Nonlinear current-voltage characteristics at quantum Hall resistance minima

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The longitudinal resistivity ρ_{xx} at the quantum Hall midplateau of a low-mobility two-dimensional electron gas in a $GaAs/Al_xGa_{1-x}As$ heterostructure has been studied for different sample widths and different sample currents. The critical current for breakdown of the quantum Hall effect was found to be proportional to the width of the samples. A subcritical rise of ρ_{xx} with current density was found to be exponential, a relation which to the best of our knowledge has not previously been reported. The power dissipation thus increases exponentially with current, leading to a critical current determined by a thermal runaway.

INTRODUCTION

The quantum Hall effect is mostly looked upon as an idealized effect in which the traditional Hall resistance is quantized precisely in units of h/e^2. Crudely speaking, this is due to a frozen commensurability between the electronic charge e and the flux quantum h/e over a finite magnetic-field range. Simultaneously the longitudinal resistivity ρ_{xx} is close to zero. In terms of the Landau quantization in a magnetic field B perpendicular to the two-dimensional electron system, one can rephrase this as a situation where the Fermi energy of the two-dimensional electron gas (2DEG) lies between Landau levels. In view of the metrological precise quantum-mechanical nature of the effect it is somewhat paradoxical that the quantum Hall effect is, as we demonstrate below, also a strongly nonlinear effect.

The metrological accuracy of quantum Hall effect is related to the smallness of ρ_{xx} and to the electrical current used. It is therefore of interest to study the current dependence of ρ_{xx} and the breakdown current. The breakdown of the quantum Hall effect has been experimentally investigated repeatedly.[1-8] The current dependence of ρ_{xx} can be divided into two qualitatively different regions, below and above a critical current I_c. Below the critical current the voltage along the quantum Hall resistor increases gradually as a function of current. At I_c the voltage rises by many orders of magnitude. For rectangular quantum Hall samples normally used for quantum metrology the critical current lies in the range from 100 to 600 μA.

Above the critical current the samples exhibit hysteresis, excessive noise, and steplike structure. Theoretically, suggestions such as Joule heating,[2,3,9] current generated thermal excitation across the Landau-level separation,[4] Zener tunneling across tilted Landau-level gaps,[10] and a matching of electron and sound velocity[11] have been the favored explanations. The current-dependent quantum Hall effect and the breakdown is however still not well understood and calls for more experiments.

In this paper we present new experiments at the midplateau quantum Hall state, which for the first time correlate the nonlinear current-voltage characteristic at low current with the width of the rectangular samples. We also find that I_c scales with the width of our Hall bars, a result which has recently also been observed by Kawaji and co-workers.[8] The critical current may be understood as a thermal runaway due to Joule heating as pointed out by Komiyama *et al.*[3] We improve this model by taking the observed subcritical rise in the resistivity with current into account.

SAMPLES AND EXPERIMENTS

The $GaAs/Ga_{1-x}Al_xAs$ heterostructures were grown by the molecular-beam epitaxy (MBE) technique in the traditional HEMT (high electron mobility transistor) configuration. Samples with a rectangular Hall-bar geometry were formed by photolithography and mesa-etching. Six voltage probes were placed in pairs on each side of the Hall bar and with a neighboring distance of four times the width of the sample. Finger-shaped contact pads were covered with Au-Ge-Ni-Au and alloyed to obtain good Ohmic contact to the 2DEG. The contact resistances were in all cases much lower than 1 Ω and of the type used for the quantum Hall metrology samples we distribute to various metrology laboratories.[12] The rectangular Hall-bar geometry was scaled to obtain different widths of the samples. The samples were placed at He temperatures ($T=1.2$ K) in a superconducting magnet.

The 2DEG density, $n=2.5\times10^{15}$ m^{-2}, and the purposely chosen low mobility, $\mu=5$ m^2/V s, were obtained from the zero-field resistivity and the Shubnikov–de Haas oscillations. Our MBE-grown HEMT wafers showed consistent behavior to that described in the following, although we emphasize that samples with very high mobilities were experimentally difficult to investigate, due to the required high-voltage resolution. Measurements on samples with the same width but different lengths were also studied and showed that the quantum Hall effect in our geometry was not influenced by the Joule heating at the current contacts. Figure 1 shows a plot of the square resistivity ρ_{xx} versus the magnetic field. The sample investigated in Fig. 1 has a width of 300 μm, which is typical for many samples used for quantum Hall metrology. The excitation current is 10 μA and has a negligible influence on the amplitude of the

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Shubnikov–de Haas oscillations.

In this paper we study the effect on the midplateau resistivity of the $i=2$ quantum Hall plateau at the magnetic field $B=5.35$ T. The middle of the quantum Hall plateau was defined as the interpolated minimum in the longitudinal resistivity ρ versus magnetic field as exemplified in Fig. 1(b). It turns out that the relative change of resistivity with current is by far largest on the plateau, although this cannot be seen with the resolution of Fig. 1(a). This nonlinearity is exemplified in Figs. 1(c) and 1(d), where the voltage at the $i=2$ midplateau is plotted as a function of current in a linear (c) and a semilogarithmic (d) plot. The extrapolated resistivity at midplateau ($B=5.35$ T; $T=1.2$ K) to zero current is constant, $\rho_{xx}=0.0002\ \Omega$, independent of the length and the width of the sample. This resistivity increases exponentially as the magnetic field is detuned from the middle of the plateau. The extrapolated midplateau resistivity varied roughly with temperature as $\rho_{xx}=\rho_0 e^{-\Delta/kT}$, where $\Delta=2$ meV for the $i=2$ plateau at $T=1.2$ K, but with a considerably slower variation at lower temperatures.[2,3] This value of Δ is considerably smaller than the Landau-level spacing, which is about 9 meV. This difference may be related to the relatively low electron mobility in our samples.

NONLINEAR AND SIZE-DEPENDENT CURRENT-VOLTAGE CHARACTERISTIC

Figure 2 shows the resistivity on a logarithmic scale plotted versus current in the full current regime for five samples with different widths. As seen the resistivity varies exponentially with current, $\rho_{xx}=e^{\alpha I}$, right up to

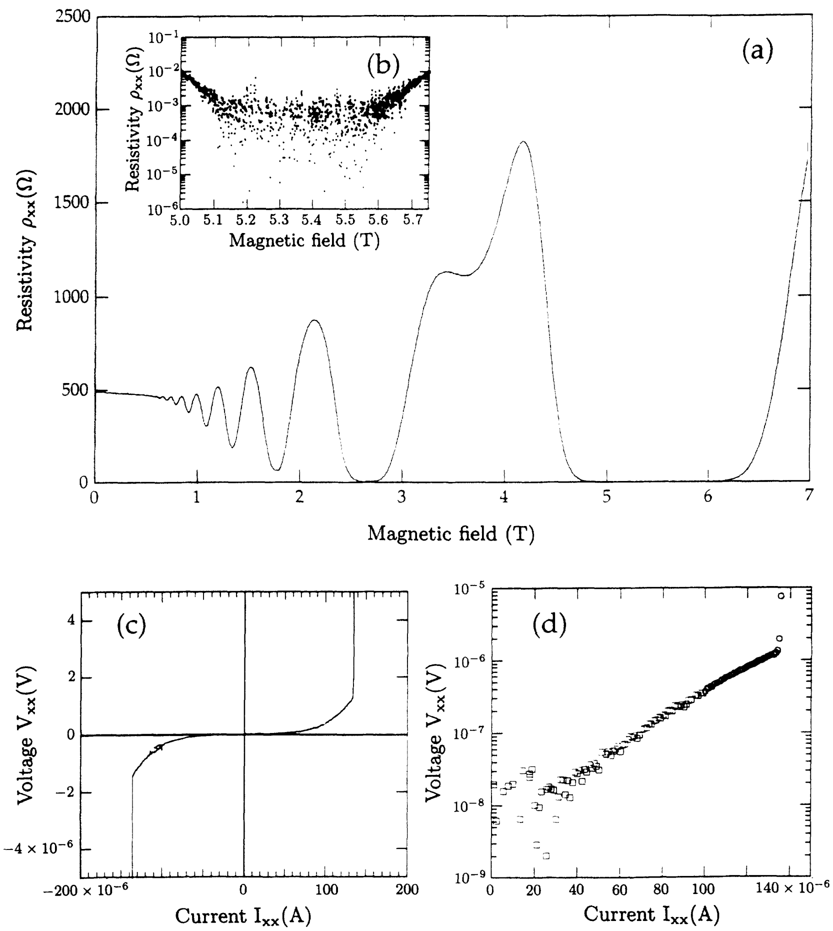

FIG. 1. (a) Magnetoresistance at 1.2 K of a 2DEG in a $Ga_{1-x}Al_xAs$ heterostructure sample with mesa width $W=300\ \mu$m and with an applied excitation dc current of 10 μA. The amplitude of the Shubnikov–de Haas oscillations is only slightly influenced by the current. Our subsequent results are referring to the middle of the $i=2$ plateau (here $B_0=5.35$ T), where the resistivity ρ_{xx} has a minimum. (b) shows the resistivity on a logarithmic scale for a small magnetic-field range around the minimum at $B_0=5.35$ T. This inset illustrates how the minimum is determined. (c) shows the voltage V_x (in μV) at the minimum in resistivity as a function of current for both polarities of the applied current. At a critical current (here 135 μA) the voltage changes by many orders of magnitude. The current-voltage characteristic below 135 μA is seen to be highly nonlinear. (d) shows the same data as (c), but plotted on a semilogarithmic scale. There is a slight curvature in the semilogarithmic plot. When instead the resistivity (voltage divided by current) is plotted the semilogarithmic plot becomes linear and the resistivity indeed has an exponential variation with current (see Fig. 2).

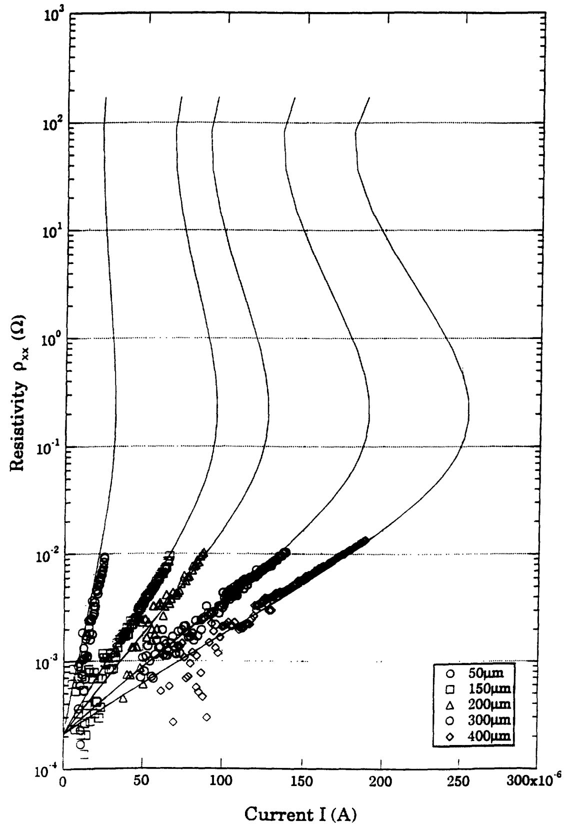

FIG. 2. The longitudinal resistivity ρ_{xx} for five samples with different width W of the mesa (50, 150, 200, 300, 400 μm) plotted as a function of current I. $T=1.2$ K. The semilogarithmic plot confirms the empirical relation $\rho_{xx} \sim e^{\alpha I}$, where the constant α is inversely proportional to the width W of the samples. At the critical current (where the experimental points stop) the resistivity abruptly changes by several orders of magnitude. The critical current is seen to grow proportional to W. Extrapolation of the resistivity to zero current gives a wafer-specific resistivity (0.0002 Ω) which does not depend on the dimensions of the samples. The experimental plots are well described by Eq. (2) as shown by the solid curves. A critical current is reached as soon as there is a solution of the theoretical curve at a higher resistivity for the same current. At that particular current the calculated curve predicts a jump to a 3–4 orders of magnitude higher resistivity.

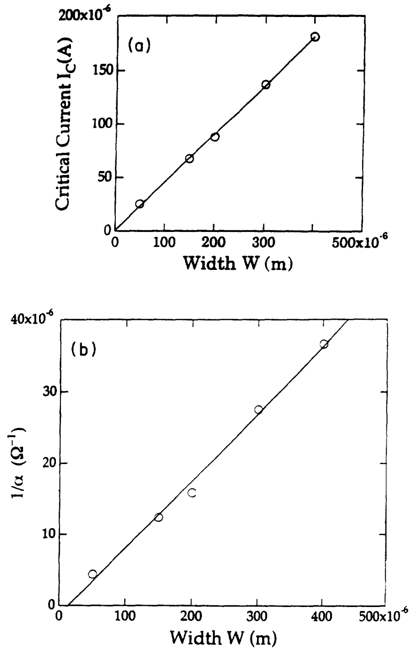

FIG. 3. (a) The critical current for five quantum Hall samples plotted as a function of the width of the samples. $T=1.2$ K. The critical currents are found as an average over several sweeps of resistivity vs current. (b) The inverse coefficient α^{-1} (in units of A) determined from the empirical relation $\rho_{xx} \sim e^{\alpha I}$ plotted as a function of the sample width at $T=1.2$ K.

the critical current, where the resistivity abruptly increases by many orders of magnitude. Due to the current-independent voltage noise, the uncertainty in the resistivity measurements diverges as the current approaches zero.

The most striking observation is connected to the size of the Hall bars. For five samples with different width the resistivity has been plotted as a function of the current in Fig. 2. The narrower a sample the faster the resistivity increases (exponentially), and the smaller is I_c. The resistivity at breakdown has about the same value (0.01 Ω) for all samples from the same wafer. In the semilogarithmic plot, Fig. 2, the resistivities all extrapolate back to the same resistivity at zero current, $\rho_{xx\min}=0.0002$ Ω, independent of the sample size. In Fig. 3(a) we plot the critical current (see Fig. 2) as a function of the width of the samples. A strictly proportional variation of the critical current I_c with width is found. Samples with higher mobility have a smaller extrapolated zero current resistivity, but only slightly smaller resistivity and higher current at breakdown. The important observation we report here is that the exponential factor α is inverse proportional to the widths of the samples. In Fig. 3(b) we plot $1/\alpha$ versus the width W of the samples. Reversal of the current did not influence the results [see Fig. 1(b)]. Measurements were taken for different voltage contacts along the Hall bar. The results were independent of the distance to the end contact and consequently independent of the heating, which is known to exist at the current contacts.[13] Samples, which had accidental scratches or variation in the width of the mesa, could not be plotted in either Fig. 3(a) or 3(b) but did still exhibit the same ratio between I_c and $1/\alpha$ for a given temperature. Measurements for different temperatures revealed that $1/\alpha$ is roughly proportional to the temperature, whereas I_c has a much weaker temperature dependence. Our experiments lead us to the following simple empirical expression for the midplateau resistivity as a function of temperature (T), current (I), and width (W):

$$\rho_{xx}(I)=\rho_{xx\min}\exp[ae\rho_{xy}I/(Wk_BT)]\ , \qquad (1)$$

where experimentally we find $a=100$ nm and $\rho_{xx\min}=0.0002$ Ω, since $\rho_{xy}=h/2e^2=12.9$ kΩ. The resistivity at breakdown is $\rho_{xx}(I_c)=0.01$ Ω.

THEORY OF THE BREAKDOWN OF THE QUANTUM HALL EFFECT

The resistivity as given by Eq. (1) depends exponentially on the current. Therefore the Joule heating in a quantum Hall sample increases faster with current than the current squared. This leads to an S-shaped relation between current and temperature, which may be readily calculated from the following heat balance equation:

$$\frac{\partial(T^*-T)}{\partial t}=\rho_{xx}\frac{(I/W)^2}{C_p}-\frac{(T^*-T)}{\tau_{ep}}=0 . \qquad (2)$$

Contrary to earlier calculations by Komiyama *et al.*[3] we include the current dependence $\rho_{xx}(I)$ in Eq. (2). If we use the free-electron value for the heat capacity: $C_p=10^{-9}$ J/m^2 K^{-2}T, the electron-phonon relaxation time becomes $\tau_{ep}=3.6\times10^{-10}$ s in order to fit the numerical solution of Eq. (2) to the data in Fig. 2 for $W=50$, 150, 200, 300, and 400 μm. There is a critical current for the calculated curves, where the resistivity increases by many orders of magnitude, and the calculation suggests a large hysteresis. The hysteresis is not experimentally equally pronounced. This is due to the well-known heating close to the current contacts[14] creating a temperature inhomogeneity, which will always trigger a runaway if possible. In order to illustrate that small fluctuations in temperature may be important, we have calculated the temperature of our quantum Hall samples as a function of current. The temperature, at which the critical current is reached, is found in our calculation to be only 5 mK above the lattice temperature $T=1.2$ K, and the temperature corresponding to the theoretical maximum current before switching (see Fig. 2) is 100 mK above the lattice temperature of 1.2 K. The resistivity increases four orders of magnitude after the abrupt jump at the critical current and the temperature of the electron system is then 5 K. On the basis of Eq. (2) we may calculate the critical current for different values of the mobility gap Δ used in the expression $\rho_{xx}=\rho_0 e^{-\Delta/kT}$, taken for one particular sample width (400 μm) and with all other parameters fixed. The critical current is found to be weakly dependent on ρ_{xx} and T. This is indeed found in many experiments.[1,2,3] Critical currents scaled to a 400-μm-wide sample and a temperature of 1.2 K for four of our own samples as well as data drawn from Refs. 1, 2, and 3 vary only within a factor of 2, while the plateau resistivity at a small current varies five orders of magnitude.

CONCLUSION

In this paper we have experimentally demonstrated that for a sample with a longitudinal quantum Hall resistivity of only 0.0002 Ω, the midplateau resistivity grows exponentially with current divided by the width of the Hall bar. The resistivity grows abruptly at a certain critical current density. The fact that current density rules the behavior is a surprise in view of the celebrated edge state model of the quantum Hall effect. The critical point corresponds roughly to the same resistivity for different samples from one particular wafer. The critical current is the result of a thermal runaway due to the Joule heating combined with the nonlinear resistivity. The subcritical nonlinear behavior of the resistivity may be connected to a hopping mechanism between edge states,[14] but cannot be quantitatively understood on the basis of current theories of the quantum Hall effect.

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[1]M. E. Cage, R. F. Dziuba, B. F. Field, E. R. Williams, S. M. Girvin, A. C. Gossard, D. C. Tsui, and R. J. Wagner, Phys. Rev. Lett. **51**, 1374 (1983).

[2]G. Ebert, K. von Klitzing, K. Ploog, and G. Weimann, J. Phys. C **16**, 5441 (1983).

[3]S. Komiyama, T. Takamasu, S. Hiyamizu, and S. Sasa, Solid State Commun. **54**, 479 (1985).

[4]M. E. Cage, G. M. Reedtz, D. Y. Yu, and C. T. van Degrift, Semicond. Sci. Technol. **5**, 351 (1990).

[5]H. Bruus and P. E. Lindelof, IEEE Trans. Instrum. Meas. **40**, 225 (1991).

[6]M. E. Cage, Semicond. Sci. Technol. **7**, 1119 (1992).

[7]C. Breitlow, G. Nachtwei, J. Breitlow-Hertzfeldt, J. H. Seyfarth, P. Svoboda, L. Bliek, and F.-J. Ahlers, Superlatt. Microstruct. **12**, 141 (1992).

[8]S. Kawaji, K. Hirakawa, and M. Nagata, Physica B **184**, 17 (1993); S. Kawaji, K. Hirakawa, M. Nagata, T. Okamoto, T. Fukase, and T. Gotoh (unpublished).

[9]O. Heinonen, P. L. Taylor, and S. M. Girvin, Phys. Rev. B **30**, 3016 (1984).

[10]L. Eaves and F. W. Sheard, Semicond. Sci. Technol. **1**, 346 (1986).

[11]P. Streda and K. von Klitzing, J. Phys. C **17**, L483 (1984).

[12]A. Kristensen, H. Bruus, H. D. Jensen, M. B. Jensen, C. B. Sørensen, and P. E. Lindelof (unpublished).

[13]U. Klass, W. Dietsche, K. von Klitzing, and K. Ploog, Z. Phys. B **82**, 351 (1991).

[14]D. G. Polyakov and B. I. Shklovski, Phys. Rev. Lett. **70**, 3796 (1993).