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Gauge-invariant response functions of fermions coupled to a gauge field

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We study a model of fermions interacting with a gauge field and calculate gauge-invariant two-particle Green's functions or response functions. The leading singular contributions from the self-energy correction are found to be canceled by those from the vertex correction for small q and Ω. As a result, the remaining contributions are not singular enough to change the leading-order results of the random-phase approximation. It is also shown that the gauge-field propagator is not renormalized up to two-loop order. We examine the resulting gauge-invariant two-particle Green's functions for small q and Ω, but for all ratios of $\Omega/v_F q$, and we conclude that they can be described by Fermi-liquid forms without a diverging effective mass.

I. INTRODUCTION

The problem of two-dimensional fermions coupled to a gauge field has been a recent subject of intensive research. This problem appears as a low-energy effective model of two different strongly correlated electronic systems, i.e., electrons in the fractional quantum Hall (FQH) regime and the high-temperature superconductors (HTSC), both of which have been considered as one of the most important problems in modern condensed-matter physics.

As the first example, this problem arises in a theory of the half-filled Landau level[1–3] (HFLL) in connection with the composite fermion theory of the FQH effect.[4] A composite fermion is generated by attaching an even number of flux quanta to an electron.[4] The transformation from the electron to the composite fermion can be realized by introducing an appropriate Chern-Simons gauge field.[1,5] Especially, at the filling fraction $\nu=\frac{1}{2}$, composite fermions see effectively zero magnetic field at the mean-field level[1–4] because of the cancellation between the average of the Chern-Simons gauge field (from the attached magnetic-flux quanta) and the external magnetic field. Thus, at the mean-field level, the system can be described as a Fermi liquid of composite fermions. The fluctuation of the gauge field beyond the mean-field level has been studied within the random-phase approximation (RPA),[1,3] which explains qualitative features of the recent experiments.[6–11]

The other source comes from the recent gauge theory of the normal state of high-temperature superconductors.[12–15] The gauge field arises as a fluctuation of the spin chirality[12] above the uniform resonating-valence-bond mean-field state of the t-J model, which is supposed to be an effective model of HTSC. The origin of the gauge-field fluctuation can be traced back to the constraint that the doubly occupied sites are not allowed because of the strong on-site Coulomb repulsion.[12,13] It has been suggested that the gauge-field fluctuations play important roles in explaining anomalous transport properties of the normal state of HTSC.[12,15,16]

Besides these real examples, the problem of fermions interacting with a gauge field has been studied as a potential example of non-Fermi liquids.[17–28] In contrast to the usual long-ranged interactions such as the Coulomb interaction, the transverse part of the gauge field cannot be screened because the gauge invariance requires the gauge field to be massless in the absence of symmetry breaking.[17,19] Thus, one can expect that the long-range interaction due to the transverse part of the gauge field gives rise to non-Fermi-liquid-like behaviors. In fact, some singular behaviors appear in the lowest-order self-energy correction of fermions by the gauge-field fluctuation.[14,17–20] The singular self-energy correction makes perturbative calculation unreliable at low energies. This motivated several nonperturbative calculations of one-particle Green's function of fermions, which show highly non-Fermi-liquid-like behaviors.[21–24,26] It turns out that, even in the lowest order, the singular self-energy correction makes the effective mass of the fermion divergent so that the usual single-particle picture breaks down.[1]

However, recent experiments on the electrons in the half-filled Landau level showed essentially Fermi-liquid-like behaviors[6–11] and also measured finite effective mass of composite fermions.[10] Therefore, we are in a situation that experiments apparently contradict the insight we got from the one-particle Green's function of the fermions. However, the one-particle Green's function for the fermions is not gauge invariant. The singular self-energy correction in the one-particle Green's function (which leads to divergent effective mass[1]) may be an artifact of the gauge choice rather than a property of physical quasiparticles. Since it is not a gauge-invariant quantity, the one-particle Green's function for the fermions cannot be directly measured in experiments. It is possible that some singularities in the gauge-dependent one-particle Green's function simply do not appear in gauge-invariant correlation functions. One purpose of this paper is to examine some gauge-invariant response functions in order to determine whether the singular behaviors in the one-particle Green's function appear in gauge-invariant correlation functions or not.

The importance of the gauge invariance in calculating correlation functions can be also seen in the following ex-

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ample. The leading-order corrections (two-loop order) to the transverse polarization function (or current-current correlation function) are given by the diagrams in Fig. 1. Note that the sum of contributions from Figs. 1(a)–1(d) is not gauge invariant because they contain only self-energy corrections but do not contain the vertex correction. For concreteness, let us consider the case of $\eta=2$ in the model given by Eq. (8), which corresponds to the case of HTSC and the short-range interaction between fermions in HFLL. We also consider $\Omega \ll v_F q$ and $q \ll k_F$ limits. In this case, it can be shown that the correction to the transverse polarization function due to the self-energy corrections [given by Figs. 1(a)–1(d)] has the following form:

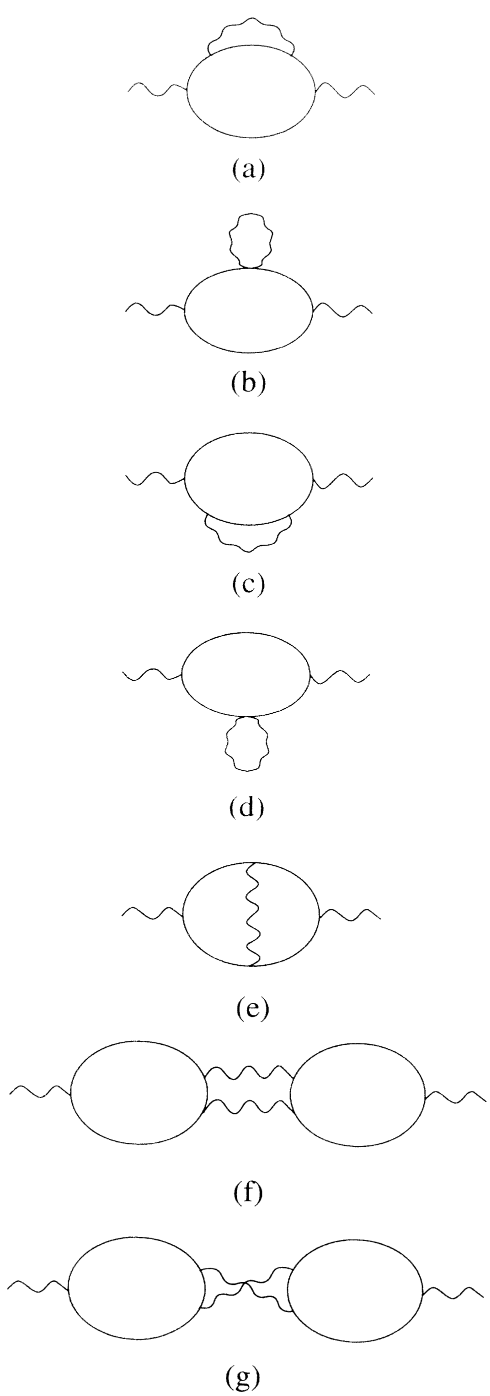

FIG. 1. The diagrams that correspond to the $[(1/N)^0]$th-order contributions to Π_{11} in the $1/N$ expansion.

$$\delta\,\mathrm{Im}\Pi^{s}_{11}(\mathbf{q},\Omega) \approx \frac{m^2 v_F^3}{2\pi\gamma}\frac{\Omega}{v_F q}\frac{(\gamma\Omega/\chi)^{2/3}}{k_F q}\,, \tag{1}$$

while the contribution from the free fermions is given by

$$\mathrm{Im}\Pi^{0}_{11}(\mathbf{q},\Omega) = -\frac{m v_F^2}{2\pi}\frac{\Omega}{v_F q}\,, \tag{2}$$

where 1 denotes the direction which is perpendicular to $\mathbf{q}$. One can see that the correction $\delta\,\mathrm{Im}\Pi^{s}_{11}$ would be more singular than the free-fermion contribution $\mathrm{Im}\Pi^{0}_{11}$ if $q,\Omega \to 0$ limit was taken with fixed $\Omega/v_F q < 1$. This result suggests that the perturbative expansion breaks down at low energies and the Fermi-liquid criterion are violated. Thus, the gauge-dependent correction (which comes from the self-energy correction) to the transverse polarization function provides a similar picture as that from the singular one-particle Green's function.[29]

Nevertheless, the perturbative corrections to the correlation functions should be calculated in a gauge-invariant way, thus one has to include the contributions from the vertex correction. The contribution to the transverse polarization function $\delta\,\mathrm{Im}\Pi^{v}_{11}$ coming from the vertex correction contains a singular term, which exactly cancels the singular contribution from the self-energy correction. Thus, the remnant terms in $\delta\,\mathrm{Im}\Pi^{v}_{11}$ provide the lowest-order corrections to the transverse polarization function and have the following form:

$$\delta\,\mathrm{Im}\Pi^{s}_{11} + \delta\,\mathrm{Im}\Pi^{v}_{11} \approx \frac{m^2 v_F^2}{\gamma}\frac{\Omega}{v_F q}\left[a\frac{(\gamma\Omega/\chi)^{2/3}}{k_F^2} + b\frac{(\gamma\Omega/\chi)}{k_F^2 q}\right], \tag{3}$$

where a and b are dimensionless constants. One can see that the corrections calculated in a gauge-invariant way are always much less than the free-fermion contribution as far as $\Omega \ll v_F q$ and $q \ll k_F$ limits are concerned. Therefore, the perturbative expansion works quite well in this regime, at least up to the leading-order gauge-field corrections, and there is no need to go beyond the perturbation theory at this order. The validity of the perturbative expansion also indicates that the transverse polarization function is well described by the Fermi-liquid theory in the region of $\Omega \ll v_F q$ and $q \ll k_F$. This provides a very different picture from that obtained through the gauge-dependent one-particle Green's function.

In this paper, we examine several gauge-invariant two-particle Green's functions or response functions in the limit of low frequency and long wavelength. It is shown that all the leading singular contributions from the self-energy correction are canceled by the contributions from the vertex correction in systematic perturbation theories (which guarantee the gauge invariance in each order of the perturbative expansion). This cancellation is essentially due to the Ward identity. It is found that singular corrections to the two-particle Green's function do not appear for all ratios of $\Omega/v_F q$ as far as the limit of low frequency and long wavelength limit is concerned. This kind of cancellation was also discussed by Ioffe and Kalmeyer[30] for a static gauge field. Recently, Khveshchenko and Stamp[23] performed nonperturbative calculations of one-particle and two-particle Green's functions using the so-called eikonal approximation. Even though they obtained a highly singular one-particle Green's function, the singularity does not show up in two-particle Green's functions for small q and Ω in this approximation.

We also show explicitly that the gauge-field propagator is not renormalized by the fluctuations beyond RPA up to the two-loop order. Nonrenormalization of the gauge-field propagator was first discussed by Polchinski[28] in the framework of a self-consistent approach. In this approach, it is assumed that the dispersion relation of fermions is given by $\omega \propto \xi_{\mathbf{k}}^{3/2}$ ($\xi_{\mathbf{k}}=k^2/2m-\mu$) and that of the gauge field is given by $\Omega \propto iq^3$, which are the results of one-loop corrections. Ignoring vertex correction by assuming the existence of a Migdal-type theorem, he showed that the assumed one-particle Green's function is self-consistent, and the polarization function is given by the same form as that of free fermions $\mathrm{Im}\Pi_{11}^0=-(mv_F^2/2\pi)(\Omega/v_F q)$ for $\Omega<\gamma^{1/3}\chi^{2/3}q^{3/2}$. As a result, the gauge-field propagator is not renormalized because the dispersion relation of the gauge field is given by $\Omega \propto iq^3$. However, we would like to remark that his result is quite different from those obtained in this paper. One can check that the polarization function in the self-consistent approach has a different form compared to that of Fermi liquid for $\Omega>\gamma^{1/3}\chi^{2/3}q^{3/2}$. However, in our perturbative calculation, the cancellation of anomalous terms from self-energy and vertex corrections leads to the result that the polarization functions have Fermi-liquid forms for all q and Ω as far as both are small.

We have made several explicit calculations of two-particle Green's functions. In particular, we consider a model given by Eq. (8) with $v(\mathbf{q})=V_0/q^{2-\eta}$ [$v(\mathbf{r})\propto V_0/r^{\eta}$, $1<\eta\le 2$], which corresponds to the interaction between fermions in the problem of HFLL. We will present the nonanalytic contributions (due to the gauge-field fluctuations) to the two-particle Green's functions. The transverse polarization function $\Pi_{11}(\mathbf{q},\Omega)$ up to two-loop order is found to have the following form. For $\Omega \ll v_F q$, we get

$$\mathrm{Im}\Pi_{11}(\mathbf{q},\Omega)\approx -\frac{mv_F^2}{2\pi}\frac{\Omega}{v_F q}\left[1-a\frac{mv_F}{\gamma}\frac{(\gamma\Omega/\chi)^{2/(1+\eta)}}{k_F^2} -b\frac{mv_F}{\gamma}\frac{(\gamma\Omega/\chi)^{3/(1+\eta)}}{k_F^2 q}\right], \tag{4}$$

while for $\Omega \gg v_F q$,

$$\mathrm{Im}\Pi_{11}(\mathbf{q},\Omega)\approx -\frac{1+\eta}{8\pi^2(5+\eta)}\frac{1}{\sin[2\pi/(1+\eta)]}\frac{v_F}{m}\frac{\gamma^{(3-\eta)/(1+\eta)}}{\chi^{4/(1+\eta)}}\Omega^{(3-\eta)/(1+\eta)}\left[1+cmv_F^3\left[\frac{\chi}{\gamma}\right]^{1/(1+\eta)}\frac{q^2}{\Omega^{(2\eta+3)/(\eta+1)}}\right], \tag{5}$$

where a,b,c are positive dimensionless constants.

The density-density correlation function $\Pi_{00}(\mathbf{q},\Omega)$ is also calculated. We have a formula valid for any ratio of $\Omega/v_F q$ as long as Ω and q are small [see Eq. (70)], but here we just discuss limiting cases. For $\Omega \ll v_F q$, we have

$$\mathrm{Im}\Pi_{00}(\mathbf{q},\Omega)\approx -\frac{m}{2\pi}\frac{\Omega}{v_F q}\left[1-\frac{1+\eta}{4\pi(5+\eta)}\frac{1}{\cos[(\eta-1)/(\eta+1)\pi]}\frac{1}{k_F m}\frac{\gamma^{(3-\eta)/(1+\eta)}}{\chi^{4/(1+\eta)}}\Omega^{(3-\eta)/(1+\eta)}\left[\frac{\Omega}{v_F q}\right]^2\right]. \tag{6}$$

On the other hand, for $\Omega \gg v_F q$,

$$\mathrm{Im}\Pi_{00}(\mathbf{q},\Omega)\approx -\frac{1+\eta}{8\pi^2(5+\eta)}\frac{1}{\sin[(2\pi)/(1+\eta)]}\frac{1}{k_F}\times\frac{\gamma^{(3-\eta)/(1+\eta)}}{\chi^{4/(1+\eta)}}\Omega^{(3-\eta)/(1+\eta)}\left[\frac{v_F q}{\Omega}\right]^2. \tag{7}$$

Note that $\mathrm{Im}\Pi_{11}(q\to 0,\Omega)=(\Omega^2/v_F^2q^2)\mathrm{Im}\Pi_{00}(q\to 0,\Omega)$ is satisfied as it should be. Equations (4)–(7) are the main results of this paper.

From the above gauge-invariant correlation functions, one can see the following.

(1) The corrections are irrelevant in the small q and Ω limit regardless of the way q and Ω approach zero (for example, $q\to 0$ limit may be taken first or $\Omega\to 0$ first, etc.). Therefore, *nonperturbative calculations are not necessary.* However, the subleading contributions are in general nonanalytic due to the long-range nature of the gauge interaction. The nonanalytic subleading terms may have some experimental consequences. For example, the NMR relaxation rate $1/T_1$ in the problem of HTSC can be determined from $\Pi_{00}(\mathbf{q},\Omega)$. At low temperatures, we have

$$\frac{1}{T_1 T}\propto \lim_{\Omega\to T} -\frac{1}{\Omega}\sum_{\mathbf{q}}\mathrm{Im}\Pi_{00}(\mathbf{q},\Omega),$$

where Π_{00} plays the role of spin susceptibility in HTSC. Equation (6) implies the following nonanalytic correction to the free-fermion result (only contributions from small $\mathbf{q}$ are considered): $1/T_1 T\propto 1-AT^{(5+\eta)/(1+\eta)}$, where A is a constant and the first term is the result of Fermi liquid. Notice that this result is in disagreement with a result based on a renormalization-group approach obtained in

Ref. 26, even near $\eta=1$. For HTSC $\eta=2$ and $1/T_1T\propto 1-AT^{7/3}$. Note that the nonanalytic correction is very small so that the Fermi-liquid form is preserved.

(2) $q\rightarrow 0$ limit of the transverse polarization function indicates that the transport scattering rate $\Gamma_{\rm tr}$ (which determines the dc conductivity) scales as $\Gamma_{\rm tr}\propto\Omega^{4/(1+\eta)}$ at low frequencies [see Eq. (45) for more details]. This result can also be obtained from the coefficient of the term which is proportional to q^2 in $\mathrm{Im}\Pi_{00}(\mathbf{q},\Omega)$, and the relation $\mathrm{Im}\Pi_{11}(q\rightarrow 0,\Omega)=(\Omega^2/v_F^2q^2)\mathrm{Im}\Pi_{00}(q\rightarrow 0,\Omega)$. This result exactly agrees with those obtained by different approaches.[12,16] Note that $\Gamma_{\rm tr}<\Omega$ for $1<\eta\le 2$.

(3) From Eq. (4), one can see that the gauge-field corrections are smaller than the result of free fermions along the curve $\Omega\propto q^{1+\eta}$, which is the dispersion relation of the gauge field. Therefore, the gauge-field propagator is not renormalized. As mentioned above, nonrenormalization of the gauge-field propagator was first discussed in Ref. 28 within a self-consistent argument.

(4) For $\eta\le 2$, the gauge-field corrections to the polarization functions are less singular than the result of the free fermions for $\Omega<v_Fq$. In particular, the edge of the particle-hole continuum in $\mathrm{Im}\Pi_{11}$ and $\mathrm{Im}\Pi_{00}$ still occurs at $\Omega\approx\tilde{v}_Fq$, where $\tilde{v}_F$ is finite and shifted from the bare Fermi velocity as in the usual Fermi-liquid theory. We conclude that the two-particle Green's functions are consistent with those of a Fermi liquid with a finite effective mass. However, a combination of a divergent mass and divergent Fermi-liquid parameters cannot be ruled out.

The remainder of the paper is organized as follows. In Sec. II, we introduce the model and review some one-particle properties. In Sec. III, the transverse polarization function for $q\rightarrow 0$ case is calculated. The cancellation of anomalous terms (coming from the self-energy and the vertex correction) up to $[(1/N)^0]$th order is explicitly shown (where N is the number of species of fermions). We also discuss the optical conductivity using the information of the calculated transverse polarization function. In Sec. IV, we calculate the transverse polarization function for the finite $q\ll k_F$ case. It is also argued that the gauge-field propagator is not renormalized up to two-loop order. In Sec. V, the density-density correlation function is calculated up to two-loop order for finite $q\ll k_F$. In Sec. VI, the results are compared to the conventional Fermi-liquid theory and their implication is discussed. We conclude this paper in Sec. VII.

II. THE MODEL AND THE ONE-PARTICLE PROPERTIES

The model is motivated by the above-mentioned two strongly correlated electronic systems. It is constructed such that it includes the most important infrared singular behaviors of the one-particle Green's function. In this paper, we consider the zero temperature limit and use the Euclidean space formalism. The mode in the Euclidean space is given by

$$Z=\int D\psi D\psi^* Da_\mu e^{-\int d\tau d^2r\mathcal{L}} ,$$

where

$$\mathcal{L}=\psi^*(\partial_0-ia_0-\mu)\psi-\frac{1}{2m}\psi^*(\partial_i-ia_i)^2\psi+ia_0n_f$$
$$+\frac{\alpha^2}{2}\int d^2r'[\nabla\times\mathbf{a}(\mathbf{r})]v(\mathbf{r}-\mathbf{r}')[\nabla\times\mathbf{a}(\mathbf{r}')] . \quad (8)$$

Here $v(\mathbf{q})=V_0/q^{2-\eta}$ $[v(\mathbf{r})\propto V_0/r^\eta,\ 1<\eta\le 2]$, m is the bare mass of the fermion, and n_f is the average density of fermions. We choose the Coulomb gauge $\nabla\cdot\mathbf{a}=0$. Note that this model is incomplete for the problem of HFLL because of the absence of the Chern-Simons term. However, one may expect that it contains possible low-energy singular behaviors because the most singular contribution to the one-particle properties comes from the transverse part of the gauge field. In the problem of HFLL, $\alpha=1/(2\pi\tilde{\phi})$ and $\tilde{\phi}=2$, which is the number of flux quanta attached to the electron.[1] For the case of HTSC, one can take $\alpha=0$.[12,13]

After integrating out fermions and including gauge-field fluctuations up to one-loop order (RPA), the effective Lagrangian density of the gauge field is given by[1,12,13]

$$\mathcal{L}_{\rm eff}=\frac{1}{2}\int\frac{d^2q}{(2\pi)^2}\frac{d\omega}{2\pi}a^*_\mu(\mathbf{q},\omega)D^{-1}_{\mu\nu}(\mathbf{q},i\omega)a_\nu(\mathbf{q},\omega) , \quad (9)$$

where

$$D^{-1}_{\mu\nu}=\begin{bmatrix}\Pi^0_{00} & 0\\ 0 & \Pi^0_{11}+\alpha^2v(q)q^2\end{bmatrix} . \quad (10)$$

Here $\mu,\nu=0,1$, and 1 represents the direction that is perpendicular to $\mathbf{q}$. Π^0_{00} and Π^0_{11} are given by the one-loop diagrams in Figs. 2(a) and 2(b) respectively. In the limit of $\omega\ll v_Fq$, one can find that[1,12,13]

$$\Pi^0_{00}=-\frac{m}{2\pi}\left[1-\frac{|\omega|}{v_Fq}\right] ,$$
$$\Pi^0_{11}=\frac{2n}{k_F}\frac{|\omega|}{q}+\frac{q^2}{24\pi m}$$
$$\equiv\gamma\frac{|\omega|}{q}+\chi_0q^2 . \quad (11)$$

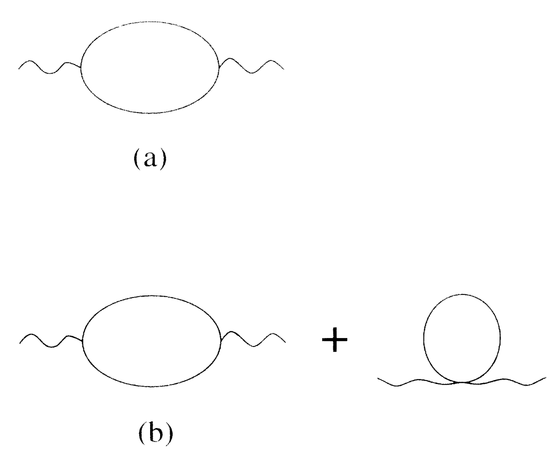

FIG. 2. The one-loop diagrams for Π^0_{00} (a) and for Π^0_{11} (b). The solid line is the bare electron propagator and the wavy line represents the gauge-field propagator. These are the leading-order diagrams of Π_{00} and Π_{11} in the $1/N$ expansion.

Therefore, the gauge-field propagator can be expressed as

$$D_{00}^{-1}=-\frac{m}{2\pi}\left[1-\frac{|\omega|}{v_F q}\right],$$
$$D_{11}^{-1}=\gamma\frac{|\omega|}{q}+[\chi_0+\alpha^2 v(q)]q^2 \approx\gamma\frac{|\omega|}{q}+\chi q^{\eta}, \tag{12}$$

where $\chi=\chi_0+\alpha^2 V_0$ for $\eta=2$ and $\chi=\alpha^2 V_0$ for $\eta\neq 2$.

Since the longitudinal part of the gauge field is screened, the transverse part of the gauge field dominates the physics. The one-loop self-energy correction due to the transverse part of the gauge field is calculated as[1,12,20] (Fig. 3)

$$\Sigma(\mathbf{k},i\omega)=\int\frac{d^2q}{(2\pi)^2}\frac{d\nu}{2\pi}\left|\frac{\mathbf{k}\times\hat{\mathbf{q}}}{m}\right|^2 \times G_0(\mathbf{k}+\mathbf{q},i\omega+i\nu)D_{11}(\mathbf{q},i\nu) \approx -i\lambda|\omega|^{2/(1+\eta)}\mathrm{sgn}(\omega), \tag{13}$$

where

$$\lambda=\frac{v_F}{4\pi\sin[2\pi/(1+\eta)]\gamma^{(\eta-1)/(\eta+1)}\chi^{2/(1+\eta)}},$$

and $G_0^{-1}(\mathbf{k},i\omega)=i\omega-\xi_{\mathbf{k}}$ ($\xi_{\mathbf{k}}=k^2/2m-\mu$). The self-energy as a function of real frequency Ω (in the Minkowski space) can be obtained from the analytic continuation of $\Sigma(\mathbf{k},i\omega)$, i.e., $\Sigma(\mathbf{k},\Omega)=\Sigma(\mathbf{k},i\omega\to\Omega+i\delta)$. Note that $|\mathrm{Im}\Sigma(\mathbf{k},\Omega)|\propto|\Omega|^{2/(1+\eta)}\gg|\Omega|$ for sufficiently small Ω or $|\Omega|\ll\lambda^{(\eta+1)/(\eta-1)}$. Therefore, the quasiparticle (the dressed fermion) is not well defined.

This can be also seen from the spectral function of fermions. The spectral function can be obtained from the imaginary part of the retarded Green's function: $A(\mathbf{k},\Omega)=-(1/\pi)\mathrm{Im}G_R(\mathbf{k},\Omega)=-(1/\pi)\mathrm{Im}G(\mathbf{k},i\omega\to\Omega+i\delta)$, where $G^{-1}(\mathbf{k},i\omega)=G_0^{-1}(\mathbf{k},i\omega)-\Sigma(\mathbf{k},i\omega)$. In the low-frequency limit,

$$A(\mathbf{k},\Omega)\approx\frac{1}{\pi}\frac{\lambda_2|\Omega|^{2/(1+\eta)}\mathrm{sgn}(\Omega)}{(\lambda_1|\Omega|^{2/(1+\eta)}-\xi_{\mathbf{k}})^2+(\lambda_2|\Omega|^{2/(1+\eta)})^2}, \tag{14}$$

where

$$\lambda_1=\lambda\cos\{\pi/2[(\eta-1)/(\eta+1)]\}$$

and

$$\lambda_2=\lambda\sin\{\pi/2[(\eta-1)/(\eta+1)]\}.$$

Note that the maximum of $A(\mathbf{k},\Omega)$ appears at $\Omega\sim(\xi_{\mathbf{k}}/\lambda_1)^{(1+\eta)/2}$. However, the width of the broad peak is also order $\Delta\Omega\sim(\xi_{\mathbf{k}}/\lambda_1)^{(1+\eta)/2}$. Therefore, the Landau criterion for the existence of quasiparticles ($\Delta\Omega\ll\Omega$) is marginally violated.

If we assumed that there is a well-defined Fermi wave vector $k_F=(4\pi n_f)^{1/2}$ and tried to fit the result to the usual quasiparticle picture, the energy spectrum of the quasiparticle would be[1]

$$\epsilon_{\mathbf{k}}\propto|k-k_F|^{(1+\eta)/2} \tag{15}$$

for k sufficiently close to k_F. From $k_F/m^*=\partial\epsilon_{\mathbf{k}}/\partial k|_{k=k_F}$, the effective mass diverges as

$$m^*\propto|k-k_F|^{-(\eta-1)/2}\propto|\epsilon_{\mathbf{k}}|^{-(\eta-1)/(\eta+1)}. \tag{16}$$

This suggests that at least some modifications to the conventional Fermi-liquid theory are necessary as far as the one-particle Green's function is concerned.

There have been also some nonperturbative calculations of the one-particle Green's function,[21–24] which were motivated by the singular perturbative correction at low energies. The results look very different from that obtained by the lowest-order perturbative calculation and even exponentially decaying one-particle Green's function is found in the so-called eikonal limit.[23]

From these results, one may doubt the validity of the quasiparticle picture although a modified Fermi-liquid description is proposed for the case of the HFLL.[1] However, one should also remember that the one-particle Green's function is not gauge invariant. This can be easily seen in the path integral representation of the one-particle Green's function[12,21] of a fermion interacting with a gauge field, i.e., each path acquires a phase factor $e^{i\int_0^t dt'\,\mathbf{a}(\mathbf{r},t')\cdot d\mathbf{r}/dt'}$, which is manifestly not gauge invariant. Therefore, it is very important to examine gauge-invariant quantities. As the first example, we will calculate the polarization function for $q\to 0$ case in the next section.

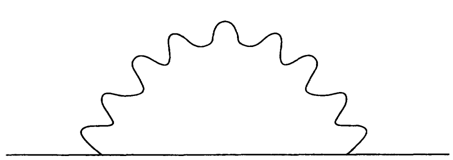

FIG. 3. The diagram that corresponds to the one-loop correction to the fermion self-energy. The solid line is the bare electron propagator and the wavy line represents the gauge-field propagator.

III. TRANSVERSE POLARIZATION FUNCTION FOR $q\to 0$ AND OPTICAL CONDUCTIVITY

Let us consider a large N generalized model of Eq. (8), where N is the number of species of fermions. In this model, each fermion bubble carries a factor of N and each gauge-field line gives a factor of $1/N$. Thus, for example, Π_{00}^0 and Π_{11}^{00} obtained in the previous section should be multiplied by N.

In this section, we consider only the $q\to 0$ case of the transverse polarization function: $\Pi_{11}(\mathbf{q}\to 0,i\nu)$. However, the relevant diagrams are the same even for the $q\neq 0$ case. The leading-order contribution is Π_{11}^0, which is proportional to N. The relevant diagrams in the next order (i.e., $[(1/N)^0]$th order) are given by Figs. 1(a)–1(g). For convenience let us define the following quantities: $\Pi_{11}^{(1)}=$ 1(a) + 1(b) and $\Pi_{11}^{(2)}$ = 1(c) + 1(d). The formal expressions of these quantities for the $q \to 0$ case are given by

$$\Pi_{11}^{(1)} = -\int \frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}})^2}{m^2}\right]\Sigma(\mathbf{k},i\omega) \times [G_0(\mathbf{k},i\omega)]^2 G_0(\mathbf{k},i\omega+i\nu) \tag{17}$$

and

$$\Pi_{11}^{(2)} = -\int \frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}})^2}{m^2}\right]\Sigma(\mathbf{k},i\omega+i\nu) \times [G_0(\mathbf{k},i\omega+i\nu)]^2 G_0(\mathbf{k},i\omega) . \tag{18}$$

These two equations can be rewritten as

$$\Pi_{11}^{(1)} = -\int \frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}})^2}{m^2}\right]\frac{\Sigma(\mathbf{k},i\omega)}{i\nu} \times \{[G_0(\mathbf{k},i\omega)]^2 - G_0(\mathbf{k},i\omega)G_0(\mathbf{k},i\omega+i\nu)\} , \tag{19a}$$

$$\Pi_{11}^{(2)} = \int \frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}})^2}{m^2}\right]\frac{\Sigma(\mathbf{k},i\omega+i\nu)}{i\nu} \times \{[G_0(\mathbf{k},i\omega+i\nu)]^2 - G_0(\mathbf{k},i\omega+i\nu)G_0(\mathbf{k},i\omega)\} . \tag{19b}$$

If we add (19a) and (19b), the first terms in each polarization bubble are canceled by each other and the remaining parts give us

$$\Pi_{11}^{(1)}+\Pi_{11}^{(2)} = \int \frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}})^2}{m^2}\right] \times \frac{\Sigma(\mathbf{k},i\omega)-\Sigma(\mathbf{k},i\omega+i\nu)}{i\nu} \times G_0(\mathbf{k},i\omega)G_0(\mathbf{k},i\omega+i\nu) . \tag{20}$$

From the above expression, it can be easily seen that the contributions from Figs. 1(b) and 1(d) are automatically canceled because the self-energy corrections in these diagrams are just the same constants.

Next we consider the diagram given in Fig. 1(e). Here we have to include the vertex correction for the $q \to 0$ case (Fig. 4),

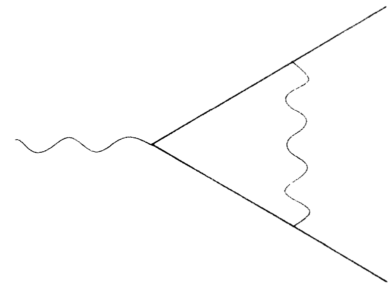

FIG. 4. The diagram that corresponds to the lowest-order vertex correction $\Gamma_0(\mathbf{k},\mathbf{q},i\omega,i\nu)$ or $\Gamma_1(\mathbf{k},\mathbf{q},i\omega,i\nu)$.

$$\Gamma_1(\mathbf{k},\mathbf{q}\to 0;i\omega,i\nu) = \int \frac{d^2q'}{(2\pi)^2}\frac{d\nu'}{2\pi}\left[-\frac{k_1+q_1'}{m}\right]\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}}')^2}{m^2}\right] \times G_0(\mathbf{k}+\mathbf{q}',i\omega+i\nu')G_0(\mathbf{k}+\mathbf{q}',i\omega+i\nu'+i\nu)D_{11}(\mathbf{q}',i\nu') . \tag{21}$$

Then $\Pi_{11}^{(3)}(\mathbf{q}\to 0,i\nu)$ can be written as

$$\Pi_{11}^{(3)} = -\int \frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}\left[-\frac{k_1}{m}\right]\Gamma_1(\mathbf{k},\mathbf{q}\to 0;i\omega,i\nu)G_0(\mathbf{k},i\omega)G_0(\mathbf{k},i\omega+i\nu) = \Pi_{11}^{(3,1)}+\Pi_{11}^{(3,2)} , \tag{22}$$

where

$$\Pi_{11}^{(3,1)} = -\int \frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}})^2}{m^2}\right]G_0(\mathbf{k},i\omega)G_0(\mathbf{k},i\omega+i\nu) \times \int \frac{d^2q'}{(2\pi)^2}\frac{d\nu'}{2\pi}\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}}')^2}{m^2}\right]G_0(\mathbf{k}+\mathbf{q}',i\omega+i\nu')G_0(\mathbf{k}+\mathbf{q}',i\omega+i\nu'+i\nu)D_{11}(\mathbf{q}',i\nu') , \tag{23}$$

and

$$\Pi_{11}^{(3,2)} = -\int \frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}G_0(\mathbf{k},i\omega)G_0(\mathbf{k},i\omega+i\nu)\int \frac{d^2q'}{(2\pi)^2}\frac{d\nu'}{2\pi}\left[\frac{q_1'k_1}{m^2}\right]\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}}')^2}{m^2}\right] \times G_0(\mathbf{k}+\mathbf{q}',i\omega+i\nu')G_0(\mathbf{k}+\mathbf{q}',i\omega+i\nu'+i\nu)D_{11}(\mathbf{q}',i\nu') . \tag{24}$$

Here we would like to point out that $\Pi_{11}^{(3,1)}$ is more singular than $\Pi_{11}^{(3,2)}$. This can be easily seen from the fact that $\Pi_{11}^{(3,2)}$ can be obtained by replacing $k_1^2/m^2=[k^2-(\mathbf{k}\cdot\hat{\mathbf{q}})^2]/m^2$ in the integrand of Eq. (23) by $q_1'k_1/m^2$. Using $q_1'=q_\parallel'\sin\theta_{\mathbf{kq}}+q_\perp'\cos\theta_{\mathbf{kq}}$ and $\xi_{\mathbf{k}+\mathbf{q}}\approx\xi_{\mathbf{k}}+v_F q_\parallel+q_\perp^2/2m$, one can do the integrals over $q_\parallel'$ and $q_\perp'$ in Eq. (24). Since the contribution from the $q_\perp'\cos\theta_{\mathbf{kq}}$ term becomes an odd function of $q_\perp'$, this term vanishes. By a formal manipulation, one can replace $q'_\parallel$ by $q'^2_\perp/k_F$ so that the q'_1 factor becomes effectively $(q'^2_\perp/k_F)\sin\theta_{\mathbf{kq}}$. Since the integrand is dominated by $|\nu|\sim(\chi/\gamma)|q_\perp|^{1+\eta}$ scaling given by the pole of the gauge-field propagator, replacing k_1 by q'_1 gives rise to an additional factor, which is proportional to $|\nu|^{2/(1+\eta)}$. Therefore, $\Pi^{(3,2)}_{11}$ should be less singular than $\Pi^{(3,1)}_{11}$ by the factor $|\nu|^{2/(1+\eta)}$ in the-low frequency limit.

Note that $\Pi^{(3,1)}_{11}$ can be rewritten as

$$\Pi^{(3,1)}_{11}=-\int\frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}})^2}{m^2}\right]\Gamma_0(\mathbf{k},\mathbf{q}\to 0;i\omega,i\nu)G_0(\mathbf{k},i\omega)G_0(\mathbf{k},i\omega+i\nu)\;, \tag{25}$$

where Γ_0 is the scalar vertex,

$$\Gamma_0(\mathbf{k},\mathbf{q};i\omega,i\nu)=\int\frac{d^2q'}{(2\pi)^2}\frac{d\nu'}{2\pi}\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}}')^2}{m^2}\right]G_0(\mathbf{k}+\mathbf{q}',i\omega+i\nu')G_0(\mathbf{k}+\mathbf{q}'+\mathbf{q},i\omega+i\nu'+i\nu)D_{11}(\mathbf{q}',i\nu')\;. \tag{26}$$

From the relation,

$$\begin{aligned}\Sigma(\mathbf{k},i\omega)-\Sigma(\mathbf{k},i\omega+i\nu)&=\int\frac{d^2q'}{(2\pi)^2}\frac{d\nu'}{2\pi}\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}}')^2}{m^2}\right][G_0(\mathbf{k}+\mathbf{q}',i\omega+i\nu')-G_0(\mathbf{k}+\mathbf{q}',i\omega+i\nu'+i\nu)]D_{11}(\mathbf{q}',i\nu')\\&=\int\frac{d^2q'}{(2\pi)^2}\frac{d\nu'}{2\pi}\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}}')^2}{m^2}\right]i\nu G_0(\mathbf{k}+\mathbf{q}',i\omega+i\nu')G_0(\mathbf{k}+\mathbf{q}',i\omega+i\nu'+i\nu)D_{11}(\mathbf{q}',i\nu')\;,\end{aligned} \tag{27}$$

we get the following identity:

$$\frac{\Sigma(\mathbf{k},i\omega)-\Sigma(\mathbf{k},i\omega+i\nu)}{i\nu}=\Gamma_0(\mathbf{k},\mathbf{q}\to 0;i\omega,i\nu)\;. \tag{28}$$

This is nothing but the Ward identity. From Eqs. (20), (25), and (28), we have

$$\Pi^{(1)}_{11}+\Pi^{(2)}_{11}+\Pi^{(3,1)}_{11}=0\;. \tag{29}$$

Now the remaining piece is just $\Pi^{(3,2)}_{11}$. Following the procedures of integration mentioned above, in the low-frequency limit, we get

$$\Pi^{(3,2)}_{11}\approx-\frac{1+\eta}{4\pi^2(5+\eta)\sin[(3-\eta)\pi/(1+\eta)]}\times\frac{v_F}{m}\frac{\gamma^{(3-\eta)/(1+\eta)}}{\chi^{4/(1+\eta)}}|\nu|^{(3-\eta)/(1+\eta)}\;. \tag{30}$$

Here it is worthwhile to compare this result with $\Pi^{(1)}_{11}+\Pi^{(2)}_{11}$ and $\Pi^{(3,1)}_{11}$, i.e., the results before cancellation. By a straightforward calculation, one can get

$$\Pi^{(1)}_{11}+\Pi^{(2)}_{11}\approx-\frac{2(1+\eta)}{\pi(3+\eta)}mv_F^2\lambda|\nu|^{-(\eta-1)/(\eta+1)}\;. \tag{31}$$

In order to calculate $\Pi^{(3,1)}_{11}$, the vertex correction should be calculated. The vertex correction $\Gamma_0(\mathbf{k},\mathbf{q}\to 0;i\omega,i\nu)$ is found to be

$$\Gamma_0\approx-\frac{v_F}{\gamma}\frac{1}{2\pi\sin[2\pi/(1+\eta)]}\frac{1}{\nu}\times\left[\left[\frac{|\omega|\gamma}{\chi}\right]^{2/(1+\eta)}\operatorname{sgn}(\omega)-\left[\frac{|\omega+\nu|\gamma}{\chi}\right]^{2/(1+\eta)}\operatorname{sgn}(\omega+\nu)\right]\;. \tag{32}$$

Using Eqs. (25) and (32), $\Pi^{(3,1)}_{11}$ can be calculated as

$$\Pi^{(3,1)}_{11}\approx\frac{mv_F^3}{2\pi^2\sin[2\pi/(1+\eta)]}\left[\frac{1+\eta}{3+\eta}\right]\times\frac{1}{\gamma^{(\eta-1)/(\eta+1)}\chi^{2/(1+\eta)}}|\nu|^{-(\eta-1)/(\eta+1)}\;. \tag{33}$$

Note that, as mentioned above, $\Pi^{(1)}_{11}+\Pi^{(2)}_{11}$ and $\Pi^{(3,1)}_{11}$ are more singular than $\Pi^{(3,2)}_{11}$ by $|\nu|^{-2/(1+\eta)}$ in the low-frequency limit. The important point is that these singular terms are canceled by each other due to the Ward identity.

Now let us look at the diagrams of Figs 1(f) and 1(g). Let $\Pi^{(4)}_{11}=1(\mathrm{f})$ and $\Pi^{(5)}_{11}=1(\mathrm{g})$. The formal expressions of these diagrams for $q\to 0$ case are given by

$$\begin{aligned}\Pi^{(4)}_{11}=\int&\frac{d^2q'}{(2\pi)^2}\frac{d\nu'}{2\pi}\frac{d^2k'}{(2\pi)^2}\frac{d\omega'}{2\pi}\frac{d^2k''}{(2\pi)^2}\frac{d\omega''}{2\pi}\left[\frac{\mathbf{k}'\cdot\mathbf{k}''-(\mathbf{k}'\cdot\hat{\mathbf{q}}')(\mathbf{k}''\cdot\hat{\mathbf{q}}')}{m^2}\right]^2D_{11}(\mathbf{q}',i\nu')D_{11}(\mathbf{q}',i\nu'+i\nu)\\&\times G_0(\mathbf{k}',i\omega')G_0(\mathbf{k}',i\omega'+i\nu)G_0(\mathbf{k}'-\mathbf{q}',i\omega'-i\nu')G_0(\mathbf{k}'',i\omega'')G_0(\mathbf{k}'',i\omega''+i\nu)G_0(\mathbf{k}''-\mathbf{q}',i\omega''-i\nu')\;,\end{aligned} \tag{34}$$

and

$$\begin{aligned}\Pi^{(5)}_{11}=\int&\frac{d^2q'}{(2\pi)^2}\frac{d\nu'}{2\pi}\frac{d^2k'}{(2\pi)^2}\frac{d\omega'}{2\pi}\frac{d^2k''}{(2\pi)^2}\frac{d\omega''}{2\pi}\left[\frac{\mathbf{k}'\cdot\mathbf{k}''-(\mathbf{k}'\cdot\hat{\mathbf{q}}')(\mathbf{k}''\cdot\hat{\mathbf{q}}')}{m^2}\right]^2D_{11}(\mathbf{q}',i\nu')D_{11}(\mathbf{q}',i\nu'+i\nu)\\&\times G_0(\mathbf{k}',i\omega')G_0(\mathbf{k}',i\omega'+i\nu)G_0(\mathbf{k}'-\mathbf{q}',i\omega'-i\nu')G_0(\mathbf{k}'',i\omega'')G_0(\mathbf{k}'',i\omega''+i\nu)G_0(\mathbf{k}''+\mathbf{q}',i\omega''+i\nu'+i\nu)\;.\end{aligned} \tag{35}$$

By changing variables as $\mathbf{q}'\to-\mathbf{q}'$, $\nu'\to-\nu'-\nu$, and using $D_{11}(-\mathbf{q}',-i\nu')=D_{11}(\mathbf{q}',i\nu')$, we get

$$\Pi_{11}^{(4)}+\Pi_{11}^{(5)}=\frac{1}{2}\int\frac{d^2q'}{(2\pi)^2}\frac{d\nu'}{2\pi}D_{11}(\mathbf{q}',i\nu')D_{11}(\mathbf{q}',i\nu'+i\nu)\left|\frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}\frac{k_1}{m}\left|\frac{k\sin\theta_{\mathbf{kq}'}}{m}\right|^2 G_0(\mathbf{k},i\omega)G_0(\mathbf{k},i\omega+i\nu)\right.$$
$$\left.\times[G_0(\mathbf{k}+\mathbf{q}',i\omega+i\nu'+i\nu)+G_0(\mathbf{k}-\mathbf{q}',i\omega-i\nu')]\right|^2, \tag{36}$$

where $\theta_{\mathbf{kq}'}$ is the angle between $\mathbf{k}$ and $\mathbf{q}'$. In the low-frequency limit, we get

$$\Pi_{11}^{(4)}+\Pi_{11}^{(5)}\approx-c_1\frac{v_F}{m}\frac{\gamma^{(3-\eta)/(1+\eta)}}{\chi^{4/(1+\eta)}}|\nu|^{(3-\eta)/(1+\eta)}, \tag{37}$$

where c_1 is a constant. One can also show that

$$\Pi_{11}^{(4)}\approx-c_0\frac{mv_F^3}{\gamma^{(\eta-1)/(\eta+1)}\chi^{2/(1+\eta)}}|\nu|^{-(\eta-1)/(\eta+1)},$$
$$\Pi_{11}^{(5)}\approx c_0\frac{mv_F^3}{\gamma^{(\eta-1)/(\eta+1)}\chi^{2/(1+\eta)}}|\nu|^{-(\eta-1)/(\eta+1)} \tag{38}$$
$$-c_1\frac{v_F}{m}\frac{\gamma^{(3-\eta)/(1+\eta)}}{\chi^{4/(1+\eta)}}|\nu|^{(3-\eta)/(1+\eta)},$$

where c_0 is a constant. That is, there is also a cancellation between the singular parts of $\Pi_{11}^{(4)}$ and $\Pi_{11}^{(5)}$.

Gathering all the previous informations and using $\Pi_{11}^{0}(\mathbf{q}\to0,i\nu)=Nn/m$, we can conclude that

$$\Pi_{11}\approx\frac{Nn}{m}-c_2\frac{k_F}{m^2}\frac{\gamma^{(3-\eta)/(1+\eta)}}{\chi^{4/(1+\eta)}}|\nu|^{(3-\eta)/(1+\eta)} \tag{39}$$

up to $[(1/N)^0]$th order, where c_2 is constant.

In order to calculate the optical conductivity, we have to consider the bubble diagrams with two external lines that represent the coupling to the external vector potential A_μ while the internal gauge-field lines are due to a_μ. There are additional diagrams generated by $\psi^\dagger a_\mu A^\mu\psi$ vertex. All the additional diagrams except one [shown in Fig. 5(a)] vanish due to the symmetry of the integrand. A typical diagram that vanishes is shown in Fig. 5(b). It turns out that the diagram represented by Fig. 5(a) gives an imaginary part, which is higher order in frequency compared to $|\nu|^{(3-\eta)/(1+\eta)}$ so that it is irrelevant in the low-frequency limit. Now we can use the imaginary part of the transverse polarization function in the Minkowski space $\Pi_{11}(\mathbf{q}\to0,\Omega)=\Pi_{11}(\mathbf{q}\to0,i\nu\to\Omega+i\delta)$ to calculate the real part of the optical conductivity

$$\mathrm{Re}\sigma(\Omega)=-e^2\frac{\mathrm{Im}\Pi_{11}(\Omega)}{\Omega}. \tag{40}$$

From Eq. (39), $\mathrm{Re}\sigma(\Omega)$ is given by

$$\mathrm{Re}\sigma(\Omega)\propto\frac{e^2k_F}{m^2}\frac{\gamma^{(3-\eta)/(1+\eta)}}{\chi^{4/(1+\eta)}}\Omega^{-2[(\eta-1)/(\eta+1)]}. \tag{41}$$

If there were no cancellation, the result would look quite different. For example, if we did not consider the vertex correction, the result from $\Pi_{11}^{(1)}+\Pi_{11}^{(2)}$ would be

$$\mathrm{Re}\sigma_{nv}(\Omega)\propto\frac{e^2mv_F^3}{\gamma^{(\eta-1)/(\eta+1)}\chi^{2/(1+\eta)}}\Omega^{-2\eta/(1+\eta)}, \tag{42}$$

where σ_{nv} represents the conductivity without vertex correction.

Now we are going to show that the right answer given by Eq. (41) is consistent with a modified Drude formula if we assume that the transport scattering rate (which is the inverse of the transport time τ_{tr}) of the fermion is given by

$$\Gamma_{tr}(\Omega)\propto(1/N)(1/mk_f)[\gamma^{(3-\eta)/(1+\eta)}/\chi^{4/(1+\eta)}]\Omega^{4/(1+\eta)}.$$

First of all, for later convenience, let us calculate the inverse of the transport time τ_{tr}^0 of the fermion[12] using the imaginary part of the self-energy $\Sigma(\mathbf{k},\Omega)$. For this purpose, we can just include the factor $1-\cos\theta=2\sin^2(\theta/2)$ in the integrand of the expression for $\mathrm{Im}\Sigma(\mathbf{k},\Omega)$, where θ is the angle between the wave vector of the fermion and that of the gauge field.[12] Using the fact that $\sin(\theta/2)\approx q/2k_F$ and $q\sim(\gamma\Omega/\chi)^{1/(1+\eta)}$ inside the integral,[12] we get

$$\frac{1}{\tau_{tr}^0}\propto\frac{1}{N}\frac{1}{mk_F}\frac{\gamma^{(3-\eta)/(1+\eta)}}{\chi^{4/(1+\eta)}}\Omega^{4/(1+\eta)}. \tag{43}$$

Therefore, we will essentially show that our result of the optical conductivity is consistent with the identification of $\Gamma_{tr}=1/\tau_{tr}^0$ or $\tau_{tr}=\tau_{tr}^0$ in a modified Drude formula.

The Drude formula that is appropriate to the large-N

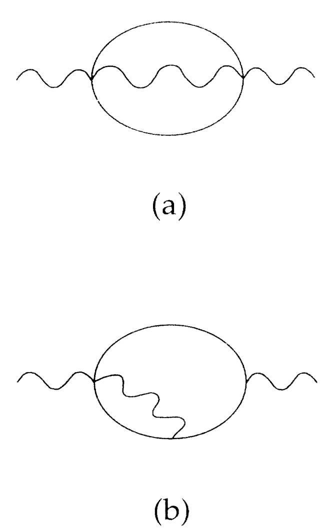

FIG. 5. (a) The nonvanishing diagram generated by $\psi^\dagger a_\mu A^\mu\psi$ vertex. (b) A typical vanishing diagram generated by $\psi^\dagger a_\mu A^\mu\psi$ vertex.

generalized model is given by

$$\mathrm{Re}\sigma(\Omega)=\frac{Nne^2}{m}\frac{\Gamma_{\rm tr}}{\Omega^2+\Gamma_{\rm tr}^2} . \tag{44}$$

In the large-N limit, if we assume $\Gamma_{\rm tr}=1/\tau_{\rm tr}^0 \propto 1/N$,

$$\mathrm{Re}\sigma(\Omega)\approx\frac{Nne^2}{m}\frac{\Gamma_{\rm tr}}{\Omega^2}\propto\frac{e^2v_F}{m}\frac{\gamma^{(3-\eta)/(1+\eta)}}{\chi^{4/(1+\eta)}} \times\Omega^{-2[(\eta-1)/(\eta+1)]} . \tag{45}$$

This is the same result as that of Eq. (41). The result of Eq. (42) can be reproduced in the same way if we assume that

$$\Gamma_{\rm tr}(\Omega)\propto(1/N)(mv_F^3)\gamma^{-(\eta-1)/(\eta+1)}\chi^{-2/(1+\eta)}\Omega^{2/(1+\eta)} ,$$

which is essentially the imaginary part of the self-energy $\Sigma(\mathbf{k},\Omega)$. Therefore, the optical conductivity is consistent with the choice of $1/\tau_{\rm tr}^0$ rather than just the naive scattering rate (given by the self-energy) as the transport scattering rate. Since the singular contribution, which gives Eq. (42), is canceled by the vertex correction, we can again say that the leading singular behaviors of one-particle properties do not show up in the optical conductivity.

For finite temperature, one can replace Ω by T in $\Gamma_{\rm tr}$. Note that the dc limit of the optical conductivity $\mathrm{Re}\sigma(\Omega\to 0)=(Nne^2/m)1/\Gamma_{\rm tr}$ cannot be obtained by the $1/N$ expansion. However, one can infer the dc limit by assuming that the full $\mathrm{Re}\sigma(\Omega)$ is given by Eq. (44) [with $\Gamma_{\rm tr}=\Gamma_{\rm tr}(T)$], which is consistent with the result of the large-N limit of the optical conductivity. If $\Gamma_{\rm tr}\propto T^{4/(1+\eta)}$ was used, one would get $\mathrm{Re}\sigma(T)\propto T^{-4/(1+\eta)}$.[12] On the other hand, one would get $\mathrm{Re}\sigma_{nv}(T)\propto T^{-2/(1+\eta)}$ if $\Gamma_{\rm tr}\propto T^{2/(1+\eta)}$ was used. In Ref. 19, the authors concluded that the resistivity of the system is proportional to $T^{2/3}$ for the short-range interaction ($\eta=2$) and this is consistent with the latter case. Therefore, our result is in disagreement with their conclusion about the resistivity.

IV. TRANSVERSE POLARIZATION FUNCTION FOR FINITE $q \ll k_F$ AND NONRENORMALIZATION OF THE GAUGE-FIELD PROPAGATOR

It is not easy to find the polarization function for arbitrary $\mathbf{q}$ and ν. However, some simplifications can be made for the $q \ll k_F$ case. In this section, we calculate $\Pi_{11}(\mathbf{q},i\nu)$ for finite $q \ll k_F$ up to two-loop order. We set $N=1$ first, and discuss the extension to the large-N case later.

First of all, $\Pi_{11}^{(1)}$ and $\Pi_{11}^{(2)}$ for finite $\mathbf{q}$ have the following form:

$$\begin{aligned}\Pi_{11}^{(1)}&=-\int\frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}})^2}{m^2}\right]\Sigma(\mathbf{k},i\omega)\\&\quad\times[G_0(\mathbf{k},i\omega)]^2G_0(\mathbf{k}+\mathbf{q},i\omega+i\nu) ,\\ \Pi_{11}^{(2)}&=-\int\frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}})^2}{m^2}\right]\Sigma(\mathbf{k}+\mathbf{q},i\omega+i\nu)\\&\quad\times[G_0(\mathbf{k}+\mathbf{q},i\omega+i\nu)]^2G_0(\mathbf{k},i\omega) .\end{aligned}\tag{46}$$

Using the similar method as that used in Sec. III, one can obtain

$$\begin{aligned}\Pi_{11}^{(1)}+\Pi_{11}^{(2)}\approx\int\frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}})^2}{m^2}\right]G_0(\mathbf{k},i\omega)\\ \times G_0(\mathbf{k}+\mathbf{q},i\omega+i\nu)\\ \times\frac{\Sigma(\mathbf{k},i\omega)-\Sigma(\mathbf{k}+\mathbf{q},i\omega+i\nu)}{i\nu-v_Fq\cos\theta_{\mathbf{kq}}} .\end{aligned}\tag{47}$$

Next we should consider the vertex correction (Fig. 4) for finite $\mathbf{q}$:

$$\begin{aligned}\Gamma_1(\mathbf{k},\mathbf{q};i\omega,i\nu)=\int\frac{d^2q'}{(2\pi)^2}\frac{d\nu'}{2\pi}A(\mathbf{k},\mathbf{q},\mathbf{q}')B(\mathbf{k},\mathbf{q},\mathbf{q}')\\ \times G_0(\mathbf{k}+\mathbf{q}',i\omega+i\nu')\\ \times G_0(\mathbf{k}+\mathbf{q}'+\mathbf{q},i\omega+i\nu'+i\nu)\\ \times D_{11}(\mathbf{q}',i\nu') ,\end{aligned}\tag{48}$$

where

$$\begin{aligned}A&=-\frac{k_1+q_1'+q_1/2}{m}=-\frac{k_1+q_1'}{m} ,\\ B&=\frac{1}{m}[(\mathbf{k}+\mathbf{q}'/2)\cdot(\mathbf{k}+\mathbf{q}+\mathbf{q}'/2)\\&\quad-(\mathbf{k}+\mathbf{q}'/2)\cdot\hat{\mathbf{q}}'(\mathbf{k}+\mathbf{q}+\mathbf{q}'/2)\cdot\hat{\mathbf{q}}'] .\end{aligned}\tag{49}$$

For $q \ll k_F$ and $|\mathbf{k}|\approx k_F$, the following approximation can be made:

$$B\approx\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}}')^2}{m} . \tag{50}$$

Using this approximation, one can show that

$$\begin{aligned}\Pi_{11}^{(3)}&=-\int\frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}\left[-\frac{k_1}{m}\right]\Gamma_1(\mathbf{k},\mathbf{q};i\omega,i\nu)\\&\quad\times G_0(\mathbf{k},i\omega)G_0(\mathbf{k}+\mathbf{q},i\omega+i\nu)\\&\approx\Pi_{11}^{(3,3)}+\Pi_{11}^{(3,4)} ,\end{aligned}\tag{51}$$

where

$$\begin{aligned}\Pi_{11}^{(3,3)}&=-\int\frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}})^2}{m^2}\right]\Gamma_0(\mathbf{k},\mathbf{q};i\omega,i\nu)G_0(\mathbf{k},i\omega)G_0(\mathbf{k}+\mathbf{q},i\omega+i\nu) ,\\ \Pi_{11}^{(3,4)}&=-\int\frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}G_0(\mathbf{k},i\omega)G_0(\mathbf{k}+\mathbf{q},i\omega+i\nu)\\&\quad\times\int\frac{d^2q'}{(2\pi)^2}\frac{d\nu'}{2\pi}\left[\frac{q_1'k_1}{m^2}\right]\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}}')^2}{m^2}\right]G_0(\mathbf{k}+\mathbf{q}',i\omega+i\nu')G_0(\mathbf{k}+\mathbf{q}'+\mathbf{q},i\omega+i\nu'+i\nu)D_{11}(\mathbf{q}',i\nu') .\end{aligned}\tag{52}$$

First, let us calculate the scalar vertex part $\Gamma_0(\mathbf{k},\mathbf{q};i\omega,i\nu)$. We use $\xi_{\mathbf{k}+\mathbf{q}'}\approx\xi_{\mathbf{k}}+v_Fq'_\parallel+q'^2_\perp/2m$ and $\xi_{\mathbf{k}+\mathbf{q}'+\mathbf{q}}\approx\xi_{\mathbf{k}}+v_Fq'_\parallel+v_Fq\cos\theta_{\mathbf{kq}}+(qq'_\perp/m)\sin\theta_{\mathbf{kq}}+q'^2_\perp/2m$ (where $q'_\parallel=q'\cos\theta_{\mathbf{kq}'}$ and $q'_\perp=q'\sin\theta_{\mathbf{kq}'}$) to perform the integral in Eq. (26). Using the fact that the important region of q' is the order of $\nu^{1/(1+\eta)}\ll 1$ so that $q'/k\approx q'/k_F\ll 1$, we conclude[23,27,28] that $q'_\parallel/k_F\approx(q'_\perp/k_F)^2$ and we can approximate the gauge-field propagator as $D_{11}(\mathbf{q}',i\nu')\approx 1/(\gamma|\nu'|/|q'_\perp|+\chi|q'_\perp|^\eta)$. After performing the $q'_\parallel$ integral, we get

$$\Gamma_0(\mathbf{k},\mathbf{q};i\omega,i\nu)\approx -iv_F\int\frac{d\nu'}{2\pi}\int\frac{dq'_\perp}{2\pi}[\mathrm{sgn}(\omega+\nu')-\mathrm{sgn}(\omega+\nu+\nu')]\times\frac{1}{i\nu-v_Fq\cos\theta_{\mathbf{kq}}-(qq'_\perp/m)\sin\theta_{\mathbf{kq}}}\,\frac{1}{\gamma|\nu'/q'_\perp|+\chi|q'_\perp|^\eta}\;. \tag{53}$$

Now the ν' integral gives

$$\Gamma_0(\mathbf{k},\mathbf{q};i\omega,i\nu)\approx -\frac{v_F}{\gamma}\frac{1}{\pi^2}\int_{-k_F}^{k_F}dq'_\perp\frac{|q'_\perp|}{\nu+iv_Fq\cos\theta_{\mathbf{kq}}+i(qq'_\perp/m)\sin\theta_{\mathbf{kq}}}\times\left[\ln\left[1+\frac{|\omega|\gamma}{|q'_\perp|^{1+\eta}\chi}\right]\mathrm{sgn}(\omega)-\ln\left[1+\frac{|\omega+\nu|\gamma}{|q'_\perp|^{1+\eta}\chi}\right]\mathrm{sgn}(\omega+\nu)\right]\;. \tag{54}$$

By changing variables, one can get the following formula:

$$\Gamma_0(\mathbf{k},\mathbf{q};i\omega,i\nu)\approx -\frac{v_F}{\gamma}\frac{1}{\pi^2}\frac{1}{\nu+iv_Fq\cos\theta_{\mathbf{kq}}}\times\left\{\left[\frac{|\omega|\gamma}{\chi}\right]^{2/(1+\eta)}F\left[\omega,\frac{(q/m)\sin\theta_{\mathbf{kq}}}{v_Fq\cos\theta_{\mathbf{kq}}-i\nu}\left[\frac{|\omega|\gamma}{\chi}\right]^{1/(1+\eta)}\right]\mathrm{sgn}(\omega)-\left[\frac{|\omega+\nu|\gamma}{\chi}\right]^{2/(1+\eta)}F\left[\omega+\nu,\frac{(q/m)\sin\theta_{\mathbf{kq}}}{v_Fq\cos\theta_{\mathbf{kq}}-i\nu}\left[\frac{|\omega+\nu|\gamma}{\chi}\right]^{1/(1+\eta)}\right]\mathrm{sgn}(\omega+\nu)\right\}\;. \tag{55}$$

Here $F(\omega,x)$ is defined as

$$F(\omega,x)=\int_{-y_c}^{y_c}dy\,|y|\frac{\ln(1+|y|^{-1-\eta})}{1+xy}\;, \tag{56}$$

where $y_c=k_F(\chi/|\omega|\gamma)^{1/(1+\eta)}$. It can be easily shown that the $q\to 0$ limit of Eq. (55) is given by Eq. (32). On the other hand, the self-energy can be written as

$$\Sigma(\mathbf{k},\omega)\approx -i\frac{v_F}{\pi^2\gamma}\left[\frac{|\omega|\gamma}{\chi}\right]^{2/(1+\eta)}\mathrm{sgn}(\omega)F(\omega,0)\;. \tag{57}$$

Collecting these results, it can be shown that

$$\Pi_{11}^{(1)}+\Pi_{11}^{(2)}+\Pi_{11}^{(3,3)}\approx -\int\frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}\left[\frac{k^2-(\mathbf{k}\cdot\hat{\mathbf{q}})^2}{m^2}\right]G_0(\mathbf{k},i\omega)G_0(\mathbf{k}+\mathbf{q},i\omega+i\nu)\times\frac{iv_F}{\pi^2\gamma}\frac{1}{v_Fq\cos\theta_{\mathbf{kq}}-i\nu}[I(\omega)-I(\omega+\nu)]\;, \tag{58}$$

where

$$I(\omega)=\left[\frac{|\omega|\gamma}{\chi}\right]^{2/(1+\eta)}\mathrm{sgn}(\omega)\times\left\{F\left[\omega,\frac{(q/m)\sin\theta_{\mathbf{kq}}}{v_Fq\cos\theta_{\mathbf{kq}}-i\nu}\left[\frac{|\omega|\gamma}{\chi}\right]^{1/(1+\eta)}\right]-F(\omega,0)\right\}\;. \tag{59}$$

The integrals in Eq. (58) can be evaluated as the following. Using $\int d^2k/(2\pi)^2=(m/2\pi)\int d\xi_{\mathbf{k}}\int d\theta_{\mathbf{kq}}/2\pi$, one can perform the $\xi_{\mathbf{k}}$ integral easily. The angular integral over $\theta_{\mathbf{kq}}$ can be done by contour integration, which requires long algebraic manipulations. The remaining ω integral and the y integral in $I(\omega)$ of Eq. (59) can be evaluated by scaling the integration variables and expanding the integrand in some limits. More details of the calculation will be demonstrated in the later evaluation of the density-density correlation function [see the discussions about Eqs. (68)–(70) in Sec. V] which can be more easily calculated. First, for $|\nu|\ll v_Fq$,

$$\Pi_{11}^{(1)}+\Pi_{11}^{(2)}+\Pi_{11}^{(3,3)}\approx c_3\frac{m^2v_F^3}{\gamma}\frac{|\nu|}{v_Fq}\frac{(\gamma|\nu|/\chi)^{4/(1+\eta)}}{k_F^3q}\ , \tag{60}$$

while, in the other limit $|\nu| \gg v_Fq$, we get

$$\Pi_{11}^{(1)}+\Pi_{11}^{(2)}+\Pi_{11}^{(3,3)} \approx c_4\frac{m^2v_F^3}{\gamma}\frac{qv_F}{|\nu|}\frac{q}{k_F}\left[\frac{(\gamma/\chi)^{2/(1+\eta)}}{m|\nu|^{(\eta-1)/(\eta+1)}}\right]^2\ , \tag{61}$$

where c_3 and c_4 are dimensionless constants.

The calculation of $\Pi_{11}^{(3,4)}$ can be also done by the similar method used in the evaluation of $\Pi_{11}^{(3,3)}$. First, for $|\nu| \ll v_Fq$, we get

$$\Pi_{11}^{(3,4)}\approx-\frac{m^2v_F^3}{\gamma}\frac{|\nu|}{v_Fq}\times\left[c_5\frac{(\gamma|\nu|/\chi)^{2/(1+\eta)}}{k_F^2}+c_6\frac{(\gamma|\nu|/\chi)^{3/(1+\eta)}}{k_F^2q}\right]\ , \tag{62}$$

whereas, in the other limit $|\nu| \gg v_Fq$,

$$\Pi_{11}^{(3,4)}\approx-\frac{1+\eta}{4\pi^2(5+\eta)}\frac{1}{\sin[4\pi/(1+\eta)]}\times\frac{v_F}{m}\frac{\gamma^{(3-\eta)/(1+\eta)}}{\chi^{4/(1+\eta)}}|\nu|^{(3-\eta)/(1+\eta)}-c_7\frac{m^2v_F^3}{\gamma}\frac{v_Fq^2}{m^2(\chi/\gamma)^{3/(1+\eta)}|\nu|^{3/(1+\eta)}}\ , \tag{63}$$

where c_5, c_6, and c_7 are dimensionless constants.

From the above results, it can be shown that $|\Pi_{11}^{(1)}+\Pi_{11}^{(2)}+\Pi_{11}^{(3,3)}| < |\Pi_{11}^{(3,4)}|$ for relevant limits. Therefore, the imaginary part of the transverse polarization function $\Pi_{11}(\mathbf{q},\Omega)$ (in the Minkowski space) up to two-loop order is given by the following formulas. For $\Omega \ll v_Fq$, we get

$$\mathrm{Im}\Pi_{11}(\mathbf{q},\Omega)\approx-\frac{mv_F^2}{2\pi}\frac{\Omega}{v_Fq}\left[1-a\frac{mv_F}{\gamma}\frac{(\gamma\Omega/\chi)^{2/(1+\eta)}}{k_F^2}-b\frac{mv_F}{\gamma}\frac{(\gamma\Omega/\chi)^{3/(1+\eta)}}{k_F^2q}\right]\ , \tag{64}$$

where a and b are dimensionless constants. Note that the correction is small as far as $1<\eta\le 2$ is concerned. On the other hand, for $\Omega \gg v_Fq$, we have

$$\mathrm{Im}\Pi_{11}(\mathbf{q},\Omega)\approx-\frac{1+\eta}{8\pi^2(5+\eta)}\frac{1}{\sin[2\pi/(1+\eta)]}\times\frac{v_F}{m}\frac{\gamma^{(3-\eta)/(1+\eta)}}{\chi^{4/(1+\eta)}}\Omega^{(3-\eta)/(1+\eta)}\times\left[1+cmv_F^3\left[\frac{\chi}{\gamma}\right]^{1/(1+\eta)}\times\frac{q^2}{\Omega^{(2\eta+3)/(\eta+1)}}\right]\ , \tag{65}$$

where c is a dimensionless constant.

For $\Omega > v_Fq$, there is no contribution to $\mathrm{Im}\Pi_{11}$ from the free-fermion bubble because the regime is outside the particle-hole continuum. Therefore, any nonzero contribution to $\mathrm{Im}\Pi_{11}$ for $\Omega \gg v_Fq$ entirely comes from the gauge-field correction. Note that the first term in Eq. (65) dominates for

$$\Omega > (mv_F^3)^{(1+\eta)/(2\eta+3)}(\chi/\gamma)^{1/(2\eta+3)}q^{(2\eta+2)/(2\eta+3)}\ .$$

On the other hand, the second term becomes more important for

$$v_Fq \ll \Omega < (mv_F^3)^{(1+\eta)/(2\eta+3)}(\chi/\gamma)^{1/(2\eta+3)}q^{(2\eta+2)/(2\eta+3)}$$

so that

$$\mathrm{Im}\Pi_{11} \propto \frac{v_F^4\gamma^{(2-\eta)/(1+\eta)}}{\chi^{3/(1+\eta)}}\frac{q^2}{\Omega^{3\eta/(1+\eta)}}$$

in this regime. As we approach the line given by $\Omega = v_Fq$, $\mathrm{Im}\Pi_{11}$ becomes

$$v_F^{(4+\eta)/(1+\eta)}\frac{\gamma^{(2-\eta)/(1+\eta)}}{\chi^{3/(1+\eta)}}q^{(2-\eta)/(1+\eta)}$$

as a function of q.

In the case of $\Omega \ll v_Fq$, the free-fermion bubble gives $\mathrm{Im}\Pi_{11}^0 = -(mv_F^2/2\pi)(\Omega/v_Fq)$. Note that

$$\mathrm{Im}\Pi_{11}(\mathbf{q},\Omega)\approx-\frac{mv_F^2}{2\pi}\frac{\Omega}{v_Fq}\left[1-a\frac{mv_F}{\gamma}\frac{(\gamma\Omega/\chi)^{2/(1+\eta)}}{k_F^2}\right]$$

for $\Omega < (\chi/\gamma)q^{1+\eta}$ and

$$\mathrm{Im}\Pi_{11}\approx-\frac{mv_F^2}{2\pi}\frac{\Omega}{v_Fq}\left[1-b\frac{mv_F}{\gamma}\frac{(\gamma\Omega/\chi)^{3/(1+\eta)}}{k_F^2q}\right]$$

for $(\chi/\gamma)q^{1+\eta} < \Omega \ll v_Fq$. It is gratifying to note that, along the line $\Omega = v_Fq$, the correction to $\mathrm{Im}\Pi_{11}$ given by the above expression agrees in its q dependence with that obtained by approaching from $\Omega \gg v_Fq$ given in the last paragraph. In any case, the corrections are small compared to the free-fermion result for $1<\eta\le 2$.

Using the result of Π_{11} for $|\nu| \ll v_Fq$, we can discuss the issue of the renormalization of the gauge-field propagator. Recall that the dispersion relation of the gauge field obtained from the one-loop correction is given by $|\nu| \sim (\chi/\gamma)q^{1+\eta}$,[1,12,13] which is below the line of $|\nu| = v_Fq$ for sufficiently small q. Along the line of $|\nu| \sim (\chi/\gamma)q^{1+\eta}$, one can easily see that the correction to Π_{11}^0 is smaller by $(mv_F/\gamma)(q/k_F)^2$. Therefore, the gauge-field propagator is not renormalized up to two-loop order. As mentioned in the Introduction, nonrenormalization of the gauge-field propagator was first discussed by Polchinski within a self-consistent argument and without vertex correction. In Ref. 19, the authors discussed the relevance of $\Gamma^{(3)}(a_\mu)$ and $\Gamma^{(4)}(a_\mu)$, which are coefficients of the a^3 and a^4 terms in the expansion of the effective action of the gauge field. They concluded that $\Gamma^{(3)}(a_\mu)$ and $\Gamma^{(4)}(a_\mu)$ are irrelevant so that the gauge field is not renormalized. Since the two-loop diagrams we considered are generated from $\Gamma^{(4)}(a_\mu)$, our calculation is consistent with their conclusion. By analogy, we expect that $\Pi_{11}^{(4)}$ and $\Pi_{11}^{(5)}$ are irrelevant for the renormalization of the gauge field because these are generated from $\Gamma^{(3)}(a_\mu)$. We also directly evaluated $\Gamma^{(3)}(a_\mu)$ and confirmed the argument of Ref. 19. Therefore, one can expect that the gauge field is not renormalized up $[(1/N)^0]$th order in the $1/N$ expansion. That is, the RPA calculation gives the leading contributions in the low-energy limit.

V. DENSITY-DENSITY CORRELATION FUNCTION FOR FINITE $q \ll k_F$

The polarization function for the density channel $\Pi_{00}(\mathbf{q},\Omega)$ can be also calculated in a similar way as used in Sec. IV. In this section, we consider the two-loop corrections given by Figs. 1(a)–1(e) and the finite $q \ll k_F$ case. The sum of the contributions from the self-energy corrections given by Figs. 1(a)–1(d) can be written as

$$\Pi_{00}^{(1)} \approx \int \frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi} G_0(\mathbf{k},i\omega)G_0(\mathbf{k}+\mathbf{q},i\omega+i\nu) \times \frac{\Sigma(\mathbf{k},i\omega)-\Sigma(\mathbf{k}+\mathbf{q},i\omega+i\nu)}{i\nu - v_F q\cos\theta_{\mathbf{kq}}}, \tag{66}$$

while the contribution given by Fig. 1(e), which comes from the vertex correction, can be also written as

$$\Pi_{00}^{(2)} = -\int \frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}\Gamma_0(\mathbf{k},\mathbf{q};i\omega,i\nu)G_0(\mathbf{k},i\omega) \times G_0(\mathbf{k}+\mathbf{q},i\omega+i\nu) . \tag{67}$$

Using Eqs. (55) and (57), it can be shown that

$$\Pi_{00}^{(1)}+\Pi_{00}^{(2)} \approx -\int \frac{d^2k}{(2\pi)^2}\frac{d\omega}{2\pi}G_0(\mathbf{k},i\omega) \times G_0(\mathbf{k}+\mathbf{q},i\omega+i\nu)\frac{iv_F}{\pi^2\gamma} \times \frac{1}{v_F q\cos\theta_{\mathbf{kq}} - i\nu}[I(\omega)-I(\omega+\nu)] , \tag{68}$$

where $I(\omega)$ is given by Eq. (59). Using $\int d^2k/(2\pi)^2 = (m/2\pi)\int d\xi_{\mathbf{k}}\int d\theta_{\mathbf{kq}}/2\pi$, one can easily perform the $\xi_{\mathbf{k}}$ integral, which generates the additional factor $v_F q\cos\theta_{\mathbf{kq}} - i\nu$ in the denominator of the integrand of Eq. (68). Recalling that $I(\omega)$ also has an angle dependence $\theta_{\mathbf{kq}}$, one can perform the angular integral over $\theta_{\mathbf{kq}}$ by contour integration, which requires long algebraic manipulations. After rescaling the ω integral by a new variable x and the y integral in $I(\omega)$ [see Eqs. (56) and (59)] by newly defined y, we get

$$\Pi_{00}^{(1)}+\Pi_{00}^{(2)} \approx \frac{2k_F^2}{\pi^3\gamma}\frac{|\nu|}{v_F^2q^2}\int_0^1 dx\int_0^1 dy\, y\ln\left|1+\frac{x\beta^{1+\eta}}{y^{1+\eta}}\right| \times \left[\frac{|\alpha|}{(1+\alpha^2)\sqrt{1+\alpha^2+y^2}} - \frac{|\alpha|}{(1+\alpha^2)^{3/2}}\right], \tag{69}$$

where $\alpha = \nu/v_F q$ and $\beta = (1/k_F)(|\nu|\gamma/\chi)^{1/(1+\eta)}$. In the small frequency ν limit, the parameter integrals can be done, yielding

$$\Pi_{00}^{(1)}+\Pi_{00}^{(2)} \approx -\frac{a_1}{k_F^{\eta-2}}\frac{|\alpha|^3}{(1+\alpha^2)^{3/2}} - \frac{1+\eta}{4\pi^2(5+\eta)}\frac{1}{\sin[4\pi/(1+\eta)]}\frac{1}{k_F\gamma}\frac{1}{v_F q} \times \left[\frac{\gamma|\nu|}{\chi}\right]^{4/(1+\eta)}\frac{\alpha^2}{(1+\alpha^2)^{5/2}}, \tag{70}$$

where a_1 is an undetermined constant. This formula is valid for all ratios of q and ν, as long as both are small. Note that the first term gives only an analytic contribution, which also arises in the usual Fermi-liquid theory. Similar methods can be used to produce a somewhat more complicated formula valid for all α for the transverse polarization function Π_{11} [for example, Eqs. (52) and (58) can be evaluated by a similar method].

After dropping the analytic contribution, we combine the free-fermion contribution and perform analytic continuation to get, for $\Omega \ll v_F q$,

$$\mathrm{Im}\Pi_{00}(q,\Omega) \approx -\frac{m}{2\pi}\frac{\Omega}{v_F q}\left[1-\frac{1+\eta}{4\pi(5+\eta)}\frac{1}{\cos[(\eta-1)\pi/(\eta+1)]}\frac{1}{k_F m}\frac{\gamma^{(3-\eta)/(1+\eta)}}{\chi^{4/(1+\eta)}}\Omega^{(3-\eta)/(1+\eta)}\left[\frac{\Omega}{v_F q}\right]^2\right], \tag{71}$$

and for $\Omega \gg v_F q$,

$$\mathrm{Im}\Pi_{00}(q,\Omega) \approx -\frac{1+\eta}{8\pi^2(5+\eta)}\frac{1}{\sin[2\pi/(1+\eta)]}\frac{1}{k_F} \times \frac{\gamma^{(3-\eta)/(1+\eta)}}{\chi^{4/(1+\eta)}}\Omega^{(3-\eta)/(1+\eta)}\left[\frac{v_F q}{\Omega}\right]^2 . \tag{72}$$

Note that $\mathrm{Im}\Pi_{11}(q\to 0,\Omega) = (\Omega^2/v_F^2q^2)\mathrm{Im}\Pi_{00}(q\to 0,\Omega)$ is satisfied. Therefore, both of $\mathrm{Im}\Pi_{11}(q\to 0,\Omega)$ and $\mathrm{Im}\Pi_{00}(q\to 0,\Omega)$ give the same answer for the optical conductivity given by Eq. (41).

VI. COMPARISON TO THE FERMI-LIQUID THEORY

In Sec. III, it was shown that the resulting conductivity is consistent with a modified Drude formula. In this section, we try to fit this result to the Fermi-liquid theory framework to extract information about the Fermi-liquid parameters and examine whether the gauge field induces some singular or divergent parameters. In the Fermi-liquid theory, the conductivity for N species of fermions is given by[31]

$$\sigma(\Omega)=\frac{Nne^2}{m^*}\frac{\tau}{1-i\Omega\tau(m/m^*)} \tag{73}$$

or

$$\mathrm{Re}\sigma(\Omega)=\frac{Nne^2}{m}\frac{\Gamma_{\mathrm{tr}}}{\Omega^2+\Gamma_{\mathrm{tr}}^2}, \tag{74}$$

where $\Gamma_{\mathrm{tr}}=\Gamma_{\mathrm{sc}}(m^*/m)$, $\Gamma_{\mathrm{sc}}=1/\tau$ is the scattering rate, and τ is the scattering time. Here m^* is the effective mass of the fermion. Using the fact that $\Gamma_{\mathrm{tr}}\propto 1/N$ in the large-N limit, we get

$$\mathrm{Re}\sigma(\Omega)\approx\frac{Nne^2}{m}\frac{\Gamma_{\mathrm{tr}}}{\Omega^2}. \tag{75}$$

Comparing the above result with Eq. (41), which is a result of the $1/N$ expansion, we can again identify Γ_{tr} with $1/\tau_{\mathrm{tr}}^0$ given in Eq. (43). Therefore, we can conclude that $\Gamma_{\mathrm{tr}}=\Gamma_{\mathrm{sc}}(m^*/m)$ scales as $\Omega^{4/(1+\eta)}$ after including $1/N$ corrections due to the gauge-field fluctuations.

In the following we will directly compare our perturbative result for Π_{00} with the density-density correlation function in the Fermi-liquid theory. Our goal is to find out whether the perturbative result can be consistent with a Fermi-liquid theory made up of quasiparticles with a divergent effective mass m^* as suggested, for example, by Eq. (16). First we consider the limit $\Omega=0, q\to 0$, where it is well known that the Fermi-liquid theory predicts

$$\Pi_{00}(\mathbf{q}\to 0,\Omega=0)=\frac{\Pi_{00}^*(\mathbf{q}\to 0,\Omega=0)}{1+f_{0s}\Pi_{00}^*(\mathbf{q}\to 0,\Omega=0)}, \tag{76}$$

where $\Pi_{00}^*=-\int[d^2p/(2\pi)^2](n_{\mathbf{p}}^0-n_{\mathbf{p}-\mathbf{q}}^0)/[\Omega-(\epsilon^{\mathbf{p}}-\epsilon_{\mathbf{p}-\mathbf{q}}^*)]$ is the free-fermion response function with an effective mass m^* and f_{0s} is the angular average of the Fermi-liquid interaction parameter $f_{\mathbf{pp}'}$. In two dimensions, for the small-q limit,

$$\Pi_{00}^*(\mathbf{q},\Omega)=-\frac{m^*}{2\pi}\left[1-\frac{x}{\sqrt{x^2-1}}\theta(x^2-1)+i\frac{x}{\sqrt{1-x^2}}\theta(1-x^2)\right], \tag{77}$$

where $x=\Omega/v_F^*q$. In Euclidean space, the above formula can be reduced to

$$\Pi_{00}^*(\mathbf{q},i\nu)=-\frac{m^*}{2\pi}\left[1-\frac{|\alpha|}{\sqrt{1+\alpha^2}}\right], \tag{78}$$

where $\alpha=\nu/v_F^*q$. Since $\Pi_{00}^*(\mathbf{q}\to 0,\Omega=0)\propto m^*$, the fact that $\Pi_{00}(\mathbf{q}\to 0,\Omega=0)$ is not enhanced implies that f_{0s} is a finite constant. However, this does not imply that the leading-order term in the perturbative expansion of f_{0s} is finite. In fact, it is clear from an expansion of Eq. (76) that if the leading-order correction to m is singular, then the contribution to f_{0s} at the same order should be also singular since Π_{00} has no singular correction in the lowest-order perturbation theory.

Next we consider the full q,Ω dependence of Π_{00} for small q and Ω. We are motivated by the belief that, in the Fermi-liquid theory, $\mathrm{Im}\Pi_{00}(\mathbf{q},\Omega)$ should exhibit the edge of the particle-hole continuum along the line $\Omega=v_F^*q$. However, when $\Omega\neq 0$, a simple formula such as Eq. (76) does not exist for $\Pi_{00}(\mathbf{q},\Omega)$. In particular, $\Pi_{00}(\mathbf{q},\Omega)$ in general depends on the higher moment angular average of the Landau functions, and not just f_{0s}. Nevertheless, the Fermi-liquid theory makes a precise prediction for $\Pi_{00}(\mathbf{q},\Omega)$ for all q,Ω in terms of m^* and the interaction parameter $f_{\mathbf{pp}'}$. This is given by the quantum Boltzmann equation for the quasiparticle distribution function $n_{\mathbf{p}}=n_{\mathbf{p}}^0+\delta n_{\mathbf{p}}$ in the Fermi-liquid theory, where $n_{\mathbf{p}}^0$ is the distribution function for the free-fermion system with an effective mass m^*,

$$[\Omega-(\epsilon_{\mathbf{p}+\mathbf{q}/2}^*-\epsilon_{\mathbf{p}-\mathbf{q}/2}^*)]\delta n_{\mathbf{p}}-(n_{\mathbf{p}+\mathbf{q}/2}^0-n_{\mathbf{p}-\mathbf{q}/2}^0)\left[U(\mathbf{q},\Omega)+\int\frac{d^2p'}{(2\pi)^2}f_{\mathbf{pp}'}\delta n_{\mathbf{p}'}(\mathbf{q},\Omega)\right]=0. \tag{79}$$

Here $\epsilon_{\mathbf{p}}^*$ is the quasiparticle energy, $U(\mathbf{q},\Omega)$ is the external potential, and $f_{\mathbf{pp}'}$ is the Fermi-liquid interaction parameter. The linear response of $\delta n_{\mathbf{p}}$ to the external potential can be calculated form Eq. (79) (to first order in $f_{\mathbf{pp}'}$),

$$\begin{aligned}\delta n_{\mathbf{p}}(\mathbf{q},\Omega)&=\left[c_{\mathbf{p}}+\int\frac{d^2p'}{(2\pi)^2}c_{\mathbf{p}}f_{\mathbf{pp}'}c_{\mathbf{p}'}\right]U(\mathbf{q},\Omega),\\ c_{\mathbf{p}}&=\frac{n_{\mathbf{p}+\mathbf{q}/2}^0-n_{\mathbf{p}-\mathbf{q}/2}^0}{\Omega-(\epsilon_{\mathbf{p}+\mathbf{q}/2}^*-\epsilon_{\mathbf{p}-\mathbf{q}/2}^*)}.\end{aligned} \tag{80}$$

The change in the density of the fermions $\delta\rho(\mathbf{q},\Omega)=\int d^2p/(2\pi)^2\delta n_{\mathbf{p}}(\mathbf{q},\Omega)$ is given by

$$\begin{aligned}\frac{\delta\rho(\mathbf{q},\Omega)}{U(\mathbf{q},\Omega)}&=-\Pi_{00}(\mathbf{q},\Omega)\\ &=\int\frac{d^2p}{(2\pi)^2}\frac{n_{\mathbf{p}}^0-n_{\mathbf{p}-\mathbf{q}}^0}{\Omega-(\epsilon_{\mathbf{p}}^*-\epsilon_{p-q}^*)}\\ &\quad+\int\frac{d^2pd^2p'}{(2\pi)^4}c_{\mathbf{p}}f_{\mathbf{pp}'}c_{\mathbf{p}'}+\cdots,\end{aligned} \tag{81}$$

where the ellipses represent the higher-order terms in $f_{\mathbf{pp}'}$. The second term is just the diagram given in Fig. 1(e), but with a frequency-independent interaction $f_{\mathbf{pp}'}$.

Let us now examine what happens to the edge in the particle-hole continuum according to our perturbative results. The gauge interaction may induce nonzero Fermi-liquid interaction function $f_{\mathbf{pp}'}$ and a change in the Fermi velocity δv_F. From Eqs. (78) and (81), a change in the Fermi velocity δv_F and the appearance of the Fermi-liquid interaction parameter induce the following change in the density-density correlation function:

$$\delta\Pi_{00} = -\frac{\delta v_F}{v_F}\left[-\Pi^*_{00} + \frac{k_F}{2\pi v_F}\frac{|\alpha|}{(1+\alpha^2)^{3/2}}\right] - \int \frac{d^2p\, d^2p'}{(2\pi)^4} c_{\mathbf{p}} f_{\mathbf{p}\mathbf{p}'} c_{\mathbf{p}'} \; . \tag{82}$$

If we assume a power-law behavior for $f_{\mathbf{pp}'} \sim 1/|\mathbf{p}-\mathbf{p}'|^{\lambda}$, with $\lambda<1$ (i.e., finite f_{0s}), one can show that the second term in Eq. (82) cannot produce the singular term $(1+\alpha^2)^{-3/2}$ near $\alpha^2=-1$. To prove this argument, let us perform the integration over $|\mathbf{p}|$ and $|\mathbf{p}'|$ in the small-q limit, yielding

$$\int \frac{d^2p\, d^2p'}{(2\pi)^4} c_{\mathbf{p}} f_{\mathbf{pp}'} c_{\mathbf{p}'} = \frac{4k_F^2}{(2\pi)^4}\int d\theta_{\mathbf{pq}}\, d\theta_{\mathbf{p}'\mathbf{q}} \frac{q^2\cos\theta_{\mathbf{pq}}\cos\theta_{\mathbf{p}'\mathbf{q}} f_{\mathbf{pp}'}}{(\Omega - v_F q\cos\theta_{\mathbf{pq}})(\Omega - v_F q\cos\theta_{\mathbf{p}'\mathbf{q}})} \; , \tag{83}$$

where $\theta_{\mathbf{pq}}$ ($\theta_{\mathbf{p}'\mathbf{q}}$) is the angle between $\mathbf{p}$ and $\mathbf{q}$ ($\mathbf{p}'$ and $\mathbf{q}$). In order to obtain the leading singularity near $\Omega=v_Fq$, the above expression can be further simplified,

$$\int \frac{d^2p\, d^2p'}{(2\pi)^4} c_{\mathbf{p}} f_{\mathbf{pp}'} c_{\mathbf{p}'} = \frac{4k_F^2}{(2\pi)^4 v_F^2}\int d\theta_{\mathbf{pq}} d\theta_{\mathbf{p}'\mathbf{q}} \frac{f_{\mathbf{pp}'}}{[(\Omega/v_Fq-1)+\frac{1}{2}\theta^2_{\mathbf{pq}}][(\Omega/v_Fq-1)+\frac{1}{2}\theta^2_{\mathbf{p}'\mathbf{q}}]} \; . \tag{84}$$

For $f_{\mathbf{pp}'} \propto 1/|\theta_{\mathbf{pq}}-\theta_{\mathbf{p}'\mathbf{q}}|^{\lambda}$ with $\lambda<1$, the above integral can be estimated through a scaling argument. We find

$$\int \frac{d^2p\, d^2p'}{(2\pi)^4} c_{\mathbf{p}} f_{\mathbf{pp}'} c_{\mathbf{p}'} \propto \frac{1}{(\Omega/v_Fq-1)^{(2+\lambda)/2}} \; , \tag{85}$$

which is less divergent than $(1+\alpha^2)^{-3/2}$ term that leads to $(\Omega/v_Fq-1)^{-3/2}$ divergence. Thus, there is no cancellation between the first and the second terms in Eq. (82). If δv_F diverges at small frequencies, we can conclude that $\delta\Pi_{00}$ will diverge in the limit $\nu\to 0$ with ν/v_Fq fixed, which contradicts our two-loop result from Eq. (71) that shows no such divergent term. Similar results also hold for the transverse current-current response function.

The argument above assumes a power-law behavior for $f_{\mathbf{pp}'} \propto 1/|\theta_{\mathbf{pq}}-\theta_{\mathbf{p}'\mathbf{q}}|^{\lambda}$. As $\lambda\to 1$, another possibility needs to be considered, namely, $f_{\hat{\mathbf{p}}\hat{\mathbf{p}}'} \propto \delta(\hat{\mathbf{p}}-\hat{\mathbf{p}}')$. This satisfies the condition that f_{0s} is finite. From Eq. (84) it is clear that this will lead to a term of order $(1+\alpha^2)^{-3/2}$, which may cancel the first term in Eq. (82). However, in this case, we shall argue that, at least at zero temperature, $f_{\hat{\mathbf{p}}\hat{\mathbf{p}}'}=\zeta\delta(\hat{\mathbf{p}}-\hat{\mathbf{p}}')$ is equivalent to a shift in the Fermi velocity by $v_F\to v_F+\zeta k_F/(2\pi)^2$. At zero temperature the excitation can be described by a distortion of the Fermi surface in the direction $\hat{\mathbf{p}}$ by an amount $\delta\nu_{\hat{\mathbf{p}}}=\int d|\mathbf{p}|\delta n_{\mathbf{p}}$. The original Landau's expression of the free-energy density takes the form

$$\begin{aligned}\delta F &= \int \frac{d^2p}{(2\pi)^2} v_F(|\mathbf{p}|-k_F)\delta n_{\mathbf{p}} \\ &\quad + \frac{1}{2}\int \frac{d^2p\, d^2p'}{(2\pi)^4} f_{\mathbf{pp}'}\delta n_{\mathbf{p}}\delta n_{\mathbf{p}'} \\ &= \int \frac{k_F d\hat{\mathbf{p}}}{(2\pi)^2}\frac{1}{2}v_F(\delta\nu_{\hat{\mathbf{p}}})^2 \\ &\quad + \frac{1}{2}\int \frac{k_F^2 d\,\hat{\mathbf{p}}\, d\hat{\mathbf{p}}'}{(2\pi)^4} f_{\hat{\mathbf{p}}\hat{\mathbf{p}}'}\,\delta\nu_{\hat{\mathbf{p}}}\delta\nu_{\hat{\mathbf{p}}'} \; .\end{aligned} \tag{86}$$

It is then clear that $f_{\hat{\mathbf{p}}\hat{\mathbf{p}}'}=\zeta\delta(\hat{\mathbf{p}}-\hat{\mathbf{p}}')$ is equivalent to $v_F\to v_F+\zeta k_F/(2\pi)^2$. The same result can also be obtained by performing an integral over $|\mathbf{p}|$ in Eq. (79), which leads to

$$(\Omega - v_Fq\cos\theta)\delta\nu_{\hat{\mathbf{p}}} - q\cos\theta\left[U(\mathbf{q},\Omega) + \int \frac{k_F d\hat{\mathbf{p}}'}{(2\pi)^2} f_{\hat{\mathbf{p}}\hat{\mathbf{p}}'}\delta\nu_{\hat{\mathbf{p}}'}\right] = 0 \tag{87}$$

in the small-q limit. Thus we see that, at zero temperature, all response functions to an external perturbation can be described by a Landau theory with a nondivergent effective mass in the small-q limit. However, it is also possible that the same response function can be described by a Landau-Fermi-liquid theory of which both effective mass and $f_{\mathbf{pp}'}$ have divergent perturbative corrections.

An examination of Eq. (70) shows that after analytic continuation, the factor $(1+\alpha^2)^{-5/2}$ diverges at $\Omega=v_Fq$, even though its coefficient vanishes for $\Omega\to 0$. In the following we attempt an interpretation of the result. We can write our perturbative result Eq. (70) as, near $\Omega=v_Fq$,

$$\begin{aligned}\mathrm{Im}\Pi_{00}(\mathbf{q},\Omega) &= \mathrm{Im}\Pi^0_{00}(\mathbf{q},\Omega) + \alpha_0\frac{\partial\, \mathrm{Im}\Pi^0_{00}(\mathbf{q},\Omega)}{\partial\Omega} \\ &\quad + \gamma_0\frac{\partial^2 \mathrm{Im}\Pi^0_{00}(\mathbf{q},\Omega)}{\partial\Omega^2} \; ,\end{aligned} \tag{88}$$

where Π^0_{00} is given by Eq. (77) with $m^*\to m$, and

$$\begin{aligned}\alpha_0 &= \frac{a_2}{k_F^{\eta-2}\chi}q \; , \\ \gamma_0 &= \frac{1+\eta}{8\pi^2(5+\eta)}\frac{1}{\cos[2\pi/(1+\eta)]}\frac{1}{k_F\gamma}\frac{1}{v_Fq} \\ &\quad \times \left[\frac{\gamma\Omega}{\chi}\right]^{4/(1+\eta)} q^2 \; ,\end{aligned} \tag{89}$$

where a_2 is a constant. The existence of the $\partial\, \mathrm{Im}\Pi^0_{00}(\mathbf{q},\Omega)/\partial\Omega$ term in Eq. (88) signifies that there is a finite nonsingular [see α_0 in Eq. (89)] shift in v_F, which also arises in the usual Fermi-liquid theory. To interpret the second-derivative term, we note that Eq. (88) is consistent with (apart from the term proportional to α_0)

$$\mathrm{Im}\Pi_{00}(\mathbf{q},\Omega)=\tfrac{1}{2}[\mathrm{Im}\Pi^0_{00}(\mathbf{q},\Omega+\Gamma)+\mathrm{Im}\Pi^0_{00}(\mathbf{q},\Omega-\Gamma)]\ , \tag{90}$$

if $\Gamma=\sqrt{2\gamma_0}$. We recall that $\mathrm{Im}\Pi^0_{00}(\mathbf{q},\Omega)$ has a discontinuity at $\Omega=v_F q$, corresponding to the edge of the particle-hole continuum. Equation (90) has the natural interpretation of a smearing of the discontinuity at a shifted (due to a shift in v_F) edge of the particle-hole continuum by the amount Γ. Setting $v_F q \propto \Omega$, we find that

$$\Gamma \propto \Omega^{1+(3-\eta)/(2+2\eta)}\ . \tag{91}$$

Note that for $\eta<3$, $\Gamma<\Omega$ so that the above picture is a self-consistent one. We also note that Γ is proportional to the square root of the coupling constant or $1/N$, and is therefore nonanalytic. We are not certain if any further physical meaning can be ascribed to the energy scale Γ.

VII. CONCLUSION

In this paper we studied properties of gauge-invariant correlation functions in a two-dimensional fermion system coupled to a gauge field. We find the physical picture emerged from those gauge-invariant correlation functions to be very different from those obtained from gauge-dependent one-particle Green's function. The corrections to the Fermi-liquid two-particle correlation functions are found to be nondivergent and subleading to the Fermi-liquid contributions up to the two-loop order, and there is no need to go beyond the perturbation theory at this order.

However, it is still possible that singular corrections to the gauge-invariant two-particle correlation functions may appear in some special cases, such as $q=2k_F$. Also, since we do not have quasiparticles to serve as the underpinning of the Fermi-liquid-like behavior for Π_{00} and Π_{11}, it is possible that singularity shows up in some other response functions. Nevertheless, the perturbative result should serve as a test for any theory such as renormalization-group analysis,[26] which attempts to go beyond perturbation theory.

Finally, we would like to comment on the implication of our results to the HTSC. Even though our results suggest that the two-particle Green's functions of fermions are Fermi-liquid-like for small q and Ω, it does not mean that the gauge-field formulation of the t-J model (in relation to the normal-sate properties of HTSC) leads to the Fermi-liquid interpretation of the normal state of HTSC. In the problem of the t-J model, there are bosons as well as fermions that are interacting with a gauge field.[12] In fact, the presence of fermions and bosons in this problem came from the non-double-occupancy constraint on the electrons. It has been also regarded as a way of describing the spin-charge separation induced by the strong correlation effects. In the papers of Nagaosa and Lee,[12] they clearly demonstrated that the anomalous transport properties are due to the bosons. That is, the presence of the bosons plays an important role in the non-Fermi-liquid behaviors of the normal state of HTSC. However, in this paper we considered only the fermions interacting with a gauge field.

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[1] B. I. Halperin, P. A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993).
[2] V. Kalmeyer and S. C. Zhang, Phys. Rev. B **46**, 9889 (1992).
[3] S. H. Simon and B. I. Halperin, Phys. Rev. B **48**, 17 368 (1993); **50**, 1807 (1994); Song He, S. H. Simon, and B. I. Halperin, *ibid.* **50**, 1823 (1994).
[4] J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989); Phys. Rev. B **41**, 7653 (1990); Adv. Phys. **41**, 105 (1992).
[5] A. Lopez and E. Fradkin, Phys. Rev. B **44**, 5246 (1991); Phys. Rev. Lett. **69**, 2126 (1992).
[6] R. L. Willet, M. A. Paalanen, R. R. Ruel, K. W. West, L. N. Pfeiffer, and D. J. Bishop, Phys. Rev. B **65**, 112 (1990).
[7] R. W. Willet, R. R. Ruel, M. A. Paalanen, K. W. West, and L. N. Pfeiffer, Phys. Rev. B **47**, 7344 (1993).
[8] R. R. Du, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **70**, 2944 (1993).
[9] W. Kang, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. **71**, 3850 (1993).
[10] D. R. Leadley, R. J. Nicholas, C. T. Foxon, and J. J. Harries, Phys. Rev. Lett. **72**, 1906 (1994).
[11] V. J. Goldman, B. Su, and J. K. Jain, Phys. Rev. Lett. **72**, 2065 (1994).
[12] N. Nagaosa and P. A. Lee, Phys. Rev. Lett. **64**, 2550 (1990); Phys. Rev. B **46**, 5621 (1992).
[13] L. B. Ioffe and A. I. Larkin, Phys. Rev. B **39**, 8988 (1989).
[14] P. A. Lee, Phys. Rev. Lett. **63**, 680 (1989).
[15] L. B. Ioffe and P. B. Wiegmann, Phys. Rev. Lett. **65**, 653 (1990).
[16] L. B. Ioffe and G. Kotliar, Phys. Rev. B **42**, 10 348 (1990).
[17] T. Holstein, R. E. Norton, and P. Pincus, Phys. Rev. B **8**, 2649 (1973).
[18] M. Yu Reizer, Phys. Rev. B **39**, 1602 (1989); **40**, 11 571 (1989).
[19] Junwu Gan and Eugene Wong, Phys. Rev. Lett. **71**, 4226 (1993).
[20] B. Blok and H. Monien, Phys. Rev. B **47**, 3454 (1993).
[21] B. L. Altshuler and L. B. Ioffe, Phys. Rev. Lett. **69**, 2979 (1992).
[22] D. V. Khveshchenko, R. Hlubina, and T. M. Rice, Phys. Rev. B **48**, 10 766 (1993).
[23] D. V. Khveshchenko and P. C. E. Stamp, Phys. Rev. Lett. **71**, 2118 (1993); Phys. Rev. B **49**, 5227 (1994).
[24] H.-J. Kwon, A. Houghton, and J. B. Marston, Phys. Rev.

Lett. **73**, 284 (1994).
[25]Y. B. Kim and X.-G. Wen, Phys. Rev. B **50**, 8078 (1994).
[26]C. Nayak and F. Wilczek, Nucl. Phys. B **417**, 359 (1994).
[27]L. B. Ioffe, D. Lidsky, and B. L. Altshuler, Phys. Rev. Lett. **73**, 472 (1994).
[28]J. Polchinski, Nucl. Phys. B **422**, 617 (1994).
[29]We would like to remark that $\delta\,\mathrm{Im}\Pi^{s}_{11}$ does not modify the RPA dressed gauge-field propagator even though this gauge-dependent correction violates the Fermi-liquid criterion. This is due to the fact that $\delta\,\mathrm{Im}\Pi^{s}_{11}$ becomes less important than the free-fermion result $\mathrm{Im}\Pi^{0}_{11}$ along the line $\Omega \propto q^3$ (which corresponds to the dispersion relation of the gauge field) for small q and Ω (Refs. 19, 23, and 28).
[30]L. B. Ioffe and V. Kalmeyer, Phys. Rev. B **44**, 750 (1991).
[31]D. Pines and P. Nozieres, *The Theory of Quantum Liquids* (Benjamin, Reading, MA, 1966), Vol. 1.