Theory of heat transfer by evanescent electromagnetic waves

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We present a macroscopic, phenomenological theory for the heat flow between two material half-spaces of differing temperatures whose surfaces are separated by a gap of width l. Our calculation parallels Liftshitz's calculation of the van der Waals force between two dielectric slabs. For l sufficiently small, the heat flow is enhanced by a contribution from evanescent waves, and in the limit of a very small gap varies as l^{-2}.

I. INTRODUCTION

As early as 1930, London[1] realized that quantum-mechanical fluctuations of electric dipole moments could give rise to a force between bodies separated by macroscopic distances (the van der Waals force). London showed that the force between two neutral atoms which did not possess permanent dipole moments and which were separated by a distance R varied as R^{-7}. His result did not take into account retardation effects, and hence was valid only when R was much less than the wavelength that corresponds to a transition between the ground and excited states of the atom. In 1948, Casimir and Polder[2] investigated the effect of retardation and showed that when this was included the force between two atoms varies as R^{-8} for large distances. One can also consider the corresponding forces between closely spaced macroscopic bodies. Casimir[3] showed that these forces could be considered to arise from the variation of the energy of the zero-point fluctuations of the electromagnetic field with respect to changes in the distance between the bodies. In 1954 Lifshitz[4] in a classic paper derived an expression for the attraction between two semi-infinite dielectric slabs using a theory of electromagnetic fluctuations developed by Rytov.[5] He found the force between the two materials by evaluating the Maxwell stress tensor for the fluctuating electromagnetic field in the gap between the slabs. His result is expressed in terms of the dielectric properties of the two materials. Thus, the theory is a macroscopic theory, and does not take account of nonlocal phenomena, such as the anomalous skin effect. In principle, the van der Waals force depends on temperature through the temperature dependence of the magnitude of the fluctuating electromagnetic field. However, Lifshitz shows that the temperature-dependent correction to the force is very small at normal laboratory temperatures.

Abrikosova and Derjaguin[6] made the first direct measurements of the van der Waals force between macroscopic bodies. They measured the force between two parallel plates of quartz separated by 1000–4000 Å. Several groups[7] made further measurements of this type, and the results are in reasonable agreement with the Lifshitz theory. In 1941, Schiff[8] observed that formation of films of superfluid helium on the walls of containers is due to the van der Waals attraction between the substrate and the helium. The theory of Lifshitz was generalized by Dzyaloshinskii, Lifshitz, and Pitaevskii[9] to include the interaction between a substrate and thin liquid films. Measurements by Sabisky and Anderson[10] of the thickness of liquid-helium films are in excellent agreement with this generalized Lifshitz theory. The existence of the fluctuating field outside of a dielectric has also been directly observed[11] in studies of inelastic scattering of low-energy electrons. Finally, we mention that a recent experiment by Sukenik *et al.*[12] has demonstrated the van der Waals force between an atom and two parallel plates.

While the theory of the van der Waals force is well established, little attention has been given to the role these fluctuations might play in heat transfer between two solids. Let us begin by considering the free surface of a single semi-infinite solid. Fluctuating currents inside the material will produce a fluctuating electromagnetic field, which extends beyond the surface of the material. This external field can be divided into propagating black-body radiation together with evanescent waves, which do not satisfy the radiation conditions. If a second material is placed at a large distance from the surface of the first material, the energy transfer will come solely from the radiation part of the field, and the rate of heat transfer between the two solids will be given by Stefan's law incorporating the emissivity and absorptivity of the materials. If the gap between the two materials is sufficiently small, however, it is possible for energy to be transferred across the gap by the evanescent waves, and it is the calculation of this energy flow that is the subject of this paper.

II. CALCULATION OF THE EVANESCENT HEAT FLOW

One can consider that the heat flow arises from the emission of a photon by an electron in one material, followed by the absorption of the photon in the other material. In this regard it is important to recognize that there is an essential incompatibility between the momentum scales for electrons and photons. In a typical metal, the momentum scale is set by the Fermi momentum p_F. However, the excess energy of the electron is $k_B T$ and so if the electron is to emit a real photon, the photon momentum must be no larger than $k_B T/c$. At room

temperature, this is typically smaller than p_F by a factor of the order of 10^{-5}. Thus, one can consider that the vast majority of photons emitted by an electron are virtual and end up being reabsorbed by the same or another electron that is nearby. It is when the second electron is in the other material that energy is transferred between the two bodies.

Our calculation of the energy flow between two half-spaces is similar to the method used by Lifshitz[4] in his derivation of the van der Waals force. In any material there will be spontaneous electrical and magnetic moments that originate from quantum and thermal fluctuations. These fluctuating moments produce fluctuating electromagnetic fields inside and outside the material. These fluctuating fields can be calculated by adding *extraneous fluctuating inductions*[13] to Maxwell's equations in much the same way that a random force is added to Newton's equations to solve the problem of Brownian motion. Of course, these extraneous inductions are random variables and cannot be completely specified. However, correlation functions for these variables can be found by using the fluctuation-dissipation theorem. Lifshitz solved Maxwell's equations including these source terms to find the fields both in the materials and in the gap. The force between the dielectrics was found by averaging the xx component of the Maxwell stress tensor, where the x direction is taken to be normal to the surfaces of the two materials. Since the field amplitudes are proportional to the extraneous fluctuating inductions, the stress tensor contains averages over the products of the components of the inductions. The fluctuation-dissipation theorem can be used to calculate these averages in terms of the dielectric constant and the temperature of the material. To calculate the heat flow we can follow a similar procedure, but, since we are interested in energy transfer, we calculate the Poynting vector rather than the Maxwell stress tensor.

We consider two semi-infinite materials with plane-parallel surfaces separated by a vacuum of thickness l (Fig. 1). The coordinate system is chosen so the yz plane is parallel to the surfaces of the two materials, the surface of the left slab is at $x=0$, and the surface of the right slab is at $x=l$. For convenience, the three regions will be labeled as follows: region 1 corresponds to the left half-space, region 2 corresponds to the right half-space, and the gap is denoted as region 0. The materials in regions 1 and 2 are characterized by complex dielectric constants $\epsilon_1(\omega)$ and $\epsilon_2(\omega)$, respectively. We assume that the materials are nonmagnetic, although it would not be difficult to extend the calculation to consider magnetic effects.

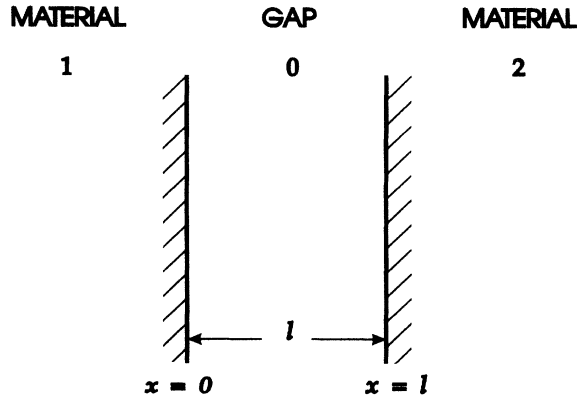

FIG. 1. Coordinate system used in the calculations.

It is convenient to begin by considering the electric and magnetic fields that are generated by the component of the fluctuating induction $\mathbf{g}(\mathbf{r},t)$ that has a particular frequency ω. To calculate these fields we have to solve Maxwell's equations in the form

$$\nabla\times\mathbf{E}(\mathbf{r},\omega)=i\frac{\omega}{c}\mathbf{H}(\mathbf{r},\omega)\ , \tag{1}$$

$$\nabla\times\mathbf{H}(\mathbf{r},\omega)=-i\frac{\omega}{c}\epsilon\mathbf{E}(\mathbf{r},\omega)-i\frac{\omega}{c}\mathbf{g}(\mathbf{r},\omega)\ , \tag{2}$$

where $\mathbf{g}(\mathbf{r},\omega)$ is the Fourier component of the extraneous electrical induction with frequency ω.[13] The fluctuation-dissipation theorem gives the average value of the product of different components of $\mathbf{g}$ at two different points in space. Since we are describing the materials by a local dielectric constant, there is no correlation between the extraneous inductions at different space points. Explicitly, we have[13]

$$\overline{g_i(\mathbf{r}_1,\omega)g_j^*(\mathbf{r}_2,\omega')}=A\epsilon''(\omega)\delta_{ij}\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\omega-\omega')\ , \tag{3}$$

where

$$A=4\hbar\left[\frac{1}{2}+\frac{1}{\exp(\hbar\omega/k_BT)-1}\right]=2\hbar\coth\frac{\hbar\omega}{2k_BT}\ . \tag{4}$$

T is the temperature and ϵ'' is the imaginary part of the dielectric constant ($\epsilon=\epsilon'+i\epsilon''$). The use of this form of the fluctuation-dissipation theorem restricts our results to materials, which can be described by a local dielectric constant.

The extraneous inductions in Eq. (2) act as sources of electromagnetic waves; these waves propagate some distance and are then reabsorbed in the medium. The net energy transfer P from region 1 to region 2 is the difference between the rate P_{12} at which waves generated by extraneous inductions in 1 are absorbed in 2, and the rate P_{21} at which waves generated in 2 are absorbed in 1. We first calculate P_{12} and then find P_{21} by symmetry. To calculate P_{12} we begin by considering an infinite volume of material with a spatially independent dielectric constant ϵ_1, in which the amplitude of the fluctuating inductions in the right half-space is set equal to zero. Thus, waves are generated only in the region $x<0$, and these waves are able to propagate into the region $x>0$ without undergoing reflection. This is an artificial situation that cannot be realized physically but it is nonetheless useful for our calculations. Subsequently, we add the effect of the gap and the second medium on the propagation of these waves.

To solve Maxwell's equations in the form of Eqs. (1) and (2) we write the fields in terms of Fourier components as

$$\mathbf{E}=\int_{-\infty}^{\infty}\mathbf{a}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}}d\mathbf{k},\quad \mathbf{H}=\frac{c}{\omega}\int_{-\infty}^{\infty}\mathbf{k}\times\mathbf{a}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}}d\mathbf{k}\ . \tag{5}$$

Additionally, we write $\mathbf{g}$ as

$$\mathbf{g}(\mathbf{r})=\int_{-\infty}^{\infty}\mathbf{g}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}}d\mathbf{k}\ . \tag{6}$$

By substituting Eqs. (5) and (6) into (1) and (2), $\mathbf{a}(\mathbf{k})$ can be expressed in terms of $\mathbf{g}(\mathbf{k})$ as follows:

$$\mathbf{a}(\mathbf{k})=\frac{1}{(k^2-\omega^2\epsilon/c^2)\epsilon}\left[\frac{\omega^2}{c^2}\epsilon\mathbf{g}(\mathbf{k})-(\mathbf{k}\cdot\mathbf{g}(\mathbf{k}))\mathbf{k}\right]\ . \tag{7}$$

The absence of fluctuations in the right half-space has the consequence that it must be possible to express the fields in this region as a sum of waves propagating to the right. To do this we first note that as a function of the x component of $\mathbf{k}$, $\mathbf{g}(\mathbf{k})$ cannot have any poles in the upper half of the complex plane since $\mathbf{g}(\mathbf{r})$ is zero for $x>0$. Thus, $\mathbf{a}(\mathbf{k})$ has a simple pole in the upper half of the complex plane at $k_x=s_1$, where $s_1\equiv(\omega^2\epsilon_1/c^2-q^2)^{1/2}$, and $\mathbf{q}=k_y\hat{\mathbf{y}}+k_z\hat{\mathbf{z}}$. The sign of s_1 is chosen so that the imaginary part of s_1 will be positive. The integration over k_x can, therefore, be carried out by completing the contour in the upper half plane. After integration the fields in the right half-space can be expressed as

$$\mathbf{E}(\mathbf{r})=\int\frac{i\pi\omega^2}{c^2s_1}\exp[i\mathbf{q}\cdot\boldsymbol{\rho}+is_1x]\times[\mathbf{g}(\mathbf{k})-(\mathbf{k}\cdot\mathbf{g}(\mathbf{k}))\mathbf{k}/k^2]d^2\mathbf{q}\ , \tag{8}$$

$$\mathbf{H}(\mathbf{r})=\int\frac{i\pi\omega}{cs_1}\exp[i\mathbf{q}\cdot\boldsymbol{\rho}+is_1x][\mathbf{k}\times\mathbf{g}(\mathbf{k})]d^2\mathbf{q}\ , \tag{9}$$

where $\boldsymbol{\rho}=y\hat{\mathbf{y}}+z\hat{\mathbf{z}}$, $\mathbf{k}=\mathbf{q}+s_1\hat{\mathbf{x}}$, and the integrals are over all of $\mathbf{q}$ space. Having written the fields $\mathbf{E}$ and $\mathbf{H}$ in terms of waves propagating to the right, we can now introduce the gap and second dielectric into our calculation.

As the next step, we create a surface at $x=0$ by setting the dielectric constant equal to 1 for $x\geq 0$. With the introduction of the discontinuity in the dielectric constant at $x=0$, the waves which originate in the left half-space will be only partially transmitted into the region $x>0$ and, in addition, refraction will occur at the interface. The transmission coefficients depend on the orientation of the waves relative to the plane of incidence. Consequently, we divide the electric field in the region $x>0$ into two contributions, ${}^{\perp}\mathbf{E}$ and ${}^{\parallel}\mathbf{E}$, which arise from waves polarized perpendicular and parallel to the plane of incidence, respectively. These fields are given by

$${}^{\perp}\mathbf{E}=\int\frac{i\pi\omega^2{}^{\perp}t_{10}}{c^2q^2s_1}\exp[i\mathbf{q}\cdot\boldsymbol{\rho}+ipx]\times[\mathbf{g}(\mathbf{k})\cdot(\mathbf{q}\times\hat{\mathbf{x}})](\mathbf{q}\times\hat{\mathbf{x}})d^2\mathbf{q}\ , \tag{10}$$

$${}^{\parallel}E=\int\frac{i\pi\,{}^{\parallel}t_{10}}{\sqrt{\epsilon_1}q^2s_1}\exp[i\mathbf{q}\cdot\boldsymbol{\rho}+ipx]\times[q^2g_x(\mathbf{k})-s_1(\mathbf{q}\cdot\mathbf{g}(\mathbf{k}))](q^2\hat{\mathbf{x}}-p\mathbf{q})d^2\mathbf{q}\ , \tag{11}$$

where ${}^{\perp}t_{10}$ and ${}^{\parallel}t_{10}$ are the transmission coefficients when the electric field is perpendicular and parallel to the plane of incidence, respectively, and $p\equiv(\omega^2/c^2-q^2)^{1/2}$ is the x component of the wave vector in vacuum. The electric field transmission coefficients are given by

$${}^{\perp}t_{10}=\frac{2s_1}{p+s_1},\quad {}^{\parallel}t_{10}=\frac{2s_1\sqrt{\epsilon_1}}{\epsilon_1p+s_1}\ . \tag{12}$$

The magnetic field can be divided in a similar way.

The gap is now introduced by setting the dielectric constant equal to ϵ_2 for $x>l$. A plane wave of unit amplitude incident on the surface of dielectric 1 from the left produces a field in the gap given by

$$t_{10}(e^{ipx}+r_{02}e^{ip(2l-x)}+r_{01}r_{02}e^{ip(2l+x)}+r_{01}r_{02}^2e^{ip(4l-x)}+r_{01}^2r_{02}^2e^{ip(4l+x)}+\cdots)\ , \tag{13}$$

where r_{01} and r_{02} are the reflection coefficients for a wave incident from vacuum on material 1 and 2, respectively. When the electric field is perpendicular to the plane of incidence r_{01} and r_{02} are

$${}^{\perp}r_{01}=\frac{p-s_1}{p+s_1},\quad {}^{\perp}r_{02}=\frac{p-s_2}{p+s_2}\ , \tag{14}$$

and when the electric field is in the plane of incidence these coefficients are

$${}^{\perp}r_{01}=\frac{\epsilon_1p-s_1}{\epsilon_1p+s_1},\quad {}^{\perp}r_{02}=\frac{\epsilon_2p-s_2}{\epsilon_2p+s_2}\ , \tag{15}$$

where $s_2\equiv(\omega^2\epsilon_2/c^2-q^2)^{1/2}$ is the x component of the wave vector in material 2. It follows from Eq. (13) that the perpendicular and parallel components of the electric field in the gap are

$${}^{\perp}\mathbf{E}_{\rm gap}=\int\frac{i\pi\omega^2}{c^2q^2s_1}{}^{\perp}t_{10}\exp(i\mathbf{q}\cdot\boldsymbol{\rho})\frac{e^{ipx}+{}^{\perp}r_{02}e^{2ipl}e^{-ipx}}{1-{}^{\perp}r_{01}{}^{\perp}r_{02}e^{2ipl}}\times[\mathbf{g}(\mathbf{k})\cdot(\mathbf{q}\times\hat{\mathbf{x}})](\mathbf{q}\times\hat{\mathbf{x}})d^2\mathbf{q}\ , \tag{16}$$

$${}^{\parallel}\mathbf{E}_{\rm gap}=\int\frac{i\pi}{\sqrt{\epsilon_1}q^2s_1}{}^{\parallel}t_{10}\exp(i\mathbf{q}\cdot\boldsymbol{\rho})\times\frac{(q^2\hat{\mathbf{x}}-p\mathbf{q})e^{ipx}+(q^2\mathbf{x}+p\mathbf{q})^{\parallel}r_{02}e^{2ipl}e^{-ipx}}{1-{}^{\parallel}r_{01}{}^{\parallel}r_{02}e^{2ipl}}\times[q^2g_x(\mathbf{k})-s_1(\mathbf{q}\cdot\mathbf{g}(\mathbf{k}))]d^2\mathbf{q}\ . \tag{17}$$

With expressions for fields in the gap we can calculate the Poynting vector in the gap. This vector is

$$\frac{c}{2\pi}\mathrm{Re}[\mathbf{E}_{\rm gap}\times\mathbf{H}^*_{\rm gap}]\ . \tag{18}$$

Into this expression we insert our results for the fields, now using the full expressions including the integral over the frequency. The result for the Poynting vector is then a double integral over products of the Fourier transforms of the fluctuating induction. This result can be simplified by averaging over these fluctuations. It follows from Eq. (3) that

$$\overline{g_i(\mathbf{k},\omega)g_j^*(\mathbf{k}',\omega')}=\frac{Ai\epsilon''}{(2\pi)^4}\frac{1}{k_x-k_x'^*}\times\delta_{ij}\delta(\mathbf{q}-\mathbf{q}')\delta(\omega-\omega')\ . \tag{19}$$

Using this result we can find the average value of the

Poynting vector. As expected, the result is independent of x and is a vector in the $\hat{\mathbf{x}}$ direction. The magnitude of this vector is the rate at which energy is transferred to medium 2 by the waves launched by fluctuating inductions in 1. To calculate the net energy flow between 1 and 2, we then subtract the flow due to waves generated in 2 being absorbed in 1. The final result for the power P per unit area flowing from 1 to 2 is

$$P=\frac{\hbar}{8\pi^2}\int_0^\infty \omega\, d\omega \left[\frac{1}{\exp(\hbar\omega/k_BT_1)-1}-\frac{1}{\exp(\hbar\omega/k_BT_2)-1}\right] \times\int_0^\infty q\, dq\left\{\left[\frac{1}{s_1}+\frac{1}{s_1^*}\right]\mathrm{Re}\{p[1-{}^{\perp}r_{02}\exp(2ipl)][1+{}^{\perp}r_{02}^*\exp(-2ip^*l)]\}\frac{|{}^{\perp}t_{10}|^2}{|1-{}^{\perp}r_{01}\,{}^{\perp}r_{02}e^{2ipl}|^2} +\frac{1}{|\epsilon_1|}\left[\frac{\epsilon_1}{s_1}+\frac{\epsilon_1^*}{s_1^*}\right]\mathrm{Re}\{p[1-{}^{\parallel}r_{02}\exp(2ipl)][1+{}^{\parallel}r_{02}^*\exp(-2ip^*l)]\}\frac{|{}^{\parallel}t_{10}|^2}{|1-{}^{\parallel}r_{01}\,{}^{\parallel}r_{02}e^{2ipl}|^2}\right\}. \tag{20}$$

For computational purposes it is convenient to change the integration variable from q to p and to replace p by $\omega p/c$ and s by $\omega s/c$ to give

$$P=-\frac{\hbar}{c^2\pi^2}\int_0^\infty \omega^3 d\omega\left[\frac{1}{\exp(\hbar\omega/k_BT_1)-1}-\frac{1}{\exp(\hbar\omega/k_BT_2)-1}\right] \times\int p^3dp\left\{\frac{(s_1+s_1^*)(s_2+s_2^*)}{|(p+s_1)(p+s_2)-(p-s_1)(p-s_2)e^{2ip\omega l/c}|^2} +\frac{(\epsilon_1^*s_1+\epsilon_1s_1^*)(\epsilon_2^*s_2+\epsilon_2s_2^*)}{|(\epsilon_1p+s_1)(\epsilon_2p+s_2)-(\epsilon_1p-s_1)(\epsilon_2p-s_2)e^{2ip\omega l/c}|^2}\right\}|e^{2ip\omega l/c}|\,, \tag{21}$$

where $s_1=\sqrt{\epsilon_1-1+p^2}$ and $s_2=\sqrt{\epsilon_2-1+p^2}$. The p integration goes from 1 to 0 and then along the imaginary axis to ∞. The portion of the integration along the real axis corresponds to the familiar black-body radiation while the integration over the positive imaginary axis gives the contribution from the evanescent waves.

III. SMALL GAP SIZE

Clearly, for very large gaps the evanescent waves will make a negligible contribution to the heat flow, and are not of importance. In the limit of a very small gap the evanescent contribution will dominate. However, even in the limit of very small l it is not easy to give a rigorous discussion of the variation of the evanescent heat flow with l. The dielectric constant may have a complicated dependence on frequency, and this makes it difficult to derive general results.

We first separate out the evanescent part P_{ev} of the heat flow in Eq. (21) and make the substitution of ip for p to get

$$P_{ev}=\frac{\hbar}{c^2\pi^2}\int_0^\infty \omega^3 d\omega\left[\frac{1}{\exp(\hbar\omega/k_BT_1)-1}-\frac{1}{\exp(\hbar\omega/k_BT_2)-1}\right] \times\int_0^\infty p^3dp\left\{\frac{(S_1-S_1^*)(S_2^*-S_2)}{|(p+S_1)(p+S_2)-(p-S_1)(p-S_2)e^{-2p\omega l/c}|^2} +\frac{(\epsilon_1^*S_1-\epsilon_1S_1^*)(\epsilon_2S_2^*-\epsilon_2^*S_2)}{|(\epsilon_1p+S_1)(\epsilon_2p+S_2)-(\epsilon_1p-S_1)(\epsilon_2p-S_2)e^{-2p\omega l/c}|^2}\right\}e^{-2p\omega l/c}\,, \tag{22}$$

where $S_{1,2}\equiv\sqrt{p^2-\epsilon_{1,2}+1}$, and the sign is chosen so that the real part of $S_{1,2}>0$.

Consider first how the integrand varies with p for a particular fixed value of ω. The effective upper limit of the p integration will be set by the factor $\exp(-2p\omega l/c)$ as

$$p_{\max}\sim\frac{c}{2\omega l}\,. \tag{23}$$

For sufficiently small l, $p_{\max}$ must become $\gg 1$. If we examine the remaining part of the integrand (i.e., excluding the exponential factor), we find that it increases with increasing p, both for $p\ll 1$ and for $p\gg 1$. Consequently, provided

that the condition $p_{\max} \gg 1$ is satisfied, the major contribution to the integral over p will come from the range in which $p \gg 1$. Thus, we can make the approximation (see further discussion below)

$$S_{1,2} \approx p + \frac{1-\epsilon_{1,2}}{2p} + \cdots . \tag{24}$$

If we insert this approximation to $S_{1,2}$ into Eq. (22) and retain only the terms of highest order in p we obtain

$$P_{\rm ev} \approx \frac{\hbar}{\pi^2 l^2} \int_0^\infty \omega \, d\omega \left[\frac{1}{\exp(\hbar\omega/k_B T_1)-1} - \frac{1}{\exp(\hbar\omega/k_B T_2)-1} \right] \times \int_0^\infty \gamma \, d\gamma \frac{\epsilon_2'' \epsilon_2''}{|(\epsilon_1+1)(\epsilon_2+1)-(\epsilon_1-1)(\epsilon_2-1)e^{-\gamma}|^2} e^{-\gamma} . \tag{25}$$

Thus, for small gap sizes the heat flow varies as l^{-2}. In order for the expansion (24) to be valid in the dominant range of the integration it is necessary that

$$\sqrt{|1-\epsilon_{1,2}|} \ll p_{\max} .$$

This is equivalent to the condition

$$l \ll \frac{c}{2\omega\sqrt{|1-\epsilon_{1,2}|}} . \tag{26}$$

Hence, a *sufficient* condition is that (26) holds for all frequencies up to $k_B T/\hbar$, where T is the larger of T_1 and T_2. Above this frequency the condition need not be satisfied because the Planck functions in Eq. (22) cut off the integral over ω. For some materials this condition may not be necessary. For example, the form of $\epsilon(\omega)$ may be such that the heat flow is dominated by frequencies much less than $k_B T/\hbar$. Then the l^{-2} dependence of the heat flow could hold out to larger gap sizes.

It is clear that if the real and imaginary parts of ϵ_1 and ϵ_2 are of the order of unity or smaller, the heat flow will always be proportional to l^{-2} when the gap is less than the thermal wavelength $\hbar c/k_B T$ divided by 2π. The heat flow due to the evanescent waves in this regime is larger than the radiation term by a factor of the order of $(\hbar c/lk_B T)^2$.

For small gap size the main contribution to the heat flow comes from evanescent waves with polarization vector perpendicular to the plane of incidence. It is interesting that it is also these waves that make the largest contribution to the van der Waals force in the limit of small gap size.[4]

For very small gaps other factors must enter to limit the increase in the heat flow. In the first place, we have treated the fluctuations as being pointlike in Eq. (3), i.e., we have assumed that it is sufficient to use a local dielectric constant. Second, by assigning each material a single temperature we are assuming that the thermal conductivity of the materials is sufficiently large that there are negligible temperature gradients in the surface regions where the evanescent waves are generated.

IV. HEAT TRANSFER FOR SMALL GAP SIZE USING RYTOV'S FORMULA

In his book,[5] Rytov derived an expression for the energy density of evanescent waves close to the surface of a material. His result can be written as

$$u_{\rm ev}(\omega) \simeq \frac{u_0(\omega)}{4} \frac{\epsilon''}{|\epsilon+1|^2} \frac{1}{(\omega x/c)^3} . \tag{27}$$

In this expression $u_{\rm ev}(\omega)$ is the contribution to the energy density of the evanescent waves from waves of frequency ω, and $u_0(\omega)$ is the energy density of black-body radiation, which is given by

$$u_{0\omega} = \frac{\hbar\omega^3}{\pi^2 c^3} \left[\frac{1}{2} + \frac{1}{\exp(\hbar\omega/k_B T)-1} \right] . \tag{28}$$

Rytov's formula is valid when $x \ll c/\omega$. It can be derived by calculating the average energy density of the electric and magnetic fields starting from Eqs. (10) and (11).

Rytov's expression can be used to find a simple expression for the heat flow between two materials when one of the materials has a sufficiently small dielectric susceptibility that it only weakly perturbs the fluctuating fields generated by the other. Let material 2 be the weak dielectric. The average rate of energy dissipation in material 2 is

$$\int dV \, {\rm Re}|\mathbf{E}\cdot j^*| = \int dV \int d\omega \frac{\omega\epsilon_2''}{2\pi} \langle E^2(\omega) \rangle , \tag{29}$$

where $\langle E^2(\omega) \rangle$ is the contribution to the average squared electric field from fluctuations at frequency ω. It follows from Rytov's results that for the evanescent wave field close to a surface the energy density is primarily in the electric field, rather than in the magnetic field. Thus,

$$\langle E^2(\omega) \rangle \approx 8\pi u_{\rm ev}(\omega) . \tag{30}$$

When this result is inserted into Eq. (29) we obtain the rate at which 2 absorbs energy from the fluctuating fields generated in 1. The result varies as l^{-2}. Then by using the fact that there must be no net energy flow when the two media are at the same temperature zero, we arrive at the following result for the net flow P from 1 to 2:

$$P \simeq \frac{\hbar}{4\pi^2 l^2} \int_0^\infty \omega \, d\omega \left[\frac{1}{\exp(\hbar\omega/k_B T_1)-1} - \frac{1}{\exp(\hbar\omega/k_B T_2)-1} \right] \frac{\epsilon_1'' \epsilon_2'}{|\epsilon_1+1|^2} . \tag{31}$$

This formula is valid for a small gap in the sense discussed in the previous section. It is also necessary that the absorption length of all evanescent waves, which make a significant contribution to the energy transfer, be large compared to the gap and, in addition, that the real part of ϵ_2-1 be sufficiently small that the waves are not appreciably reflected on going from the gap into material 2. These conditions are required so that the presence of the material 2 does not significantly perturb the evanescent field of material 1.

The result (31) is in agreement with our general formula for a small gap when the susceptibility of material 2 is made very small.

V. HEAT TRANSFER FOR DRUDE MATERIALS

In this section we consider heat transfer between two materials with the same dielectric constant ϵ given by

$$\epsilon=1+\frac{4\pi i\sigma}{\omega}\ , \tag{32}$$

where σ is independent of frequency and temperature. For simplicity, we will set $T_1=T$ and $T_2=0$, and calculate the power P flowing as a function of T. This involves no loss of generality since we can clearly use our result for $P(T)$ to calculate the net heat flow between the two materials for arbitrary values of the temperatures T_1 and T_2.

It is convenient to introduce dimensionless variables σ', l', ω', and P' defined by the following relations:

$$\sigma=\frac{\sigma' k_B T}{\hbar},\quad l=\frac{l'\hbar c}{k_B T},\quad \omega=\frac{\omega' k_B T}{\hbar},\quad P=P' P_{\rm bb}\ , \tag{33}$$

where $P_{\rm bb}$ is the energy transfer rate for black-body radiation, given by

$$P_{\rm bb}=\pi^2 k_B^4 T^4/60\hbar^3 c^2\ . \tag{34}$$

Figure 2 shows the reduced heat flow P' as a function of the reduced gap l' for a number of different values of the reduced conductivity σ'. These results include both the radiation and the evanescent component and are calculated from Eq. (21). The dependence of the power on the gap and on the conductivity is complex and is summarized in Fig. 3. The σ'-l' plane can be divided into the following regimes.

(1) In this regime the dominant contribution to the heat flow comes from the radiation contribution, i.e., the part of the integral over p in Eq. (21) for which p is real. For sufficiently large gap the heat flow is independent of gap size. For $\sigma'\ll 1$, i.e., $\sigma\ll k_B T/\hbar$, the power is independent of σ. However, for $\sigma>k_B T/\hbar$ the heat flow decreases with increasing σ because the emissivity decreases.

(2) This regime corresponds to the small gap limit, which we discussed earlier. The heat flow varies as $\sigma' l'^{-2}$. For a sufficiently small gap this region is always reached regardless of the value of σ'. The value of the gap along the boundary between regions 1 and 2 i.e., the gap at which evanescent waves begin to dominate the heat flow) depends on the value of σ' because the evanescent wave contribution to the heat flow varies as σ'^{-1}, while the radiation contribution is independent of σ' in this region. Thus, the boundary between 1 and 2 is given by the condition $l'\sim\sigma'^{1/2}$. In this regime the main contribution to the heat flow comes from evanescent waves with electric field vectors lying in the plane of incidence.

(3) In this region the power transmitted is proportional to the conductivity and independent of the gap. The heat flow is dominated by evanescent waves with polarization perpendicular to the plane of incidence, i.e., parallel to the surfaces of the materials on either side of the gap. The boundary between this region and region 2 is set by

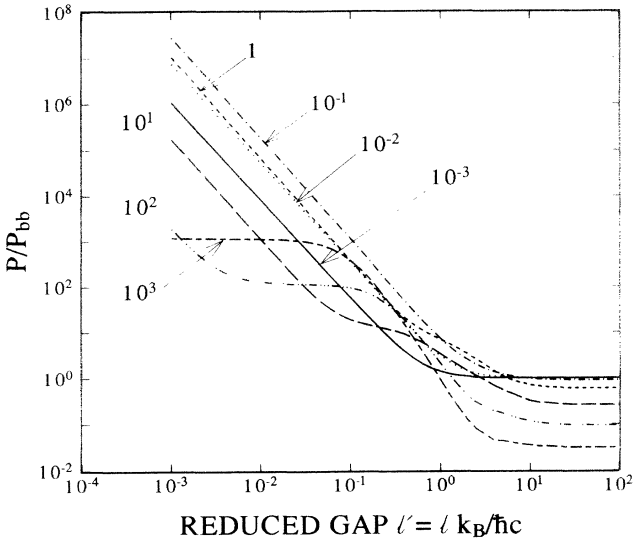

FIG. 2. Heat flow between two Drude materials as a function of conductivity σ and gap l. The quantities plotted are the power P divided by the black-body power $P_{\rm bb}$ and the dimensionless gap l' equal to $lk_B T/\hbar c$. The different curves are labeled by the dimensionless conductivity σ' equal to $\sigma\hbar/k_B T$.

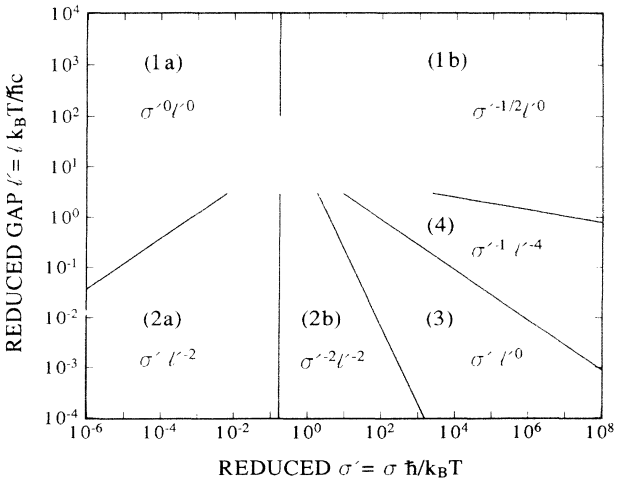

FIG. 3. Plot showing the different regions of the gap-conductivity plane. l' and σ' are the dimensionless gap and conductivity, respectively, as defined in Eq. (33). The dependence of the power on l' and σ' in the different regions is indicated.

the condition $l \sim \sigma^{-3/2}$.

(4) In this region the heat flow varies as $l'^{-4}\sigma'^{-1}$ and comes from waves polarized perpendicular to the plane of incidence. The boundary between regions 3 and 4 occurs for $l \sim c/(\sigma\omega)^{1/2}$, i.e., essentially when the gap becomes equal to the classical skin depth.

VI. RELATION TO EXPERIMENT

For a measurement of the evanescent heat flow it is clearly desirable to have the contribution from evanescent waves dominate over the contribution from propagating radiation. To achieve this condition it is necessary to have the gap between the materials significantly less than the thermal wavelength $\hbar c/k_B T$. Thus, at room temperature it is desirable to have a gap of 1 μm or less. One practical way to produce a small gap is through the use of a scanning tunneling microscope, although the gap in such a device is not a planar gap of uniform width that we have considered here. Williams and Wickramasinghe[14–16] developed a scanning thermal profiler for imaging the thermal properties of materials. This device consists of a conical tungsten tip (radius $\sim$1000 Å) coated with an insulating film except at the very end. A nickel film covers the insulator and the exposed tungsten tip, producing a thermocouple junction with linear dimensions of only a few hundred angstroms. Measurements of the heat flow across a gap between the substrate and the tungsten tip gave interesting results.[17] When the spacing was greater than the mean free path of the air molecules ($\sim$700 Å), the heat flow could be understood in terms of classical conduction through the air. In this regime the power increases steadily as the gap decreases. For smaller gaps the power became independent of gap size. This is the expected behavior for heat flow through a gas when the gap is less than the mean free path. However, when the gap was decreased to about 100 Å the heat flow began to increase very rapidly.

Dransfeld and Xu[18] proposed that in this experiment the heat transfer was mediated by thermally excited evanescent electromagnetic waves. As a specific example, they discussed heat transfer from a metal tip to a substrate that is an ionic crystal. They argued that the optical phonons in the ionic crystal produce an evanescent electric field above the surface with an energy density of the order of magnitude

$$\frac{E^2}{8\pi} \sim \frac{k_B T}{x^3} , \tag{35}$$

where x is the distance from the surface. They then estimate the heat transfer, $\dot{Q}$, to a conical metallic tip inserted into this evanescent field using

$$\dot{Q} = \frac{E^2}{8\pi} Ac(1-R) = \frac{k_B T}{l^3} Ac(1-R) , \tag{36}$$

where A is the area of the flat part of the conical tip, c is the speed of light, l is the distance of the tip from the surface, and $(1-R)$ is the fraction of the incident infrared radiation which is absorbed. Thus, they calculate the power absorbed by treating the evanescent waves as though they were propagating waves incident on the surface of the tip. For the optical reflectivity R they used the value 0.98 taken from other experimental measurements. When this is done the heat flow is found to be comparable to the heat flow through air at a gap of around 1000 Å. Thus, the power that they calculate is considerably larger than that measured by Williams and Wickramasinghe. However, this discrepancy is perhaps not surprising since Dransfeld and Xu have performed their calculation for an ionic substrate outside of which one may expect there to be stronger evanescent fields than for other materials.

To confirm that the heat flow that is measured is indeed associated with evanescent waves one could remove the air. This would be very interesting, but apparently has not been done. In addition, one could study the effect of different choices for the substrate. The calculation of Dransfeld and Xu is specifically for ionic crystals; they mention that there should also be a significant heat flow when the substrate is a metal, but the effect should be much smaller for a nonionic insulator. Since it is not yet certain that the experiment is actually measuring evanescent heat flow and because of the complicated tip geometry that is involved, we have not attempted to make a comparison of our theory with these experiments.

To perform a room-temperature test of the theory it would be desirable to perform an experiment with a gap down to a few hundred Å and with reasonably flat and parallel surfaces. This appears to be feasible. However, it may be easier to perform a low-temperature experiment where the thermal wavelength Λ_T is much larger and it is easier to make the gap size much less than Λ_T. For example, at 0.1 K, the effect of evanescent waves should become apparent when the gap is decreased into the millimeter range. The main experimental difficulty in a low-temperature experiment appears to be finding materials that have a reduced conductivity not too far removed from unity in this temperature range, i.e., which have conductivities within a few order of magnitudes of 10^{10} s^{-1}. The most promising candidates appear to be highly doped semiconductors.

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