PHYSICAL REVIEW B VOLUME 50, NUMBER 3 15 JULY 1994-I

Acoustic-mode coupling and electron heating in thin metal films

Kris Johnson and M. N. Wybourne
Department of Physics, University of Oregon, Eugene, Oregon 97403

N. Perrin
Laboratoire de Physique de la Matière Condensée de l'Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris CEDEX 05, France and Université Pierre et Marie Curie, 4 Place Jussieu, 75252 Paris CEDEX 05, France

(Received 14 April 1994)

We discuss the heating of electrons by a steady-state electric field in an unbounded thin metal film by considering the energy flow from the electron system to the acoustic modes of the film. Using the dispersion relationships and the resulting density of states for the acoustic modes calculated from a non-Debye model, and a deformation potential coupling between the electrons and the acoustic modes, it is shown that the relationship between the electric field and the electron temperature is dependent on the coupling between acoustic modes at the surface of the film. At low excitation we show the energy relaxation is dominated by the antisymmetric Rayleigh mode.

The acoustic-mode spectrum in an unbounded elastic, isotropic plate has been known and studied for a long time.[1–3] Two distinct families of modes exist.[3,4] The first is a set of purely transverse modes that have polarization vectors in the plane of the plate and consequently do not couple to other modes at the free surface of the plate. These modes are described by the straightforward dispersion relationship $\omega^2=v_t^2[\beta^2+(n\pi/d)^2]$, where v_t is the transverse wave velocity, d is the plate thickness, n is an integer ($n\geq 0$) and β is the wave vector in the plane of the plate. At the Brillouin zone center, one mode goes linearly to zero frequency while all the other modes tend toward a finite limit. At large β or in thick films, all the modes tend asymptotically to the dispersion relationship for a bulk sample. The second family of modes results from the coupling between longitudinal and transverse partial waves that occurs at the free surfaces. These are the Lamb waves,[1] the lowest two of which go to zero frequency at the zone center and are the Rayleigh modes of the plate.[2] Precise details of the Lamb wave dispersion relationships depend on the ratio of the longitudinal v_l and transverse wave velocities v_t, but in general, as the plate is made thinner the energy separation between the modes increases. At the Brillouin zone center the energy separation of the modes is approximately $\hbar\pi v/d$ where v can be either v_l or v_t. In most situations the energy separation between the modes is much smaller than the thermal energy of the lattice kT; hence many modes are occupied. In this case the acoustic spectrum and density of states may be considered to be Debye-like. However, the Debye model is not a good approximation when the mode separation exceeds kT, that is, $Td<\hbar\pi v/k$. Thus, as the product Td is reduced the change in the effective character of the acoustic spectrum is expected to be manifest in a variety of phenomena associated with the thermal excitations in thin plates.

In this paper we discuss the heating of electrons caused by a voltage applied across an unbounded metal film. The voltage produces a field E in the film. We consider the balance between the energy gained by the electrons from the field and the energy lost to the lattice. We use a density of states appropriate for the two families of acoustic modes supported by the film, and a deformation potential coupling between the electrons and the acoustic modes; we do not assume a linear dispersion for the acoustic modes. We show the relationship between the electron temperature T_e and the electric field depends on the coupling between the longitudinal and transverse partial waves that occurs at the free surfaces. The high field results are consistent with previous work that used an inelastic electron scattering rate due to phonons as a power law of T_e.[5–7] In the present work we demonstrate that the acoustic-mode coupling at the free surfaces of the film strengthens the electric field dependence of the electron temperature.

The electric field heating of electrons in thin metallic films depends on the electron-phonon interaction and on the resulting nonequilibrium phonon distribution. Assuming an inelastic electron-electron scattering time that is sufficiently small to thermalize the electron system, neglecting the slight anisotropy induced by the current flow, and considering the film to be immersed in helium,[8] the electron system can be considered as an equilibrium system. In the steady state, a current density $\vec{J}$ flows in the film and the power $P_e=\vec{J}\cdot\vec{E}$ provided to the electron system is equal to the rate of energy transfer to the phonons:[9]

$$\vec{J}\cdot\vec{E}=\sum_{\text{modes}}\int\hbar\omega g(\omega)\frac{N(\omega,T_e)-N(\omega)}{\tau_{ep}}\,d\omega, \qquad (1)$$

where $N(\omega,T_e)$ is the Bose distribution at T_e, $g(\omega)$ is the phonon density of states, and τ_{ep} is the electron-phonon scattering time of the mode considered. The spatially uniform nonequilibrium phonon distribution in the steady state $N(\omega)$ is deduced from the Boltzmann equation, the rate of energy loss from the phonons is characterized by the escape time τ_p from the film to the helium. We use $\tau_p=\eta d/|v_g|$ where v_g is the group velocity for a particular mode and η is a mismatch parameter related to the Kapitza resistance.[10] Introducing into Eq. (1) the steady-state expression of $N(\omega)$

0163-1829/94/50(3)/2035(4)/$06.00

deduced from the Boltzmann equation, we obtain the following relation between E and T_e:[9]

$$\vec{J}\cdot\vec{E}=\sum_{\text{modes}}\int \hbar\omega g(\omega)\frac{N(\omega,T_e)-N(\omega,T_h)}{\tau_p+\tau_{ep}}\,d\omega, \qquad (2)$$

where T_h is the temperature of the surrounding helium.

In the analysis of Kanskar *et al.*[7] the electron-phonon scattering time τ_{ep} is given by the Pippard result[11] calculated in the Debye approximation. Schmid[12] has shown that the Pippard expressions of τ_{ep} for the longitudinal and transverse polarizations can be deduced from the phonon self-energy in the isotropic model introduced by Abrikosov *et al.*[13] Following the above procedures,[12,13] beyond the Debye model and introducing the phonon dispersion, we again find the Pippard result[11] for both the longitudinal and transverse phonons, with $\tau_{ep}(\beta)=1/[2\alpha(\beta)v_g(\beta)]$ where α is the sound attenuation.

In previous work on electron heating in films,[7,14] the coupling between the longitudinal and transverse waves at the film surfaces has been neglected. In order to analyze the effect of coupling we first calculate the complete dispersion relationship of the modes. This relationship can be obtained from the Rayleigh-Lamb equations for an isotropic unbounded film,

$$\frac{\tan q_t d/2}{\tan q_l d/2}=-\left[\frac{4\beta^2 q_l q_t}{(q_t^2-\beta^2)^2}\right]^{\pm 1} \qquad (3)$$

for symmetric $(+1)$ and antisymmetric (-1) solutions, where symmetric and antisymmetric refer to the boundary displacement.[4] The magnitudes of the wave vector components q_l and q_t perpendicular to the film are related to the frequency $\omega(\beta)$ by

$$\beta^2+q_{l,t}^2=\left(\frac{\omega}{v_{l,t}}\right)^2, \qquad (4)$$

where l is for the longitudinal and t for the transverse mode. By solving Eqs. (3) and (4) numerically, we obtain the dispersion relation $\omega(\beta)$.

The density of states for the coupled modes is obtained from the dispersion curves by calculating the group velocity $v_g=d\omega(\beta)/d\beta$.

For each mode the number of states $g(\omega)$ per unit volume and per unit frequency is

$$g(\omega)=\frac{1}{2\pi d}\,\frac{\beta}{|v_g|}. \qquad (5)$$

In deriving this result, we have considered the energy surface of revolution generated by the curve $\omega(\beta)$ in a two-dimensional isotropic phase space; the constant energy surfaces are disks as in a Debye model but the group velocity depends on β.

The electron-phonon time in Eqs. (1) and (2) for the uncoupled transverse mode τ_{ep} is given by the Pippard expression. In the case of coupled modes, we use the relaxation times for the transverse τ_{ep}^T and longitudinal τ_{ep}^L modes weighted by the square of the amplitude ratio $|A_L/A_T|^2=\gamma$ of the partial longitudinal and transverse waves according to the expression

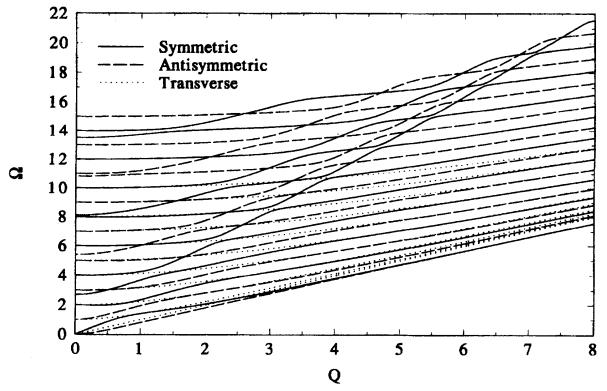

FIG. 1. Acoustic-mode dispersion relationship.

$$\frac{1}{\tau_{ep}}=\frac{\gamma}{\gamma+1}\,\frac{1}{\tau_{ep}^L}+\frac{1}{1+\gamma}\,\frac{1}{\tau_{ep}^T}. \qquad (6)$$

The square of the amplitude ratio has been used because the electron-phonon matrix element depends linearly on displacement, thus the rate depends on the square of displacement.[15]

To investigate the electron heating characteristics we have applied the model to a 25-nm-thick gold film. The parameters we have used are an electron density of 5.9×10^{28} m^{-3}, longitudinal and transverse velocities of sound $v_l=3.2\times10^3$ ms^{-1} and $v_t=1.2\times10^3$ ms^{-1}, respectively. The elastic mean-free path of the electrons was taken to be $l_e=17$ nm.

The dispersion relationship for the first eleven acoustic uncoupled transverse, eleven symmetric, and eleven antisymmetric modes of the unbounded film are shown in Fig. 1 in terms of a reduced frequency $\Omega=\omega d/\pi v_t$ and a reduced wave vector $Q=\beta d/\pi$. The density of states for the coupled and uncoupled modes has been calculated from the dispersion curves, and is shown in Fig. 2 in terms of $G(\Omega)=2d^2v_t g(\omega)$. At $\beta=0$, $G(\Omega)$ for the uncoupled transverse mode has steps at integer values of Ω. Between the steps, $G(\Omega)$ has a linear frequency dependence which is characteristic of a two-dimensional acoustic spectrum. The density of states for the symmetric (antisymmetric) modes has steps at even (odd) values of Ω and at $\Omega=mv_l/v_t$ where m is odd (even). The steps at $\Omega=mv_l/v_t$ are small and are indicated by arrows on Fig. 2. At $Q=0$, half of the symmetric and antisymmetric modes are degenerate with the uncoupled transverse modes. Unlike the case for the uncoupled modes, a consequence of mode coupling close to $Q=0$ is an enhancement of $G(\Omega)$ on the high frequency side of the steps. Moreover, at wave vectors where the coupling causes v_g to become zero van Hove singularities occur as seen in Fig. 2 for the symmetric mode at $\Omega=1.98$. The total density of states for the gold film is the sum of the density of states for the uncoupled transverse waves and the symmetric and antisymmetric waves as shown in Fig. 2. In the high frequency limit the total density of states converges to the frequency dependence and magnitude expected from a Debye model of bulk gold.

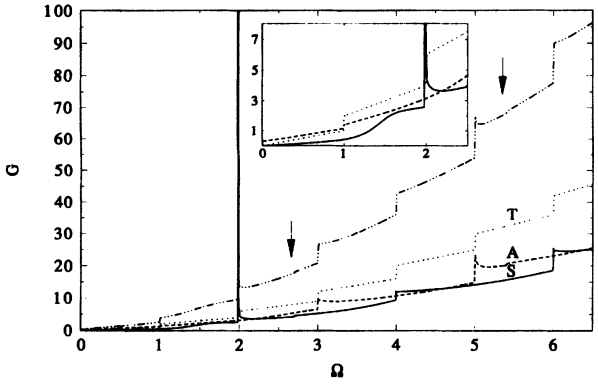

FIG. 2. The density of states for the uncoupled transverse (T), symmetric (S), and the antisymmetric (A) acoustic modes. The upper curve is the total density of states. The arrows indicate the steps at v_l/v_t and $2v_l/v_t$. The inset is an expanded view of the low frequency region.

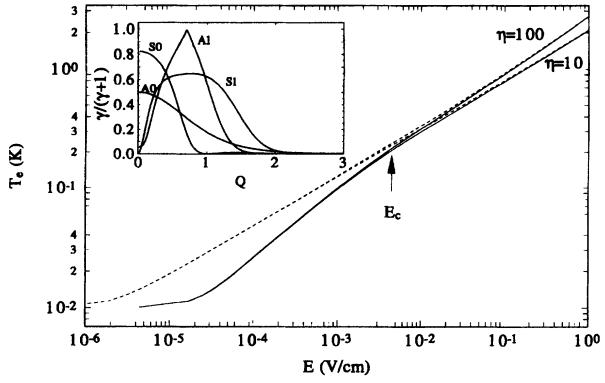

FIG. 4. The electron temperature as a function of the electric field with $\eta=10$ and $\eta=100$. The solid curves are with mode coupling, the dashed curves are without mode coupling. The inset shows the amplitude ratios for the first two symmetric $S0$, $S1$ and antisymmetric $A0$, $A1$ modes.

The electron-acoustic wave relaxation times for the coupled symmetric and antisymmetric modes were calculated as a function of Q using the Pippard expressions[11] and are shown in the inset to Fig. 3. The ratio γ in Eq. (6) was calculated as a function of Q using the Rayleigh-Lamb equations (3). Examples of the amplitude ratios for the first two symmetric and antisymmetric modes are shown in the inset to Fig. 4. As expected, it is seen in the inset to Fig. 3 that in the limit $\beta l_e<1$ the relaxation rate for all the modes increases as Q^2 and when $\beta l_e>1$ the rate becomes proportional to Q and independent of l_e.[11] To visualize which modes dominate the electron energy relaxation, we have used the dispersion to plot the relaxation time as a function of Ω, as shown in Fig. 3. At frequencies up to $\Omega=1$ the antisymmetric Rayleigh mode dominates the relaxation with a time that is inversely proportional to the frequency. In this frequency range the times for the symmetric Rayleigh and transverse waves both have a power law dependence

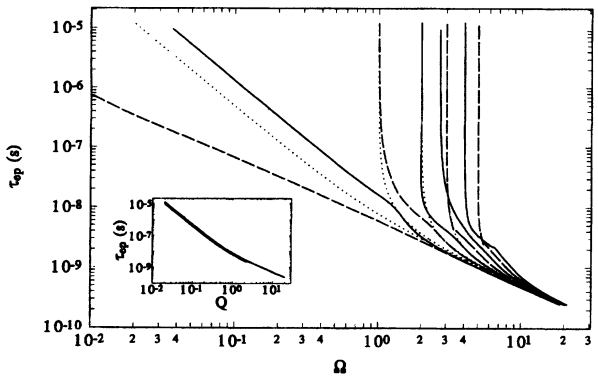

FIG. 3. The electron-phonon relaxation time for the first four transverse (dotted curves), symmetric (solid curves), and antisymmetric (dashed curves) modes as a function of the reduced frequency. The inset shows the electron-phonon relaxation time as a function of the reduced wave vector. The increased width of the curve indicates the range of times for the different modes. At high Q the times for all modes converge to a single value.

$\tau_{ep}\sim\Omega^{-2}$. At higher frequencies, relaxation is possible to higher energy acoustic modes of the film by the onset of additional relaxation at $\Omega=1,2,3,\ldots$ and $\Omega=v_l/v_t, 2v_l/v_t, \ldots$ as seen in Fig. 3. For clarity, we have only plotted the times up to $\Omega=5$. Above the onset frequencies all the relaxation times asymptotically approach an Ω^{-1} frequency dependence.

The electric field dependence of the electron temperature shown in Fig. 4 has been determined by numerically evaluating Eq. (2) at a substrate temperature $T_h=10$ mK, and with $\eta=10$ and 100. The number of modes that need to be considered depends on the electric field. In the present work the thirty-three modes discussed above were found to be sufficient for convergence. For comparison we have also performed the calculation assuming no coupling between the longitudinal and transverse waves at the film surfaces. At high electric fields the same power law relationship is found for both the coupled and uncoupled calculations. In agreement with Kanskar *et al.*,[7] the power law depends on τ_p, and in the limit of small τ_p, $T_e\sim E^{2/5}$.[5,6] With mode coupling the electric field dependence of T_e is found to strengthen below a field E_c at which $T_e\approx\hbar\pi v_t/5kd$ and becomes approximately $T_e\sim E^{0.6}$ as seen in Fig. 4. In these conditions, the mode separation is about five times the thermal energy kT_e. Since for all the modes $G(\Omega)$ are very similar below $\Omega=1$, they can be factored out of the mode summation in Eq. (2) leaving a summation within the integral over the rates $(\tau_{ep}+\tau_p)^{-1}$. Furthermore, the mean value of τ_p is less than τ_{ep} in this frequency range; so the energy relaxation is dominated by the smallest relaxation time which from Fig. 3 is the antisymmetric Rayleigh mode.

The power ($\propto E^2$) required to start heating the electrons above T_h is seen in Fig. 4 to be one hundred times larger for the coupled modes than for the uncoupled modes. This result shows that the electron and phonon systems are more strongly interacting in the case of acoustic-mode coupling at the surfaces.

Electron heating in thin films where the acoustic-mode spectrum is modified by spatial quantization has been studied. We have used a non-Debye model for the acoustic den-

sity of states. It has been shown that the electron temperature becomes more strongly dependent on the electric field as theresult of acoustic-mode coupling.

This work was partly supported by the National Science Foundation under Grant No. DMR-9019525, and a grant from NATO.

[1] H. Lamb, Proc. R. Soc. London **93**, 114 (1917).

[2] Lord Rayleigh, Proc. London. Math. Soc. **20**, 225 (1889).

[3] B. A. Tavger and V. Ya. Demikhovkij, Izv. Vyssh. Uchebn. Zaved. Fiz. **4**, 130 (1966).

[4] B. A. Auld, *Acoustic Fields and Waves in Solids* (Wiley, New York, 1973).

[5] P. W. Anderson, F. Abrahams, and T. V. Ramakrishnan, Phys. Rev. Lett. **43**, 718 (1979).

[6] M. R. Arai, Appl. Phys. Lett. **42**, 906 (1983).

[7] M. Kanskar, M. N. Wybourne, and Kris Johnson, Phys. Rev. B **47**, 13 769 (1993).

[8] S. I. Rokhlin, D. E. Chimenti, and A. H. Nayfeh, J. Acoust. Soc. Am. **85**, 1074 (1989), and references therein; the perturbation of the Lamb wave spectrum by plate loading with liquid helium of density $\rho_0=0.145$ g cm^{-3} is shown to be negligible.

[9] N. Perrin and H. Budd, Phys. Rev. Lett. **28**, 1701 (1972); J. Phys. (Paris) **33**, C4-33 (1972).

[10] E. T. Swartz and R. O Pohl, Rev. Mod. Phys. **61**, 605 (1989).

[11] A. B. Pippard, Philos. Mag. **46**, 1104 (1955).

[12] A. Schmid, Z. Phys. **259**, 421 (1973).

[13] A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Dover, New York, 1975).

[14] Y. K. Kwong, K. Lin, M. S. Isaacson, and J. M. Parpia, J. Low Temp. Phys. **93**, 7 (1993).

[15] J. M. Ziman, *Electrons and Phonons* (Clarendon, Oxford, 1960).