Spin-gap fixed points in the double-chain problem

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Applying the bosonization procedure to weakly coupled Hubbard chains, we discuss the fixed points of the renormalization-group procedure where all spin excitations are gapful and a singlet pairing becomes the dominant instability.

I. INTRODUCTION

Recently the field of non-Landau–Fermi-liquid states in various quasi-one-dimensional systems has been very active. Although basic properties of purely one-dimensional systems (chains) are quite well known by now, it still remains to be understood how these change under coupling between chains. In particular, if an infinite number of chains is coupled to form a two-dimensional array then some kind of a dimensional crossover occurs. Despite numerous intensive studies of these questions, it still remains open how the non-Landau–Fermi-liquid one-dimensional features evolve into isotropic two-dimensional behavior.

On the other hand one may expect new physics in a system with a finite number of coupled chains which may exhibit an unusual amalgamation of both one- and two-dimensional features. Besides a general theoretical interest, such systems are also attractive because they can be found in real materials. Recently it was pointed out that some substances such as $Sr_2Cu_4O_6$ provide a physical realization of weakly coupled double chains.[1] Moreover, higher stoichiometric compounds in the series $Sr_{n-1}Cu_{n+1}O_{2n}$ present examples of coupled n-chain ladders.

This expectation is reinforced by the behavior of $S = 1/2$ Heisenberg multichain or ladder systems. Whereas the single spin chain shows quasi-long-range order with gapless spinon excitations, the double-chain system shows spin liquid behavior[2–4] with strictly short-range order and finite gap in the spin excitation spectrum. This contrast in behavior has led to the conjecture that a lightly doped double-chain system should preserve the spin gap and become superconducting.[1] Further in view of the insensitivity of the spin liquid state to the ratio of interchain to intrachain coupling,[4] one expects that it is robust and should occur also for a Hubbard model even in the weak coupling regime. This expectation is supported by recent numerical studies of Hubbard[5] as well as t-J (Ref. 6) double-chain models. This is our motivation to examine the renormalization-group (RG) theory of weakly coupled Hubbard double chains and to look for a spin liquid fixed point.

Recent weak coupling RG studies of the double-chain Hubbard model[7] revealed some strong coupling fixed points characterized by enhanced singlet pairing. However, the analysis performed in Ref. 7 is essentially restricted to the case of weak fermion correlations. These authors did not examine the half-filled case with weak interchain hopping where the umklapp processes on the individual chains become relevant.

In the present paper we undertake an attempt to construct a description of the spin-gap fixed point by using a bosonic representation. This is expected to be an adequate tool to demonstrate the development of the strong coupling regime in both cases of weakly coupled Hubbard chains and of a strongly correlated double-chain t-J model. The latter case will be considered elsewhere.[8]

To clarify the essence of the double-chain physics in the presence of strong correlations it is worthwhile to begin with a review of well-known properties of the single-chain Hubbard model.

Away from half filling the model can be only found in the so-called Tomonaga-Luttinger (TL) regime which corresponds to both gapless spin and charge excitations.[9] It is customary to describe the TL behavior in terms of spin and charge correlation exponents K_s and K_c.

The spin exponent K_s equals unity everywhere in the TL regime while K_c gradually increases from the value $K_c = 1/2$ at values of the on-site repulsion $U = \infty$ and any electron density $\rho \neq 1$ [as well as at $\rho \to 1$ and arbitrary U/t (t is the intrachain hopping)] as U increases or ρ gets smaller.

In the regime of strong correlation at ρ close to unity one can argue that gapless spin fluctuations drive the coupling constants of the charge sector to the repulsive region ($K_c < 1$). In this case we can expect a change of behavior when the exchange coupling between chains is introduced. The spin gap, which we have argued is a robust feature of the single rung ladder at $\rho = 1$, acts to cut off the spin fluctuation spectrum at low energies so that these may not renormalize the charge couplings significantly. As a result there can be an effective attraction at $\rho \lesssim 1$ without a threshold value of the ratio U/t. This will manifest itself in a finite spin gap also at $\rho \lesssim 1$ and a scaling to the Luther-Emery line rather than

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the Tomonaga-Luttinger line. Such behavior has been reported for single-chain models with a spin gap caused by frustrated or modulated exchange couplings on the single chain.[10,11] In the present case of a single rung ladder or double chain, we will choose the regime with the interchain hopping $t_\perp$ weak ($t_\perp \ll U$) so that as $\rho \to 1$ real interchain kinetic energy processes characterized by $t_\perp$ will scale to zero. In this limit the induced interchain exchange processes will remain finite and dominate.

A mean field analysis,[12] as well as an exact diagonalization[6] of the t-J model predicts that the spin gap remains upon doping and the resonant valence bond (RVB) state at $\rho = 1$ evolves into a superconductor with approximate d-wave symmetry. Noack *et al.*[5] found a similar behavior in their numerical studies of moderately coupled Hubbard ladders.

In this paper we investigate the case of a weakly coupled Hubbard ladder with $\rho \lesssim 1$ and $t_\perp < U < t$ using RG methods and look for a strong coupling fixed point with the same characteristics.

II. BOSONIZED WEAK COUPLING LIMIT OF THE DOUBLE-CHAIN PROBLEM

To get a first insight into the problem we start with a conventional bosonization of the small U/t Hubbard model on two weakly coupled chains:

$$\begin{aligned} H = & -t\sum(u^\dagger_{i\sigma}u_{i+1,\sigma} + d^\dagger_{i\sigma}d_{i+1,\sigma} + \text{H.c.}) \\ & -t_\perp\sum(d^\dagger_{i\sigma}u_{i\sigma} + u^\dagger_{i\sigma}d_{i\sigma}) \\ & +U\sum(u^\dagger_{i\sigma}u_{i\sigma}u^\dagger_{i,-\sigma}u_{i,-\sigma} + d^\dagger_{i\sigma}d_{i\sigma}d^\dagger_{i,-\sigma}d_{i,-\sigma}). \end{aligned} \tag{2.1}$$

Here $u_{i\sigma}$ and $d_{i\sigma}$ denote fermions on upper (u) and lower (d) chains.

Apparently, at $U \ll t_\perp, t$ the interaction term has to be treated as a small perturbation to the rest of the Hamiltonian (2.1) and the bare transverse hopping leads to the formation of two [bonding (B) and antibonding (A)] bands: $(A,B) = \frac{1}{\sqrt{2}}(u \mp d)$. Thus at $U \ll t_\perp$ a proper starting point is provided by the two-band model which was previously studied in the framework of a general weak coupling g-ology.[13,14] The analysis carried out in Ref. 7 was also based on the two-band picture.

However, if the opposite condition $t_\perp \ll U$ is satisfied, then the effect of band splitting is completely suppressed due to the requirement to avoid a double on-site occupancy. This behavior persists down to quarter filling ($\rho = 1/2$). In the framework of the RG approach this phenomenon manifests itself as a vanishing of the renormalized $t_\perp$. In view of this we suppose that in the case $t_\perp \ll U$ one has to start from the picture of two degenerate bands to implement correctly the fermion correlations.

The preceding RG analysis of the general two-band model in absence of umklapp processes[13,14] already shows many technical complexities. For this reason the results of these studies are not simply physically transparent. Moreover, it turns out that all nontrivial fixed points are located far in the strong coupling regime where the lowest order RG calculations cease to be valid. So one might expect that a more informative investigation can be done on the basis of a bosonic representation which is usually capable of giving a correct evolution toward strong coupling and even of providing an asymptotically exact solution of the Luther-Emery type.[15] Recently the method of bosonization was applied to the double-chain problem in the context of a special model which includes only forward scattering.[16] This analysis[16] led to the prediction that coupled chains provide a proper basis for the occurrence of singlet pairing.

In this paper we will perform a more general analysis than that of Ref. 16 to see whether the above statement holds for a wider class of models.

To proceed with a bosonic representation we introduce a conventional set of bosonic fields ϕ^f_c, ϕ^f_s where the "flavor index" f has one of two values u or d. These fields describe fluctuations of charge (c) and spin (s) densities, respectively. In the continuum limit the fermion operators can be written in terms of these variables as follows:

$$\Psi^f_\sigma(x) \sim \sum_\mu \exp\left(i\mu k_F x + \frac{i}{\sqrt{2}}(\mu\phi^f_c + \theta^f_c + \mu\sigma\phi^f_s + \sigma\theta^f_s)\right), \tag{2.2}$$

where $\mu = R, L$ is the chirality index and the fields $\theta^f_{c,s}$ are dual to the $\phi^f_{c,s}$ [$\theta^f_{c,s} = \int_{-\infty}^x \pi^f_{c,s}(x')dx'$, where $\pi^f_{c,s}$ is a momentum variable conjugated to $\phi^f_{c,s}$].

Applying the formula (2.2) and introducing the linear combinations $\phi^\pm_{c,s} = \frac{1}{\sqrt{2}}(\phi^u_{c,s} \pm \phi^d_{c,s}), \theta^\pm_{c,s} = \frac{1}{\sqrt{2}}(\theta^u_{c,s} \pm \theta^d_{c,s})$ corresponding to total (+) and relative (−) charge or spin density fluctuations, one can readily obtain the bosonic form of the Hamiltonian (2.1),

$$\begin{aligned} H_B = & \frac{1}{2}\sum_\pm\left(v_c K^\pm_c(\partial\theta^\pm_c)^2 + \frac{v_c}{K^\pm_c}(\partial\phi^\pm_c)^2 + v_s K^\pm_s(\partial\theta^\pm_s)^2 + \frac{v_s}{K^\pm_s}(\partial\phi^\pm_s)^2\right) \\ & +t_\perp[\cos\phi^-_c\cos\phi^-_s + \cos(\phi^+_c + \delta x)\cos\phi^+_s]\cos\theta^-_c\cos\theta^-_s + g_{BS}\cos 2\phi^+_s\cos 2\phi^-_s + g_U\cos(2\phi^+_c + 2\delta x)\cos 2\phi^-_c\,, \end{aligned} \tag{2.3}$$

where last two cosine terms represent spin backscattering and umklapp processes, respectively. Each of these terms is, in fact, a sum of two contributions $\cos 2\sqrt{2}\phi^f_{c,s}$ coming from u and d species. The umklapp term becomes relevant when the doping $\delta(= \frac{\pi}{2} - k_F)$ vanishes. As usual, the bare values of the correlation exponents $K^\pm_{c,s} = \frac{v_F}{v_{c,s}}$ can be changed by short wavelength renormalizations. Neglecting these corrections we obtain that the bare correlation exponents governing the charge dynamics $K_c^{\pm}[= (1+\frac{U}{\pi t})^{-1/2}]$ are smaller than unity while the spin exponents $K_s^{\pm}[= (1+\frac{U}{\pi t})^{1/2}]$ are opposite and $K_s^{\pm} > 1$. In addition, the bare values of the amplitudes g_{BS} and g_U are equal to $\frac{U}{\pi t}$.

To perform a renormalization procedure we divide up all variables on slow and fast components and then integrate out the fast variables. Using the bare values of correlation exponents one can estimate scaling dimensions of various terms in (2.3) according to the conventional formula[17] (γ and γ' are arbitrary)

$$\Delta(\cos\gamma\phi_{c,s}^{\pm}\cos\gamma'\theta_{c,s}^{\pm}) = \frac{1}{4}\left(\gamma^2 K_{c,s}^{\pm} + \frac{\gamma'^2}{K_{c,s}^{\pm}}\right). \tag{2.4}$$

The Hamiltonian (2.3) has to be supplemented by extra terms which are generated in the course of renormalization. Indeed, performing an expansion of the partition function $Z = \mathrm{Tr}\exp(-\beta H_B)$ in $t_\perp$ one immediately observes that the interchain hopping produces the following relevant terms (the new couplings g_i should not be confused with the traditional g-ological notations):

$$\begin{aligned}\Delta H = {} & g_1\cos 2\phi_c^-\cos 2\theta_s^- + g_2\cos 2\theta_c^-\cos 2\phi_s^- + g_3\cos 2\theta_c^-\cos 2\phi_s^+ \\ & + g_4\cos(2\phi_c^+ + 2\delta x)\cos 2\theta_s^- + g_5\cos 2\phi_c^-\cos 2\phi_s^- + g_6\cos 2\phi_s^+\cos 2\theta_s^- \\ & + g_7\cos(2\phi_c^+ + 2\delta x)\cos 2\theta_c^- + g_8\cos(2\phi_c^+ + 2\delta x)\cos 2\phi_s^+ + g_9\cos 2\theta_c^-\cos 2\theta_s^- .\end{aligned} \tag{2.5}$$

All these terms have scaling dimensions not greater than 2 and result from the second-order perturbation corrections to the single-chain Hamiltonian $\Delta H \sim t_\perp^2\langle(\sum_i u_i^\dagger d_i + d_i^\dagger u_i)^2\rangle$.

Physically these terms correspond to processes of coherent interchain particle-hole and particle-particle hopping triggered by the single-particle one. The crucial importance of these processes was previously pointed out by many authors (see, for instance, Refs. 18,19).

In the second order in $t_\perp$ the RG equations derived by the use of the method of Ref. 20 have the following form ($\xi = \ln x$):

$$\frac{dg_1}{d\xi} = \left(2 - K_c^- - \frac{1}{K_s^-}\right)g_1 + \frac{t_\perp^2}{2}\left(K_c^- + \frac{1}{K_s^-} - K_s^- - \frac{1}{K_c^-}\right) - g_4 g_U, \tag{2.6}$$

$$\frac{dg_2}{d\xi} = \left(2 - K_s^- - \frac{1}{K_c^-}\right)g_2 + \frac{t_\perp^2}{2}\left(K_s^- + \frac{1}{K_c^-} - K_c^- - \frac{1}{K_s^-}\right) - g_3 g_{BS}, \tag{2.7}$$

$$\frac{dg_3}{d\xi} = \left(2 - K_s^+ - \frac{1}{K_c^-}\right)g_3 + \frac{t_\perp^2}{2}\left(K_s^+ + \frac{1}{K_c^-} - K_c^- - \frac{1}{K_s^+}\right) - g_2 g_{BS}, \tag{2.8}$$

$$\frac{dg_4}{d\xi} = \left(2 - K_c^+ - \frac{1}{K_s^-}\right)g_4 + \frac{t_\perp^2}{2}\left(K_c^+ + \frac{1}{K_s^-} - K_s^+ - \frac{1}{K_c^-}\right) - g_1 g_U, \tag{2.9}$$

$$\frac{dg_5}{d\xi} = (2 - K_c^- - K_s^-)g_5 + \frac{t_\perp^2}{2}\left(K_c^- + K_s^- - \frac{1}{K_s^-} - \frac{1}{K_c^-}\right), \tag{2.10}$$

$$\frac{dg_6}{d\xi} = \left(2 - K_s^+ - \frac{1}{K_s^-}\right)g_6 + \frac{t_\perp^2}{2}\left(K_s^+ + \frac{1}{K_s^-} - K_c^+ - \frac{1}{K_c^-}\right) - g_3 g_9 - g_4 g_8, \tag{2.11}$$

$$\frac{dg_7}{d\xi} = \left(2 - K_c^+ - \frac{1}{K_c^-}\right)g_7 + \frac{t_\perp^2}{2}\left(K_c^+ + \frac{1}{K_c^-} - K_s^+ - \frac{1}{K_s^-}\right) - g_4 g_9, \tag{2.12}$$

$$\frac{dg_8}{d\xi} = \left(2 - K_c^+ - \frac{1}{K_s^+}\right)g_8 + \frac{t_\perp^2}{2}\left(K_c^+ - \frac{1}{K_s^-} + K_s^+ - \frac{1}{K_c^-}\right) - g_3 g_7, \tag{2.13}$$

$$\frac{dg_9}{d\xi} = \left(2 - \frac{1}{K_c^-} - \frac{1}{K_s^-}\right)g_9 + \frac{t_\perp^2}{2}\left(-K_c^- + \frac{2}{K_s^-} - K_s^- + \frac{2}{K_c^-} - K_s^+ - K_s^+\right) - g_4g_7 - g_3g_6, \tag{2.14}$$

$$\frac{dg_{BS}}{d\xi} = (2 - K_s^- - K_s^+)g_{BS} - g_2g_3, \tag{2.15}$$

$$\frac{dg_U}{d\xi} = (2 - K_c^- - K_c^+)g_U - g_1g_4, \tag{2.16}$$

$$\frac{d\ln K_c^-}{d\xi} = \frac{1}{2}\left(-K_c^-(g_1^2 + g_5^2 + g_U^2) + \frac{1}{K_c^-}(g_2^2 + g_3^2 + g_7^2 + g_9^2)\right), \tag{2.17}$$

$$\frac{d\ln K_s^-}{d\xi} = \frac{1}{2}\left(-K_s^-(g_2^2 + g_5^2 + g_{BS}^2) + \frac{1}{K_s^-}(g_1^2 + g_4^2 + g_6^2 + g_9^2)\right), \tag{2.18}$$

$$\frac{d\ln K_c^+}{d\xi} = -\frac{1}{2}K_c^+(g_4^2 + g_7^2 + g_8^2 + g_U^2), \tag{2.19}$$

$$\frac{d\ln K_s^+}{d\xi} = -\frac{1}{2}K_s^+(g_3^2 + g_6^2 + g_8^2 + g_{BS}^2), \tag{2.20}$$

$$\frac{d\ln t_\perp}{d\xi} = 2 - \frac{1}{4}\left(K_c^- + \frac{1}{K_c^-} + K_s^- + \frac{1}{K_s^-}\right). \tag{2.21}$$

In addition, there are two equations describing evolutions of velocities $v_{c,s}$ but one can always include these corrections into the definition of the correlation exponents.

In comparison with the equations obtained in Refs. 13, 14 our RG equations (2.6)–(2.21) are already written in terms of physically relevant combinations of original g-ological" couplings, and so one could hope that this description might appear to be more transparent. As we shall show the above system consistently demonstrates a development of the strong coupling regime in rather general conditions, and so we do not take into account next-to-leading order corrections which would be only necessary if one discussed fixed points at finite coupling.

First, in the case of spinless fermions away from half filling the only relevant couplings are g_1, g_2, and K_c^- and Eqs. (2.6)–(2.21) reduce to those found previously.[21–23].

However, in the physically relevant case of spin-$\frac{1}{2}$ fermions away from $\rho = 1$ one can only neglect the couplings g_4, g_7, g_8, and g_U associated with umklapp processes and then the number of residual couplings is large (10) and coincides with that found in Ref. 7.

The fact that Eqs. (2.6)–(2.21) originate from the repulsive Hubbard model simplifies their analysis significantly. To see that one can choose a two-step renormalization procedure to that of Ref. 23 and integrate the above equations first up to the scale $\xi_0 = \ln\frac{t}{t_\perp}$ where the renormalized amplitude of the single-particle hopping $t_\perp(\xi)$ becomes of order unity (and stops). It can be easily seen that at $\xi \sim \xi_0$ one can still neglect renormalizations of the correlation exponents $K_c^\pm = 1 - \frac{z_c^\pm}{2}$ and $K_s^\pm = 1 - \frac{z_s^\pm}{2}$ from their bare values corresponding to $z_c^\pm = -z_s^\pm = \lambda = \frac{U}{\pi t}$.

By straightforward generalization of the analysis of Refs. 22,23 one can obtain the evolution of the couplings $g_i(\xi)$ given by Eqs. (2.6)–(2.14) with only inhomogeneous terms proportional to $t_\perp^2$ kept,

$$g_i(\xi) = C_i t_\perp^2(0)\frac{e^{2\Delta_{t_\perp}\xi} - e^{\Delta_i\xi}}{2\Delta_{t_\perp} - \Delta_i}, \tag{2.22}$$

where $\Delta_i, i = 1, ..., 9,\ BS, U$ denote dimensions of relevant operators and C_i are the coefficients standing in front of terms proportional to $t_\perp^2$ in the right-hand side of (2.6)–(2.14).

It follows from (2.22) that $g_1(\xi_0) = -g_2(\xi_0) = -g_3(\xi_0) = g_4(\xi_0) = -\lambda/2$ while all the other couplings $g_i(\xi_0), i = 5, ..., 9$ are of order λ^2. On the other hand at $\xi > \xi_0$ one can also omit in (2.6)–(2.14) all inhomogeneous terms using $g(\xi_0)$ as bare values. Naively, it would mean that one has to account for the leading couplings $g_{1,2,3,4}$ plus g_{BS}, g_U first and then to treat all the rest as additional perturbations. However, it turns out that the solution is not so straightforward.

Let us consider first the case away from half filling. Then it can be shown that the couplings g_2, g_3, and g_{BS} all tend to zero though g_1 diverges. Asymptotically the following relations hold:

$$\frac{g_2(\xi)}{g_1(\xi)} \sim \exp\left[\xi\left(K_c^- - \frac{1}{K_c^-} + \frac{1}{K_s^-} - K_s^-\right)\right] \to 0,$$
$$\frac{g_3(\xi)}{g_1(\xi)} \sim \exp\left[\xi\left(K_c^- - \frac{1}{K_c^-} + \frac{1}{K_s^-} - K_s^+\right)\right] \to 0, \tag{2.23}$$
$$\frac{g_{BS}(\xi)}{g_1(\xi)} \sim \exp\left[\xi\left(K_c^- + \frac{1}{K_s^-} - K_s^- - K_s^+\right)\right] \to 0,$$
$$\frac{1}{K_c^-(\xi)} - \frac{1}{K_c^-(0)} \sim K_s^-(\xi) \sim \frac{g_1^2(\xi) - g_1^2(0)}{2 - K_c^-(\xi) - \frac{1}{K_s^-(\xi)}}.$$

Thus we infer that $K_c^-(\xi)$ vanishes while $K_s^-(\xi)$ goes to infinity. But this means that the assumption about the smallness of the couplings $g_{5,\ldots,9}$ made on the basis of their values at $\xi = \xi_0$ was not quite correct. Namely, one has to include those terms which contain one of the fields ϕ_c^- or θ_s^- which are "close" to getting locked. A simple inspection yields that the second relevant coupling (besides g_1) is g_6 while in the case of $\rho = 1$ one has to keep $g_{4,8,U}$ as well. The resulting system of equations in the range $\xi_0 < \xi < \frac{1}{\lambda}$ reads as

$$\frac{dg_1}{d\xi} = \frac{1}{2}(z_c^- - z_s^-)g_1 - g_4 g_U, \tag{2.24}$$

$$\frac{dg_6}{d\xi} = \frac{1}{2}(z_s^- - z_c^-)g_6 - g_4 g_8, \tag{2.25}$$

$$\frac{dz_c^-}{d\xi} = g_1^2 + g_U^2, \tag{2.26}$$

$$\frac{dz_s^-}{d\xi} = -g_1^2 - g_6^2 - g_4^2, \tag{2.27}$$

$$\frac{dz_s^+}{d\xi} = g_6^2 + g_8^2, \tag{2.28}$$

$$\frac{dz_c^+}{d\xi} = g_4^2 + g_8^2 + g_U^2. \tag{2.29}$$

Away from half filling all $g_{4,8,U}$ freeze at $\xi \sim \ln\frac{1}{\delta}$ (and, consequently, z_c^+ is frozen too) and there are only $g_{1,6}$ left over. Then the system (2.24)–(2.29) demonstrates a development of the strong coupling regime in all channels except the + charge one [namely, $g_1(\xi), g_6(\xi) \to -\infty$ and $z_c^-(\xi), z_s^+(\xi) \to \infty$ while $z_s^-(\xi) \to -\infty$]. As usual, these tendencies have to be undersood in such a way that at $\xi \sim \frac{1}{\lambda}$ all couplings reach values of order unity and do not vary further.

Including the couplings $g_{4,8,U}$ at $\delta \to 0$ one can see that they do not alter the behavior found for the doped case while the + charge sector is also driven to the strong coupling regime in accordance with the complete freezing of charge degrees of freedom at $\rho = 1$.

To facilitate the analysis of the leading instabilities of the complete Hamiltonian (2.3,2.5) one has to consider eight relevant order parameters where plus and minus correspond to intra- versus interchain type of ordering,

$$\mathrm{CDW}_+ = \sum \Psi^{f\dagger}_{\mu\sigma}\Psi^{f}_{-\mu,\sigma} \sim \cos(\phi_c^+ + \phi_c^-)\cos(\phi_s^+ + \phi_s^-), \tag{2.30}$$

$$\mathrm{CDW}_- = \sum \Psi^{f\dagger}_{\mu\sigma}\Psi^{-f}_{-\mu,\sigma} \sim \cos(\phi_c^+ + \theta_c^-)\cos(\phi_s^+ + \theta_s^-), \tag{2.31}$$

$$\mathrm{SDW}_+ = \sum \Psi^{f\dagger}_{\mu\sigma}\Psi^{f}_{-\mu,-\sigma} \sim \cos(\phi_c^+ + \phi_c^-)\cos(\theta_s^+ + \theta_s^-), \tag{2.32}$$

$$\mathrm{SDW}_- = \sum \Psi^{f\dagger}_{\mu\sigma}\Psi^{-f}_{-\mu,-\sigma} \sim \cos(\phi_c^+ + \theta_c^-)\cos(\theta_s^+ + \phi_s^-), \tag{2.33}$$

$$\mathrm{SS}_+ = \sum \sigma\Psi^{f}_{\mu\sigma}\Psi^{f}_{-\mu,-\sigma} \sim \cos(\theta_c^+ + \theta_c^-)\sin(\phi_s^+ + \phi_s^-), \tag{2.34}$$

$$\mathrm{SS}_- = \sum \sigma\Psi^{f}_{\mu\sigma}\Psi^{-f}_{-\mu,-\sigma} \sim \cos(\theta_c^+ + \phi_c^-)\sin(\phi_s^+ + \theta_s^-), \tag{2.35}$$

$$\mathrm{TS}_+ = \sum \sigma\Psi^{f}_{\mu\sigma}\Psi^{f}_{-\mu,\sigma} \sim \cos(\theta_c^+ + \theta_c^-)\sin(\theta_s^+ + \theta_s^-), \tag{2.36}$$

$$\mathrm{TS}_- = \sum \sigma\Psi^{f}_{\mu\sigma}\Psi^{f}_{-\mu,\sigma} \sim \cos(\theta_c^+ + \phi_c^-)\sin(\theta_s^+ + \phi_s^-). \tag{2.37}$$

Remember that if any of the fields ($\phi^{\pm}_{c,s}$ or $\theta^{\pm}_{c,s}$) gets locked, then the corresponding cosine acquires a nonzero expectation value and $\langle\cos\phi(x)\cos\phi(0)\rangle \to |\langle\cos\phi(0)\rangle|^2$ as x tends to infinity. On the other hand, fluctuations of both this variable and its dual one become gapful. Formally one can identify the state where $\phi^{\pm}_{c,s}$ is ordered with the limit $K^{\pm}_{c,s} \to 0$ while $\theta^{\pm}_{c,s}$ becomes ordered at $K^{\pm}_{c,s} \to \infty$.

Then one can easily see that in the case when θ_s^- and ϕ_s^+ are locked the only competing instabilities are the interchain charge density wave corresponding to the order parameter $\mathrm{CDW}_- \sim \cos(\phi_c^+ + \theta_c^-)$ and the interchain singlet pairing described by $\mathrm{SS}_- \sim \cos(\theta_c^+ + \phi_c^-)$. In fact, the former state can be also recognized as a counterpart of the two-dimensional flux phase. Indeed, this state is characterized by the commensurate with density flux $\Phi = 2k_F$ which is defined as a circulation of a phase of the on-rung order parameter $\langle u_i^\dagger d_i + d_i^\dagger u_i\rangle$ through a plaquette formed by two adjacent rungs of the ladder. In the case of spinless fermions this type of ordering called "orbital antiferromagnet" was first considered in Ref. 24 as a prototype of recently proposed two-dimensional flux states.

Although the flux phase can be in principle realized in some extended models we see that in our case of the double-chain Hubbard model where the field ϕ_c^- also gets locked the ground state is a spin-gapped singlet superconductor.

It is also instructive to express the above order parameters in terms of the hybridized one-particle states corresponding to the above-mentioned "bonding" and "antibonding" bands,

$$\mathrm{CDW}_- = \sum_\sigma A^\dagger_{R\sigma} A_{L\sigma} - B^\dagger_{R\sigma} B_{L\sigma},$$

$$\mathrm{SS}_- = \sum_\sigma A^\dagger_{R\sigma} A^\dagger_{L,-\sigma} - B^\dagger_{R\sigma} B^\dagger_{L,-\sigma}. \tag{2.38}$$

Considering the distribution of signs of the order parameter SS_- on the "four-point Fermi surface" $[\vec{k} = (k_F, 0), (-k_F, 0), (k_F, \pi), (-k_F, \pi)]$ we observe that it corresponds to "d-wave" type pairing. We conjecture that in a two-dimensional array of weakly coupled double chains with a continuum Fermi surface this type of ordering does transform into ordinary d-wave pairing.

III. CONCLUSIONS

In the present paper we applied the bosonization method to find further arguments in support of the recently proposed scenario of singlet superconductivity in the spin-gap state of doubled Luttinger chains. Previous results obtained in the framework of the mean field approach[12] as well as earlier numerical studies[3,5,6] also testify in favor of this picture.

We also want to stress that our conclusions contradict a recent claim about the existence of the strong coupling fixed point where some spin excitations remain gapless made in Ref. 25. These authors considered the double-chain t–J model without an interchain spin exchange ($J_\perp = 0$). Then on the bare level their Hamiltonian can be assigned to the universality class of the purely forward scattering model considered in Ref. 16. In this special case indeed the only field becoming massive is θ_s^-. In principle, it cannot be ruled out that for some specific double-chain models only part of all relevant fields acquires masses and the others remain massless. One example of this type was discussed by the authors of Ref. 26 who found only ϕ_c^- and ϕ_s^+ to be massive in the framework of the model including solely an interchain interaction of fermions with opposite spins.

However, our investigation of Hubbard-type models shows that the presence of the interchain one-particle hopping is already sufficient to generate the antiferromagnetic spin exchange term with $J_\perp \sim \frac{t_\perp^2}{U}$ which makes all spin modes gapful at $\rho \lesssim 1$. We believe that spin liquid behavior with a finite spin gap which evolves into "d-wave" pairing upon doping is a robust feature of a whole variety of spin isotropic models of strongly correlated fermions on weakly coupled double chains.

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