Anyonic ions, energy bands, and photoluminescence of fractional quantum Hall systems

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A two-dimensional electron liquid interacting with free or localized valence-band holes located a distance d away from the electron layer is studied by exactly diagonalizing the Hamiltonian of a finite-size system. For a coplanar electron-hole system, our calculation does not show the usual sign of an incompressible state or a fractional quantum Hall effect. For the case where the separation d is about 1.5 times the magnetic length, pronounced cusps are revealed in a plot of the ground-state energy versus the Landau-level degeneracy. Detailed analysis suggests that the ground state responsible for the cusps consists of anyonic ions (composed of Laughlin quasielectrons bound to a hole) weakly coupled to an incompressible fluid of the remaining electrons. The hypothesis of stable anyonic ions is further supported by the structure displayed in the photoluminescence (PL) spectrum. Three PL systems, namely, electron–free-hole (e-h), electron–localized-hole (e-A^+), and electron–neutral-acceptor (e-A^0), have been investigated as functions of separation d, filling factor ν_e, and temperature T. Multipeaked spectra are obtained for all three systems; they can be understood in terms of quasielectron-hole recombination processes and the "band structure" of two-dimensional electrons in the fractional quantum Hall effect regime. The connections between the calculated spectrum and some recent experimental results are also discussed.

I. INTRODUCTION

During the past several years, the investigation of magnetophotoluminescence of two-dimensional (2D) electron systems in the fractional quantum Hall effect (FQHE) regime[1] has attracted a great deal of attention, both experimental[2,3] and theoretical.[4,5] While the experiments have produced a number of interesting results and revealed several intriguing fine features in the photoluminescence (PL) spectrum, theoretical understanding of these observations is still at the primitive stage, and physical interpretations for many of them are not yet available. Nevertheless, it is generally believed that the features displayed in the PL spectrum reflect in one way or another the detailed electronic structure of two-dimensional systems in strong magnetic fields.

The 2D systems studied experimentally[2,3] are formed at the interface of a single GaAs/$Al_xGa_{1-x}As$ heterostructure. Several different kinds of PL systems have been investigated experimentally, viz., an electron–free-hole system (e-h) and an electron–localized-hole system (e-A). The localized hole can be bound to an acceptor ion giving an electron–neutral-acceptor system (e-A^0) or it can be bound to a neutral defect giving an electron–positively-charged-acceptor system (e-A^+). In the e-h system, an electron recombines with a photoexcited free hole in the valence band of the GaAs and emits a photon of frequency ω. The valence-band hole involved in the recombination process may be located in the same layer as the electrons or in a layer separated by a finite distance from the electron layer, depending on the structure of the system. In the e-A^0 system, on the other hand, an electron recombines with a localized hole bound to a Be acceptor that is purposely δ doped in the GaAs. After the recombination, the Be acceptor is negatively charged and acts as a perturbation on the 2D electron system. In contrast, in an e-h system the electron liquid is perturbed by the valence-band hole before the recombination. In the extreme quantum limit, both systems display some characteristic features (typically, appearance of additional peaks in the PL spectrum as the Landau-level filling factor ν becomes much smaller than 1) that have been viewed as the evidence of a Wigner solid state.

At $\nu \sim \frac{1}{3}$ the experimental e-h PL spectrum shows a doublet structure over a quite wide range of the filling factor (from $\nu = \frac{1}{2}$ to $\frac{1}{5}$). The origin of the doublet structure is still a mystery. It has been suggested that it may be connected to the internal structure of the photoexcited hole state.[6,7] Theoretical studies,[5,8] however, indicate that the PL spectrum should be very rich and informative even if such internal structure is absent. At a filling factor slightly less than $\frac{1}{3}$, it has been shown[5] that, for the case where the interaction between electrons and holes is very small, the e-h PL spectrum displays two peaks, with the higher-energy one corresponding to the recombination process in which one valence-band hole and a quasielectron are annihilated with the simultaneous creation of two quasiholes. As the filling factor is further reduced, a third peak appears at a still higher energy; the three peaks are almost equally separated by an energy of the order of the Laughlin gap.[8] In this paper, we present

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a comprehensive numerical study of the ground-state properties of the two-dimensional electron fluid in the presence of external perturbations (i.e., valence-band holes or impurities) and its PL spectrum in the FQHE regime. (Some preliminary results have been published in Refs. 8 and 9.)

The electron system studied in the present paper is assumed to be strictly two dimensional, and holes or neutral Be acceptors are confined in a 2D layer some distance away from the electrons. Introducing the finite distribution of hole density in the direction perpendicular to the layers does not change the PL spectra qualitatively. We have calculated the optical emission spectrum for both e-h and e-A^0 systems as a function of the Landau-level filling factor at several values of layer separation d and at several values of temperature. Having noticed the possibility that the valence-band holes may be bound or localized by neutral impurities, dislocations, interface roughness, etc., in a real sample, we also studied a PL system in which electrons recombine with localized valence-band holes (e-A^+ system). The PL spectrum at one particular filling factor and zero temperature has been calculated by MacDonald, Rezayi, and Keller[5] for an e-h system and by Apal'kov and Rashba[10] for e-A^+ and e-A^0 systems.

For an e-h system, at $d=0$ our results show that the low-temperature PL intensity in general is very weak except for $\nu_c=p/q$ with q being odd, where ν_c is the difference in filling factor between electrons and holes. This result, which contradicts the general belief that no PL anomalies should appear for a coplanar system, comes from the fact that the ground state of a system of N_e electrons and N_h holes ($N_e>N_h$) cannot be viewed as a simple mixture of a gas of N_h excitons of momentum zero and a fluid of N_e-N_h electrons. In other words, the ground state of the electron-hole system cannot in general be obtained from the ground state of a pure electron system by simply adding the appropriate number of zero-momentum excitons. As the layer separation d deviates from zero, the PL spectrum changes rapidly and some fine features appear. These fine features, which exist over a wide range of electron density, are closely related to the band structure[11] of 2D electron systems in the FQHE regime. Each peak in the PL spectrum results from optical transitions from low-lying states of the initial system to a group of states that belong to a certain band of the final electron system. The detailed features of the PL spectrum, nevertheless, are determined not only by the band structure of the electron system but also by the interaction between electrons and holes. When the layer separation is about the order of the magnetic length, we find that a valence-band hole can form stable bound states[8,9] with a few Laughlin quasiparticles, and the signature of these bound-state complexes (or anyonic ions) is apparent in the PL spectrum.

In the case of an e-A^0 system, at $d=0$ the perturbation of the negatively charged Be acceptor to the final electron system can be so strong that the FQHE as well as the band structure in the FQHE regime are completely destroyed in the vicinity of the acceptor. Consequently, the PL only displays a broad single peak (or band), and no fine structure is obtained except that the width and shape of the peak may depend on the value of the Landau-level filling factor. The single peak remains the only feature in the PL spectrum for small layer separations, but it starts to split into two or three smaller peaks as the separation becomes comparable to the magnetic length. At very large layer separations the PL spectrum of an e-A^0 process very much resembles that of an e-h process, as does the spectrum of an e-A^+ process. A unique character of e-A^+ systems is that even at $d=0$ the spectrum still possesses a multipeak structure (usually two major peaks), which again can be understood in terms of the electronic band structure of 2D systems.

The numerical calculations are performed in the spherical geometry.[12,13] A finite number of electrons and holes (or acceptors) are put on a sphere of radius $R=\sqrt{S}$ (in unit of magnetic length, $l=\sqrt{hc/eB}$) with a magnetic monopole at the center, where $2S$ is the total number of magnetic flux quanta passing through the sphere. The degeneracy of the first Landau level is therefore given by $2S+1$. We neglect any contribution from the higher Landau levels, assuming that the cyclotron energy is very large compared to $e^2/\epsilon l$. The Coulomb interaction between electrons is taken to be inversely proportional to the chord distance, and the interaction between electrons and holes (or negatively charged Be acceptors, in the e-A^0 system) is modulated by the layer separation d, i.e.,

$$V(|\hat{\Omega}_1-\hat{\Omega}_2|)=\pm\frac{1}{\sqrt{R^2|\hat{\Omega}_1-\hat{\Omega}_2|^2+d^2}}\ , \tag{1}$$

where $\hat{\Omega}$ is a unit vector in the radial direction denoting the position of a particle on the sphere. The negative sign in Eq. (1) corresponds to the electron-hole interaction in e-h and e-A^+ systems and the positive sign corresponds to the electron–charged-acceptor interaction in the e-A^0 system (after the recombination). The quantum states of a pure electron system as well as an electron-hole system are classified by eigenvalues $L(L+1)$ and M of the square of the total angular momentum operator and its z component $\hat{L}_z$. The linear momentum is related to the angular momentum by $k\approx L/R$. For electron systems perturbed by a localized hole or impurity, the spherical symmetry is broken; quantum states can only be classified by the eigenvalues M of $\hat{L}_z$.

The photoluminescence intensity resulting from recombination of an electron with a hole (whether localized or not) at temperature T is given by

$$I(\omega)=\frac{I_0}{Z}\sum_{i,f}e^{-E_i/k_BT}|\langle\Psi_f^{(N_e-1,0)}|\hat{L}|\Psi_i^{(N_e,1)}\rangle|^2 \times\delta(\omega-E_i+E_f)\ , \tag{2}$$

where $Z=\sum_i\exp(-E_i/k_BT)$ is the partition function of the initial system. $\Psi_i^{(N_e,1)}$ and $\Psi_f^{(N_e-1,0)}$ are the quantum states of the initial and final systems with N_e electrons and one hole, and N_e-1 electrons and no hole, respectively. $\hat{L}$ in Eq. (2) is the luminescence operator

$$\hat{L}=\int d^2\mathbf{r}\,\hat{\psi}_e(\mathbf{r})\hat{\psi}_h(\mathbf{r})\ , \tag{3}$$

with $\hat{\psi}_e(\mathbf{r})$ being the electron annihilation operator and $\hat{\psi}_h(\mathbf{r})$ being the hole annihilation operator. Each PL system has a specific luminescence operator which will be given later. Most of the calculations presented in this paper are made for systems that have seven electrons and one hole (or one neutral Be acceptor) initially, and six electrons after the recombination processes. In order to compare our numerical results with some existing experimental data, we also calculated the zero (M_0), first (M_1), and second (M_2) moments of the PL spectrum. They are defined as

$$M_0=\int I(E)dE \ , \tag{4}$$

$$M_1=\frac{1}{M_0}\int I(E)E\,dE \ , \tag{5}$$

$$M_2^2=\frac{1}{M_0}\int I(E)(E-M_1)^2dE \ . \tag{6}$$

The values of M_0, M_1, and M_2 give the integrated intensity, the centroid, and the linewidth of the spectrum.

The paper is organized as follows. In Sec. II we study the e-h PL system, starting with the discussions of the ground state and collective excitations of electron-hole systems, in particular, the hidden symmetry at $d=0$ and the stable anyonic ions at $d\sim 1.5$. The e-A^+ systems are investigated in Sec. III; the localization of the valence-band hole changes the PL spectrum of a coplanar system dramatically. Section IV discusses the e-A^0 PL spectrum. Finally, a summary of our results and some remarks are given in Sec. V.

II. e-h SYSTEMS

Before presenting our PL results for the e-h systems, we would like first to discuss some properties of a two-dimensional electron-hole gas in strong magnetic fields. It has been known for many years that, for a system with an equal number of electrons and holes in the same two-dimensional layer, the ground state can be obtained exactly and viewed as a Bose-Einstein condensed state of noninteracting electron-hole pairs.[14] This is a result of the symmetry between the electron-electron and the electron-hole interactions. As the hole layer moves away from the electron layer the interlayer correlation becomes relatively less important compared to the intralayer interaction. The system is expected to undergo a phase transition to a double FQHE state[15] or an excitonic charge-density-wave state[16] depending on the filling factor of electrons (holes). A state that is similar to the half-integer FQHE state observed in electron-electron double quantum wells[17] may also exist in electron-hole quantum wells with an appropriate well separation.

For a general electron-hole system in which the number of electrons differs from the number of holes, the nature of the ground state is still the subject of controversy. In the coplanar case it has been argued that the charged electron-hole fluid should display a FQHE when the filling factor of the excess charge $\nu_c=\nu_e-\nu_h$ is a rational fraction with an odd denominator.[18] This conclusion, obtained from a mapping between an electron-hole system and a spin-$\frac{1}{2}$ electron system, follows from the assumption that, for the spin-$\frac{1}{2}$ system with a negligible Zeeman splitting, the ground state is always maximally spin polarized. It is further predicted that the PL spectrum of the coplanar system in the FQHE regime does not exhibit anomalies associated with the FQHE.[5] Our numerical results, however, display no sign of an incompressible state, and the calculated PL intensity shows a strong peak at $\nu_c=p/q$. For the asymmetric cases, we find that, when the layer separation is of the order of the magnetic length, strong cusps appear in a plot of the ground-state energy as a function of S indicating the existence of the FQHE and of stable anyonic ions.[9]

The luminescence operator for e-h systems is the same as the annihilation operator of an exciton with zero total momentum. In the spherical geometry, $\hat{\psi}_e$ and $\hat{\psi}_h$ in Eq. (3) can be written as $\hat{\psi}_e(\mathbf{r})=\sum_m a_m\Phi_m^e(\mathbf{r})$ and $\hat{\psi}_h(\mathbf{r})=\sum_m b_m\Phi_m^h(\mathbf{r})$, where a_m (b_m) is the annihilation operator of a free electron (hole), and $\Phi_m^e(\mathbf{r})$ [$\Phi_m^h(\mathbf{r})$] is the electron (hole) wave function with z-component angular momentum m (the eigenvalue of $\hat{L}_z$) given by

$$\Phi_m^e(\mathbf{r})=\left[\frac{2S+1}{4\pi}\left[\frac{2S}{S+m}\right]\right]^{1/2} \times\cos^{S+m}(\theta/2)\sin^{S-m}(\theta/2)e^{im\phi} \ . \tag{7}$$

Here θ and ϕ are the spherical coordinates. Substituting these two expressions into Eq. (3) and noticing that $\Phi_m^e(\mathbf{r})=(-1)^{S+m}\Phi_{-m}^{h*}(\mathbf{r})$, we obtain

$$\hat{L}=\sum_m(-1)^{S+m}a_m b_{-m} \ . \tag{8}$$

Its Hermitian conjugate $\hat{L}^+$ creates an electron-hole pair with a total momentum $\mathbf{k}=0$.

A. $d=0$

For the symmetric case where electrons and holes are in the same layer, it has been shown that the system possesses a so-called hidden symmetry[18–20] which can be easily understood by noticing the following commutation relation between the creation operator $\hat{L}^+$ and the Hamiltonian of the system $\hat{H}$:

$$[\hat{H},\hat{L}^+]=E_x(0)\hat{L}^+ \ , \tag{9}$$

where $E_x(0)=-\sqrt{\pi/2}(e^2/\epsilon l)$ is the binding energy of a single bare exciton with $\mathbf{k}=\mathbf{0}$ in the strong-field limit. It follows from Eq. (9) that the introduction of an electron-hole pair with a total momentum $\mathbf{k}=\mathbf{0}$ to a coplanar system only changes the energy of the system by $E_x(0)$, the single-exciton energy. In other words, as far as the energy of the system is concerned, it appears as if a $\mathbf{k}=\mathbf{0}$ exciton does not interact with any other particles. Furthermore, Eq. (9) implies that if $|N_e,N_h\rangle$ is an eigenstate of a system of N_e electrons and N_h holes with energy E_0, then $[\hat{L}^+]^n|N_e,N_h\rangle$ will be an eigenstate of a system of N_e+n electrons and N_h+n holes with energy $E_0+nE_x(0)$. In the following we will call those electron-hole states which can be obtained by simply operating with $\hat{L}^+$ on the eigenstates of a pure electron system the multiplicative states (MS).[20] They are of particular importance in understanding the PL spectrum of a coplanar e-h system.

In Fig. 1 we have plotted the ground-state energy of

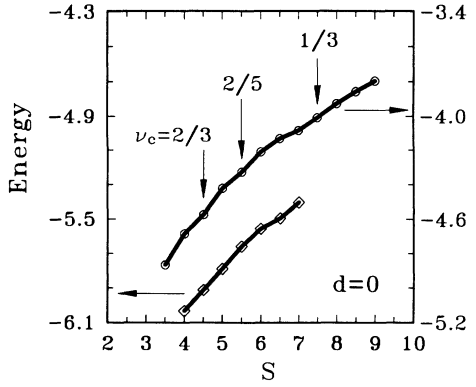

FIG. 1. (a) Ground-state energy as a function of S for electron-hole systems at $d=0$: circles, $N_e=7$ and $N_h=1$; diamonds, $N_e=8$ and $N_h=2$. Energy is in units of $e^2/\epsilon l$.

two systems (seven electrons and one hole, and eight electrons and two holes) at $d=0$ as a function of the parameter S. The energies have been shifted by $-N_c^2/2R$ because of the neutralizing positive-charge background, where $N_c=N_e-N_h$. In both cases $N_c=6$. According to the prediction of Ref. 18, one would expect downward cusps to appear at $S=7.5$, 5.5, and 4.5, corresponding to $\nu_c=\frac{1}{3}$, $\frac{2}{5}$, and $\frac{2}{3}$. Our numerical result, however, shows no apparent sign of a discontinuity in slope at these values of S. The assumption[18] that the ground state of the spin-$\frac{1}{2}$ electron system, onto which the electron-hole system maps, is maximally spin polarized is equivalent to the assertion that the ground state of the electron-hole system can be obtained from the ground state of an N_e-electron system by simply adding N_h excitons of $\mathbf{k}=\mathbf{0}$. Finite-size calculations indicate that the ground state for given S is not necessarily one of the multiplicative states $[\hat{L}^+]^{N_h}|N_c,0\rangle$, where $|N_c,0\rangle$ is a quantum state of an N_c-electron system with the same value of S. This is clearly shown in Figs. 2(a) and 2(b), in which the energy spectrum of a seven-electron and one-hole system has been plotted at $S=7.5$ ($\nu_c=\frac{1}{3}$) and $S=7$, with multiplicative states indicated by diamonds. At $S=7.5$ the lowest multiplicative state is separated from other MS's by an energy gap. In the gap there exist a number of nonmultiplicative states. The ground state ($L=1$) which is nonmultiplicative has an energy slightly lower than the lowest multiplicative state ($L=0$) and is almost degenerate with a non-MS at $L=2$. Decrease the Landau-level degeneracy by 1; a different situation is obtained at $S=7$, where the lowest MS ($L=3$) is higher than the ground state ($L=1$, nonmultiplicative) by $0.06e^2/\epsilon l$, a value which is comparable to the energy gap of the $\frac{1}{3}$ Laughlin state.

To assure ourselves that the finite energy difference ΔE between the lowest MS and the ground state is not a finite-size effect, in the inset of Fig. 3(a) we plotted ΔE of a system N_e electrons and one hole at $S=S_{\nu_c=1/3}-0.5$ as a function of $1/N_e$, where $S_{\nu_c=1/3}$ is the S value at $\nu_c=\frac{1}{3}$. The extrapolation to $N_e\to\infty$ gives $\Delta E=0.04e^2/\epsilon l$. The finite value of ΔE implies that the

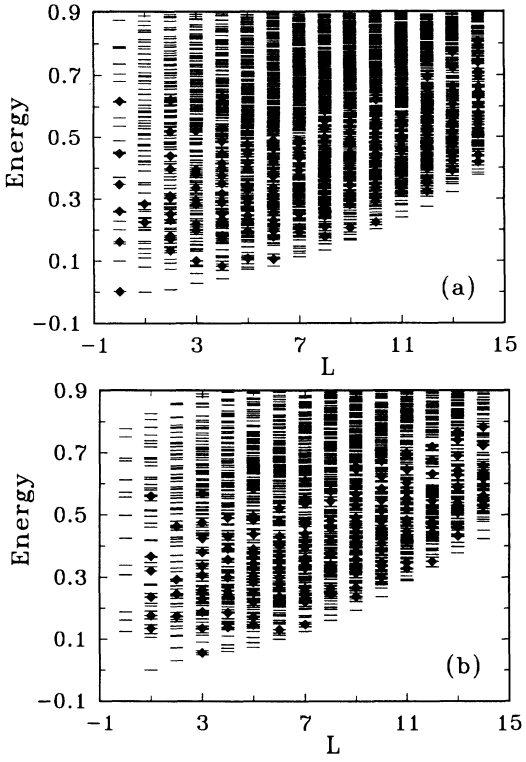

FIG. 2. Energy spectrum of an electron-hole system with $N_e=7$, $N_h=1$, and $d=0$. The states indicated by diamonds can be obtained by operating $\hat{L}^+$ onto the states belonging to a six-electron system. (a) $S=7.5$ ($\nu_c=\frac{1}{3}$); (b) $S=7.0$.

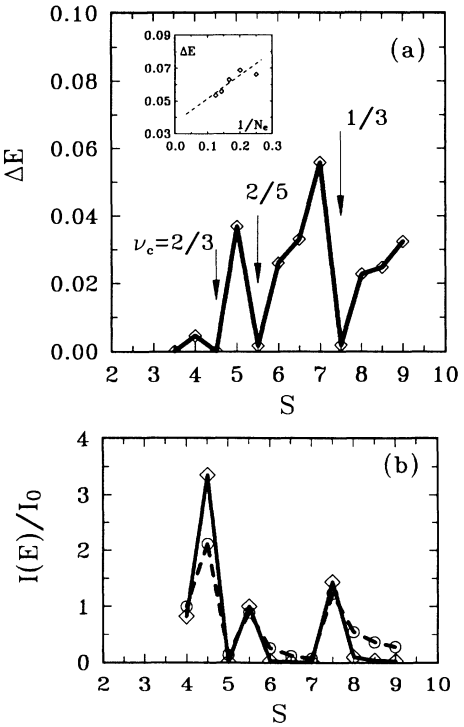

FIG. 3. (a) Energy difference ΔE between the lowest multiplicative state and the ground state of a system with $N_e=7$ and $N_h=1$ at $d=0$, as a function of S. Inset: ΔE at $S=S_{\nu_c=1/3}-0.5$ as a function of $1/N_e$ (see text). (b) PL intensity at $d=0$ ($N_e=7$ and $N_h=1$): Diamonds and solid line, $T=0.005e^2/\epsilon l$; circles and dashed line, $T=0.01e^2/\epsilon l$.

added electron-hole pair acquires a finite total momentum, and consequently an electric dipole moment. The interaction between the exciton with the dipole moment and the rest of the electrons thus lowers the energy of the electron-hole system. Figure 3(a) is a plot of ΔE as a function of S for a seven-electron and one-hole system. The energy difference of the lowest MS and the ground state becomes very small only at S values that correspond to $\nu_c=\frac{1}{3}$, $\frac{2}{5}$, and $\frac{2}{3}$ (or, in general, p/q with q being odd integers). Because of the commutation relation (9) obeyed by the luminescence operator $\hat{L}$, it is apparent that at very low temperature the PL intensity given by Eq. (2) is observable only if the lowest MS is the ground state or very close to the ground state in energy. In Fig. 3(b) we show the PL intensity at two different temperatures $T=0.005e^2/\epsilon l$ and $0.01e^2/\epsilon l$. The luminescence frequency ω is simply equal to the bare-exciton binding energy $E_x(0)$.[21] At $T=0.005$ the PL intensities at $\nu_c=\frac{1}{3}$, $\frac{2}{5}$, and $\frac{2}{3}$ are an order of magnitude stronger than the intensities at the other S values. As the temperature increases the difference in the PL intensity becomes less dramatic. We have also noticed that the excitation energy spectrum of electron-hole systems ($d=0$) at $\nu_c=\frac{1}{3}$ is very different from that at $\nu_c=\frac{2}{3}$. While at $\nu_c=\frac{2}{3}$ a quadratic dependence of low-lying excitation energy on the wave vector k is observed in the long-wave length limit, the low-energy excitation branch at $\nu_c=\frac{1}{3}$ varies as k to a higher power for small k.

B. $d\neq 0$

For the asymmetric case in which electrons and holes are in two different layers separated by a distance d, the hidden symmetry of Eq. (9) no longer holds. Shakeup processes in the PL become possible, and there will be a finite probability for the ground state of the initial system to decay into the several different states of the final system. The PL spectrum as well as the properties of the electron-hole system depend strongly on the interlayer interaction, or the location of valence-band holes relative to the electron layer. At a very large separation, where the electron-hole interaction is weak and negligible compared with the electron-electron interaction, the relative PL intensity $I(\omega)/I_0$ is simply proportional to the spectral function of a two-dimensional electron system.[22] In Fig. 4 the PL spectrum for a seven-electron and one-hole system at $d=8l$ is shown for S values ranging from 5.5 to 9.5. At $S=9.0$ (corresponding to $\nu_e=\frac{1}{3}$) the seven electrons condense into the Laughlin incompressible state, and the luminescence intensity has a single peak resulting from the recombination of an electron with a valence-band hole and the creation of three Laughlin quasiholes.

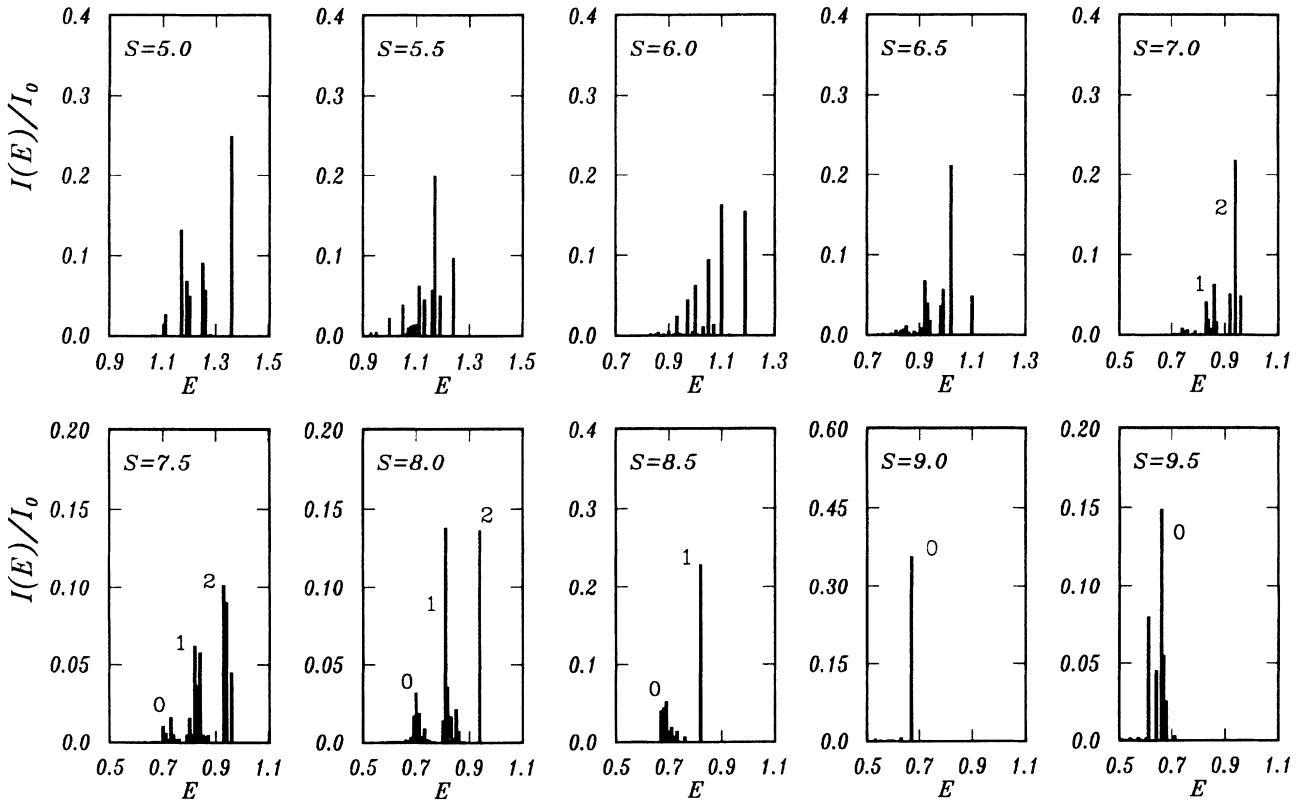

FIG. 4. Photoluminescence spectrum of an e-h system ($N_e=7$ and $N_h=1$) at $d=8.0$ and $T=0.005e^2/\epsilon l$ for S values from 5.0 to 9.5. Energy is in units of $e^2/\epsilon l$. d is in units of the magnetic length l. The number beside a PL peak indicates the quasielectron process (Fig. 5 below) from which the peak results.

In the theory of the FQHE, each quasielectron (quasihole) of the $\frac{1}{3}$ state has a charge $\pm\frac{1}{3}e$ and obeys fractional statistics.[23,24] Decreasing the Landau-level degeneracy by one unit creates a quasiparticle in the electron fluid. The PL spectrum at $S=8.5$ displays two peaks (Fig. 4) one of which is located at almost the same position as the single peak of $S=9$. The second peak with a higher energy is believed to result from a PL process which can be thought of as a quasielectron (instead of a real electron) recombining with the valence-band hole and leaving two quasiholes in the final state.[5] Generally speaking, in the presence of a number of free quasielectrons, we expect to observe a PL peak corresponding to an N-quasielectron process ($N=0,1,2,3$; see Fig. 5) in which N quasielectrons recombine with the valence-band hole and leave $3-N$ quasiholes in the final electron system. At $S=8$ three almost equally spaced peaks are obtained in the spectrum. The two low-energy peaks are located at the same positions as the two peaks for $S=8.5$. The third peak comes from the two-quasielectron process.

Further decreasing the Landau-level degeneracy to $S=7.5$ and 7.0 (increasing the number of quasielectrons in the initial system to three and four), one would anticipate a fourth peak appearing at a still higher energy. Our calculations nevertheless exhibit at most three peaks. The absence of the peak corresponding to the three-quasielectron process at a large layer separation ($d=8$) is consistent with recent experiment observations,[25] and can be interpreted as a result of the repulsive interaction among quasielectrons, which lowers the probability of quasielectrons being present at the same place and keeps three of them from recombining with the valence-band hole simultaneously. Later we will show that increasing the electron-hole interaction will enhance the PL intensity from the three-quasielectron process. When the electronic-filling factor becomes larger than $\frac{2}{5}$, new peaks start to emerge again on the high-energy side of the PL spectrum (Fig. 4, $S=6.5$ and 6.0). These peaks result from recombination processes involving the quasiparticles of the $\frac{2}{5}$ incompressible state. A general feature displayed in Fig. 4 is that, starting from $\nu_e=\frac{1}{3}$ ($S=9$), as the population of quasielectrons is increased the PL spectral intensity shifts from the lower-energy peak (zero-quasielectron process) towards the higher energy peak (two-quasielectron process). On the other side of $\nu_e=\frac{1}{3}$, i.e., $S>9.0$, all of the spectral intensity comes from the zero-quasielectron process. The small splitting at $S=9.5$ (also observed at larger values of S) may be explained in terms of the recombination processes of quasielectrons of the $\frac{1}{5}$ Laughlin state with a valence-band hole. The zero, first, and second moments [defined in Eqs. (4)–(6)] of the PL intensity are plotted in Fig. 6 as a function of S. A downward cusp in M_1 is observed at $\nu_e=\frac{1}{3}$ ($S=9.0$). Raising the temperature in general increases the spectral weights of the lower-energy peaks. The detailed discussion on the effect of temperature is deferred to Sec. III.

The energy separations between the peaks in Fig. 4 are of the order of $0.1e^2/\epsilon l$ ($\approx 0.14e^2/\epsilon l$, at $S=8$). They are related more closely to the band structure of 2D electron systems in the FQHE than the simple Laughlin energy gap at exactly $\nu_e=\frac{1}{3}$. It has been pointed out recently by several authors that the low-lying eigenstates of interacting electrons in the fractional quantum Hall phase[11] form fairly well-defined bands in the energy space. The classification of one or two lowest bands has been successful via both the hierarchy theory[12,24] of Haldane and Halperin and the composite-fermion theory[11] of Jain.

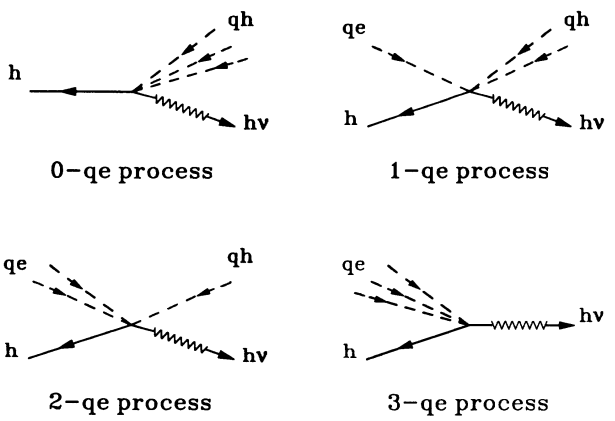

FIG. 5. Recombination processes of a valence-band hole (h) and quasielectron (qe's) near $\nu_e\sim\frac{1}{3}$.

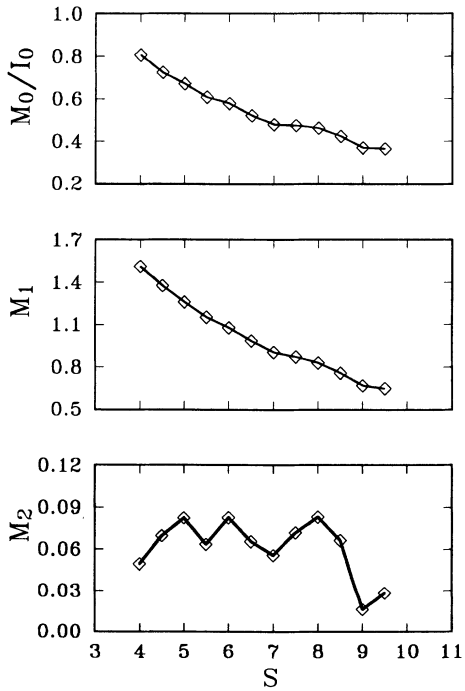

FIG. 6. Integrated intensity (M_0), centroid (M_1), and linewidth (M_2) of the PL spectrum for an e-h system ($N_e=7$ and $N_h=1$) at $d=8.0$. M_1 and M_2 are in units of $e^2/\epsilon l$.

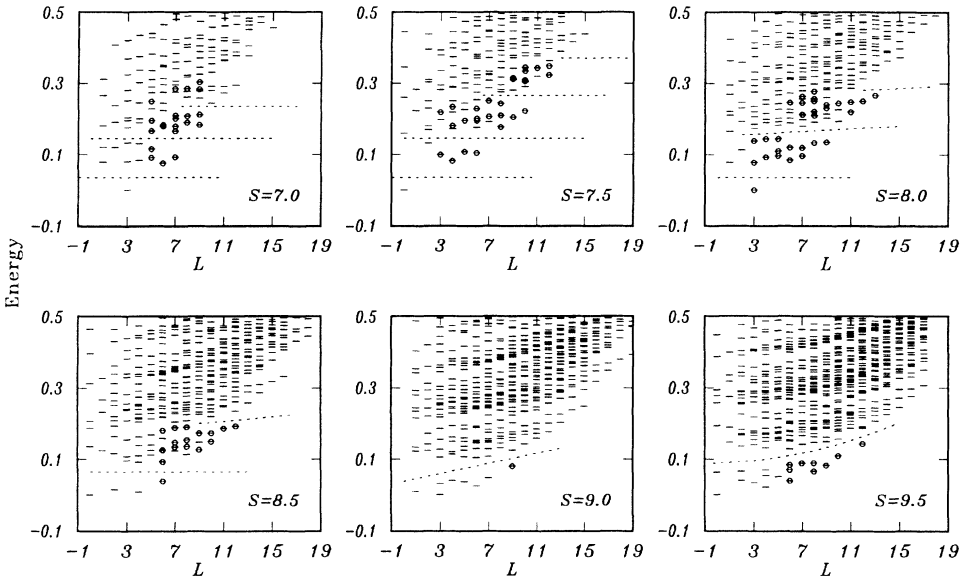

FIG. 7. Energy spectra of a six-electron system. States circled are those to which the optical transitions contribute significantly to the PL spectrum of Fig. 4.

The observation of the multipeak structure in the PL spectrum is direct evidence of the existence of energy bands in the FQHE regime. The features in the PL spectrum reflect the energy structure of the final electron system. Each peak in a spectrum is produced by a group of transitions from the low-lying energy states (or the ground state if $T=0$) of an initial system to the states that belong to a certain energy band of the final system. This is illustrated in Fig. 7 where the energy spectra of a six-electron system are shown for S from 7.0 to 9.5. The states circled in Fig. 7 are those (final) states to which the optical transitions give a finite contribution to the PL intensity. These states form several well-defined groups, and the lowest two energy groups fit well in the band picture discussed in Ref. 11. For example, at $S=7.5$ analysis shows that the highest-energy peak in Fig. 4 results from the transitions to the states of the first excited band in Fig. 7. This is consistent with the previous assertion that this peak is produced by the two-quasielectron process illustrated in Fig. 5. The initial system (seven electrons and one hole) at $S=7.5$ contains three Laughlin quasielectrons of the $\frac{1}{3}$ state. After the optical transition via the two-quasielectron process, the final system has one quasiexciton or quasielectron and quasihole pair, which is exactly what the states in the first excited band at $S=7.5$ in Fig. 7 represent. Following the same argument, the states in the higher-energy groups should belong to those of two and three quasielectron-quasihole pairs. The absence of the three-quasielectron process is also evident in Fig. 7, because no finite transition probability to the lowest band (the ground state) at $S=7$ and 7.5 is obtained. For $S>9.0$ ($\nu_e>\frac{1}{3}$), on the other hand, only the transitions to the lowest-energy band contribute to the PL intensity. The splittings at $S=9.5$ (and larger S values) apparently come from the fine structure of the lowest band. Our grouping of eigenstates according to the PL spectrum is of course quite rough for high-energy bands. A more accurate description can be obtained by using Jain's composite-fermion theory.[11]

Having studied the luminescence spectrum at a larger layer separation ($d=8$) and its relation to the band structure of the 2D electrons, we now proceed to discuss the properties of an electron-hole system at a separation which is of the order of the magnetic length. The electron-hole interaction, which is still much weaker than

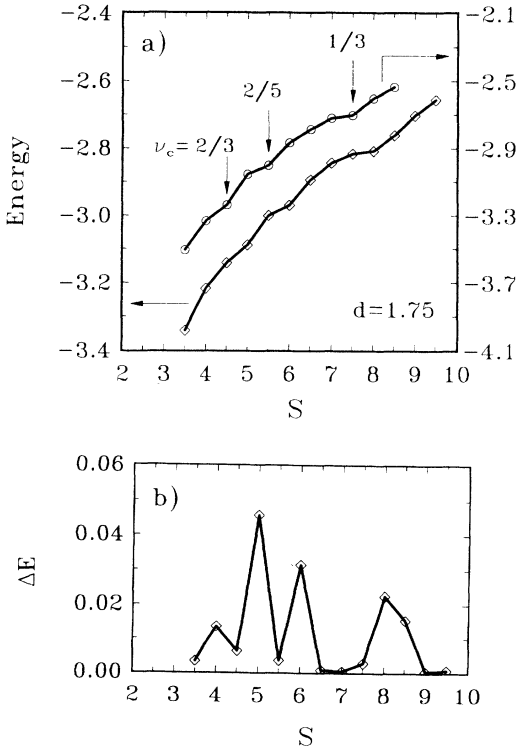

FIG. 8. (a) Ground-state energy as a function of S: diamonds, electron-hole system with $N_e=7$, $N_h=1$, and $d=1.75$; circles, system with six electrons only. (b) Energy difference between the ground state and the lowest excited state for an electron-hole system with $N_e=7$, $N_h=1$, and $d=1.75$. Note that the peaks here correspond to the cusps (kinks) in (a). Energy is in units of $e^2/\epsilon l$.

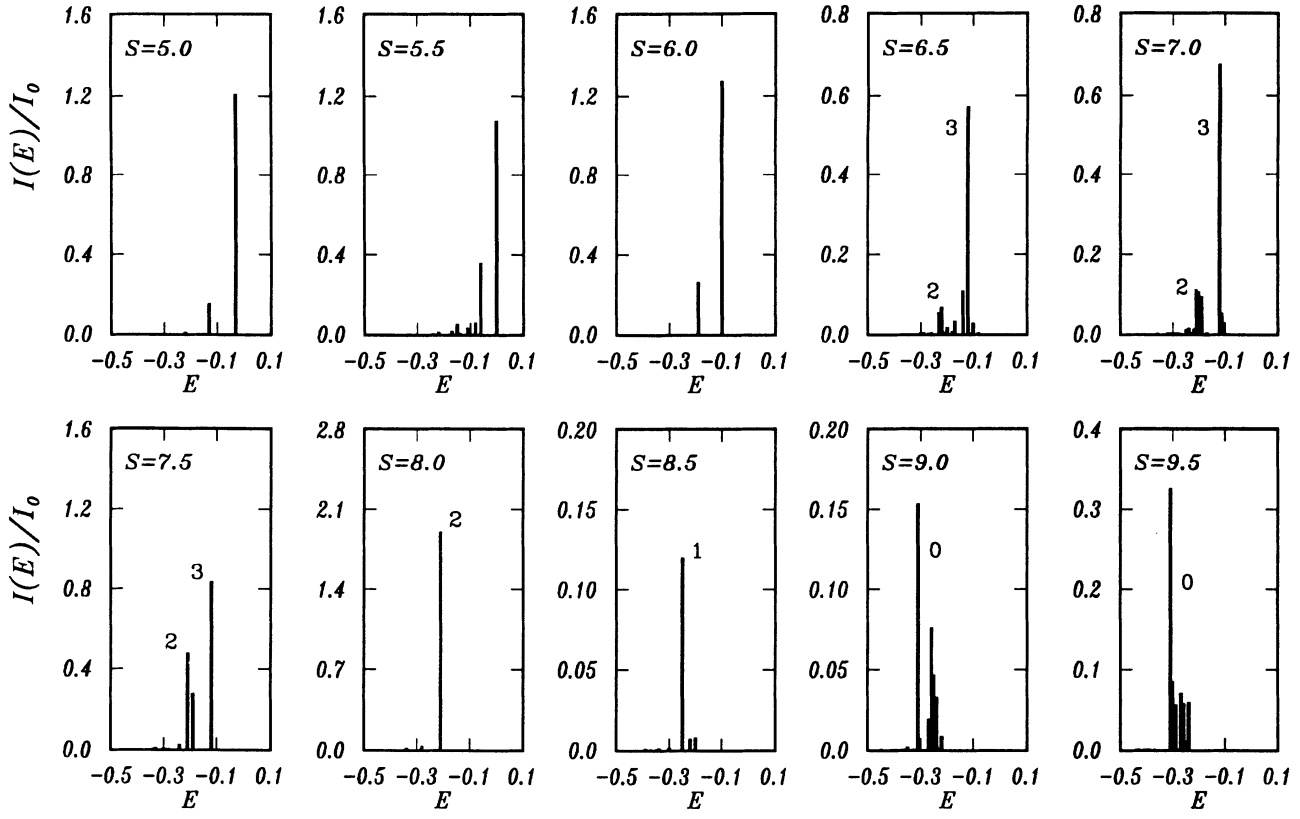

FIG. 9. Photoluminescence spectrum of an e-h system ($N_e=7$ and $N_h=1$) at $d=1.75$ and $T=0.005e^2/\epsilon l$ for S values from 5.0 to 9.5. Energy is in units of $e^2/\epsilon l$. The number beside a PL peak indicates the quasielectron process (Fig. 5) from which the peak results.

the electron-electron interaction, now plays an important role in understanding the PL. Because of the relatively weaker interlayer interaction, one would expect that in a certain range of layer separations each valence-band hole might be able to bind only one or two quasielectrons,[9] instead of a whole real electron. This is exactly what happens at $d\sim 1.5$. In Fig. 8(a) the ground-state energy of a system of seven electrons and one hole at $d=1.75$ is plotted as a function of S. A pronounced cusp is revealed at $S=8$. Two other weaker cusps (or kinks) appear at $S=6$ and 5. Also plotted in Fig. 8(a) is the ground-state energy of a six-electron system. As can be seen, by adding one electron-hole pair to the six-electron system the cusps (kinks) corresponding to $\nu_c=\frac{1}{3}$, $\frac{2}{5}$, and $\frac{2}{3}$ have all been

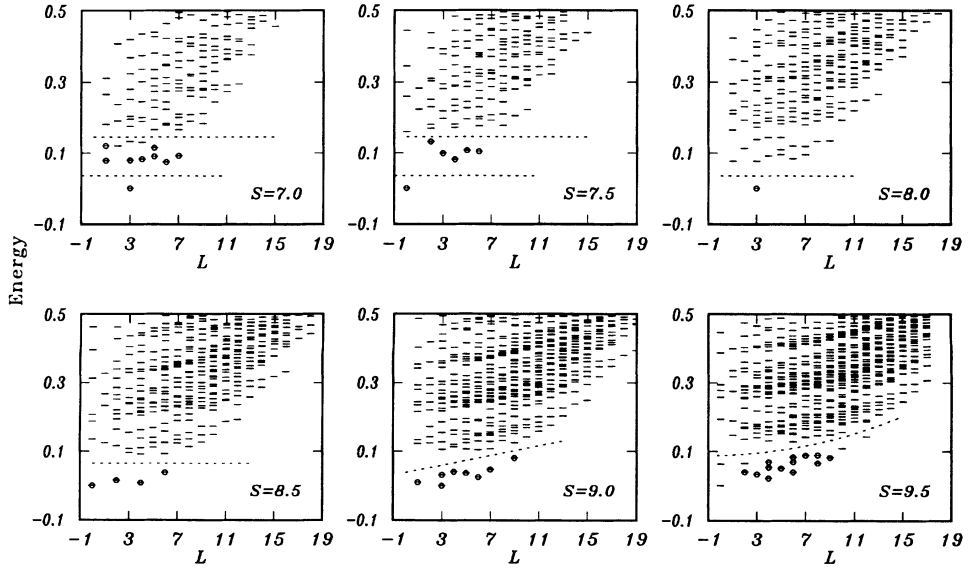

FIG. 10. Energy spectra of a six-electron system. States circled are those to which the optical transitions contribute significantly to the PL spectrum of Fig. 9.

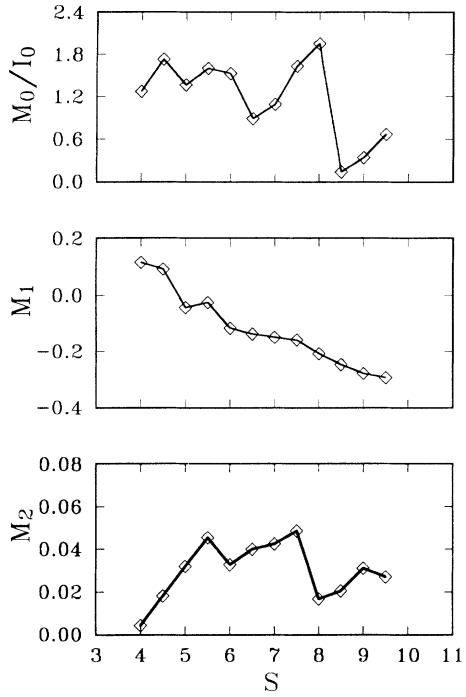

FIG. 11. Integrated intensity (M_0), centroid (M_1), and linewidth (M_2) of the PL spectrum for an e-h system ($N_e=7$ and $N_h=1$) at $d=1.75$. M_1 and M_2 are in units of $e^2/\epsilon l$.

shifted towards the right by 0.5 of the S value. We have also calculated the ground-state energy for an eight-electron and two-hole system at the same layer separation. The result shows that the positions of the cusps (kinks) are all shifted in S value by one unit towards the right relative to those in the six-electron system. In general for an N_e-electron and N_h-hole system ($N_e \gg N_h$) with $d \approx 1.5$–2.0, we would expect that cusps (kinks) appear at

$$2S = 2S_{\nu_c} + N_h \ , \tag{10}$$

where S_{ν_c} is the S value at which an incompressible state occurs for the $N_c = N_e - N_h$ electron system. For the $\nu_c = \frac{1}{3}$ state, we have $S_{\nu_c=1/3} = 3(N_c - 1)$ and therefore

$$2S = 3(N_e - 1) - 2N_h \ . \tag{11}$$

This relation suggests that the ground state at $S=8$ in Fig. 8(a) consists of a $\nu = \frac{1}{3}$ incompressible liquid of seven electrons and a bound-state complex of one hole and two Laughlin quasielectrons. In Fig. 8(b) we show the energy difference ΔE between the ground state and the lowest excited state for several S values. It is found that at the positions where the cusps appear in the ground-state energy, ΔE displays strong peaks characteristic of a dissipationless system. Calculated results for the electron-hole pair correlation function and the energy spectrum of the system also support the conjecture of the exotic bound-state complex (anyonic ion).[9]

In the thermodynamic limit, $(N_e - N_h)/2S_{\nu_c} \to p/q$, $N_e/2S \to \nu_e$, and $N_h/2S \to \nu_h$. If we introduce an effective electron filling factor

$$\tilde{\nu}_e \equiv \frac{\nu_e - \nu_h}{1 - \nu_h} \ , \tag{12}$$

the expression (10) implies a discontinuity in the chemical potential at $\tilde{\nu}_e = p/q$, and the cusp at $S=8$ in Fig. 8(a) corresponds to $\tilde{\nu}_e = \frac{1}{3}$.

The existence of the stable anyonic ions is also evident in the luminescence recombination processes. The PL spectrum of a seven-electron and one-hole system at $d=1.75$ is displayed in Fig. 9. Again we use the number of quasielectrons involved in the recombination process to label the peaks in the spectrum. Similar to the situation at $d=8$ where multipeak structure shows up only for $\nu_e < \frac{1}{3}$, the peak features appear only for $S < 8.0$ ($\tilde{\nu}_e < \frac{1}{3}$) in the present case. At $S=8$ ($\tilde{\nu}_e = \frac{1}{3}$), because of the bound state formed by two quasielectrons and the valence-band hole, only a single peak is obtained, resulting from the simultaneous recombination of the two bound quasielectrons with the free hole and creation of a single quasihole. The final state of such a transition is the single state of the lowest band in Fig. 10 (at $S=8$). Decrease the S value by one-half of a unit to $S=7.5$; a second peak emerges at the high-energy side. This peak, which is absent at $d=8$, comes from the three-quasielectron process corresponding to the transition to the ground state of the final system (Fig. 10). While the interaction between quasielectrons tends to suppress such a process, the interlayer interaction enhances it, so that it can occur at small values of d. Notice that the energy separation between peaks at $d=1.75$ is somewhat smaller than that at $d=8$. It is apparent that the anyonic ion (of two quasielectrons and one real hole) exists not only at $S=8.0$ but in a wide range of S values. This explains why two peaks at most are obtained in the PL spectrum, as well as the absence of zero- and one-quasielectron processes for $S \le 8.0$. Even at $S=9$ and larger, where no quasielectron is present in the seven-electron system (in the absence of a valence-band hole), the recombination process (at small values of d) which creates three quasiholes may also be quite different from the zero-quasielectron process discussed earlier. The interlayer interaction may first lead to a ground state in which one or two quasielectron-quasihole pairs are present in the initial system; the quasielectrons then recombine with the valence-band hole, producing the appropriate number of quasiholes. The final outcome is of course the same as that of the zero-quasielectron process. The moments of the PL spectrum at $d=1.75$ are plotted in Fig. 11 as a function of S. Notice that the average PL energy (M_1) shows both downward and upward cusps near $S=8$, 6, and 5 (corresponding to $\tilde{\nu}_e = \frac{1}{3}$, $\frac{2}{5}$, and $\frac{2}{3}$). The integrated intensity also displays a big drop near these filling factors, especially near $\tilde{\nu}_e = \frac{1}{3}$, which is in agreement with a recent experimental observation.[26]

III. e-A^+ SYSTEM

As we have mentioned in Sec. I, because of sample defects (such as impurities, dislocations, interface roughness, etc.) some valence-band holes may be localized. In

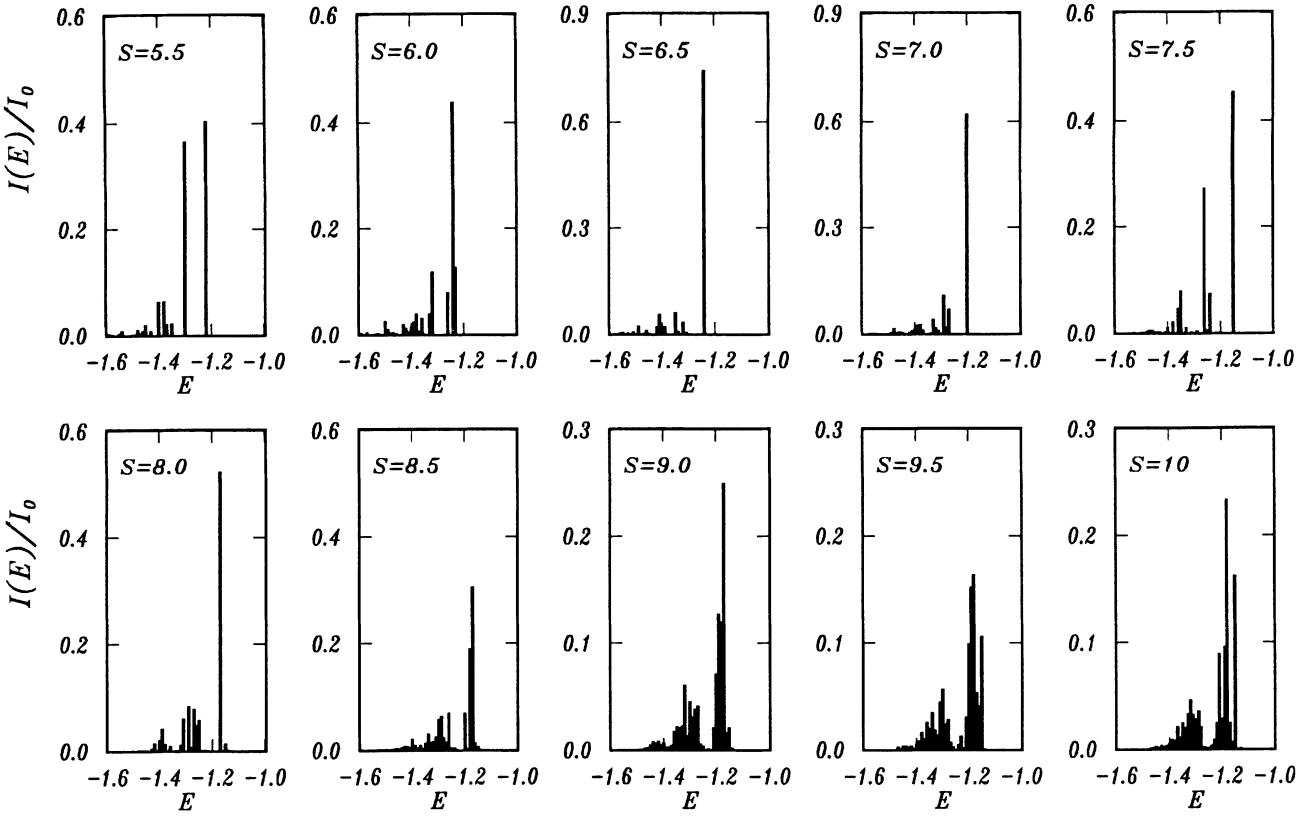

FIG. 12. Photoluminescence spectrum of an e-A^+ system ($N_e=7$ and $N_h=1$) at $d=0$ and $T=0.005e^2/\epsilon l$ for S values from 5.5 to 10. Energy is in units of $e^2/\epsilon l$.

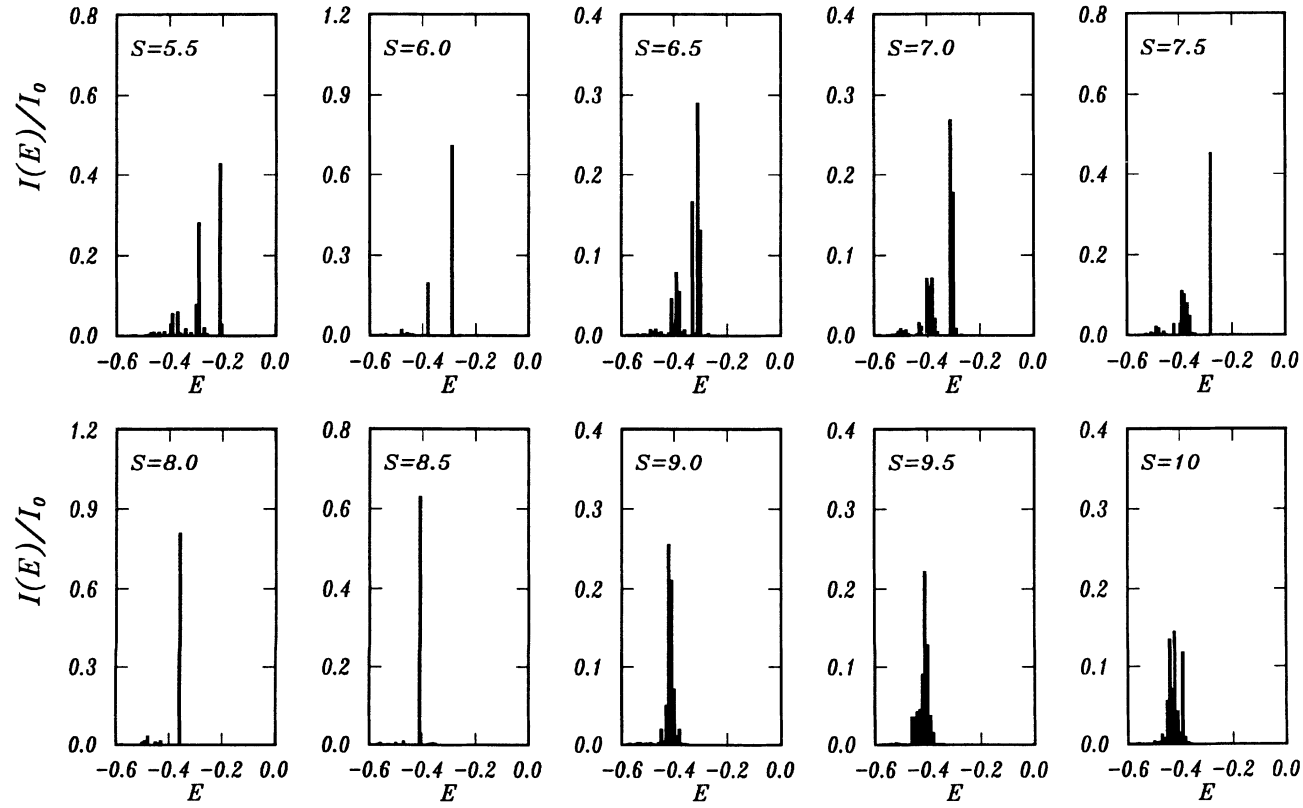

FIG. 13. Photoluminescence spectrum of an e-A^+ system ($N_e=7$ and $N_h=1$) at $d=1.3$ and $T=0.005e^2/\epsilon l$ for S values from 5.5 to 10. Energy is in units of $e^2/\epsilon l$.

this section we study the properties and the PL of an electron-A^+ system. We assume that the hole is localized at the north pole of the sphere with a wave function

$$\Phi^h(\theta,\phi)=(-1)^{2S}\left[\frac{2S+1}{4\pi}\right]^{1/2}\cos^{2S}(\theta/2)e^{-i2S\phi}\ . \quad (13)$$

It has an angular momentum (z component) $m=-S$. The luminescence operator (3) is then given by

$$\hat{L}=(-1)^{2S}a_S b_{-S}\ , \quad (14)$$

where a_S and b_{-S} are the annihilation operators of an electron and a hole with z-component angular momentum S and $-S$, respectively. The initial electron system of the PL process is perturbed by the localized hole, and the eigenstates can only be classified by the z component M of the total angular momentum. The PL spectrum of a system of seven electrons and one localized hole at $d=0$ is shown in Fig. 12 for S values ranging from 5.5 to 10. Notice that, even for this coplanar case, structure in the spectra is clearly observable. In particular at $\nu_e \le \frac{1}{3}$ ($S \ge 9$) two broad peaks are obtained. The peak with the higher energy results from the transition to the lowest band of six electrons (Fig. 10), and its width increases as the filling factor is decreased. The lower-energy peak, which is absent for the coplanar e-h system, results from the transition to the first excited band and corresponds to a shakeup process which creates three quasiholes and one quasielectron-quasihole pair. For $S<9$ the PL spectrum generally consists of a single dominant peak (coming from the transition to the lowest band of six electrons) and some small spectral weight spreading over the low-energy side, except for the case of $S=7.5$ where three well-defined peaks are revealed.

As the localized hole is moved away from the electron layer, the spectral intensity of the lower-energy peak for $S \ge 9.0$ starts to decrease, and at about $d=1$ only the high-energy peak remains. In Fig. 13 we plot the PL spectrum at $d=1.3$ for several S values with $T=0.005(e^2/\epsilon l)$. The spectrum in Fig. 13 looks very much like that in Fig. 9. Calculations of the ground-state energy for the initial system at $d=1.3$ reveal a pronounced cusp at $S=8$ and apparent kinks at $S=6$ and 5, similar to what is shown in Fig. 8(a). These cusps or kinks indicate the presence of anyonic ions consisting of a localized hole and a few Laughlin quasielectrons.[27] The PL spectrum at a much higher temperature $T=0.02e^2/\epsilon l$ (for $d=1.3$) is shown in Fig. 14. In general, raising the temperature broadens the PL peaks and increases the relative spectral weight of the low-energy peaks. For the purpose of comparison, in Fig. 15 we plot the moments of PL intensity, defined in Eqs. (4)–(6), as a function of Landau-level degeneracy for two temperatures. At $T=0.005e^2/\epsilon l$, well-defined steps are obtained in the plot of the centroid (the average PL energy M_1).

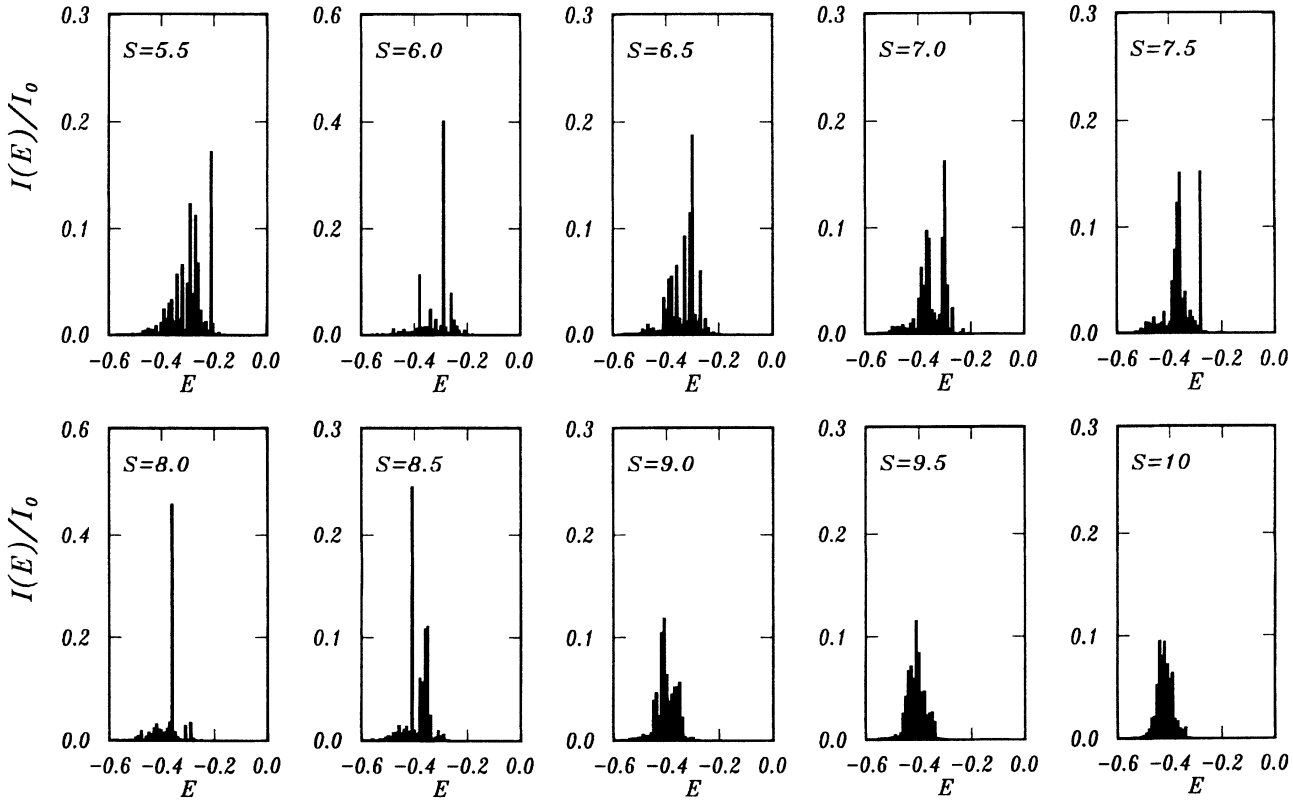

FIG. 14. Photoluminescence spectrum of an e-A^+ system ($N_e=7$ and $N_h=1$) at $d=1.3$ and $T=0.02e^2/\epsilon l$ for S values from 5.5 to 10. Energy is in units of $e^2/\epsilon l$.

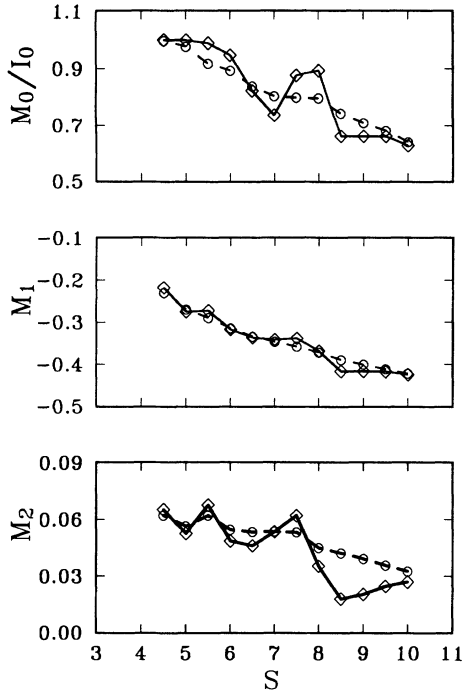

FIG. 15. Integrated intensity (M_0), centroid (M_1), and linewidth (M_2) of the PL spectrum for an e-A^+ system ($N_e=7$ and $N_h=1$) at $d=1.3$: diamonds and solid line, $T=0.005e^2/\epsilon l$; circles and dashed line, $T=0.02e^2/\epsilon l$. M_1 and M_2 are in units of $e^2/\epsilon l$.

Similar steps are observed experimentally in Refs. 2 and 28, where the e-A^0 recombination processes are believed to have the dominant contribution to the PL intensity. At very large layer separation the structure of the PL spectrum resembles that of an e-h system (Fig. 4), although the distribution of the spectral weight is somewhat different.

IV. e-A^0 SYSTEM

We now investigate the luminescence spectrum resulting from the recombination of an electron with the hole of a neutral acceptor complex. There have been quite a few experimental reports[2,28] on the photoluminescence of such systems, especially at small Landau-level filling factors, where the Wigner solid state is energetically favorable. An obvious difference between the e-A^0 system and the systems we discussed in the previous sections is that the initial state of electrons in the present system is free of perturbations while the final state is disturbed by the negatively charged acceptor ion. The perturbation to the final state of the system is treated as that of a negative point charge. The neutral acceptor ion is placed at the north pole in our calculation. The luminescence operator in the present case is simply $\hat{L}=a_S b_{\rm ac}$, where $b_{\rm ac}$ is the annihilation operator of the hole bound to the acceptor ion, and a_S is the annihilation operator of an electron with a z-component angular momentum $L_z=S$. The PL intensity (2) can be rewritten as

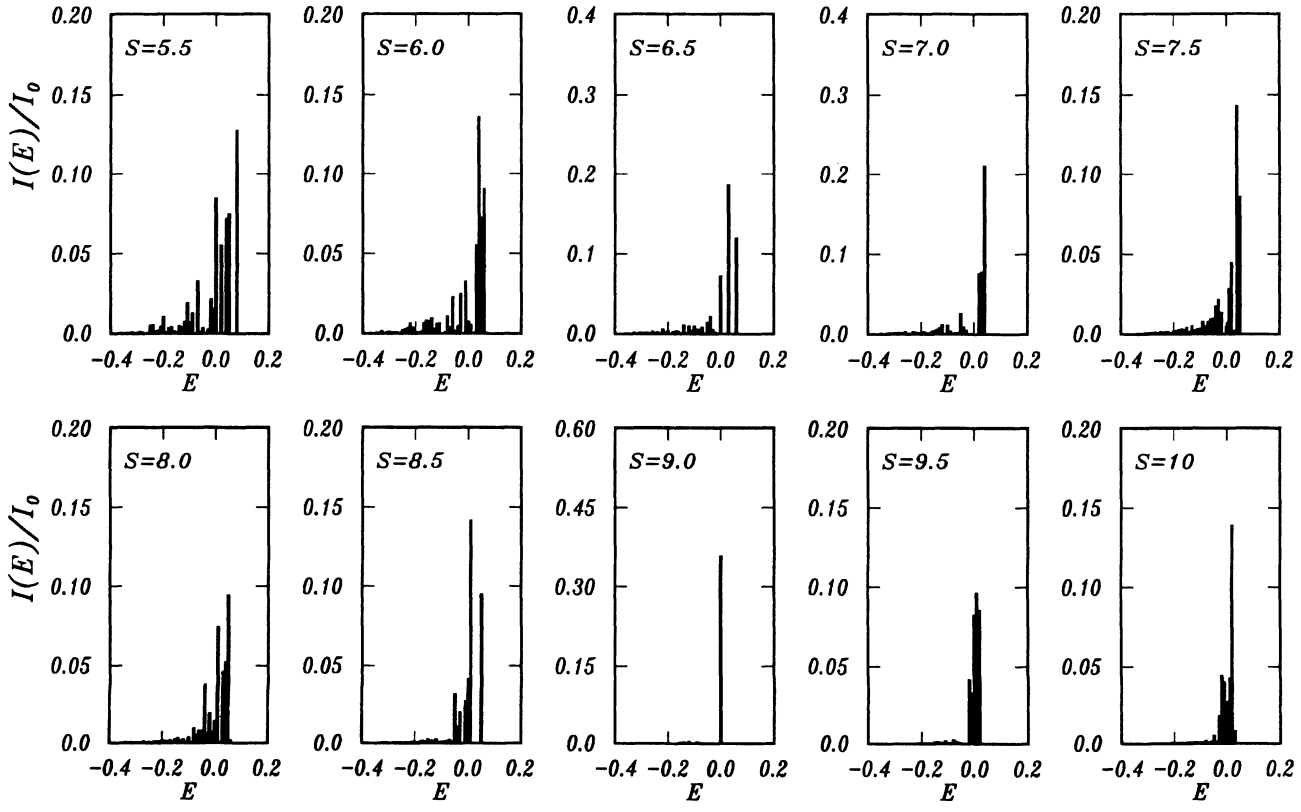

FIG. 16. Photoluminescence spectrum of an e-A^0 system ($N_e=7$ and $N_h=1$) at $d=0$ and $T=0.005e^2/\epsilon l$ for S values from 5.5 to 10. Energy is in units of $e^2/\epsilon l$.

$$I(\omega)=\frac{I_0}{Z}\sum_{i,f} e^{-E_i/k_BT}|\langle\Psi_f^{(N_e-1)}|a_S|\Psi_i^{(N_e)}\rangle|^2 \times\delta(\omega-E_i+E_f)\ . \quad (15)$$

Here $|\Psi_i^{(N_e)}\rangle$ is the initial state of N_e electrons and $|\Psi_f^{(N_e-1)}\rangle$ is the final state of N_e-1 electrons perturbed by the acceptor ion at the north pole. It is not difficult to show that the zero moment of the PL spectrum is given by a simple expression

$$M_0=\frac{N_e}{2S+1}I_0\ , \quad (16)$$

which is independent of the perturbation to the final system of the negatively charged ion. In the thermodynamic limit, we have $M_0=\nu I_0$.

In the previous sections we have shown that the features in the PL spectrum are closely related to the energy structure of the final electron system. For an e-A^0 system, at $d<1$ the perturbation due to the acceptor ion is so strong that the Laughlin incompressible state and the band structure of the final system are completely destroyed. One therefore should not expect any multipeak features in the PL spectrum. In Fig. 16 the luminescence spectra are plotted for several S values at $d=0$. While the spectrum displays a single well-defined peak for $\nu_e<\frac{1}{3}$ ($S>9$), a very broad and asymmetric PL peak (or band) is obtained for $S<9$. As the acceptor layer moves away from the 2D electrons and the perturbation of the negative ion becomes weaker, the broad peak for $S<9$ starts to split into a few narrower peaks. Figure 17 displays the PL spectra at $d=3.0$, where the multipeak structure is clearly visible for $S<9.0$. The separation between peaks increases as the layer separation is increased, and reaches the order of $0.14e^2/\epsilon l$ at a large value of d. The integrated intensity, centroid, and linewidth of the PL spectrum are shown in Fig. 18. An analytic theory developed by Apal'kov and Rashba[29] for an e-A^0 PL system shows that the first moment of the spectrum at a very large layer separation should display a downward cusp at $\nu_e=\frac{1}{3}$. Our numerical results at $d=8$ are consistent with the prediction of the analytic theory, which has been employed to explain the downward cusps observed experimentally in Ref. 28 (although both downward and upward cusps are obtained in the experiments). The experimental data for the integrated intensity as a function of magnetic field, on the other hand, display some structure, in contrast to the result of Eq. (16), where a linear dependence of M_0 on the magnetic field is predicted. The origin of this discrepancy is not known at present.

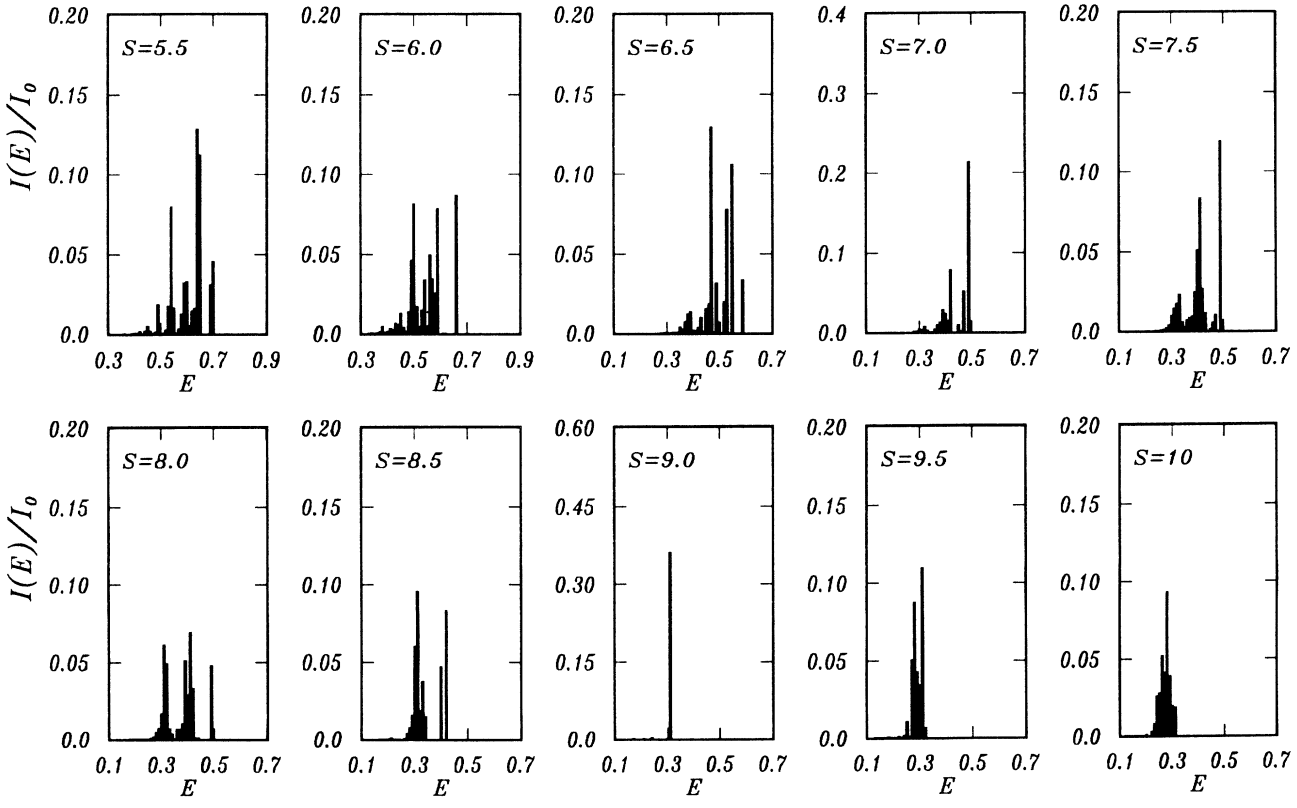

FIG. 17. Photoluminescence spectrum of an e-A^0 system ($N_e=7$ and $N_h=1$) at $d=3.0$ and $T=0.005e^2/\epsilon l$ for S values from 5.5 to 10. Energy is in units of $e^2/\epsilon l$.

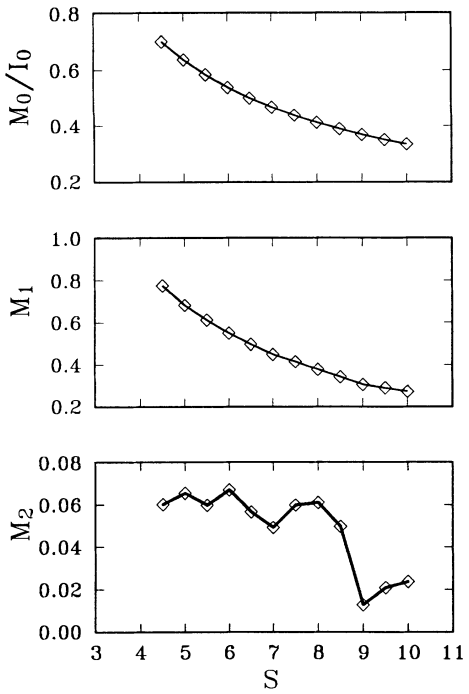

FIG. 18. Integrated intensity (M_0), centroid (M_1), and linewidth (M_2) of the PL spectrum for an e-A^0 system ($N_e=7$ and $N_h=1$) at $d=3.0$. M_1 and M_2 are in units of $e^2/\epsilon l$.

V. CONCLUSION

We have presented a comprehensive study of the photoluminescence recombination process for three fractional quantum Hall systems (e-h, e-A^+, and e-A^0) with an electronic filling factor around $\frac{1}{3}$ (from $\nu_e \sim \frac{1}{2}$ to $\nu_e \sim \frac{2}{7}$). Multipeak structure is obtained in the PL spectrum for all three systems. The two essential elements which are of particular importance in determining and understanding these peak features are the interlayer interaction and the energy-band structures of 2D electrons. While the former controls the spectral weight distribution, the latter determines the energy separation between the peaks. For a coplanar e-h system, because of the symmetry between the electron-electron interaction and electron-hole interaction, only a single peak at the exciton binding energy is obtained. The intensity of the peak depends strongly on the effective filling factor ν_c ($=\nu_e-\nu_h$) and temperature T. For a given temperature the intensity has its local maximum at $\nu_c=p/q$ with q being an odd integer. In contrast to the prediction in Ref. 18, our calculations show no sign of a FQHE at $\nu_c=p/q$.

For an e-h system with $d\neq 0$, we find that a valence-band hole can form various bound states with Laughlin quasielectrons. In particular, at around $d=1.5$ a hole binds two quasielectrons. The conjecture of the stable anyonic ions is based on the analysis of the ground-state properties of e-h systems, and is further supported by the calculated results of the photoluminescence spectra. Since the quasielectrons are neither bosons nor fermions but satisfy fractional statistics, the nature of the anyonic ion is quite different from that of an ordinary ion. We have also studied[30] a simple anyonic system consisting of a few anyons having a charge $-e/3$ and satisfying $\frac{1}{3}$ statistics and a negative point charge located at a distance d away from the anyon layer. Our calculation shows that in a strong magnetic field and at $d\sim 1.5$ a bound state of two anyons is indeed energetically favorable. Experimental detection of these ionic states will certainly be of great interest and fundamental importance, and provide further support for the present FQHE theory.

Near $\nu_e=\frac{1}{3}$ there are four possible PL recombination processes, corresponding to the annihilation of N quasielectrons and a real hole and the creation of $N-3$ quasiholes ($N=0,1,2,3$). Each of them could produce a peak in the PL spectrum. For a large layer separation only three peaks are obtained; the three-quasielectron process gives little PL intensity. This is in agreement with experimental observations reported in Ref. 25, where a broad PL spectrum of an e-h system can be viewed as a superposition of three well-defined peaks separated by an energy of the order of the Laughlin gap. At $d\sim 1.5$, the photoluminescence displays a single peak for $\tilde{\nu}_e<\frac{1}{3}$ [where $\tilde{\nu}_e\equiv(\nu_e-\nu_h)/(1-\nu_h)$], and a doublet structure for $\tilde{\nu}_e>\frac{1}{3}$. The temperature dependence and the energy separation of the doublet structure are similar to those of the twin peaks revealed in Ref. 3. Nevertheless, we are reluctant to claim that the calculated doublet structure is *the* explanation of the experimentally observed structure, since the latter exists at $\nu_e<\frac{1}{3}$ as well. A thorough understanding of the twin-peak structure in Ref. 3 will not be possible until experimentalists can accurately determine the location of the valence-band hole relative to the electron layer, as well as the possible internal structure of the hole state.

ACKNOWLEDGMENTS

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