

## Infrared-reflectivity study of ZrO<sub>2</sub>-HfO<sub>2</sub> solid solutions

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Infrared-reflectance spectra of the solid solutions Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> (0 ≤ x ≤ 1) have been measured at room temperature in the frequency region 50–4000 cm<sup>-1</sup> by Fourier-transform infrared spectroscopy in order to study the composition effect on the vibrational properties of Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub>. At most, twelve infrared-active modes of 5A<sub>u</sub> and 7B<sub>u</sub> symmetry were identified in the reflectance spectra of Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub>. The spectra of Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> varied systematically with x, with changes in the frequency, linewidth, and intensity of these modes. Particularly, the B<sub>u</sub> modes at 585 and 606 cm<sup>-1</sup> for ZrO<sub>2</sub> considerably stiffened in a nonlinear way, indicating a pronounced stiffening around x = 0.8 as x increases. It was found that the lattice parameters a, b, and c of Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> decrease linearly with x whereas β initially increases up to x = 0.6 and then decreases towards that of the end member HfO<sub>2</sub>. Hence, it was concluded that the stiffening of all other modes except a B<sub>u</sub> mode is associated with the bond reinforcement evidenced by the reduced cell volume of Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> as the Hf<sup>4+</sup> ions of smaller radius are substituted for the Zr<sup>4+</sup> ions. It was also concluded that the anomalous feature around x = 0.8 in the frequency, linewidth, and intensity of the modes as a function of x can be reconciled with two-mode behavior, which has been characterized by discontinuities at either x = 0.6 or 0.3 in the x dependence of seven Raman mode frequencies for Zr<sub>x</sub>Hf<sub>1-x</sub>O<sub>2</sub> [C. Carlone, Phys. Rev. B **45**, 2079 (1992)].

### I. INTRODUCTION

Zirconia (ZrO<sub>2</sub>) has chemical and physical properties similar to hafnia (HfO<sub>2</sub>). Both oxides crystallize in a low-symmetry crystal structure (monoclinic) with four formula units per unit cell (space group C<sub>2h</sub><sup>5</sup>; P<sub>2</sub>/c). The lattice parameters of ZrO<sub>2</sub> are a = 5.150 Å, b = 5.211 Å, c = 5.317 Å, and β = 99.23°,<sup>1</sup> whereas the unit cell of HfO<sub>2</sub> is slightly modified: a = 5.1170 Å, b = 5.1754 Å, c = 5.2915 Å, and β = 99.21°.<sup>2</sup> The same crystal structure and similar properties of both oxides mean that they can form a series of solid solutions, Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub>, over a wide concentration range (0 ≤ x ≤ 1) in the ZrO<sub>2</sub>-HfO<sub>2</sub> system.

A Raman spectroscopic study of the solid solutions Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> has been done in the context of the composition effect on optical modes of mixed crystals.<sup>3</sup> Krebs and Condrate<sup>4</sup> have found that some Raman modes of A<sub>g</sub> or B<sub>g</sub> symmetry in the high-frequency region shift almost linearly with x from one end member to another, suggesting one-mode behavior. On the other hand, Carlone<sup>5</sup> has found that the Raman spectra of Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> exhibit both one- and two-mode behavior depending on the optical modes; the two-mode behavior is characterized by discontinuities at either x = 0.3 or x = 0.6 in the x dependence of seven Raman mode frequencies for Zr<sub>x</sub>Hf<sub>1-x</sub>O<sub>2</sub>.

A group theoretical analysis yields the following irreducible representations of optical modes at the wave vector k = 0 for the monoclinic structure of ZrO<sub>2</sub> and HfO<sub>2</sub>.<sup>6,7</sup>

$$\Gamma_{\text{vib}} = 9A_g(\text{R}) + 9B_g(\text{R}) + 8A_u(\text{IR}) + 7B_u(\text{IR}).$$

The Raman spectroscopic study<sup>5</sup> on the mode behavior of Zr<sub>x</sub>Hf<sub>1-x</sub>O<sub>2</sub> was done by observing and assigning the

Raman-active modes 9A<sub>g</sub> and 9B<sub>g</sub>. As there exist 15 infrared-active modes (8A<sub>u</sub> + 7B<sub>u</sub>), it is also possible to look into the mode behavior of Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> by infrared spectroscopy. While Krebs and Condrate<sup>4</sup> have stated that the infrared bands of Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> shift as x increases, the infrared spectra were not provided, nor an analysis because of the infrared bands being broad.

The present investigation concerns the effect of composition on vibrational properties of the solid solutions Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> by infrared spectroscopy. In addition, the unit-cell dimensions of Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub>, which have not been reported to the author's knowledge, were determined as well. The unit-cell dimensions are associated with cation-anion bond lengths, such that the bond-length changes greatly affect the A<sub>u</sub> and B<sub>u</sub> modes that correspond to the atomic displacements parallel to the c axis and in the ab plane, respectively. Thus, it is essential to know how the lattice parameters of Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> change as a function of x to understand the composition effect on the vibrational properties of Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub>.

### II. EXPERIMENTAL

The solid solutions Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> (0 ≤ x ≤ 1) were prepared by the solid-state reaction method rather than starting with the mixed hydroxide of ZrO<sub>2</sub>-HfO<sub>2</sub> solid solutions.<sup>4</sup> Appropriate amounts of ZrO<sub>2</sub> (99.9%) and HfO<sub>2</sub> (99.9%) in order to change the value of x by 0.1 were thoroughly mixed and formed into pellets (10 mm in diameter and ~1 mm in thickness). These pellets were sintered at 800°C for 24 h, and then reground, pelletized, and finally sintered at 1450°C for 7 h.

X-ray diffraction (XRD) with Cu Kα radiation confirmed a series of solid solutions Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> over the entire concentration range (0 ≤ x ≤ 1). Lattice param-

ters were determined by using a computer program with the reflections in the range  $2\theta=10^\circ-80^\circ$  and Si as an internal standard.

Infrared-reflectance spectra of the as-prepared pellets were measured at room temperature by Fourier-transform infrared spectroscopy (model JEOR100), in two different frequency regions, 400–4000  $\text{cm}^{-1}$  and 50–600  $\text{cm}^{-1}$ , respectively. Resolution was 4  $\text{cm}^{-1}$  in the midinfrared region and 2  $\text{cm}^{-1}$  in the far-infrared one. Either beam splitter, KBr or Mylar, was used to measure infrared-reflectance spectra in each frequency region; a TGS (triglycine sulfate) detector was used. A mirror of an evaporated thin film of aluminum was used to measure the reference spectrum in each frequency region, and the infrared-reflectance spectra of  $\text{Zr}_{1-x}\text{Hf}_x\text{O}_2$  are represented as the intensity ratios of the measured spectra with respect to the reference spectrum.

### III. RESULTS

#### A. Lattice parameters

Powder XRD indicated no formation of any ternary compound(s), confirming a series of solid solutions  $\text{Zr}_{1-x}\text{Hf}_x\text{O}_2$  over the entire concentration range ( $0 \leq x \leq 1$ ). The lattice parameters and the unit-cell volume of  $\text{Zr}_{1-x}\text{Hf}_x\text{O}_2$  are shown as a function of  $x$  in Fig. 1. The lattice parameters  $a$ ,  $b$ , and  $c$  decrease linearly with  $x$  conforming to Vegard's law, whereas  $\beta$  initially

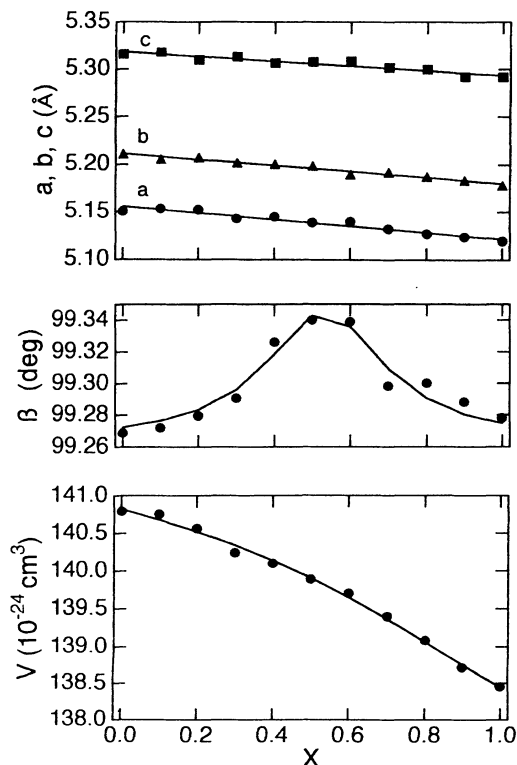


FIG. 1. Lattice parameters  $a$ ,  $b$ ,  $c$ , and  $\beta$  as a function of  $x$  for  $\text{Zr}_{1-x}\text{Hf}_x\text{O}_2$ ; the unit-cell volume  $V$  ( $V=abc \sin\beta$ ) is plotted as well.

increases up to  $x=0.6$  and then decreases toward that of the end member  $\text{HfO}_2$  as  $x$  increases. The lattice parameters  $a$ ,  $b$ , and  $c$  of  $\text{ZrO}_2$  and  $\text{HfO}_2$  ( $a=5.1501$  Å,  $b=5.2109$  Å,  $c=5.3157$  Å, and  $\beta=99.268^\circ$  for  $\text{ZrO}_2$ ;  $a=5.1197$  Å,  $b=5.1781$  Å,  $c=5.2918$  Å, and  $\beta=99.278^\circ$  for  $\text{HfO}_2$ ) are comparable to the literature data<sup>1,2</sup> although the  $\beta$ 's are slightly larger than those in Refs. 1 and 2. The concentration dependence of  $\beta$  causes a deviation from linearity above  $x=0.6$  in the unit-cell volume ( $V=abc \sin\beta$ ) vs  $x$  plot.

A neutron-diffraction study<sup>8</sup> has indicated that the lattice parameters  $a$ ,  $b$ , and  $c$  of monoclinic  $\text{ZrO}_2$  decrease whereas  $\beta$  increases while cooling from 1250 to 300 K; these lattice-parameter changes are similar to those up to  $x=0.6$  for  $\text{Zr}_{1-x}\text{Hf}_x\text{O}_2$ . Hence, the lattice contraction while cooling and the substitution for Zr by Hf of smaller radius may affect the lattice parameters of  $\text{ZrO}_2$  similarly. The substitution of Hf for Zr above  $x=0.6$  leads to a decrease in  $\beta$ .

#### B. Infrared-reflectance spectra

The infrared-reflectance spectra of  $\text{Zr}_{1-x}\text{Hf}_x\text{O}_2$ ,  $\text{ZrO}_2$ , and  $\text{HfO}_2$  for reference are shown in Fig. 2; note that each spectrum is offset vertically for clarity and displayed in two frequency regions that partly overlap. The transmission spectrum of monoclinic  $\text{ZrO}_2$  (Ref. 9) can be compared with the infrared-reflectance spectrum of  $\text{ZrO}_2$  in the present work. However, we would stress that our infrared-reflectance spectrum is more informative concerning the infrared-active modes. The frequencies and symmetries of infrared modes for monoclinic  $\text{ZrO}_2$  at 300 K are summarized as follows:<sup>10</sup> 104  $\text{cm}^{-1}$  ( $A_u$ ), 180  $\text{cm}^{-1}$  ( $A_u$ ), 192  $\text{cm}^{-1}$  ( $A_u$ ), 233  $\text{cm}^{-1}$  ( $A_u$ ), 270  $\text{cm}^{-1}$  ( $A_u$ ), 360  $\text{cm}^{-1}$  ( $B_u$ ), 373  $\text{cm}^{-1}$  ( $B_u$ ), 415  $\text{cm}^{-1}$  ( $A_u$ ), 443  $\text{cm}^{-1}$  ( $A_u$ ), 515  $\text{cm}^{-1}$  ( $2B_u$ ), 620  $\text{cm}^{-1}$  ( $2B_u$ ), and 740  $\text{cm}^{-1}$  ( $A_u$  and  $B_u$ ). Among these infrared modes, twelve infrared modes of  $5A_u$  and  $7B_u$  can be identified approximately at 220  $\text{cm}^{-1}$  ( $A_u$ ), 250  $\text{cm}^{-1}$  ( $A_u$ ), 330  $\text{cm}^{-1}$  ( $B_u$ ), 370  $\text{cm}^{-1}$  ( $B_u$ ), 420  $\text{cm}^{-1}$  ( $A_u$ ), 440  $\text{cm}^{-1}$  ( $A_u$ ), 520  $\text{cm}^{-1}$  ( $2B_u$ ), 600  $\text{cm}^{-1}$  ( $2B_u$ ), and 740  $\text{cm}^{-1}$  ( $A_u$  and  $B_u$ ) in the infrared-reflectance spectrum of  $\text{ZrO}_2$ . No  $A_u$  modes at 104, 180, and 192  $\text{cm}^{-1}$  are apparent in Fig. 2. We can notice that the  $A_u$  mode at 443  $\text{cm}^{-1}$  for  $\text{ZrO}_2$  is apparent as a shoulder and becomes invisible as the  $A_u$  mode at 420  $\text{cm}^{-1}$  increases in intensity with  $x$ . The  $B_u$  mode at 330  $\text{cm}^{-1}$  also appears as a shoulderlike feature, which becomes pronounced with  $x$  as noted for  $\text{HfO}_2$ . It is noteworthy that the infrared-reflectance spectra of  $\text{Zr}_{1-x}\text{Hf}_x\text{O}_2$  vary systematically with  $x$ , changing the position, linewidth, and intensity of these infrared modes. It is particularly noticeable that the infrared mode at 600  $\text{cm}^{-1}$  ( $2B_u$ ) stiffens considerably, changing in intensity and/or linewidth as  $x$  increases.

#### C. Analysis

In the present work, no infrared-reflectance spectra consisting of the  $5A_u+7B_u$  modes have been analyzed

based on the classical dispersion theory together with a Kramers-Kronig analysis,<sup>11,12</sup> whereby we can obtain the dispersion parameters such as resonance frequency, damping factor, and oscillator strength; we may expect that these parameters change depending on  $x$ . Instead, curve-fitting procedures have been performed to assess the exact position as well as the linewidth of each infrared mode, using a computer program installed with our infrared spectrometer.

The position, height, and width were all varied as fitting parameters so as to obtain the best fit to the experimentally observed infrared bands. It turned out that the best fit to the observed ir bands can be achieved with two Gaussian curves.

A curve fitting to the infrared mode ( $2B_u$ ) at  $600\text{ cm}^{-1}$  for  $\text{Zr}_{1-x}\text{Hf}_x\text{O}_2$  ( $x=0.7$ ) is exemplified in Fig. 3. The ir mode frequencies thus determined are plotted as a function of  $x$  for  $\text{Zr}_{1-x}\text{Hf}_x\text{O}_2$  in Fig. 4. It is reasonable to decompose the infrared modes at  $600\text{ cm}^{-1}$  ( $2B_u$ ),  $515\text{ cm}^{-1}$  ( $2B_u$ ),  $330\text{--}370\text{ cm}^{-1}$  ( $B_u$ ),  $410\text{--}445\text{ cm}^{-1}$  ( $A_u$ ), and  $740\text{ cm}^{-1}$  ( $A_u$  and  $B_u$ ) into two parts because these infrared modes are doubly degenerate or overlapping each other. In fact, the curve-fitting procedures were

successful with two Gaussian curves for these infrared modes. No successful fitting was possible with a single Gaussian curve for the  $A_u$  mode at  $220\text{ cm}^{-1}$ ; at least two Gaussian curves were necessary to attain an acceptable fit to the experiment, so that the peak frequency of the synthesis of two decomposed curves is adopted for this infrared mode. For a similar reason, the peak frequency is adopted for the  $A_u$  mode at  $250\text{ cm}^{-1}$ .

We should notice the following. All the identified ir modes would stiffen with  $x$ , except the  $B_u$  mode at  $355\text{ cm}^{-1}$  which slightly softens with  $x$ . The softening of this  $B_u$  mode with  $x$  may be seen by a subtle downshift of the peak at about  $350\text{ cm}^{-1}$  with  $x$  (see Fig. 2). It is of particular interest that the  $B_u$  modes at  $585$  and  $606\text{ cm}^{-1}$  for  $\text{ZrO}_2$  stiffen considerably in a nonlinear way as  $x$  increases, indicating a pronounced stiffening around  $x=0.8$ . This significant stiffening tendency around  $x=0.8$  can be observed in the  $x$  dependence of the other  $A_u$  or  $B_u$  modes, as shown in Fig. 3.

Each mode obtained by decomposing the infrared mode ( $2B_u$ ) at  $600\text{ cm}^{-1}$ , exhibits an  $x$  dependence of the linewidth as shown in Fig. 5 and reveals an anomalous feature around  $x=0.8$ . On the other hand, the integrat-

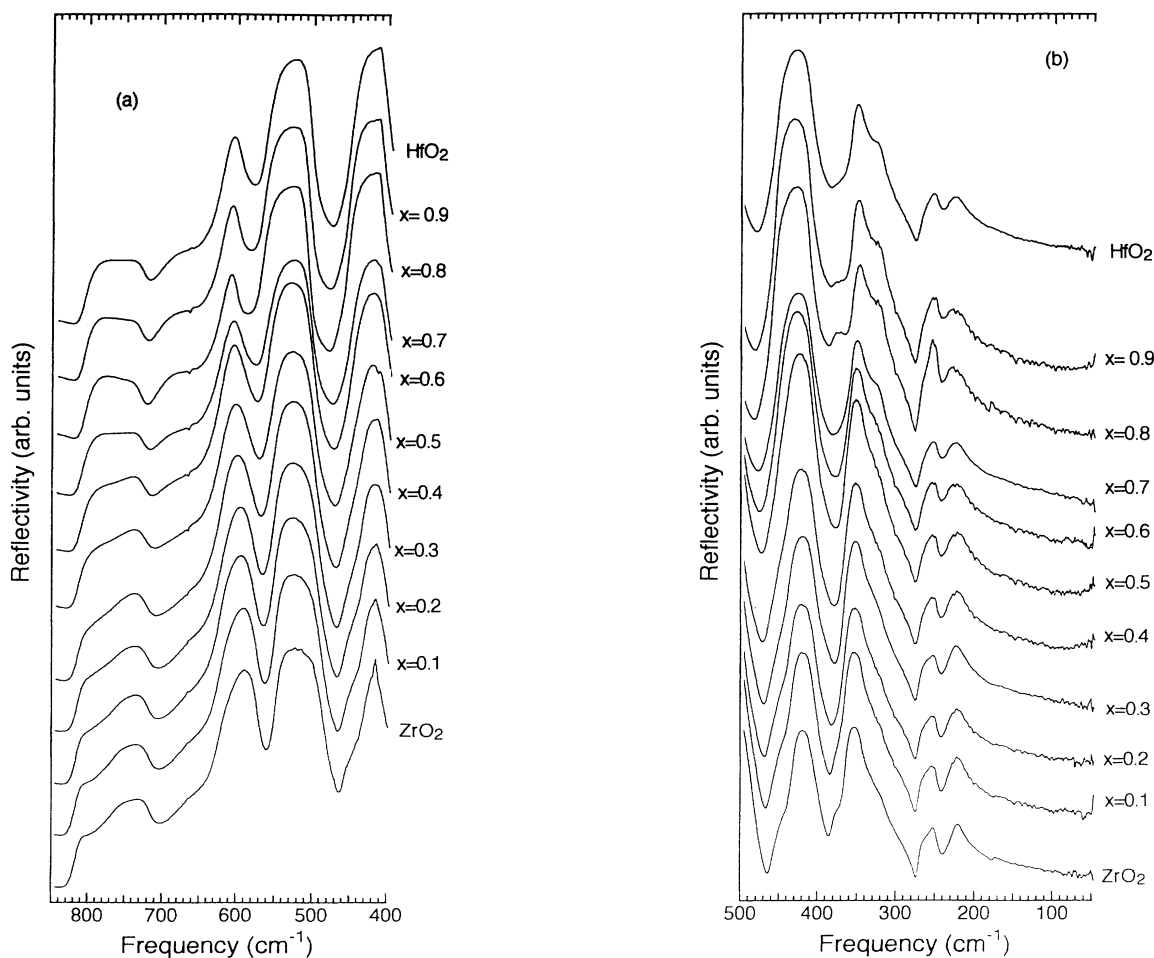


FIG. 2. Infrared-reflectance spectra of  $\text{Zr}_{1-x}\text{Hf}_x\text{O}_2$  in two frequency regions that partly overlap:  $400\text{--}850\text{ cm}^{-1}$  (a) and  $50\text{--}500\text{ cm}^{-1}$  (b). Each spectrum is offset vertically for clarity.

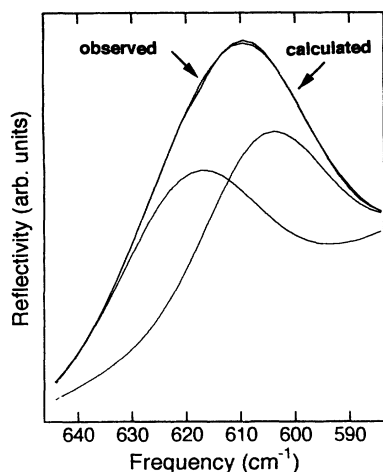


FIG. 3. Curve fitting to the infrared mode ( $2B_u$ ) at  $600\text{ cm}^{-1}$  for  $\text{Zr}_{1-x}\text{Hf}_x\text{O}_2$  ( $x=0.7$ ) with two Gaussian curves.

ed intensity of the infrared mode at  $600\text{ cm}^{-1}$  ( $2B_u$ ) decreases nonlinearly with  $x$ , whereas the two  $A_u$  modes over  $400\text{--}600\text{ cm}^{-1}$  increase overall as  $x$  increases (Fig. 6). It is interesting to note that the  $A_u$  and  $B_u$  modes change in intensity in an opposite way with respect to  $x$ , and that the anomalous feature around  $x=0.8$  is manifested even in the  $x$  dependence of the intensity.

#### IV. DISCUSSION

First, a comment should be made concerning the mode stiffening, where all the identified ir modes except the  $B_u$  mode at  $355\text{ cm}^{-1}$  increase in frequency with  $x$ . The  $A_u$  and  $B_u$  modes correspond to the atomic displacements parallel to the  $c$  axis and in the  $ab$  plane, respectively. Consequently, we can conclude that the observed mode stiffening is associated with the reinforcement of bonds responsible for the atomic displacements, which is evidenced by the decrease in the lattice parameters  $a$ ,  $b$ , and  $c$  with increasing  $x$  in  $\text{Zr}_{1-x}\text{Hf}_x\text{O}_2$ . In view of the bond

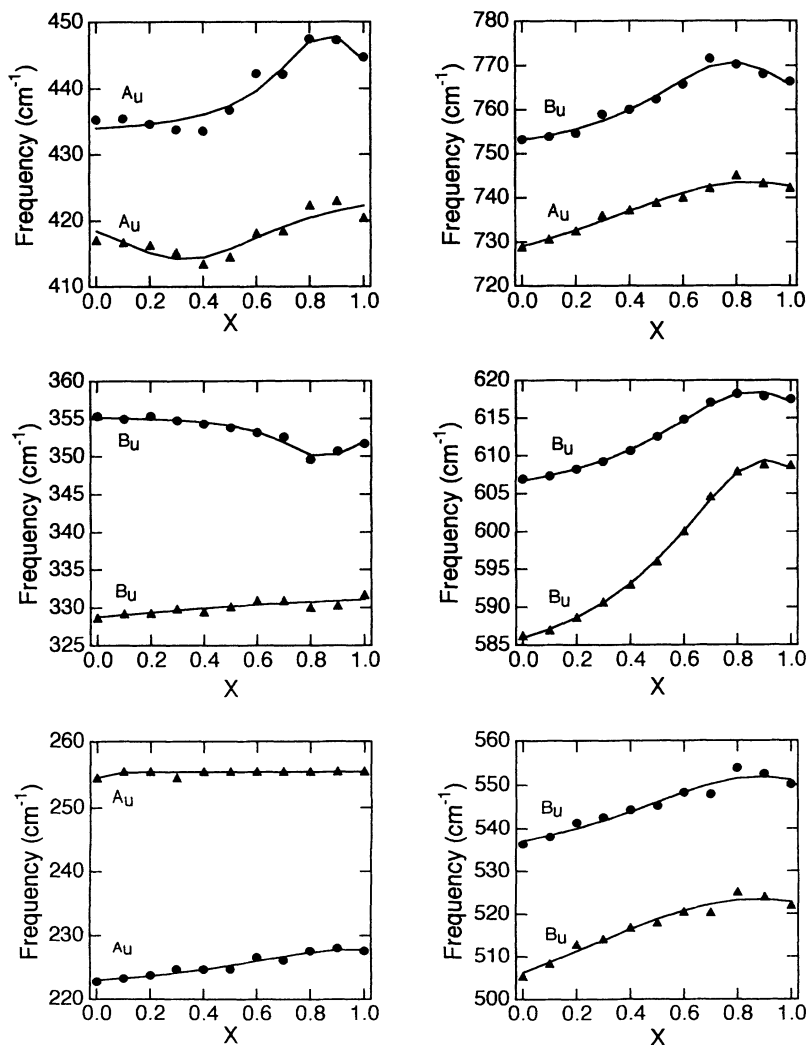


FIG. 4. The ir mode frequencies as a function of  $x$  for  $\text{Zr}_{1-x}\text{Hf}_x\text{O}_2$ ; note that the lines connecting the frequencies are only for guiding the eye.

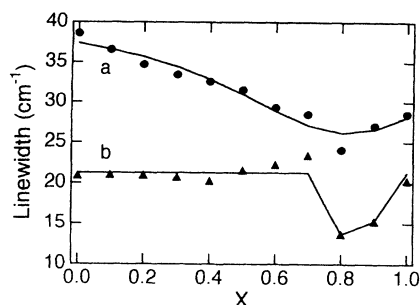


FIG. 5. Linewidths of each mode obtained by decomposing the infrared mode ( $2B_u$ ) at  $600\text{ cm}^{-1}$  as a function of  $x$  for  $Zr_{1-x}Hf_xO_2$ . Curve *a*, for the high-frequency mode; curve *b*, for the low-frequency mode. The lines are only a guide to the eye.

reinforcement, the  $x$  dependence for the  $B_u$  mode at  $355\text{ cm}^{-1}$  is exceptional; no explanation can be found for this, but it is interesting that this mode also behaves anomalously around  $x=0.8$ , i.e., yielding a minimum in the  $x$  dependence of frequency.

We could find a relation between the unit-cell volume  $V$  and the mode frequency obtained by decomposing the mode ( $2B_u$ ) at  $600\text{ cm}^{-1}$  for  $Zr_{1-x}Hf_xO_2$  as shown in Fig. 7. The observed relation (nonlinear) contradicts a linear relationship between the Sb-O bond-stretching frequency and the unit-cell volume of different  $L_3Sb_5O_{12}$  oxides ( $L = \text{Pr, Nd, Sm, etc.}$ ).<sup>13</sup>

This nonlinear relation between  $V$  and the mode frequency implies that bond reinforcement does not increase linearly as a function of  $x$  in  $Zr_{1-x}Hf_xO_2$ . Actually, both curves of Fig. 7 can be divided into three regions; the substitution effect on the bond reinforcement is relatively small and still under the control of  $ZrO_2$  in the region *c*, but it becomes pronounced in the region *b*, and saturates in the region *a* near the other end member  $HfO_2$ . The reduced unit-cell volume relevant to the bond reinforcement is explained based on the smaller ionic radius of  $Hf^{4+}$  ( $0.78\text{ \AA}$ ) than  $Zr^{4+}$  ( $0.79\text{ \AA}$ ),<sup>14</sup> as the Hf ions are substituted for the Zr ions in  $Zr_{1-x}Hf_xO_2$ .

Secondly, we pay attention to the anomalous feature

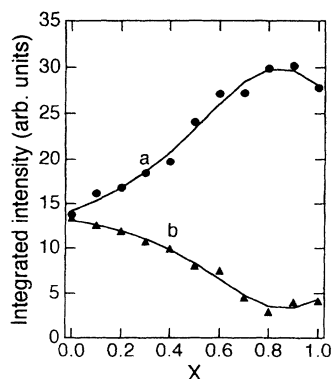


FIG. 6. Integrated intensities of the two  $A_u$  modes over  $400\text{--}460\text{ cm}^{-1}$  (curve *a*) and the  $2B_u$  modes at  $600\text{ cm}^{-1}$  (curve *b*) as a function of  $x$  for  $Zr_{1-x}Hf_xO_2$ .

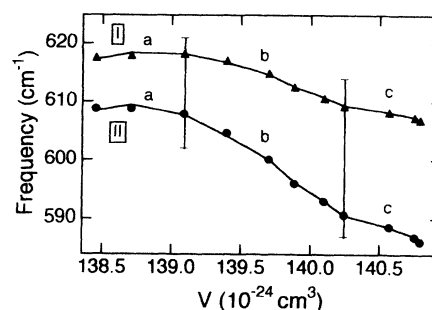


FIG. 7. Relationship between the unit-cell volume  $V$  and the mode frequency obtained by decomposing the infrared mode at  $600\text{ cm}^{-1}$  ( $2B_u$ ) for  $Zr_{1-x}Hf_xO_2$ , I, for the high-frequency mode, and II, for the lower-frequency mode.

around  $x=0.8$  in the  $x$  dependence of frequency, integrated intensity, and linewidth of the modes. At first, it was thought that the samples with high  $x$  might contain inhomogeneities. To resolve this problem, we reground the samples with  $x=0.7, 0.8$ , and  $0.9$ , and we repeated infrared-reflectivity measurements after forming pellets under the same conditions as before. These samples reproduced the anomalous feature in question around  $x=0.8$ . Hence we can conclude that the observed anomalous feature around  $x=0.8$  is inherent in the  $x$  dependence of frequency, intensity, and linewidth of the modes for  $Zr_{1-x}Hf_xO_2$ .

A plausible explanation is that the  $ZrO_2$ - $HfO_2$  solid solutions exhibit two-mode behavior as pointed out by Carlone.<sup>5</sup> He has found that the  $x$  dependence of seven Raman mode frequencies is characterized by discontinuities at either  $x=0.6$  or  $x=0.3$  in  $Zr_xHf_{1-x}O_2$ . It is of particular interest that these  $x$  values are close to the  $x$ 's ( $x=0.3$  and  $x=0.8$ ) at the borderline between each region in Fig. 7; note that Carlone<sup>5</sup> has chosen the  $x$ 's approximately ( $x=0, 0.1, 0.25, 0.5, 0.75, 0.9$ , and  $1.0$ ) in  $Zr_xHf_{1-x}O_2$ , which makes a difference in comparison with the present work.

In the random-element-isodisplacement (REI) model,<sup>15-19</sup> it is assumed that the three force constants  $f_{ij}(x)$  in a mixed crystal  $AB_{1-x}C_x$  ( $ij = AB, AC, BC$ ) have the same compositional dependence such that  $f_{ij}(x) = f_{ij}(0)[1 + \Theta x]$ , where  $f_{ij}(0)$  is the force constant at  $x=0$  and  $\Theta$  is a constant which describes the effect of lattice-parameter change on the force constants. When analyzing long-wave optical phonons in mixed crystals based on the REI model,<sup>12,18,20</sup> the constant  $\Theta$  was allowed to depend on composition. Even so, it seems unlikely that the nonlinear  $x$  dependence of frequency as well as the significant stiffening around  $x=0.8$  can be reproduced starting with  $\nu(x) = [f_{ij}(x)/\mu(x)]^{1/2}$ , where  $\nu(x)$  is the  $x$  dependence of the frequency, and  $\mu(x)$  is the reduced mass given by  $\mu^{-1}(x) = (1-x)/M_{Zr} + x/M_{Hf}$  for  $Zr_{1-x}Hf_xO_2$  with  $M_{Zr}$  and  $M_{Hf}$  being the masses of Zr and Hf atoms, respectively.

Besides, the lattice parameters  $a, b$ , and  $c$  decrease linearly with  $x$  whereas  $\beta$  increases up to  $x=0.6$  and then decreases with  $x$  for  $Zr_{1-x}Hf_xO_2$ . This change in  $\beta$  contradicts the case for  $Zr_{1-x}Hf_xS_3$  (Ref. 21) with the same

structure as Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub>, where  $\beta$  increases linearly over the entire range of  $x$  ( $0 \leq x \leq 1$ ). We do not know how this change of  $\beta$  should be adjusted to the linear dependence of force constants on the lattice parameter in the REI model.<sup>19</sup>

Under these circumstances, we are inclined to conclude that the solid solutions Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> exhibit two-mode behavior which is associated with the anomalous feature around  $x=0.8$  in the  $x$  dependence of frequency, linewidth, and intensity of the modes. Experimentally, however, it is still necessary to examine whether the samples with high  $x$  are free of any microscopic inhomogeneities in the distribution of the constituent atoms.

Finally, we note that the observed anomalous feature is manifested in the infrared spectrum of Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> with  $x=0.8$  *per se*. We can easily notice that the infrared mode ( $2B_u$ ) at  $600 \text{ cm}^{-1}$  decreases considerably in linewidth and that a small band at  $355 \text{ cm}^{-1}$  is apparent as well. The position of this small band is equal to that of the  $A_u$  mode ( $\approx 355 \text{ cm}^{-1}$ ) for ZrO<sub>2</sub>, so it does originate in the perturbation due to the substitution of Zr for Hf in HfO<sub>2</sub>; this persistent band was observed even in the infrared spectrum of the  $x=0.8$  sample that was reground and measured again.

## V. CONCLUSIONS

Infrared-reflectance spectra of the solid solutions Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> ( $0 \leq x \leq 1$ ) have been measured at room

temperature in the frequency region  $50\text{--}4000 \text{ cm}^{-1}$  by Fourier-transform infrared spectroscopy. At most, twelve infrared-active modes of  $5B_u$  and  $7A_u$  symmetry were identified in the infrared-reflectance spectra of Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub>. The spectra of Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> varied systematically with  $x$ , with changes in the frequency, linewidth, and intensity of these modes. In particular, the  $B_u$  modes at  $585$  and  $606 \text{ cm}^{-1}$  for ZrO<sub>2</sub> considerably stiffened in a nonlinear way, indicating a pronounced stiffening around  $x=0.8$  as  $x$  increases.

X-ray diffraction revealed that the lattice parameters,  $a$ ,  $b$ , and  $c$  of Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> decrease linearly with  $x$  whereas  $\beta$  initially increases up to  $x=0.6$  and then decreases with  $x$ . Hence, we concluded that the stiffening of the identified ir modes is associated with the bond reinforcement evidenced by the reduced cell volume of Zr<sub>1-x</sub>Hf<sub>x</sub>O<sub>2</sub> when the Hf<sup>4+</sup> ions of smaller radius than Zr<sup>4+</sup> are substituted for the Zr<sup>4+</sup> ions.

It is also concluded that the anomalous feature around  $x=0.8$  in the  $x$  dependence of frequency, linewidth, and intensity of the modes can be reconciled with two-mode behavior, which was characterized by discontinuities at either  $x=0.6$  or  $0.3$  in the  $x$  dependence of seven Raman mode frequencies for Zr<sub>x</sub>Hf<sub>1-x</sub>O<sub>2</sub>.<sup>5</sup>

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