## Brief Reports

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## Inelastic exchange scattering in electron-energy-loss spectroscopy: Localized excitations in transition-metal and rare-earth systems

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Cross sections for quasiatomic excitation by exchange scattering of electrons are calculated in the Born-Ochkur approximation for  $3p-3d$  and  $3d-3d$  transitions in transition-metal systems, and for  $4d-4f$ and  $4f-4f$  transitions in rare earths. The energy dependence of the spin polarization of  $3p-3d$  and  $4d-4f$ losses in reflection electron-energy-loss spectroscopy from ferromagnetic surfaces involves a balance of small-angle spin-dependent inelastic processes accompanied by a high-angle elastic scattering, and large-angle spin-flip exchange scattering without the need for elastic scattering. Both 3d-3d excitations, e.g., in transition-metal compounds, and 4f-4f excitations in rare earths involve spin-flip transition whose scattering amplitudes  $g$  fall off with momentum transfer  $q$  such that the full width at half maximum  $q_{1/2}$  (in a.u.) is given by  $q_{1/2} (r_{nl}) \sim 2$ , where  $\langle r_{nl} \rangle$  is the expectation value of r for the 3d or 4f electron. The angular width of the spin-flip differential cross section is then much greater than for dipole transitions, a pattern that helps to account for how these intra-atomic transitions compete with dipole processes for primary energies in excess of 100 eV.

For primary electron energies significantly larger than the excitation or loss energy, electron-energy-loss spectroscopy is usually dominated by dipolar processes involving scattering of the incident electrons through a small angle: observation of the loss electron in refiection mode then requires an additional large-angle elastic scattering. This single stage process [Fig. 1(a)] is generally much less probable than an inelastic  $+$  elastic scattering [Fig. 1(b}] due to the dominance of near forward inelastic scattering. When electrons interact with an atom having a net spin, the outcome of the scattering process will depend on whether the spin of the incoming electron is parallel or antiparallel to the atomic spin: if atomic spins near a surface are aligned as in a ferromagnetic system, the electron scattering cross sections will produce both spin polarization in the scattered beam if the incident beam is unpolarized (see, for example,  $3p \rightarrow 3d$  in the electron energy losses of amorphous Fe alloys<sup>1</sup>) and spin asymmetry in the scattering of spin-up and spindown electrons.

These phenomena arise from exchange terms in the inelastic-scattering cross sections. Treating the scattering of the incident electron and the promotion of an oriented electron into an empty state as a two-electron problem, i.e., neglecting multiplet effects in the partially filled shell, the scattering may be described in terms of a direct amplitude  $f(\theta)$  and an exchange amplitude  $g(\theta)$ , where  $\theta$  is the scattering angle. For  $3p \rightarrow 3d$  transitions in ferromagnetic Fe or Mn<sup>1</sup> or  $4d \rightarrow 4f$  transitions in ferromagnetic rare earths, the polarization is then given by

$$
P = \frac{|f|^2 + |g|^2 - |f - g|^2}{|f|^2 + |g|^2 + |f - g|^2}.
$$
 (1)

In systems with open shells it is also possible for incident electrons to induce dipole forbidden intermultiplet excitations, e.g.,  $3d^n-3d^n$  or  $4f^n-4f^n$  transitions, which



FIQ. 1. (a) A large-angle inelastic-scattering event; (b) a small-angle inelastic scattering followed by large-angle elastic scattering.

require a spin flip.<sup>2-5</sup> The differential cross section is then proportional to  $|g|^2$ .

## **THEORY**

In this paper, we examine the properties of  $f$  and  $g$  for  $3p \rightarrow 3d$  transitions in Fe and  $4f^7-4f^7$  transitions in Gd within a simple quasiatomic scattering model, the Born-Ochkur approximation,  $6,7$  where for incident energy  $E_p = k_i^2/2$  a.u. and incident wave vector  $k_i$ ; f and g are given by

$$
f = -\frac{2}{q^2}l(q) , \qquad (2a)
$$

$$
g = -\frac{2}{k_i^2} l(q) , \qquad (2b)
$$

where  $q = k_i - k_f$  is the momentum transfer and  $l(q)$  the transition amplitude, is given by

$$
l(q) = \int \psi_{nl'}^* e^{i\mathbf{q} \cdot \mathbf{r}} \psi_{nl} \mathbf{dr} , \qquad (3)
$$

where  $\psi_{nl(n'l')}$  is the initial (final) state wave function.<br>The excitation energy  $\Delta E = (k_i^2 - k_f^2)/2$ , and the approx imation is valid for  $E_p \gg \Delta E$ . When the scattering angle is small,

$$
P \simeq \frac{g(\theta)}{f(\theta)} = \frac{1}{4} \left[ \frac{\Delta E}{E_p} \right]^2.
$$
 (4)

Figure 2 plots P as a function of  $\theta$  for  $3p \rightarrow 3d$  transitions in Fe( $\Delta E$ =56 eV) at  $E_p$ =90, 227, and 1321 eV. At small scattering angles,  $P$  is small and arises from differences in the direct scattering amplitudes for spins parallel  $(f-g)$ or antiparallel  $(f)$  to the oriented atom direction<sup>1</sup> rather than spin-flip transitions.  $P \rightarrow 1$  in the range 60°-75°, than spin-inp transitions.  $P \rightarrow 1$  in the range 60 – *i*,<br>corresponding to conditions when  $|q| = |k_i|$  and  $f = g$ . This is an artifact of the simplification implicit in the



FIG. 2. The spin polarization  $P$  versus scattering angle for  $3p \rightarrow 3d$  transitions in ferromagnetic iron for primary energies 90, 227, and 1321 eV.

Born-Ochkur exchange correction, and higher-order theories will yield somewhat smaller values. In addition, the first Born approximation is known to be erratic in its prediction at high scattering angles. However, Born-Ochkur gives a good approximation to the spin asymmetry in H ionization<sup>8</sup> by electrons for  $E_p \geq 80$  eV, and this simple theory gives general insight into the spin dependence of scattering. When P is large  $(g \sim f)$ , spin flip transitions have assumed considerable importance, but the peak in  $P$  contrasts strongly with the rapid falloff in the conventional first-order Born cross section

$$
\frac{d\sigma}{d\Omega} \propto (\theta^2 + \theta_0^2)^{-1} \tag{5}
$$

where  $\theta_0 = \Delta E/(2E_p)$ .  $(d\sigma/d\Omega)_{3p\rightarrow 3d}^{\text{IST BORN}}$  has fallen to half its maximum value at  $\theta=18^\circ$ , 7°, and 1.2°, respective ly, at the energies shown, so that high polarization events have low cross section even at  $E_p = 90$  eV. The pattern is then one of low polarization with a high inelastic forward scattering cross section and of high polarization and low inelastic cross section at higher scattering angle. This implies that the balance between low-angle loss accompanied by a single high-angle elastic scattering [Fig. 1(b)] or multiple elastic scattering, and high-angle losses is delicate and highly dependent on both experimental geometry and the state of order at the surface. This is discussed in detail by Porter and Matthew<sup>9</sup> for  $p \rightarrow d$  and  $d \rightarrow f$  transitions in transition-metal systems and rare earths, respectively.

For spin-flip transitions within a configuration, e.g., 4f-4f transitions in the rare earths, Born-Ochkur leads to

$$
\frac{d\sigma}{d\Omega} \propto \left| \int |\psi_{nl}|^2 e^{i\mathbf{q} \cdot \mathbf{r}} \mathbf{dr} \right|^2.
$$
 (6)

Here the one-electron wave function  $\psi_{nl}$  of both the initial and final states is the same, and so  $d\sigma/d\Omega$  is proportional to the x-ray atomic scattering for the excited electron. The differential cross section peaks in the forward direction as for dipolar losses but falls off much more slowly with angle. Expanding  $e^{i\mathbf{q}\cdot\mathbf{r}}$  and approximating the matrix element by a parabolic function of q,  $d\sigma/d\Omega$ falls off to half its maximum value at  $\theta_{1/2}$  given by

$$
\theta_{1/2} \approx 2 \arcsin \left[ \left( \frac{3(1 - 2^{-1/2})}{4E_p \langle r_{nl}^2 \rangle} \right)^{1/2} \right],
$$
\n(7)

where  $\langle r_{nl}^2 \rangle$  is the expectation value of  $r^2$  for the electron in both its initial and final state. Higher order terms in the expansion increase this value slightly (by  $\sim$  10% for  $E_p = 100$  eV in Gd). Figure 3 compares the differential cross sections for  $4f-4f$  transitions in Gd at three typical primary energies.  $(d\sigma/d\Omega 0)_{\theta=0}$  varies as  $E_p^{-2}$ , and the total 4f-4f cross section varies as  $E_p^{-3}$  as noted by Joachain.<sup>10</sup> In contrast to the  $4f^7-4f^7$  G loss ( $\Delta E = 4.5$ ) eV), dipolar losses at the same  $\Delta E$  have fallen to half maximum by 1° for  $E_p = 100$  eV so that in a reflection loss experiment these will again be a balance between doubleand multiple-scattering  $(\text{loss} + \text{elastic})$  and single inelastic-scattering events. The sensitivity of the ratio of  $4f<sup>7</sup>-4f<sup>7</sup>$  losses in Gd to geometry, primary energy, and

surface order is consistent with this conclusion. Only for inelastic low-energy electron-diffraction events are the low angle exchange events, i.e., non-spin-flip processes, likely to predominate.

For the corresponding spin-flip cross sections for 3d-3d transitions in ionic Mn compounds ( $\Delta E = 3$  eV),  $(d\sigma/d\Omega)_{\theta=0}$  is lower than for Gd (five electrons instead of seven) and  $(d\sigma/d\Omega)$  falls off more quickly with angle of seven) and  $(a\sigma/a\Omega t)$  hans on more quickly with angle<br>since  $\langle r_{\text{Mn 3d}}^2 \rangle = 1.64$  a.u.<sup>2</sup> is greater then  $\langle r_{\text{Gd4f}}^2 \rangle = 0.89$ <br>a.u.<sup>2</sup>—see Froese Fischer.<sup>11</sup> In terms of momentum transfer the results may be expressed by a kind of "uncertainty" style relationship

$$
\Delta q_{1/2} \langle r_{nl} \rangle \sim 2 \tag{8}
$$

where  $\Delta q_{1/2}$  is the momentum transfer at which the transition *amplitude* g has fallen to half its  $\theta = 0$  value and  $\langle r_{nl} \rangle$  is a measure of the size of the electron orbital. These results are also relevant to  $d - d$  transitions involving spin flip where the levels are further split by a crystal field.  $12-15$  Indeed the recent results of Gorschlüter and Merz<sup>13</sup> on  $d$ - $d$  excitations in NiO(100) and CoO(100) are consistent with significant contributions from high-angle losses without elastic scattering.

Although higher order theories would modify the simple pattern outlined here, the Born-Ochkur model provides a good general guide to the angular distribution of electron exchange scattering for quasiatomic  $3p - 3d$  and  $4d - 4f$  excitations in transition metals and rare earths. Likewise,  $4f-4f$  transitions in rare earths and  $3d-3d$ transitions in ionic transition-metal compounds should be reasonably well described provided  $E_p \gg \Delta E$ . Although Stoner excitations<sup>16,17</sup> and  $d$  -d transitions in paramagne ic model systems<sup>18</sup> are less quasiatomic and show marked bandlike character, the results presented here usefully complement the dielectric descriptions of spin-flip processes developed, for example, by Modesti et  $al.^{17}$  and Mills.<sup>19</sup> It must, however, be emphasized that this simple model does not treat the behavior of the individual multiplet transitions and a more detailed theory would predict some differences in the variation of cross sections with energy and angle for different loss components, as is indeed observed by Matthew et  $al.$ <sup>3</sup> and Kolaczkiewicz and Bauer et  $al.$ <sup>4</sup> The pioneering calculations of Moser and Wendin<sup>20</sup> using the distorted-wave approximation



FIG. 3. The differential scattering cross section versus scattering angle for 4f-4f transitions in Gd at  $E_p = 100$ , 200, 300 eV.

predict L-S resolved results for La  $4d \rightarrow 4f$  and  $3d \rightarrow 4f$ transitions and Th  $4d \rightarrow 5f$  and  $4f - 5f$  transitions, but make no explicit spin-polarization predictions. Clearly such approaches provide a way forward for the problems investigated here, but the simple scaling patterns inherent in the Born-Ochkur approximation provide useful initial tests of spin-polarized loss data.

## **SUMMARY**

The Born-Ochkur approximation is shown to give useful insights into exchange scattering in electron-energyloss spectroscopy in the reflection mode. Both the polarization induced in dipolar transitions and spin-flip transitions are much less forward scattering dominated than for nonexchange scattering and as a result experimental observations will include both single inelastic-scattering events and multiple-scattering events where low-angle inelastic scattering is combined with elastic scattering.

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