbf{r}) = t\sigma_z\delta(\mathbf{r}-\mathbf{r}_1)\hat{g}(\rho_1;\rho') ,$$
$$(i\omega_n - H_{2D}\hat{g}(\rho;\rho') = t\sigma_z\delta(\rho-\rho_1)\hat{G}(\mathbf{r}_1) + \delta(\rho-\rho') , \tag{2}$$

where $\hat{G}(\mathbf{r})$ and $\hat{g}(\rho;\rho')$ are the Green's functions of the superconductor and 2D system, respectively, $\omega_n=(2n+1)\pi T$ is the Matsubara frequency, and T is the temperature.

In a magnetic field, which is larger than the low critical one, the superconducting order parameter is nonhomogeneous. While its form near the point contact is unknown, presumably it is smaller than in the volume.[11] For our purpose the fact of the Andreev reflection is important, rather than knowledge of the exact form of the order parameter near the contact. So to solve the first equation of (2), we approximate the order parameter by some effective constant $\Delta(\mathbf{r})=|\Delta(\mathbf{r})|\exp\{i\chi(\mathbf{r})\}$, where $\chi(\mathbf{r})$ is the superconducting phase magnitude near the contact, and treat the equation as a volume problem. Inserting the solution of the first equation of system (2) into the second equation, we obtain the equation for $\hat{g}(\rho;\rho')$:

$$(i\omega_n - H_{2D})\hat{g}(\rho;\rho') = -\frac{\pi\nu_0 t^2}{\sqrt{\omega_n^2+|\Delta(\mathbf{r}_1)|^2}}\delta(\rho-\rho_1)\begin{bmatrix} i\omega_n & \Delta(\mathbf{r}_1) \\ \Delta^*(\mathbf{r}_1) & i\omega_n \end{bmatrix}\hat{g}(\rho;\rho') + \delta(\rho-\rho') . \tag{3}$$

Here ν_0 is the density of states in a superconducting metal.

The Hamiltonian H_{2D} has the general form

$$H_{2D} = \begin{bmatrix} \frac{1}{2m}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}\right]^2 + eV(\rho) - \varepsilon_F & 0 \\ 0 & -\frac{1}{2m}\left[\mathbf{p}+\frac{e}{c}\mathbf{A}\right]^2 - eV(\rho) + \varepsilon_F \end{bmatrix} , \tag{4}$$

where $V(\rho)$ is the confining potential of 2D electrons and ε_F is the Fermi level. From this point it is convenient to take into account the properties of the eigenstates of the 2D system in a magnetic field. It is well known (see Ref. 8) that the spectrum of two-dimensional electrons in a magnetic field consists of bulk Landau levels and of the spectrum of the edge states. Spatially, these edge states locate near the boundary, the edge states of the lowest Landau level been nearest to the boundary. As far as we are concerned in the point contact with the surface, we restrict ourselves to MES's of the lowest Landau level only.[8] We also assume that the Fermi level lies between the bulk Landau levels, where we may neglect the coupling between MES's and the bulk.

Because of the impurity scattering, the MES's with energy that coincides with the Landau level might have a strong overlap with the bulk. When the Fermi level moves to those states, the Josephson coupling proceeds by the bulk magnetic states. We consider this case in Sec. V.

In the representation of the edge states, the Hamiltonian

$$\frac{1}{2m}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}\right]^2 + V(\rho) - \varepsilon_F$$

reduces to the Hamiltonian[10,12] $-iVd/dx-\varepsilon_F$ ($\hbar\equiv 1$ everywhere below), which describes the motion of a particle along the surface. Here x is the coordinate along side the equipotential line near the surface, where electrons drift in the magnetic field with velocity V. The wave function $\Phi(y)$ in the transverse direction is the nodeless function of the range of the magnetic length, centered near the edge. The precise form of $\Phi(y)$ depends on the details of the surface potential. In the case when the potential can be approximated as a linear one, $\Phi(y)$ is a wave function of the ground state of the oscillator. The term

$$\frac{1}{2m}\left[\mathbf{p}+\frac{e}{c}\mathbf{A}\right]^2+V(\rho)-\varepsilon_F$$

of H_{2D} corresponds to a particle moving in the opposite direction. This term includes the magnetic field that is opposite to the field in the first term and reduces to $iVd/dx-\varepsilon_F$.

The Green's function for MES's may be represented in the form

$$\hat{g}(\rho;\rho')=\Phi(y)\Phi(y')g(x;x') \ . \tag{5}$$

Multiplying (3) by $\Phi(y)$ from the left and integrating over y, we obtain, for $g(x;x')$, the equation

$$\begin{bmatrix} i\omega_n+iV\frac{d}{dx}+\varepsilon_F & 0 \\ 0 & i\omega_n+iV\frac{d}{dx}-\varepsilon_F \end{bmatrix} g(x;x') = -\mu_1\delta(x-x_1)\begin{bmatrix} i\omega_n & \Delta(\mathbf{r}_1) \\ \Delta^*(\mathbf{r}_1) & i\omega_n \end{bmatrix} g(x;x') + \delta(x-x') \ . \tag{6a}$$

Here we introduce a parameter

$$\mu_1=\frac{\pi v_0 t^2}{\sqrt{\omega_n^2+|\Delta(\mathbf{r}_1)|^2}}[\Phi(y_1)]^2 \ . \tag{6b}$$

In a case of a few point contacts, one needs to take a sum of all contacts on the right side of the system (6a). The δ function on the right side of the system (6a) describes the coupling to the superconductor through the point contact; therefore, it has to be considered as a limiting form of the smooth function.

In what we have considered above, we assumed that the superconducting leads are conventional singlet superconductors and did not distinguish between spin-up and -down states in the N region. Both states are necessary for the proximity effect. Zeeman splitting leads to a spatial separation of MES's of opposite spin directions. Generally speaking, Eq. (6a) has to be modified to incorporate the fact that MES's with up and down spin have different velocities and hopping to the leads. Our assumption means that we neglect these differences.[13]

Equation (6a) can be solved exactly. We will apply it in the next sections to describe the proximity effect and Josephson coupling by MES's.

III. PROXIMITY EFFECT ON MAGNETIC EDGE STATES

In this section we consider the penetration of Cooper pairs into the 2D system. In our case the amplitude of the Cooper pairs is determined by the nondiagonal element of the Green's function

$$F(x)=T\sum_{\omega_n}\mathrm{Sp}\{(\sigma_x+i\sigma_y)g(x;x)\} \ . \tag{7}$$

Consider the system with one contact at the point x_1. Equation (6a) can be solved by the substitution

$$g(x;x')=\frac{1}{2}\exp\left\{-\omega_n\int_{x'}^{x}\frac{dx}{V}[1+\mu_1\delta(x-x_1)]\right\} \times\left\{a(1+\sigma_z)\exp\left[i\frac{\varepsilon_F(x-x')}{V}\right] +b(1-\sigma_z)\exp\left[-i\frac{\varepsilon_F(x-x')}{V}\right]+c\sigma_+\exp\left[i\frac{\varepsilon_F(x+x')}{V}\right]+d\sigma_-\exp\left[-i\frac{\varepsilon_F(x+x')}{V}\right]\right\} \ . \tag{8}$$

The wave function of the Cooper pairs (7) is related to d^*, for which we have the equations

$$\begin{aligned} &iV\frac{d}{dx}a+\mu\Delta\exp\left[-\frac{2i\varepsilon_F x_1}{V}\right]\delta(x-x_1)d=\delta(x-x') \ , \\ &\mu\Delta^*\exp\left[\frac{2i\varepsilon_F x_1}{V}\right]\delta(x-x_1)a+iV\frac{d}{dx}d=0 \ . \end{aligned} \tag{9}$$

Here x_1 is the coordinate of the point contact. The solution (8) has to be continuous at $x=0,L$. Here L is the length of the drift path along the surface.

Solving the system (9), we obtain an expression for the wave function of the Cooper pair:

$$F(x)=\frac{2T}{V}\sum_{\omega_n>0}\frac{\eta_{\omega_n}\exp\{i\phi_1(x,x_1)+i\phi_2(x,x_1)\}}{\cosh\{(\omega_nL/V)(1+\mu/L)\}-\sqrt{1-\eta_{\omega_n}^2}\cos(\varepsilon_FL/V)} \ . \tag{10}$$

Here we introduce the parameter

$$\eta_{\omega_n}=\frac{\mu\Delta}{V}\Big/\left[1+\left[\frac{\mu\Delta}{2V}\right]^2\right] .$$

The phase $\phi_1(x,x_1)$ corresponds to circling from x_1 to x in a positive direction,

$$\phi_1(x,x_1)=\frac{\varepsilon_F}{V}\begin{cases}x-x_1 & \text{if } x\geq x_1 ,\\ L-x_1+x & \text{if } x\leq x_1 .\end{cases} \tag{11a}$$

The phase $\phi_2(x,x_1)$ describes the path from x_1 to x in the opposite direction,

$$\phi_2(x,x_1)=-\frac{\varepsilon_F}{V}\begin{cases}L-x+x_1 & \text{if } x\geq x_1 ,\\ x_1-x & \text{if } x\leq x_1 .\end{cases} \tag{11b}$$

A few consequences come from this expression. Below we consider the limit of a long junction $V/\Delta<L$.

(1) Provided a constant velocity V, the density of Cooper pairs does not depend on the distance from the contact; in fact, it depends only on the perimeter of the two-dimensional system. As follows from (5), the density decreases exponentially at the magnetic length from the surface.

(2) The coherence length is equal to $\ell_T=V/2\pi T$. This expression is reminiscent of that for a clean metal.

(3) The coherence length has to be compared to the perimeter L. When $L>\ell_T$, one may take into account only the first term in (10); therefore, the pair's wave function module is proportional to $\exp(-L/\ell_T)$. When $L<\ell_T$, the sum in (10) may be converted to an integral, and it appears that $|F(x)|\propto L^{-1}$. It follows from the above that the proximity effect disappears in the thermodynamic limit $L\to\infty$.

This can be understood from the classical point of view: The electrons combining Cooper pairs are in the states connected by the time reversal transformation, and therefore they have opposed velocities in 2D. To create a pair they need to encircle the whole surface; so it is not surprising that $|F(x)|$ depends only on L. It is worth mentioning that this dependence resembles the dependence of the Cooper pair amplitude on the distance from the boundary in usual *SN* sandwiches (see review in Refs. 1 and 2).

(4) Because of the term $\cos(\varepsilon_F L/V)$ in (10), $F(x)$ is an oscillating function of the Fermi energy. The magnetic field shifts the levels of the two-dimensional system and causes an Aaronov-Bohm-type oscillation of $F(x)$. This is in direct connection with the Aaronov-Bohm effect in quantum dots.[6,7]

(5) The momentum of the pair,

$$\frac{\partial}{\partial x}\{\phi_1(x,x_1)+\phi_2(x,x_1)\}=\frac{2\varepsilon_F}{V} ,$$

being of the same direction, is twice that of the electrons in MES's. Using this fact, it is easy to find that the velocities of the pair and electron coincide. This means that the supercurrent contributes paramagnetically to the total magnetization of the N region.[14] At the point of contact $x=x_1$, the phase experiences a jump:

$$(\phi_1+\phi_2)_{x\to+x_1}-(\phi_1+\phi_2)_{x\to-x_1}=\frac{2\varepsilon_F}{V} .$$

This points out that in the case of extended contact, where the phase will be continuous, the current changes direction on the extension of contact.

IV. JOSEPHSON COUPLING BY EDGE MAGNETIC STATES

In this section we consider the dc Josephson current between two superconducting contacts attached to the two-dimensional system at the points x_1 and x_2. The current, flowing throughout the junction, might be represented as a difference between current incoming into the point x_1 and the current outgoing from x_1:

$$I=\frac{eV}{2}T\sum_{\omega_n}\{[g(x;x)|_{x<x_1}-g(x;x)|_{x>x_1}]\sigma_z\} . \tag{12}$$

Here we took into account that, by the definition, $\Phi(y)$ is a normalized wave function.

To find the Green's function, one must use Eq. (6a), where into the right side the term

$$-\mu\delta(x-x_2)\begin{bmatrix} i\omega_n & \Delta(\mathbf{r}_2)\\ \Delta^*(\mathbf{r}_2) & i\omega_n\end{bmatrix}g(x;x') ,$$

describing the coupling to the second lead, has to be added. We assume that except the phases leads are identical and drop the index of μ_i. Solving the equation for the Green's function by the substitution (8), we find, from (12),

$$I=2eT\sum_{\omega_n}\frac{\eta_{\omega_n}^2\sin\theta}{\cosh[(\omega_n L/v)(1+2\mu/L)]-(1-\eta_{\omega_n}^2)\cos(\varepsilon_F L/V)+\eta_{\omega_n}^2\cos\theta} . \tag{13}$$

The phase θ includes the phase difference of the superconducting leads and the phases due to the circling the surface [Eqs. (11a) and (11b)]:

$$\theta=\chi_1-\chi_2+\phi_1(x_2,x_1)+\phi_2(x_2,x_1) . \tag{14}$$

θ contain the whole dependence of the current on the location of the contacts.

In the leading order in η, expression (13) gives the perturbative result of Ref. 3. From (13) it follows that at low temperature and $\cos(\varepsilon_F L/V)\approx 1$ perturbation theory fails. This is a regime where resonant transmission dominates the current.

At zero temperature from (13), we have (considering $V/L<\Delta$, we may neglect the dependence of η on ω_n)

$$I=\frac{\pi-\arccos[(1-\eta^2)\cos(\varepsilon_F L/V)-\eta^2\cos\theta]eV\eta^2}{\sqrt{1-[(1-\eta^2)\cos(\varepsilon_F L/V)-\eta^2\cos\theta]^2}}\times\frac{eV\eta^2}{\pi L}\sin\theta\ . \tag{15a}$$

Expressions (13) and (15a) are valid for any η. The case of small value of η is physically more relevant; therefore, we consider it in more detail.

The resonance value of the current is proportional to η, and it occurs for $\cos(\varepsilon_F L/V)=1$:

$$I=\left[\frac{eV\eta}{L}\right]\operatorname{sgn}\left[\frac{\cos\theta}{2}\right]\sin\left[\frac{\theta}{2}\right]\ . \tag{15b}$$

The current is dominated by the Andreev level. To show this, let us consider the spectrum. One can obtain it from the poles of expression (13) by the substitution $\omega_n\rightarrow -iE$.

Most interesting is the low-energy-lying spectrum. For $\Delta \gg E$ and small μ, neglecting $1\gg 2\mu/L$, we obtain the dispersion equation

$$\cos\left[\frac{EL}{V}\right]=(1-\eta^2)\cos\left[\frac{\varepsilon_F L}{V}\right]-\eta^2\cos\theta\ . \tag{16}$$

At the resonance the solutions of (16) are

$$\frac{EL}{V}=\pm 2\eta\cos\frac{\theta}{2}+2\pi l\ ,\quad l=0,\pm 1,\ldots\ .$$

The total energy $E(\theta)$ is the sum over all energies below zero, and it depends on θ throughout the top level:

$$\frac{E(\theta)L}{V}=-2\eta\cos\frac{\theta}{2}\quad \text{for } \theta<\pi$$

and

$$\frac{E(\theta)L}{V}=2\eta\cos\frac{\theta}{2}\quad \text{for } \theta>\pi\ .$$

Taking the derivative of $-E(\theta)$ over θ, we find (15b).

Away from resonance the current is proportional to η^2,

$$I\approx\left[\frac{eV\eta^2}{L}\right]\sin\theta\ . \tag{15'}$$

Many levels contribute to the current in this case. It can be easily seen for $\cos(\varepsilon_F L/V)=-1$. The solution of (16) is

$$\frac{EL}{V}=\pm 2\eta\sin\frac{\theta}{2}\pm(2l+1)\pi\ .$$

A shift of the energy levels for small θ is proportional to η; therefore, only the cancellation of the contributions of many individual levels leads to (15').

Let us point out that at the resonance pair amplitude $F(x)$ is nonzero in the limit $\eta\rightarrow 0$ for $T=0$.

If the temperature is large, the coherence length is smaller than L; in expression (13), the term with the smallest frequency dominates and the current is equal to $I=8e\eta^2 T\exp(-L/\ell_T)\sin\theta$. The dependence of the current on L is determined by the amplitude $F(x)$.

For the sake of completeness, let us mention the results for a short junction: $V/L>\Delta$. In the nonresonant case in (15), Δ has to be substituted for V/L. The resonance current will be modified only if $\eta^V/\Lambda>L$, and its value will be given by (15b) where η^V/L must be changed to $\Delta/2$.

We have considered Josephson coupling by MES's of a closed 2D system; we call that a closed *SNS* junction. To conclude this section, let us mention another kind of Josephson coupling by MES's; we call that an open SNS junction. Schematically, the device is shown in Fig. 2. The MES's of sample 1 are located far enough from that of sample 2 so that the direct tunneling between them is strongly suppressed. The transitions can proceed by the superconducting leads only. The tunneling distance has to be of the order of or smaller than the coherence length of the superconductor; otherwise, the coupling will be small. We may use perturbation theory. Estimations show that the current is proportional to $\sin\theta$, and its dependence on the distance in a long junction has the form

$$I\propto T\sum_{\omega_n>0}\exp\left[-\frac{R\omega_n}{V}\right]\ ,$$

being proportional to R^{-1} for $\ell_T>R$ and exponentially small otherwise. R is the sum of paths along the MES's of samples 1 and 2.

There are several reasons why the open *SNS* junction could be interesting, First, this type of junction may prove useful in the case of MES's of spin-polarized electrons. Spin-flip-assisted coupling between the superconductor and MES's has to be included in consideration in this case. The proximity effect in a closed system is very weak due to the condition that the orbital wave function of the pair constructed from MES's must be antisymmetric al. It turns out that the latter condition is much less restrictive in the geometry of Fig. 2, and consequently, the Josephson coupling through spin-polarized MES's

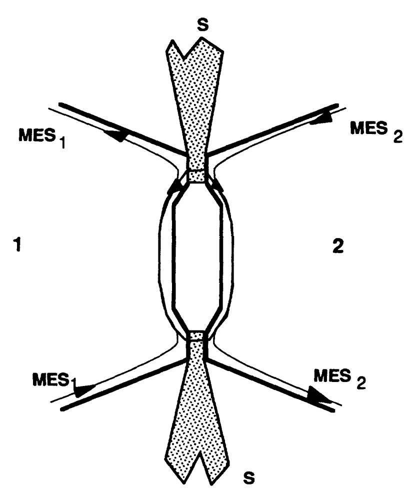

FIG. 2. Schematic picture of the open *SNS* junction. The electrons of the pair tunnel from (say) the upper superconducting lead to the magnetic edge states of sample 1 (MES_1) and sample 2 (MES_2), from which they finally tunnel to the second lead, thus providing the Josephson coupling. The thickness of the superconducting barrier is of the order of or less than the coherence length of the superconductors.

may be stronger in an open than in a closed system. Second, this type of junction may prove useful in the investigation of the dependence of I-V characteristics on the electron states in the region of the junction. For instance, one can consider the effect of the interaction of Josephson oscillations with edge magnetoplasmons. The frequencies of magnetoplasmon modes can range from far infrared to radio frequencies,[15] depending on the magnetic field and system size. The system size can be easily varied in the open geometry which allows for an easy adjustment of the resonance between the Josephson oscillations and magnetoplasmon modes.

V. JOSEPHSON COUPLING BY BULK MAGNETIC STATES

It is natural to consider the system where superconducting contacts attach the bulk states of a 2D system. It is necessary to take into account the scattering of electrons on the impurities. The most reliable estimation of the current can be made in the case of short-range scatterers. Transport is diffusive on scales less than a localization length, and it happens by the hopping of the centers of the cyclotron orbits of electrons. We may apply a conventional diagrammatic approach.[16–18] For the mean-square value of the current, it shows that the supercurrent may be represented the form

$$I=I(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,H)\sin[\chi_1-\chi_2+\phi(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,H)] ,$$

where $\chi_1-\chi_2$ is the phase difference of the superconducting leads, located at the points $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$, and $\phi(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,H)$ is an additional random phase which appears in the magnetic field. At the large distance $|\boldsymbol{\rho}_1-\boldsymbol{\rho}_2|$ this random phase becomes of the order π, and so after averaging of I over the impurity configurations, the current becomes exponentially small. On the other hand, a typical value of the current is

$$I(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,H)\propto e\left[\frac{\nu_0|t|^2}{|\boldsymbol{\rho}_1-\boldsymbol{\rho}_2|}\right]^2\nu_{2\mathrm{D}}(\varepsilon_F,H) , \tag{17}$$

for

$$\sqrt{D(\varepsilon_F)/\Delta}<|\boldsymbol{\rho}_1-\boldsymbol{\rho}_2|<\sqrt{D(\varepsilon_F)/T} .$$

Here $\nu_{2\mathrm{D}}(\varepsilon_F,H)$ and $D(\varepsilon_F)$ are the density of states and diffusion constant of 2D electrons. We assume that $\Delta>T$ and that the Fermi level is close enough to the center of the Landau level, and so $|\boldsymbol{\rho}_1-\boldsymbol{\rho}_2|$ is less than the localization length.

In (17) all the dependence on the magnetic field enters only through the density of states. This is because we neglect Zeeman splitting, which suppresses the current when the splitting becomes larger than the broadening of Landau levels due to the disorder.

Equation (17) predicts Shubnikov–de Haas oscillations and the enhancement of the current in a magnetic field. It can be real only for a system where the cyclotron frequency is much larger than the Zeeman splitting.[13]

The MES's of those energies that coincide with the bulk Landau level may have a strong coupling to the bulk due to impurity scattering. In this case Eq. (17) describes the current of the devise considered in the previous sections in the case where the Fermi level coincides with the bulk Landau level. At low temperature, when $\sqrt{D(\varepsilon_F)/T}$ is larger than the geometrical size L of the system, one must substitute L for $|\boldsymbol{\rho}_1-\boldsymbol{\rho}_2|$ in (17). Comparing the coupling by MES's, given by Eq. (15b) and (15′), and the bulk, we see that the first is likely to be larger.

VI. CONCLUSION

This paper deals with the proximity effects and SNS supercurrent in a magnetic field which is extremely large for weak superconductivity. Equations (10), (13), and (17) constitute the main results of this paper.

We consider two limiting situations.

First, the case when the Josephson coupling proceeds by edge magnetic states. We show that the proximity effect forms a thin paramagnetic superconducting strip near the boundary of the N region. We consider multiple Andreev scattering of MES's, which allows us to obtain a nonperturbative expression for the supercurrent. A detailed analysis of the current at low temperature in parallel to the properties of the Adreev spectrum is given.

Second, we consider the coupling by bulk magnetic states. We calculate the supercurrent in the case of the short-range scatterers and show that for a small Zeeman energy the coupling undergoes Shubnikov–de Haas oscillations and increases with magnetic field.

ACKNOWLEDGMENTS

I thank R. Serota for helpful discussions. This work was supported by the Russian Academy of Sciences Grant No. 93-02-2567. The hospitality of the Department of Physics of the University of Cincinnati is greatly appreciated.

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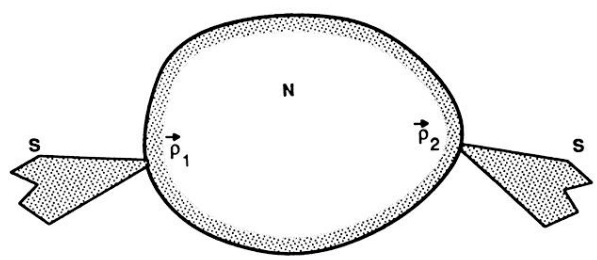

FIG. 1. Schematic picture of a *SNS* junction. Shaded at the left and right sides are the superconducting leads *S*. The point contacts located at $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ connect them to the 2D system (*N*), the shaded area inside of which denotes the region of the proximity-induced superconductivity. The thickness of this strip is of the order of the magnetic length.

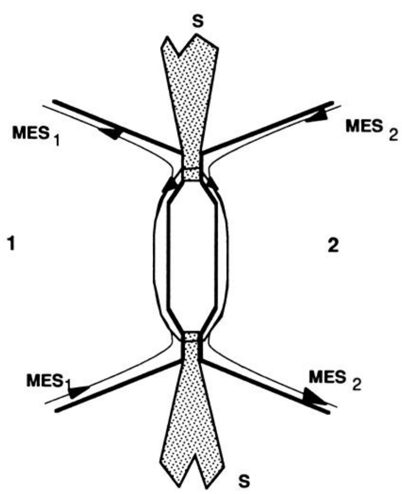

FIG. 2. Schematic picture of the open *SNS* junction. The electrons of the pair tunnel from (say) the upper superconducting lead to the magnetic edge states of sample 1 (MES_1) and sample 2 (MES_2), from which they finally tunnel to the second lead, thus providing the Josephson coupling. The thickness of the superconducting barrier is of the order of or less than the coherence length of the superconductors.