Slave-boson calculation of the Landau parameters of the one-band Hubbard model

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We present a microscopic slave-boson calculation of the three Landau parameters F_0^s, F_0^a, and F_1^s of the Hubbard model for any strength of the interaction U and any filling $\delta=1-n$. The Landau parameter F_1^a is then obtained by using the s-p approximation on the forward-scattering sum rule for the Landau parameters. General analytic expressions are given for the four Landau parameters and related observables. Simple asymptotic expressions for these quantities in four interesting regimes on the U,δ manifold are presented. We also show the results of numerical calculations of these quantities for the full range of U and δ for a system with a flat density of states, which are then compared with the experimental results for normal ^{3}He. We find good agreement with the experimentally deduced Landau parameters for both the half-filled-band model and the δ-dependent model at reasonable values of U and δ.

I. INTRODUCTION

In 1956 Landau[1] constructed an elegant semiphenomenological theory to describe the macroscopic behavior of normal Fermi liquids in the low-temperature limit. Landau's Fermi-liquid theory has been successfully applied to the understanding of the thermodynamic and transport properties of quantum liquids such as liquid ^{3}He,[2-4] as well as to the description of their collective modes. On the other hand, considerable efforts[5-13] (for the earlier work see Ref. 14) have been made to obtain a reliable microscopic calculation of the effective interaction, the scattering amplitude, and the Landau parameters of an isotropic Fermi liquid.[4,9,10] Such microscopic calculations are beyond the scope of Landau's theory itself. But since the Landau parameters offer a means of parametrizing normal Fermi liquids, a first-principles calculation of these quantities would indeed by very useful. Several attempts have been made along these lines.[6-8] However, these works have not proven very successful. Better results were obtained in Vollhardt's calculation using Gutzwiller's method[15] in his "almost localized" approach.[10] In the late 1960s the variational functional approach, based on the Jastrow wave function, was used by Feenberg and his collaborators[8] with some success compared with the earlier works using the Brueckner and Gammel K-matrix approach,[6] which led to poor results for the effective interaction. However, later work by Ostgaad[7] improved the estimates of the Landau parameters along the K-matrix scheme. Babu and Brown's attempt[9] to compute the interaction function is interesting but it does not give good quantitative results. de Châtel[12] calculated explicitly the Fermi-liquid parameters of the Anderson model for an infinite f-f Coulomb interaction, and his calculations can be shown to be exact for $N_d \to \infty$ (to lowest order in $1/N_d$, if N_d is the orbital degeneracy). de Châtel's calculation has been extended to a finite f-f Coulomb interaction and corrections for order $1/N_d$ were studied by Li *et al.*[13]

One of the best candidates for a microscopic model calculation of the Landau parameters in a strongly correlated fermion system is certainly the Hubbard model, which in its simple form or through its extensions is believed to contain the essence of the physics of strongly interacting fermions such as liquid ^{3}He,[10] transition metals, and heavy-fermion systems. In fact this model has been proposed to describe both the strongly interacting Fermi liquids (of which liquid ^{3}He is the prototype) and the metal-insulator transitions.[16] The Hubbard model is also considered to be relevant[17] to the high-temperature superconductors.[18] It is widely believed that understanding the controversial properties of the normal state of the cuprates, which behave sometimes very differently from a Fermi liquid,[19] is a key step towards explaining the mechanism of high-T_c superconductivity itself. The current debate on this subject, which centers on the existence and the nature of the quasiparticles in the two-dimensional (2D) Hubbard model, as well as the attempts to construct a new quantum-liquid theory for these systems,[20,21] have brought the understanding of strongly correlated fermion systems to the forefront of condensed matter physics. We will restrict ourselves to presenting a microscopic calculation of the Landau parameters, assuming that they can be obtained from the various susceptibilities in the usual manner.

The main problem in developing a microscopic theory that goes beyond perturbation theory has been how to construct a reliable and manageable many-body theory that can deal with a strongly interacting quantum liquid at high densities. Amongst the methods that have been proposed (see, e.g., the review by Vollhardt[10]), Gutzwiller's approach is particularly appealing because

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of its simplicity and because it yields better results than other approximations. Furthermore, Gutzwiller's approach covers almost the whole range of interactions and band fillings. A great success of this method has been the description of the Mott-Hubbard transition, although it has been shown recently that the transition disappears when Gutzwiller's wave function[14] is used on the Hubbard model without Gutzwiller's approximation.[22] Kotliar and Ruckenstein[23] (KR) have developed a functional-integral approach, the field-theory representation of the slave-boson (SB) technique, in which Gutzwiller's results are reproduced at the static paramagnetic saddle point. Their approach not only bridges two fundamentally different methods, the variational method and the functional-integral one, but also offers a systematic way to improve and extend Gutzwiller's results. It also allows for the study of finite-temperature effects.[24-31]

In this paper we present a microscopic calculation of the first four Landau parameters (F_0^s, F_0^a, F_1^s, and F_1^a) for the Hubbard model using the spin-rotationally-invariant version of KR's slave-boson approach. We have calculated all of the two-point dynamical correlation functions and structure factors.[28] The agreement between the quantum Monte Carlo results and the slave-boson approach is impressive, considering that there are no adjustable parameters in this approach.[26,29-31] With this method, the calculation of F_0^a, which amounts to inverting a 2×2 submatrix of the fluctuation matrix $S(k)$, is much simpler than that of F_0^s, which involves the inversion of a 5×5 submatrix $S(k)$. We will adopt the same approach and notations previously used in Ref. 28. The Landau parameters depend in general on the dimension of the system. We will restrict ourselves mainly to the flatband case. As will be seen below, there are no difficulties in extending the calculation to d-dimensional Hubbard lattices. The van Hove singularity in the 2D case brings an extra difficulty into the calculations, but there are ways to avoid it by using a t-matrix approach or by adding next-nearest-neighbor hopping to displace the singularity from half filling. The flatband case is especially appealing since it corresponds to the density of states of a 2D liquid with a free-particle spectrum for the bare particles. Furthermore, it is only in this case that simple analytic expressions for the Landau parameters and related observables may be obtained. In fact in the flatband case all of the Landau parameters can be expressed in terms of a scaled interaction U/U_c, where U_c corresponds to the critical interaction for the Brinkman-Rice transition at half filling. The dimension dependence of the Landau parameters is only explicit in the expression for U_c. Otherwise the expressions for the Landau parameters themselves are explicitly independent of the dimension of the system, which is a scaling property of sorts. In the case of F_0^a, our expression in the flatband case reduces to a previously obtained result,[11] as expected. While F_0^s can be obtained through the long-wavelength limit of the full charge response, we have used instead a much simpler self-consistent approach involving the calculation of the compressibility. In the 2D case, our result for the flatband case reproduces the one previously published[28] without introducing an ambiguous "average density of states" in the calculation.[28] The second symmetric Landau parameter F_1^s comes directly from the saddle-point value of the renormalized hopping factor of KR's slave-boson theory. The last Landau parameter F_1^a is obtained through the use of the forward-scattering sum rule and a cutoff procedure called the s-p approximation. All of these calculations are based on the assumption that the Fermi liquid is stable (which corresponds to the paramagnetic phase of the slave-boson theory). The study of the stability of the Fermi liquid involves a detailed study of the phase diagram of the system, which is beyond the scope of this work.

This paper is organized as follows. In Sec. II we present the general microscopic calculation of the three leading Landau parameters F_0^s, F_0^a, and F_1^s for the Hubbard model. In Sec. III we discuss the isotropic Fermi liquid in the normal phase. For the case of a flatband, we give concise analytic expressions in which the four Landau parameters are expressed as functions of the scaled interaction U/U_c and of the doping factor δ. We then present the analytic expressions for the important thermodynamic, transport, and collective-mode properties of the correlated fermion system using Landau's well-known formulas. Simplified expressions for the Landau parameters and the observables in four special but interesting limits are given in the Appendix. In Sec. IV we present the numerical results for the Landau parameters in the whole range of U/U_c and δ for the flatband. We then compare these results with the experimentally deduced Landau parameters for normal ^{3}He by fitting the pressure dependence of the physical properties[10,11] and of the critical temperature of superfluid ^{3}He with appropriate values of U/U_c (and of δ for the δ-dependent model). We then discuss our results and conclude in Sec. V.

II. CALCULATION OF THE LANDAU PARAMETERS IN KR'S SLAVE-BOSON APPROACH

In the last section we have mentioned that, in some sense, the model calculation of the Landau parameter F_0^s (or F_0^a) amounts to the calculation of the dynamical charge (or spin) correlation function. Since the dynamical charge (spin) correlation function can usually be reduced to the random-phase approximation (RPA) form in the long-wavelength limit, as proven by the slave-boson approach,[28] the value of F_0^s (F_0^a) follows directly. Concise expressions have only been obtained for F_0^a in the flatband case. Before proceeding to the detailed derivation of the Landau parameters, let us briefly recall Kotliar and Ruckenstein's slave-boson theory and its approach to the calculation of the dynamical correlation functions. We assume that the system has space-time inversion symmetry, as is usually the case in quantum liquids with no external magnetic fields. It has been shown[28] that there are only four independent two-point (in the space-time manifold) correlation functions which can be formed out of the occupation number operator $n_{i\sigma}(\tau)$ for fermions of spin s at site i and at imaginary time τ. The dynamical charge and spin correlation functions as well as the other two independent correlation functions are quite complex, as there are as many as four to eight creation and annihilation operators $c^\dagger_\sigma(i,\tau)$ and $c_\sigma(i,\tau)$ involved in the calculation of the expectation value. At first sight it would hardly seem believable that such complicated dynamical correlation functions may be calculated simply, but in fact they can be readily obtained from two-point boson propagators. Following these lines, analytic expressions for those four independent dynamical correlation functions have been obtained.[28] The reliability of the substitution of Fermi operators by boson operators has been tested numerically. Numerical results have also shown reasonable agreement with Monte Carlo simulations[29–31] (note that no adjustable parameters are introduced in the slave-boson calculations). The crucial noninteracting limit is recovered in the $U\to 0$ limit. Particle-hole symmetry, as well as the various symmetry relations between the dynamical correlation functions, have also been proven to be satisfied.[30]

Let us now turn to the microscopic calculation of the first four Landau parameters for the Hubbard model. In the usual notation,

$$H=\sum_{i,j,\sigma} t_{ij}c^\dagger_{i\sigma}c_{j\sigma}+U\sum_i n_{i\uparrow}n_{i\downarrow} \ . \tag{1}$$

For the nearest-neighbor Hubbard model, $t_{ij}=-t$ for neighboring sites, 0 elsewhere. For a quantum liquid, the kinetic part of the Hamiltonian is simply $\hbar^2k^2/2m$ in the k representation. Unlike most other approaches, where the complication caused by the Hubbard interaction term is handled directly, Kotliar and Ruckenstein use a different point of view. They associate the repulsive Hubbard interaction with the suppression of the hopping process. Following KR we rewrite the Hubbard Hamiltonian in terms of the slave-boson operators e_i, $\underline{p}_i$, and d_i (for empty, singly occupied, and doubly occupied sites) and of the fermion operators $f_{i\sigma}$:

$$H=\sum_{\langle i,j\rangle}\sum_{\sigma,\sigma'} f^\dagger_{i\sigma}z^\dagger_{i\sigma\sigma'}z_{j\sigma'\sigma}f_{j\sigma}+U\sum_i d^\dagger_i d_i \ , \tag{2}$$

where the renormalized hopping factor z is defined by

$$\underline{z}_i=[(1-d^\dagger_i d_i)\underline{\tau}_0-\underline{p}^\dagger_i\underline{p}_i]^{-1/2}(e^\dagger_i\underline{p}_i+\underline{\tilde{p}}^\dagger_i d_i)$$
$$\times[(1-e^\dagger_i e_i)\underline{\tau}_0-\underline{\tilde{p}}^\dagger_i\underline{\tilde{p}}_i]^{-1/2} \ . \tag{3}$$

The underbar denotes a 2×2 matrix in spin space, $\underline{\tau}_0$ is the unit matrix, $\underline{\tilde{p}}_i$ is the time-reserved form of the operator $\underline{p}_i$. In order to stay within the physical subspace, the following local constraints must be enforced:

$$e^\dagger_i e_i+d^\dagger_i d_i+\mathrm{tr}(\underline{p}^\dagger_i\underline{p}_i)=1 \ , \tag{4}$$

$$(\underline{p}^\dagger_i\underline{p}_i)_{\alpha\beta}+d^\dagger_i d_i\delta_{\alpha\beta}=f^\dagger_{i\alpha}f_{i\beta} \ . \tag{5}$$

As mentioned before, the key point in the slave-boson approach is that, within the functional-integral formulation, all of the dynamical correlation functions for the fermion occupation numbers $n_{i\sigma}(\tau)$ may be approximated by the two-point dynamical correlation functions of the slave-boson fields. Indeed, it suffices to replace doubly and empty site-occupation projectors by their corresponding boson projectors, i.e.,

$$D_i(\tau)\equiv n_{i\sigma}(\tau)n_{i-\sigma}(\tau)\to d^\dagger_i(\tau)d_i(\tau) \ , \tag{6a}$$

$$E_i(\tau)\equiv[1-n_{i\sigma}(\tau)][1-n_{i-\sigma}(\tau)]\to e^\dagger_i(\tau)e_i(\tau) \ , \tag{6b}$$

and the single-occupancy operator by the state with spin projection s along a quantization axis $\hat{s}$:

$$P_{i\sigma}(\tau)\equiv n_{i\sigma}(\tau)[1-n_{i-\sigma}(\tau)]$$
$$\to \mathrm{Tr}[(\underline{\tau}_0+\sigma\hat{s}\cdot\underline{\tau})\underline{p}^\dagger_i(\tau)\underline{p}_i(\tau)] \ , \tag{6c}$$

where $\underline{\tau}$ is the vector of Pauli matrixes. In order to calculate the correlation functions one must go beyond mean-field theory. The first useful approximation is obtained by calculating the effect of Gaussian fluctuations about the mean field, which provides the leading corrections in a loop expansion. The partition function is given by

$$\int D[f^*,f]D[\text{boson}]\exp\left[\int_0^\beta \partial\tau\, L_{\text{eff}}(\tau)\right] \ , \tag{7}$$

where $\beta=1/k_BT$. We use the same letter to symbolize both the Grassman variables and the corresponding fermion operators in the functional integral. The effective Lagrangian will not be repeated here [see Ref. 24]. After integrating the Grassman variables, which appear only quadratically in the functional integral, one can expand around the saddle point to second order in the boson fields to obtain their fluctuation matrix $S(k)$ about their mean-field values. This introduces the 11-component fluctuation vector of the boson fields $\psi(k)$, defined as the deviation of the fields about their mean-field values:

$$\psi(k)=(\delta e,\delta d,\delta p_0,\delta\beta_0,\delta\alpha;\delta p_1,\delta\beta_1,\delta p_2,\delta\beta_2,\delta p_3,\delta\beta_3) \ , \tag{8}$$

where the boson fields a and $\beta_{i\mu}$ are the Lagrange multipliers enforcing the local constraints (4) and (5). The argument k of the fluctuation matrix and the fluctuation vector stands for $(\mathbf{k},\omega_n)$, where ω_n is a Matsubara boson frequency. The calculation of the dynamical correlation functions is then straightforward: it suffices to invert $S(k)$ and then take suitable linear combinations of the matrix elements $S^{-1}_{\alpha\beta}(k)$. Since the spin and charge degrees of freedom are decoupled in the fluctuation matrix, $S(k)$ is reduced to a block-diagonal $(5\times5)\otimes(2\times2)^{\otimes 3}$ matrix.[28] We need only invert a 2×2 and a 5×5 matrix to obtain respectively the spin and charge dynamical correlations.

In the spin-rotationally-invariant version[25] of KR's slave-boson theory, all 11 slave-boson saddle-point values are expressed through two parameters, the dopant concentration δ and the parameter x, defined as[11]

$$x=e+d \ . \tag{9}$$

The saddle-point values of the fields are

$$d=(x^2-\delta)/2x \ , \tag{10a}$$

$$e=(x^2+\delta)/2x \ , \tag{10b}$$

$$p_0^2=1-(x^4+\delta^2)/2x^2 \ , \tag{10c}$$

$$\alpha=\frac{U_c}{2}x^2p_0^2\left[\frac{1}{1-\delta}+\frac{1}{x^2+\delta}\right], \tag{10d}$$

$$\beta_0=\frac{U_c}{2}x^2\left[\frac{p_0^2}{x^2+\delta}+\frac{p_0^2\delta}{1-\delta^2}-\frac{1}{2}\right], \tag{10e}$$

and

$$\mathbf{p}=\boldsymbol{\beta}=0 . \tag{10f}$$

The value of x is found through Vollhardt's optimization equation, which corresponds to one of the saddle-point equations:

$$(1-x^2)x^4/(x^4-\delta^2)=U/U_c , \tag{11}$$

where

$$U_c=-8\frac{\bar{\varepsilon}_0}{1-\delta^2} \tag{12}$$

corresponds to the critical interaction for the Brinkman-Rice transition when $\delta=0$. Here $\bar{\varepsilon}_0$ is the difference between the average kinetic energy and the kinetic energy at the Fermi level at half filling.

The fermion spectrum $E_{\mathbf{k},\sigma}$ is given by the expression[28]

$$\begin{aligned}E_{\mathbf{k},\sigma}=E_{\mathbf{k}}&=z^2[\varepsilon_{\mathbf{k}}(\delta)-\varepsilon_{\mathbf{k}_F}^{(0)}(\delta=0)]-(\mu-\beta_0)\\ &\equiv z^2\Delta\varepsilon_{\mathbf{k}}-(\mu-\beta_0) ,\end{aligned} \tag{13}$$

where $\varepsilon_{\mathbf{k}}$ is the kinetic energy of the bare particles and $\varepsilon_{\mathbf{k}_F}^{(0)}$ is the corresponding quantity at the Fermi level at half filling. This fermion spectrum for the Hubbard model holds for any dopant concentration and for any strength of the interaction. It is consistent with Methfeseel and Mattis's well-known exact result[32] which states that at half filling the chemical potential is given by

$$\mu(\delta=0,U)=\frac{U}{2} . \tag{14}$$

From the slave-boson approach we also have[28]

$$\beta_0(\delta=0,U)=\frac{U}{2} . \tag{15}$$

Thus

$$E_{\mathbf{k}_F}(\delta=0,U)=0 , \tag{16}$$

which is in conformity with the convention that energy is measured from the Fermi level at half filling. A useful definition of the effective chemical potential is[28]

$$\mu_{\text{eff}}=(\mu-\beta_0)/z^2 , \tag{17}$$

which, given the filling factor, can be determined through

$$\begin{aligned}n&=\frac{2}{L}\sum_{\mathbf{k}}f(E_{\mathbf{k}})=\frac{2}{L}\sum_{\mathbf{k}}f(z^2\Delta\varepsilon_{\mathbf{k}}-(\mu-\beta_0))\\ &=\frac{2}{L}\sum_{\mathbf{k}}\int d\varepsilon\,\delta(\varepsilon-\Delta\varepsilon_{\mathbf{k}})f(z^2\Delta\varepsilon_{\mathbf{k}}-(\mu-\beta_0))\\ &=\int d\varepsilon\,N(\varepsilon)f(z^2\varepsilon-(\mu-\beta_0)) ,\end{aligned} \tag{18}$$

where $N(\varepsilon)$ is the density of states of a given hopping matrix t_{ij} for the bare particles of both spins, calculated using

$$N(\varepsilon)=\frac{2}{L}\sum_{\mathbf{k}}\frac{\partial f}{\partial\Delta\varepsilon_{\mathbf{k}}} . \tag{19}$$

In particular, at zero temperature the Fermi function becomes $\theta(z^2\varepsilon-(\mu-\beta_0))$, so that

$$n=\int_{-\infty}^{\mu_{\text{eff}}}d\varepsilon\,N(\varepsilon) . \tag{20}$$

Thus the effective chemical potential is band-structure dependent. Similarly, the average kinetic energy $\bar{\varepsilon}_0$ is also band-structure dependent. In the energy representation, Eq. (12) is given by the expression[28]

$$U_c=-8\frac{1}{1-\delta^2}\int_{-\infty}^{\mu_{\text{eff}}}d\varepsilon\,\varepsilon N(\varepsilon) . \tag{21}$$

Combining the slave-boson expression for mass enhancement with the corresponding formula from the Landau theory we find

$$1/z^2=m^*/m=1+\frac{F_1^2}{3} . \tag{22}$$

Hereafter we will use the 3D formulas for the relations between the Landau parameters and the normalized observables or scattering amplitudes. All the derivations of the formulas can also be done for the 1D and 2D cases by substituting the relations between the Landau parameters and the normalized observables or scattering amplitudes by those applicable in the 2D and 1D cases. Substituting the saddle-point value of the renormalized hopping factor z [Eq. (3)] in Eq. (22),

$$F_1^s=3(1-y^2)^2/(2y-y^2-\delta^2) , \tag{23}$$

where $y\equiv x^2$. The corresponding forward-scattering amplitude is

$$A_1^s=2(-1+y)^2/(1-\delta^2) . \tag{24}$$

Note that, in the slave-boson approach, the above formulas are general results, independent of the flatband assumption. The dependence of U_c on dimension implies that the solution of the optimization equation x and thus the Landau parameters are generally dimension dependent.

In the long-wavelength limit, the expression for the dynamical spin susceptibility reduces to the RPA form,[24,28] so one can easily extract the Landau parameter

$$F_0^a=\frac{N_F}{2p_0^2}\left\{\alpha-\beta_0+\bar{\varepsilon}_0\left[z\frac{\partial^2 z}{\partial p_1^2}+\left[\frac{\partial z_\uparrow}{\partial p_1}\right]^2\right]+4p_0t_Fz\frac{\partial z_\uparrow}{\partial p_1}\right\}, \tag{25}$$

where N_F is the renormalized density of states $N_F=N_F^0/z^2$. In what follows, we use the subscript F and the superscript (0) to denote the value at the Fermi level and the value at half filling, respectively. With this notation t_F is the value of $\Delta\varepsilon_{\mathbf{k}}=\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}_F}^{(0)}(\delta=0)$ at the Fermi level:

$$t_F=\mu_{\rm eff}=(\mu-\beta_0)/z^2 \,. \tag{26}$$

We also have the derivatives

$$\frac{\partial z_\uparrow^2}{\partial p_1}=4\delta\eta^2 p_0^2(1-2\eta^2 p_0^2 x^2) \tag{27}$$

and

$$\frac{\partial z_\uparrow^2}{\partial p_1^2}=2\sqrt{2}\eta^3 p_0\{x[1+2\eta^2 p_0^2(1+2\delta^2)]-2(e-d)\delta\} \,, \tag{28}$$

where $\eta=1/(1-\delta^2)$. Using (12) and (25), F_0^a can be rewritten as

$$F_0^a=\frac{N_F U_c}{2p_0^2}\left\{(\alpha-\beta_0)/U_c -(1-\delta^2)\left[z\frac{\partial z^2}{\partial p_1}+\left[\frac{\partial z_\uparrow}{\partial p_1}\right]^2\right]\Big/8 +4p_0 z\frac{\partial z_\uparrow}{\partial p_1}\mu_{\rm eff}/U_c\right\} . \tag{29}$$

Substituting the boson fields, the hopping factor, and the first and second derivatives of z with respect to the boson fields, we obtain

$$(\alpha-\beta_0)/U_c=-(y^2-3y+y\delta^2+\delta^2)/4(1-\delta^2) \,, \tag{30}$$

$$z^2=(-\delta^2+2y-y^2)/(1-\delta^2) \,, \tag{31}$$

$$\frac{\partial z_\uparrow}{\partial p_1}=-\left[\frac{2}{y}\right]^{1/2}\frac{\delta(1-y)^2}{(1-\delta^2)^{3/2}} \,, \tag{32}$$

and

$$\frac{\partial^2 z_\uparrow}{\partial p_1^2}=-\frac{2\sqrt{2y-y^2-\delta^2}(y^2+2\delta^2 y^2-3y-3\delta^2 y+3\delta^2)}{y(1-\delta^2)^{5/2}} \,. \tag{33}$$

Thus the general expression for F_0^a can be rewritten as

$$F_0^a=\frac{N_F U_c}{4(2y-y^2-\delta^2)}[16\mu_{\rm eff}z^2\delta(1-y)^2/U_c-y(3y-\delta^2-y\delta-y^2) +z(3\delta^2-3y-3\delta^2 y+2\delta^2 y^2+y^2)/y-\delta^2(1-y^4)/(1-\delta^2)] \,. \tag{34}$$

We can now examine the Landau parameter F_0^s. Vollhardt has shown[10] that the two Landau parameters F_0^a and F_0^s obey the symmetry relation $F_0^s(U)=F_0^a(-U)$ at half filling. We will show that, in the slave-boson approach, this symmetry is retained at half filling at the mean-field level. By inverting the 5×5 submatrix of the fluctuation matrix $S(k)$ and by taking the long-wavelength limit, the full charge correlation function can be obtained and then reduced to a RPA form. The Landau parameter F_0^s follows directly from that RPA expression

$$F_0^s=\frac{N_F\delta}{\overline{N}_0 z^2}\frac{\partial z}{\partial\delta}-N_F^0\frac{1}{z^2}\frac{\partial\beta_0}{\partial\delta} \,. \tag{35}$$

This expression was derived in Ref. 28. $\overline{N}^0$ is an ambiguous "average density of occupied states," used as an intermediate step in the calculations, in order to avoid inverting the 5×5 submatrix of the fluctuation matrix. The derivatives are given by

$$\frac{\partial z^2}{\partial\delta}=\frac{4\delta p_0^2 y}{(1-\delta^2)^2}+\frac{2}{(1-\delta^2)}\left[p_0^2\frac{\partial y}{\partial\delta}+y\frac{\partial p_0^2}{\partial\delta}\right] \tag{36}$$

and

$$\frac{\partial\beta_0}{\partial\delta}=\frac{U_c}{2}y\left[-\frac{p_0^2}{(y+\delta)^2}+\frac{1+\delta^2}{(1-\delta^2)^2}p_0^2\right] +\frac{\partial y}{\partial\delta}\left[y-\frac{U_c}{2}\frac{yp_0^2}{(y+\delta)^2}\right] +\frac{\partial p_0^2}{\partial\delta}\frac{U_c}{2}y\left[\frac{1}{y+\delta}+\frac{\delta}{1-\delta^2}\right] . \tag{37}$$

The derivatives of p_0^2 and $y=x^2$ are[28]

$$\frac{\partial p_0^2}{\partial\delta}=-\frac{\delta}{y}+\frac{1}{2}\left[\frac{\delta^2}{y^2}-1\right]\frac{\partial y}{\partial\delta} \tag{38}$$

and

$$\frac{\partial y}{\partial\delta}=\frac{2u\delta}{y(2u-2+3y)} \,. \tag{39}$$

So far we have followed Ref. 28 in the derivation of the expressions for the Landau parameters. Henceforth, we rederive F_0^s in a simple, self-consistent way, without introducing an ambiguous "average density of states." The compressibility of a Fermi liquid can be written as

$$\frac{\partial\mu}{\partial n}=\frac{1+F_0^s}{(m^*/m)N_0^F} \,, \tag{40}$$

hence

$$F_0^s=-1+\frac{N_0^F}{z^2\partial n/\partial\mu} . \quad (41)$$

One thus needs to calculate $\partial n/\partial\mu$. Rewriting $\partial n/\partial\mu$ as

$$\frac{\partial n}{\partial\mu}=\frac{1}{L}\sum_{k,\sigma}\frac{\partial f_{\mathbf{k},\sigma}}{\partial\mu}=\frac{1}{L}\sum_{\mathbf{k},\sigma}\frac{\partial f_{\mathbf{k},\sigma}}{\partial E_{\mathbf{k},\sigma}}\frac{\partial E_{\mathbf{k},\sigma}}{\partial\mu} , \quad (42)$$

where $f_{\mathbf{k},\sigma}=1/[1+\exp(\beta E_{\mathbf{k},\sigma})]$ is the Fermi distribution, the right-hand side of Eq. (42) can be expressed in terms of $\partial n/\partial\mu$ as

$$\frac{\partial E_{\mathbf{k},\sigma}}{\partial\mu}=1-\frac{\partial}{\partial\mu}(\beta_0-z^2\Delta\varepsilon_{\mathbf{k}})$$
$$=1-\left[\Delta\varepsilon_{\mathbf{k}}\frac{\partial z^2}{\partial\delta}-\frac{\partial\beta_0}{\partial\delta}\right]\frac{\partial n}{\partial\mu} . \quad (43)$$

Inserting the above expression into (42), a self-consistent equation for $\partial n/\partial\mu$ then emerges, and its formal solution is

$$\frac{\partial n}{\partial\mu}=\frac{(-2/L)\sum_{\mathbf{k}}\partial f_{\mathbf{k}}/\partial E_{\mathbf{k}}}{1-(\partial\beta_0/\partial\delta)(2/L)\sum_{\mathbf{k}}\partial f_{\mathbf{k}}/\partial E_{\mathbf{k}}+(\partial z^2/\partial\delta)(2/L)\sum_{\mathbf{k}}(\partial f_{\mathbf{k}}/\partial E_{\mathbf{k}})\Delta\varepsilon_{\mathbf{k}}} . \quad (44)$$

At zero temperature the above expression simplifies considerably, since we have then $\partial f_{\mathbf{k}}/\partial E_{\mathbf{k}}=-\delta(E_{\mathbf{k}})$. Within the range of temperature in which Landau's theory is applicable to the normal ^{3}He liquid, the Landau parameters are almost temperature independent. We can thus compute them at zero temperature, considerably simplifying the calculation. Assuming that Luttinger's theorem is valid, as is usually the case in a normal Fermi liquid, we have

$$\frac{-2}{L}\sum_{\mathbf{k}}\frac{\partial f_{\mathbf{k}}}{\partial E_{\mathbf{k}}}=\frac{2}{L}\sum_{\mathbf{k}}\delta(E_{\mathbf{k}})=\int d\varepsilon\, N_0(\varepsilon)\delta(z^2\varepsilon-(\mu-\beta_0))$$
$$=N_0(\mu_{\text{eff}})/z^2=N_F^0/z^2 , \quad (45)$$

where $N_0(\varepsilon)$ is the bare-particle density of states.

Similarly, we find

$$\frac{2}{L}\sum_{\mathbf{k}}\delta(E_{\mathbf{k}})\Delta\varepsilon_{\mathbf{k}}=\mu_{\text{eff}}\frac{N_F}{z^2} . \quad (46)$$

For finite temperatures one simply has to replace (45) and (46) by the integrals

$$\frac{-2}{L}\sum_{\mathbf{k}}\frac{\partial f_{\mathbf{k}}}{\partial E_{\mathbf{k}}}=\beta\int d\varepsilon_{\mathbf{k}} f_{\mathbf{k}}(E_{\mathbf{k}})[1-f_{\mathbf{k}}(E_{\mathbf{k}})]N(\varepsilon_{\mathbf{k}}) \quad (47)$$

and

$$-\frac{2}{L}\sum_{\mathbf{k}}\frac{\partial f_{\mathbf{k}}}{\partial E_{\mathbf{k}}}=\beta\int d\varepsilon_{\mathbf{k}} f_{\mathbf{k}}(E_{\mathbf{k}})[1-f_{\mathbf{k}}(E_{\mathbf{k}})]\varepsilon_{\mathbf{k}}N(\varepsilon_{\mathbf{k}}) , \quad (48)$$

where $E_{\mathbf{k}}=z^2\varepsilon_{\mathbf{k}}-(\mu-\beta_0)$. Hence, at $T=0$,

$$F_0^s=-1+\frac{N_F^0}{z^2\partial n/\partial\mu}$$
$$=-N_F^0\left[\frac{\partial\beta_0}{\partial\delta}+\mu_{\text{eff}}\frac{\partial z^2}{\partial\delta}\right]\Big/z^2$$
$$=-N_F^0U_c\left[\frac{1}{U_c}\frac{\partial\beta_0}{\partial\delta}+\frac{\mu_{\text{eff}}}{U_c}\frac{\partial z^2}{\partial\delta}\right]\Big/z^2 . \quad (49)$$

Clearly, this is the same result as Eq. (35) obtained from the charge correlation function. Following the derivation of the formula for F_0^a, the boson-field solutions are inserted into Eqs. (36)–(39), leading to

$$\frac{\partial z^2}{\partial\delta}=2\delta(1-y)^2(-2\delta^2+2y+\delta^2y-y^3)/[(1-\delta^2)^2(2\delta^2-3\delta^2y+y^3)] , \quad (50)$$

$$\frac{1}{U_c}\frac{\partial\beta_0}{\partial\delta}=\frac{(-1+y)(2\delta^2+2\delta^4-9\delta^2y-\delta^4y+2y^2+3\delta^2y^2+y^3+\delta^2y^3-y^4-\delta^2y^4)}{4(1-\delta^2)^2(2\delta^2-3\delta^2y+y^3)} . \quad (51)$$

As in the general expression for F_0^a, one only needs to calculate two parameters, N_FU_c and μ_{eff}/U_c, which are dimension and t_{ij} dependent. In the next section we will present formulas for these two parameters in the isotropic liquid. There is no problem in calculating these two parameters for any band structures or lattices. It suffices to compute the density of states from the given band structure, and to solve Eq. (26), which yields the effective chemical potential. Then U_c can be obtained from (12), and N_F is easily obtained since it is just equal to $N_0(\mu_{\rm eff})/z^2$.

III. THE NORMAL ISOTROPIC QUANTUM LIQUID AND THE CASE OF THE FLATBAND

On a lattice, there are generally more Landau parameters to take into account[33] than for an isotropic system because of the actual shape of the Fermi surface. As we mentioned earlier, there are no intrinsic difficulties in computing the response functions in the presence of a lattice. The Landau parameters are then simply defined using the long-wavelength limit of these response functions (reducing the rotation-group representation according to the point-group representation of the lattice, i.e., a further decomposition for the interactions between quasiparticles).

However, we will restrict ourselves in this paper to isotropic systems, particularly to the Hubbard model with a free-particle-like spectrum for the bare particles, which is relevant to the case of a normal Fermi liquid, e.g., to liquid ^{3}He. It was shown in the last section that one needs to calculate two parameters, $N_F U_c$ and $\mu_{\rm eff}/U_c$, in order to obtain a general analytic expression for the Landau parameters.

In order to calculate the Landau parameters for the isotropic Hubbard liquid, we need the familiar results for the density of states and the Fermi momentum, namely,

$$N^0_{F,\rm 1D}=\frac{m}{\pi k_{F,\rm 1D}} \quad \text{with } k_{F,\rm 1D}=n\pi \ , \tag{52a}$$

$$N^0_{F,\rm 2D}=\frac{m}{\pi} \quad \text{with } k_{F,\rm 2D}=(2n\pi)^{1/2} \ , \tag{52b}$$

$$N^0_{F,\rm 3D}=\frac{m}{\pi^2}k_{F,\rm 3D} \quad \text{with } k_{F,\rm 3D}=(3\pi^2 n)^{1/3} \ . \tag{52c}$$

At $T=0$, the effective chemical potential can be written

$$\mu_{\rm eff}\equiv\frac{\mu-\beta_0}{z^2}=\Delta\varepsilon_{\mathbf k}=\frac{1}{2m}=\{k_F^2(\delta)-[k_F(\delta=0)]^2\} \ . \tag{53}$$

The effective chemical potential is given by

$$\mu_{\rm eff,1D}=-\frac{\delta(2-\delta)}{2(1-\delta)N^0_{F,\rm 1D}} \ , \tag{53a}$$

$$\mu_{\rm eff,2D}=\frac{-\delta}{N^0_{F,\rm 2D}} \ , \tag{53b}$$

and

$$\mu_{\rm eff,3D}=\frac{3(1-\delta)^{1/3}[1-(1-\delta)^{3/2}]}{2N^0_{F,\rm 3D}} \ . \tag{53c}$$

Substituting these results in (17) the chemical potential emerges immediately as

$$\mu_{\rm 1D}=\beta_0-z^2\frac{\delta(2-\delta)}{2(1-\delta)N_{F,\rm 1D}} \ , \tag{54a}$$

$$\mu_{\rm 2D}=\beta_0-z^2\frac{\delta}{N^0_{F,\rm 2D}} \ , \tag{54b}$$

and

$$\mu_{\rm 3D}=\beta_0-z^2\frac{3(1-\delta)^{1/3}[1-(1-\delta)^{3/2}]}{2N^0_{F,\rm 3D}} \ . \tag{54c}$$

Assuming that the bare particles have a free-particle-like spectrum, we obtain

$$\bar{\varepsilon}_0=\overline{\Delta\varepsilon_{\mathbf k}}=\frac{2}{2mL}\sum_{\mathbf k}[k^2-(k_F^0)^2] \ . \tag{55}$$

Substituting Eqs. (52a)–(52c) and (55) into (12), we find

$$U_{c,\rm 1D}=\frac{4\pi^2[3-(1-\delta)^2]}{3m(1+\delta)} \ , \tag{56a}$$

$$U_{c,\rm 2D}=\frac{4\pi}{m} \ , \tag{56b}$$

and

$$U_{c,\rm 3D}=\frac{12(3\pi^2)^{2/3}}{m(1+\delta)}\left[\frac{1}{3}-\frac{(1-\delta)^{2/3}}{5}\right] \ . \tag{56c}$$

Combining (52a)–(52c) and (56a)–(52c), we obtain

$$N_F U_c|_{\rm 1D}=\frac{4[3-(1-\delta)^2]}{3(1-\delta^2)} \ , \tag{57a}$$

$$N_F U_c|_{\rm 2D}=4 \ , \tag{57b}$$

and

$$N_F U_c|_{\rm 3D}=\frac{36}{(1+\delta)}\left[\frac{(1-\delta)^{1/3}}{3}+\frac{(1-\delta)}{5}\right] \ , \tag{57c}$$

as well as

$$\mu_{\rm eff}/U_c|_{\rm 1D}=-\frac{3(1+\delta)[(1-\delta)^2-1]}{8(2+\delta)^2} \ , \tag{58a}$$

$$\mu_{\rm eff}/U_c|_{\rm 2D}=-\frac{\delta}{4} \ , \tag{58b}$$

and

$$\mu_{\rm eff}/U_c|_{\rm 3D}=\frac{(1+\delta)[(1-\delta)^{2/3}-1]}{24[\frac{1}{3}-(1-\delta)^{2/3}/5]} \ . \tag{58c}$$

Inserting the above expressions into the general expressions for the Landau parameters (34) and (49), we find that they can be expressed as functions of two variables, y and δ. For a given scaled interaction $u=U/U_c$ and doping factor δ, y can be obtained by solving the optimization equation. We will not present these lengthy analytic expressions for the Landau parameters. The 2D case is simple, as it reduces to the flatband case. We thus reproduce the results of Ref. 11, as shown below. There are no difficulties in dealing with the more complex 3D and 1 D cases since all the necessary ingredients are given in Eqs. (34) for F_0^a and (49) for F_0^s, as well as in Eqs. (57a)–(58c) for the parameters $N_F U_c$ and $\mu_{\rm eff}/U_c$.

Let us now consider the case of a flatband. Assuming a bandwidth equal to $2W$ for the bare particles, the density of states of the bare particles for both spin up and spin down is simply

$$N(E)=1/W \ , \tag{59a}$$

the effective chemical potential

$$\mu_{\rm eff}=-W\delta z^2 \ , \tag{59b}$$

the critical interaction

$$U_c=4W \ , \tag{59c}$$

the average kinetic energy

$$\bar{\varepsilon}=-\frac{W}{2}(1-\delta^2) \ , \tag{59d}$$

and the kinetic energy at the Fermi level

$$t_F=\mu_{\rm eff}/z^2 \ . \tag{59e}$$

Inserting these results into the expressions for F_0^a and F_0^s, we obtain the following expressions:

$$F_0^a=-1+(1-\delta^2)(y^2-\delta^2)/(2y-y^2-\delta^2)^2 \tag{60}$$

and

$$F_0^s=\frac{(1-y)(2\delta^2-5\delta^2y+2y^2+\delta^2y^2+y^3-y^4)}{[(\delta^2-2y+y^2)(-2\delta^2+3\delta^2y-y^3)]} \ . \tag{61}$$

Expression (60) is in exact agreement with the results of Ref. 11. The corresponding scattering amplitudes are

$$A_0^a=\frac{(1-y)(y^3-3y^2+3\delta^2y-\delta^2)}{[(1-\delta^2)(y^2-\delta^2)]} \tag{62}$$

and

$$A_0^s=\frac{(1-y)(2\delta^2-5\delta^2y+2y^2+\delta^2y^2+y^3-y^4)}{[(-1+\delta^2)(-2\delta^2+3\delta^2y-2y^2+y^3)]} \ . \tag{63}$$

Using the forward-scattering sum rule and the *s-p* approximation, i.e., taking a cutoff $l<2$, a tedious calculation yields

$$A_1^a=\frac{(y-1)^2(-10\delta^4+19\delta^4y-8\delta^2y^2-2\delta^2y^3+2y^4-y^5)}{[(-1+\delta^2)(\delta^2-y^2)(-2\delta^2+3\delta^2y-2y^2+y^3)]} \tag{64}$$

and

$$F_1^a=3(y-1)^2(-10\delta^4+19\delta^4y-8\delta^2y^2-2\delta^2y^3+2y^4-y^5)/(16\delta^4-6\delta^6-48\delta^4y+9\delta^6y+8\delta^2y^2+48\delta^4y^2 -8\delta^2y^3-25\delta^4y^3-8y^4+10\delta^2y^4 +8y^5-\delta^2y^5-4y^6+y^7) \ . \tag{65}$$

Since the effective mass is simply related to the Landau parameter F_1^s, we have derived the four Landau parameters. With these we may now calculate a number of important observable thermodynamic and transport properties, as well as the collective-mode properties. According to Landau's theory of Fermi liquids, the specific heat at low temperature is proportional to m^*/m,

$$C_v=C_v^0\frac{m^*}{m} \quad \text{with} \quad \frac{m^*}{m}=1+\frac{F_1^2}{3} \ .$$

(From now on we will use the superscript 0 to mark the free Fermi-gas results.) The mass enhancement m^*/m is thus equal to the normalized specific heat C_v/C_v^0. We thus immediately obtain the following expression for these quantities:

$$C_v/C_v^0=m^*/m=(-1+\delta^2)/(\delta^2-2y+y^2) \ . \tag{66}$$

The above expressions for the low-temperature specific heat, the mass enhancement, and F_1^s are valid for any density of states, independently of the flatband assumption. Similarly, we can obtain the formula for the normalized spin susceptibility. In Landau's theory the static magnetic susceptibility is expressed as

$$\chi_s=\frac{m^*/m}{1+F_0^a}\chi_s^0 \ . \tag{67}$$

Thus we have

$$\chi_s/\chi_s^0=(\delta^2-2y+y^2)/(\delta^2-y^2) \ . \tag{68}$$

Landau's theory gives for the compressibility

$$K=\frac{m^*/m}{1+F_0^s}K^0 \ . \tag{69}$$

Substituting the mass enhancement factor Eq. (66) and the Landau parameter F_0^s into the preceding equation, we obtain the following result for the normalized compressibility:

$$K/K^0=\frac{(-2\delta^2+3\delta^2y-y^3)}{(-2\delta^2+3\delta^2y-2y^2+y^3)} \ . \tag{70}$$

The Wilson ratio, defined as

$$W_r=\frac{(\chi_s/\chi_s^0)}{(C_v/C_v^0)} \ , \tag{71}$$

has played an interesting role in the study of Kondo and heavy-fermion systems. It highlights some of the behavior peculiar to these strongly interacting systems. Given the expressions for m^*/m and χ_s/χ_s^0, the Wilson ratio can be reexpressed in terms of the Landau parameters as $W_r=1/(1+F_0^a)$. Thus we have

$$W_r=(\delta^2-2y+y^2)^2/[(1-\delta^2)(y^2-\delta^2)] \ . \tag{72}$$

Finally, using the Landau theory formula for the ratio between the first-sound velocity and the Fermi velocity

$$c_1/v_F=(c_1/v_F)_0\left[\frac{(1+F_0^s/3)}{(1+F_1^s/3)}\right]^{1/2}, \tag{73}$$

we can find the normalized ratio of the first-sound velocity

$$\frac{c_1}{(c_1)_0}=\left[\frac{(-2\delta^2+3\delta^2 y-2y^2+y^3)}{(-2\delta^2+3\delta^2 y-y^3)}\right]^{1/2}. \tag{74}$$

Note that the dopant concentration only appears as δ^2 in the optimization equation (11) and in the formulas we have just derived for the Landau parameters and related observables. Thus the particle-hole symmetry, which can be expressed as an invariance under the transformation $\delta\to-\delta$, holds in the above expressions. Note also that in all of the expressions for the Landau parameters and the observables, the bandwidth W and the dimension d do not appear explicitly. They only affect implicitly the value of y (the solution of the optimization equation) through U_c. Thus in the case of the flatband we have some form of scaling: all of the Landau parameters and observables are expressed simply through two explicit parameters, the filling factor δ and the scaled interaction $u=U/U_c$.

Equations (60)–(74) reduce to the correct noninteracting limit. When $U\to 0$, all Landau parameters vanish and all the normalized observables are equal to 1 as expected. This limit can only be assured at the mean-field level, the next-order correction in the loop expansion having met enormous trouble[34,35] when attempts were made to recover the correct noninteracting limit.

The optimization equation in the opposite limit, the strong-coupling limit $U\to\infty$, has only one physical solution, $y=|\delta|$. Inserting this result into our expressions for the Landau parameters and the normalized observables, we find

$$\begin{aligned}
&F_0^s\to 1/|\delta|,\ \ F_0^a\to -1,\ \ F_1^s\to -\tfrac{3}{2}+\frac{3}{2|\delta|},\ \ F_1^a\to -3\ ;\\
&A_0^s\to 1/(1+|\delta|),\ \ A_0^a\to -\infty\ ,\\
&A_1^s\to 3(1-|\delta|)/(1+|\delta|),\ \ A_1^a\to -\infty\ ;\\
&m^*/m\to(1+|\delta|)/2|\delta|,\ \ K/K^0\to\tfrac{1}{2},\ \ c_1/(c_1)_0\to\sqrt{2}\ ;\\
&\chi_s/\chi_s^0\to\infty;\ \text{and}\ W_r\to\infty\ .
\end{aligned} \tag{75}$$

The fact that the antisymmetric Landau parameters approach, in the strong-coupling limit, the critical values for the stability of the Landau Fermi-liquid phase $[F_l^{s,a}>-(2l+1)]$ suggests that the ferromagnetic phase should become more stable than the Landau Fermi liquid. When $\delta=0$ the above limiting behavior might be explained in term of Nagaoka's theorem.[36] It is still an open question if Nagaoka's theorem holds for the Hubbard model in the small-doping limit. The strong-coupling results also show that at finite, even infinitesimal, doping the Brinkman-Rice localization is forbidden.

In the half-filled case ($\delta=0$) there is a simple physical solution to the optimization equation, $y=1-u$, where $u=U/U_c$. The Landau parameters and the normalized observables discussed earlier then simply reduce to the well-known Gutzwiller results obtained in the investigation of the normal ^{3}He liquid by the "almost localized approach" of Vollhardt:[10]

$$F_0^s=(2-u)u/(1-u)^2\ , \tag{76a}$$

$$F_0^a=-u(2+u)/(1+u)^2\ , \tag{76b}$$

$$F_1^s=3u^2/(1-u^2)\ , \tag{76c}$$

$$F_1^a=-3u^2/(3+u^2)\ , \tag{76d}$$

$$A_0^s=(2-u)u\ , \tag{76e}$$

$$A_0^a=-u(2+u)\ , \tag{76f}$$

$$A_1^s=3u^2\ , \tag{76g}$$

$$A_1^a=-u^2\ , \tag{76h}$$

$$m^*/m=1/(1-u^2)\ , \tag{76i}$$

$$K/K^0=(1-u)/(1+u)\ , \tag{76j}$$

$$c_1/(c_1)_0=\left[\frac{1+u}{1-u}\right]^{1/2}, \tag{76k}$$

$$\chi_s/\chi_s^0=(1+u)/(1-u)\ , \tag{76l}$$

and

$$W_r=(1+u)^2\ . \tag{76m}$$

In the above expressions, the scaled interaction u is restricted to be less than 1. When u approaches 1 (i.e., $U\to U_c$), F_0^s, F_1^s, the mass enhancement and the normalized sound velocity diverge, and the normalized compressibility vanishes. This corresponds to the Brinkman-Rice localization transition.[10,12,16] The other parameters take the values

$$F_0^a\to-\tfrac{3}{4},\ \ F_1^a\to-\tfrac{3}{4},\ \ W_r\to 4,\ \ K/K^0\to 0\ . \tag{77}$$

Comparing these results with those obtained in the strong-coupling limit, we find that the two limits $\delta\to 0$ and $U/U_c\to\infty$ do not commute.

In order to obtain general expressions in terms of the Hubbard model parameters U and δ (we will use units such that $k_B=\hbar=t=1$ from now on), we have to solve the optimization equation, which is a cubic equation. The standard formulas for a cubic equation yield solutions that are too complex to give useful explicit analytic expressions for the Landau parameters and their byproducts, the observables. In the following special but interesting cases[27,28] we can calculate simple expressions for the first four Landau parameters and their byproducts.

Case I: $u<1$, $\delta\ll(1-u)^{3/2}$. Taking δ as a small expansion parameter, we obtain

$$y=1-u+u\delta^2/(1-u)^2+O(\delta^3)\ . \tag{78}$$

Case II: $u>1$, $|\delta|\ll 1$. We have

$$y=\frac{\delta}{\zeta}-\frac{\delta^2}{2u\zeta^4}\ \ \text{where}\ \zeta=\sqrt{1-1/u}\ . \tag{79}$$

Case III: $u \ll 1$, the weak-coupling limit for any dopant concentration δ. Taking u as a small expansion parameter, we find

$$y=1-(1-\delta^2)u+2u^2\delta^2(1-\delta^2)+O(u^3) . \tag{80}$$

Case IV: $u \gg 1$, the strong-coupling limit at any filling. Taking $1/u$ as the small expansion parameter, we have

$$y=\delta(1+(1-\delta)/(2u))+O\left[\frac{1}{u^2}\right] . \tag{81}$$

The results of these expansions for the four Landau parameters, the related scattering amplitudes, and the normalized observables are presented in the Appendix.

IV. NUMERICAL STUDIES OF THE LANDAU PARAMETERS AND RELATED OBSERVABLES AND COMPARISON WITH EXPERIMENTS ON LIQUID ^{3}He

In this section we present some numerical results for the first four Landau parameters using their slave-boson expressions. Given the Landau parameters, the numerical results of the transport and thermodynamic properties, as well as the behavior of the collective mode, can be obtained through Landau's theory of Fermi liquids. The numerical work that we present here is restricted to the Hubbard model. As we mentioned in the previous section, there is no difficulty in principle in extending these calculations to other models of strongly correlated Fermi systems. There is no restriction in the numerical calculations on the strength of the Hubbard interaction U and the doping factor δ. We assume a flat density of states for simplicity. It has been shown that the flatband model works fairly well for the normal ^{3}He liquid. However, numerical calculations can be worked out for any fermionic spectrum using the results of the preceding section.

The parameters of the system are the strength of the Hubbard interaction U, the doping factor δ, the hopping matrix t_{ij} for the lattice models, the dimension of the system, and its temperature T. Extending Gutzwiller's variational approach to finite temperatures is one of the important pieces of progress brought on by KR's slave-boson approach.[30] Since we will compare our numerical result with the experimental ones for normal ^{3}He, we restrict our numerical work to the zero-temperature case. The whole framework presented in this paper may be used to consider finite-temperature effects for some other fermion system where temperature effects may be crucial. In the slave-boson formulation at the saddle point [Eqs. (9)–(12)], the effect of the dimension of the system appears only in the calculation of U_c and of the effective chemical potential through the density of states [Eqs. (12) and (20)]. The kinetic part t_{ij} acts similarly. In fact, together they determine the fermionic spectrum and thus the density of states of the bare particles. The flatband formulation is appealing not only because of its simplicity, but also because it corresponds to the interesting case of a 2D fermion liquid. However, the effective mass is then related to the Landau parameter F_1^s through $m^*/m=1+F_1^s$ instead of Eq. (22). Furthermore, the scattering amplitudes have different expressions in 2D than in 3D. Anyway, the flatband approximation has been widely used for liquid ^{3}He. It has been used in conjunction with the Hubband model on a lattice[10] because it is simple and can deal adequately with properties which are not sensitive to the details of the density of states. If the slave-boson approach is applied directly to the Hubbard model on a lattice to calculate the Landau parameters at the mean-field level, we encounter difficulties due to the van Hove singularity at half filling. As discussed in the Introduction this can be avoided by pushing the calculations to higher orders, as in the t-matrix approach in which the singularity is rounded, or by removing it from the fillings of interest by including next-nearest-neighbor hopping. Note that the Hubbard liquid and the Hubbard lattice greatly differ on one important point. The latter offers the possibility of a far richer magnetic phase diagram. However, as mentioned earlier, we restrict ourselves to the study of the paramagnetic phase.

The numerical calculations proceeded as follows. For a given value of the scaled interaction $u=U/U_c$ and of the doping factor δ, we solve numerically the optimization equation. We then insert the solution into Eqs. (23), (60), (61), and (65), which yield the first four Landau parameters. Note that in the derivation of Eqs. (60)–(65) we used the following approximations: the flatband approximation, the *sp* approximation on the forward-scattering sum rule, and the saddle-point approximation on the Lagrangian. In order to calculate the Landau parameters for a general fermionic spectrum, the same algorithm can be used, but it is complicated by the absence of such simple relations as Eqs. (59a)–(59e). In this case we must first use the bare-particle spectrum of the Hamiltonian in order to obtain the density of states (DOS) of the bare particles. Using this DOS and Eq. (20), the effective chemical potential μ_{eff} can then be calculated for a given δ. With this effective chemical potential and using Eq. (21), we can then find the critical interaction U_c.

In the finite-temperature case, the numerical calculations are more complicated, since the Fermi distribution function $f(E_{\mathbf{k}})$ cannot be reduced to a theta function $\theta(-E_{\mathbf{k}})$. Indeed, the θ function has the scaling property $\theta(|a|x)=\theta(x)$, which can be used to simplify the equations. At finite temperature, the renormalized hopping factor must be calculated in a self-consistent way because of its dependence on the saddle-point values of the fields; in other words, it must be iterated until the saddle-point values converge. Then by using the expressions derived earlier relating the Landau parameters to the saddle-point values and their derivatives, the Landau parameters F_0^s, F_0^a, and F_1^s can be found. Finally F_1^a is obtained with the forward-scattering sum rule and the s-p approximation.

As in the case of solid ^{3}He, the original lattice-gas model[10] of ^{3}He is based on the explicit assumption of a half-filled lattice. The pressure dependence of the observables was ascribed to the Hubbard interaction U. Within this model a comprehensive investigation of the pressure dependence of the two leading Landau parameters and the normalized susceptibility and compressibility have

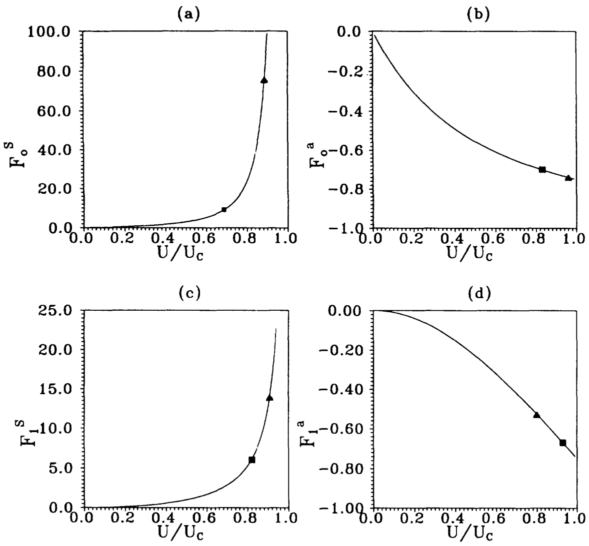

FIG. 1. The four leading symmetric Landau parameters for the half-filled Hubbard model, as a function of the scaled interaction $u=U/U_c$. The Landau parameters were calculated using the saddle-point values of the slave-boson fields. The squares and the triangles mark the experimental results (Ref. 37) at respectively 0 and 27 bars. The Landau parameter F_1^a was obtained using the *s-p* approximation in the forward-scattering sum rule.

been calculated and compared with the experimental results for the normal ^{3}He liquid.[37,38] The Hubbard liquid, on the other hand, has also been used in the investigation of the pressure dependence of the superfluid transition.[24] Reasonably good agreement between experimental and theoretical results was obtained for both normal and superfluid ^{3}He,[24] when the experimental pressure dependence of the mass enhancement was used to obtain the pressure dependence of the scaled interaction $u=U/U_c$.

TABLE I. The values of U/U_c used for the half-filled model. The method of the least-square errors was used to fit the four leading Landau parameters simultaneously. Hereafter, we use the symbol T to denote our theoretical values; the symbols G and W correspond to Greywall's (Ref. 38) and Wheatley's (Ref. 37) data, respectively.

Pressure (bars)	U/U_c $(G\text{-}T)$	U/U_c $(W\text{-}T)$
0.00	0.710	0.725
3.00	0.772	0.782
6.00	0.805	0.812
9.00	0.827	0.832
12.00	0.842	0.847
15.00	0.855	0.860
18.00	0.865	0.867
21.00	0.872	0.875
24.00	0.877	0.882
27.00	0.885	0.887
30.00	0.890	0.892
33.00	0.895	0.897
34.36	0.897	0.900

TABLE II. The values of U/U_c and δ for the δ-dependent model. The notations and the fitting method were discussed in the caption of Table I.

Pressure (bars)	U/U_c $(G\text{-}T)$	$\delta(G\text{-}T)$	U/U_c $(W\text{-}T)$	δ $(W\text{-}T)$
0.00	0.740	0.0420	0.745	0.0340
3.00	0.805	0.0325	0.845	0.0405
6.00	0.835	0.0265	0.880	0.0345
9.00	0.850	0.0210	0.905	0.0310
12.00	0.865	0.0185	0.915	0.0270
15.00	0.875	0.0165	0.930	0.0250
18.00	0.885	0.0150	0.940	0.0235
21.00	0.890	0.0135	0.940	0.0210
24.00	0.895	0.0120	0.945	0.0195
27.00	0.900	0.0110	0.945	0.0175
30.00	0.905	0.0100	0.945	0.0160
33.00	0.910	0.0095	0.950	0.0155
34.36	0.915	0.0100	0.945	0.0140

It was thus concluded that ^{3}He is a strongly correlated Fermi system, a so-called "almost localized liquid," because the scaled interaction parameter u used in the fitting procedure had to be set between 0.8 and 1.

Figures 1(a)–1(d) show the dependence of the first four Landau parameters on u, assuming a half-filled band ($\delta=0$). The solid triangles and squares correspond to the values of the experimental data at 0 and 27 bars, respectively.[37] We find that the difference between the scaled

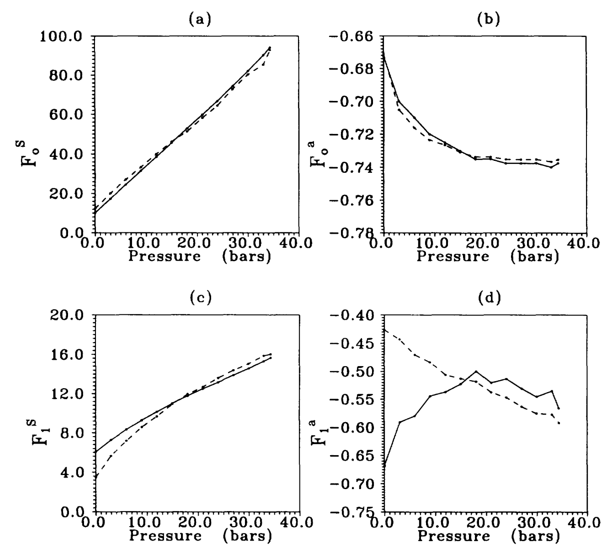

FIG. 2. The experimental (full line) and theoretical (dashed line) pressure dependence of the leading symmetric Landau parameters for the Hubbard model. Both the filling factor δ and the scaled interaction u are taken as free parameters in the fitting procedure, in which the least-squares method is applied simultaneously to the four leading Landau parameters. The experimental data for F_0^s, F_0^a, and F_1^s are taken from Wheatley's compilation (Ref. 37). The experimental and theoretical data for F_1^a are obtained by using the forward-scattering sum rule and the *s-p* approximation. (a), (b), (c), and (d) show the results for F_0^s, F_0^a, F_1^s, and F_1^a, respectively.

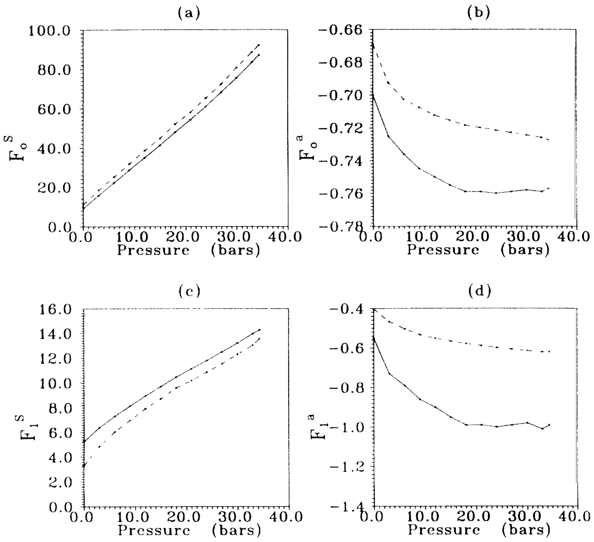

FIG. 3. As in Fig. 2, but the experimental data are taken from Greywall (Ref. 38). In his experiments, F_1^a was obtained without the *s*-*p* approximation, which we use to obtain the theoretical results.

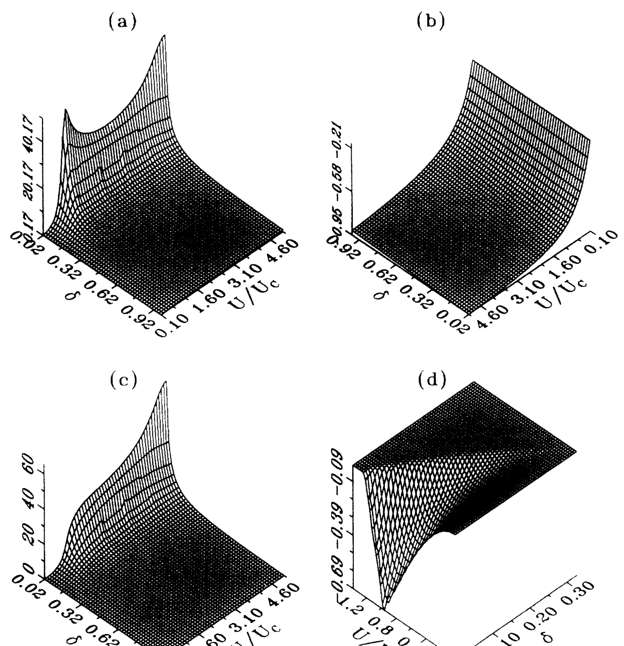

FIG. 4. The 3D diagram of the four leading Landau parameters for the Hubbard model in the SB approach. The flatband assumption is used. The Landau parameters are plotted as a function of the scaled interaction U/U_c and the doping factor δ. F_1^a was obtained with the *s*-*p* approximation. When the values of F_1^a became unstable (<-3) or unphysical, they were set to zero. Thus from this figure one can find the regime of validity of the SB calculation of F_1^a on the $(U/U_c, \delta)$ manifold.

interactions corresponding to the experimental pressures is small. In Table I we present values of u corresponding to a fit with the experimental data.[37,38] The fits were made using the least-squares fitting procedure on the first four Landau parameters simultaneously. One value of u was ascribed to each of the 13 pressures, covering the whole pressure range before the melting pressure of ^{3}He is reached. In the half-filled-band model the effect of pressure is attributed solely to the Hubbard interaction. Table I shows that the theoretical results give the correct tendency for the effects of pressure: the higher the pressure, the larger the scaled interaction. The best-fit values of u are close to the values previously used to fit the pressure dependence of the superfluid transition temperature.[24]

An improvement over the half-filled model, the so-called compressible model, was proposed[11] in which the

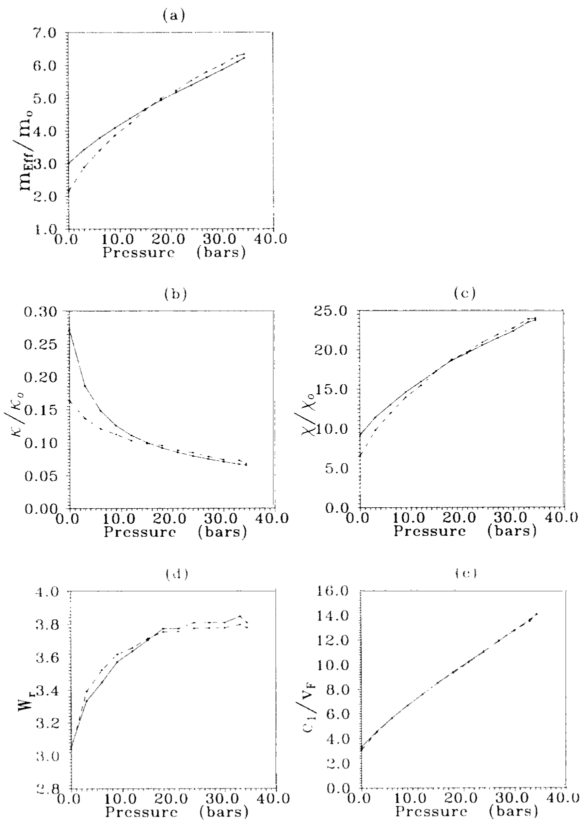

FIG. 5. The pressure dependences of the experimental (full lines) and theoretical (dashed lines) values of some observables. The theoretical values were obtained from the 3D Fermi-liquid theory formulas relating the Landau parameters (obtained from the slave-boson approach) to the observables. We assumed a flat density of states. Both the filling factor δ and the scaled interaction U/U_c were taken as free parameters in the fitting procedure. The experimental data was taken from Wheatley's compilation (Ref. 37). (a), (b), (c), (d), and (e) show, respectively, the results for the mass enhancement factor, the normalized compressibility, the normalized spin susceptibility, the Wilson ratio, and the ratio between the first-sound velocity and the Fermi velocity.

TABLE III. The pressure dependence of the Landau parameter F_1^a. The symbols T^* and T denote the results for the half-filled and the δ-dependent models, respectively. Wilson's data, obtained through the use of the *s-p* approximation in the forward-scattering sum rule, are listed in the column labeled *W* (*s-p*). Greywall's data are listed in the column labeled *G*. The symbol *s-p* indicates that the results were obtained using the *s-p* approximation in the forward-scattering sum rule. The other symbols, as well as the fitting procedure, were discussed in the caption of Table I.

Pressure (bars)	*W* (*s-p*)	*W-T*	*W-T**	*G*	*G* (*s-p*)	*G-T*	*G-T**
0.00	−0.67	−0.43	−0.45	−0.55	−0.41	−0.40	−0.43
3.00	−0.59	−0.44	−0.51	−0.73	−0.31	−0.47	−0.50
6.00	−0.58	−0.47	−0.54	−0.79	−0.27	−0.50	−0.53
9.00	−0.54	−0.48	−0.56	−0.86	−0.22	−0.53	−0.56
12.00	−0.54	−0.51	−0.58	−0.90	−0.20	−0.55	−0.57
15.00	−0.52	−0.51	−0.59	−0.95	−0.18	−0.57	−0.59
18.00	−0.50	−0.52	−0.60	−0.99	−0.15	−0.58	−0.60
21.00	−0.52	−0.54	−0.61	−0.99	−0.18	−0.59	−0.61
24.00	−0.51	−0.55	−0.62	−1.00	−0.20	−0.60	−0.61
27.00	−0.53	−0.56	−0.62	−0.99	−0.24	−0.61	−0.62
30.00	−0.55	−0.58	−0.63	−0.98	−0.27	−0.61	−0.63
33.00	−0.54	−0.58	−0.64	−1.01	−0.28	−0.62	−0.63
34.36	−0.57	−0.59	−0.64	−0.99	−0.32	−0.62	−0.64

filling factor itself becomes pressure dependent. Following the same line of thought, we have fitted simultaneously the four Landau parameters to the experimental pressures using both u and δ as fitting parameters (which we call the δ-dependent model). The values of u and δ used to fit the pressure are presented in Table II. We find that

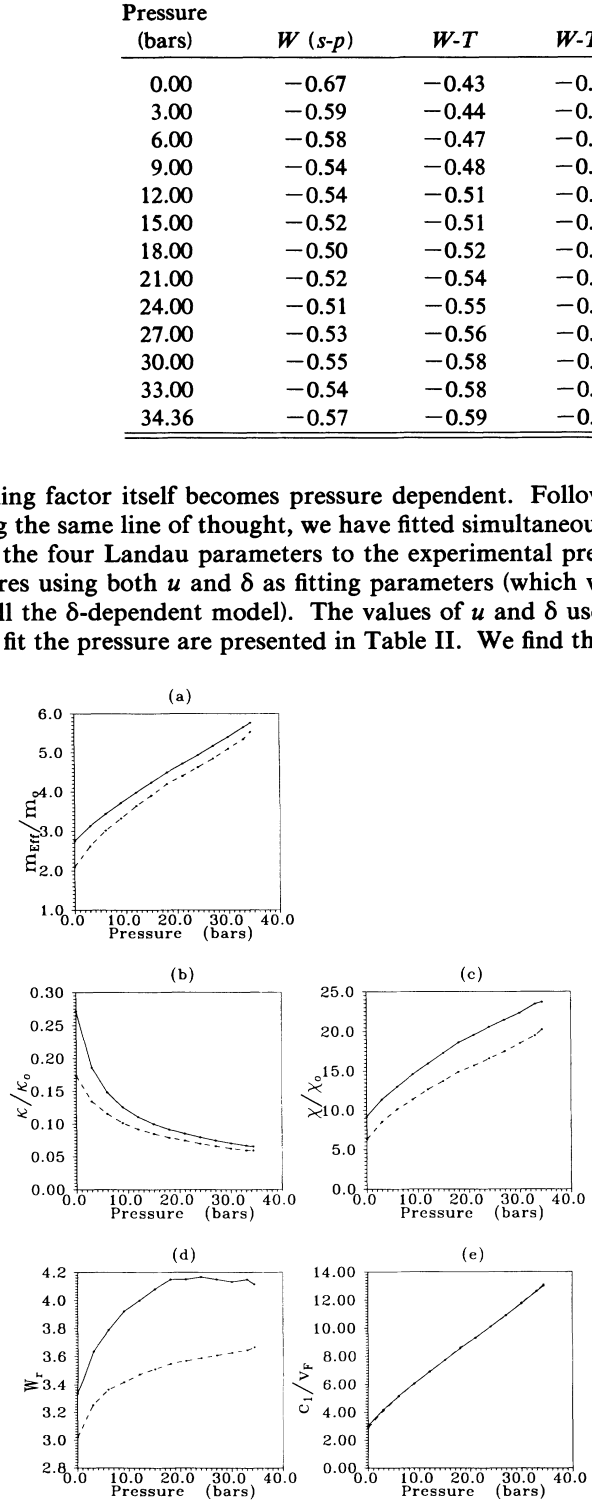

FIG. 6. As in Fig. 5, but Greywall's data (Ref. 38) are used for the comparison instead of Wheatley's (Ref. 37).

the parameters behave as expected: the higher the pressure, the stronger the scaled interaction, and/or the smaller the doping factor. The values of u are still in the strong-coupling regime. The filling factors have values of the same order as those deduced from the experiments. As mentioned by Vollhardt, Wölfle and Anderson,[11] in the incompressible model the localization transition occurs precisely at half filling, whereas in the compressible model the Brinkman-Rice transition can no longer occur. This, in turn, removes the strong pressure dependence of the effective mass and of the spin susceptibility found in the case of an incompressible lattice and yields a smooth increase with pressure, as qualitatively observed in the experiments.

In Figs. 2(a)–2(d) we present the pressure dependence of the four leading Landau parameters and compare them with the data of Wheatley[37] (F_1^a was deduced from the forward-scattering sum rule using the *s-p* approximation). For each pressure the value of u and δ is taken from the fit in Table II. Excellent agreement with the experimental data is found for F_0^s, F_0^a, and F_1^s. Good agreement is also obtained at high pressures for F_1^a.

Corresponding results for Greywall's experimental data[38] are presented in Figs. 3(a)–3(d). The agreement is not as good as for Wheatley's results. In Greywall's experiments, F_1^a was deduced from the coefficient of the $T^3 \ln(T)$ term of the specific heat in the low-temperature range. Greywall's experimental measurements for the normal ^{3}He liquid thus show that the *s-p* approximation on the forward-scattering sum rule does not seem to give satisfactory results (see Table III). However, this cutoff procedure has been widely adopted for the normal ^{3}He liquid in the literature.[4]

Figures 4(a)–4(d) give the 3D diagrams for the four leading Landau parameters as a function of u and d. Figure 4(d) shows that our calculation of F_1^a is only sensible in a restricted regime on the u and d manifold. F_1^a was set to zero on the figure when it corresponded to an unstable system ($F_1^a < -3$) or was unphysical.

Figures 5(a)–5(e) and 6(a)–6(e) compare our results for the pressure dependence of some important normalized observables with both sets of experimental data.[37,38] The agreement between our results and Wheatley's is better than with Greywall's. The reason for these differences obviously stems from our use of the *s-p* approximation. Figures 5(e) and 6(e) (the ratio between the first-sound velocity and the Fermi velocity) show, however, excellent agreement in both cases.

V. DISCUSSION AND CONCLUSION

We have presented in this paper a systematic investigation of the Landau parameters in the framework of Kotliar and Ruckenstein's slave-boson approach. Kotliar and Ruckenstein have shown that the saddle-point values of the slave-boson fields reproduce Gutzwiller's results. However, it is not trivial to show that the long-wavelength and static limits of KR's spin and charge dynamical correlation functions also reproduce part of the results of Gutzwiller's approach for the Landau parameters. We have shown in this paper that, in the case of the flatband, both approaches agree on the values of F_0^s and F_1^s. Also, F_0^s can be obtained through either fluctuations or static quantities (such as $\partial n/\partial\mu$).

The more interesting aspect of the slave-boson approach, even at its mean-field level, is that it greatly extends the applicability of the Gutzwiller variational approach. One can apply this functional-integral approach to a variety of models, including the extended Hubbard model. It can also be used to study finite-temperature effects and can be applied to different geometries of the physical system under consideration (for example, any dimension, different symmetries, arbitrary bare-particle spectra, or finite system sizes).

The calculation of the spin and charge dynamical correlation functions assumes a $U(1)^{\otimes 4}$ gauge symmetry of the action. It has been argued that the correct gauge symmetry in KR's slave-boson scheme is $\mathrm{U}(1)^{\otimes 3}$.[37] Since the saddle-point values of slave-boson fields are not affected by whether one uses the $\mathrm{U}(1)^{\otimes 4}$ or the $\mathrm{U}(1)^{\otimes 3}$ representation, and our calculation of the Landau parameters gives the same results whether they are obtained through limits of dynamical correlation functions or from purely static quantities, we conclude that the static limit of the dynamical and charge correlation functions should be correct even in the $\mathrm{U}(1)^{\otimes 4}$ formalism.

KR's slave-boson scheme offers the possibility to consider in a systematic way fluctuation effects, i.e., it may be pushed to higher order in the loop expansion, beyond the mean-field level. There have been successful works done along these lines,[24,26,29–31] but important difficulties were encountered[34,35] in the attempt to improve the mass enhancement value, corresponding to the Landau parameter F_1^s. These difficulties may be related to the high-frequency aspects of the theory.[35] Even if KR's approach can treat finite-temperature effects, the temperature cannot be pushed too high. One of the present authors[39] has shown that the classical limit of the specific heat cannot be recovered in KR's slave-boson approach. This is not surprising since the starting point of this scheme is the Gutzwiller approach, which is a variational approach to the ground-state properties.

Analytic mean-field expressions for the Landau parameters F_0^s, F_0^a, and F_1^s were presented in this paper. All the expressions for the Landau parameters and related observables have particle-hole symmetry, and reduce to the correct noninteracting and strong-coupling limits. At zero temperature all of the Landau parameters and corresponding observables may be expressed simply as functions of two variables: the scaled interaction $u = U/U_c$ and the doping factor δ. In the flat-band case, detailed analytic expressions for the first four Landau parameters and corresponding observables were presented. Asymptotic expressions for these quantities were also given in four regimes of interest on the u and δ manifold.

Our numerical results show that reasonable agreement may be obtained between the experimental results and our theoretical results, for either the half-filled band of the δ-dependent model, with realistic values of the Hubbard interaction and filling factors.

It is well known that little direct information is available for the Landau parameter F_1^a for liquid ^{3}He, especially at high pressures. On the theoretical side, the direct calculation of F_1^a is also difficult. Previous calculations of this quantity[40] showed poor agreement and were quite sensitive to the method used. In Fig. 4(d) we give the results of the flatband calculation of F_1^a, based on the forward-scattering sum rule and the *s-p* approximation as a function of u and δ. One may argue about the validity of the *s-p* approximation in some regime on the u and δ manifold: it is anyway obvious that a more refined approach to the calculation of this quantity is called for.

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APPENDIX

In this appendix we give the expressions of the Landau parameters, of the forward-scattering amplitudes, and of the normalized observables for the four limiting cases mentioned in Sec. III. In the expressions for y, the doping factor δ should be taken as its absolute value $|\delta|$, since it appears only as δ^2 in the optimization equation. Thus the expansion procedure will not break particle-hole symmetry.

Case I: $u<1,\ \delta \ll (1-u)^{3/2}$

Substituting the approximate value of y for case I and the saddle-point values for the slave-boson fields, a straightforward but lengthy calculation leads to the following results for the first three Landau parameters:

$$F_0^s|_{\rm I} = -1 + \frac{1}{(1-u)^2} + \frac{u^2(u^2-2u-7)\delta^2}{(1+u)(1-u)^5}, \quad \text{(A1a)}$$

$$F_0^a|_{\rm I} = \frac{-u(2+u)}{(1+u)^2} + \frac{u^2\delta^2}{(1+u)^2(1-u)}, \quad \text{(A1b)}$$

and

$$F_1^s|_{\rm I} = \frac{3u^2}{1-u^2} - \frac{3u^2\delta^2(1+2u-u^2)}{(1+u)^2(1-u)^4}. \quad \text{(A1c)}$$

The scattering amplitudes[28] are given by

$$A_0^s|_{\rm I} = (2-u)u + u^2\delta^2\frac{(-7-2u+u^2)}{1-u^2}, \quad \text{(A1d)}$$

$$A_0^a|_{\rm I} = -(2+u)u + u^2\delta^2\frac{1+u}{1-u}, \quad \text{(A1e)}$$

and

$$A_1^s|_{\rm I} = 3u^2 + 3u^2\delta^2\frac{-1-2u+u^2}{(1-u)^2}. \quad \text{(A1f)}$$

Expressions (A1a) and (A1b) show that the symmetry property $F_0^a(U/U_c) = F_0^s(-U/U_c)$ is only satisfied at half filling.

The Landau parameter F_1^a may be obtained by using the cutoff $l<2$ in the forward scattering sum rule $\sum_l (A_1^s + A_1^a) = 0$, where

$$A_1^{s(a)} = F_1^{s(a)}/[1+F_1^{s(a)}/(2l+1)]$$

in 3D, which is equivalent to the *s-p* approximation.[40] The forward-scattering sum rule has been shown to be in good agreement with experimental data for liquid ^{3}He at least at low pressures. We find

$$F_1^a|_{\rm I} = -\frac{3u^2}{3+u^2} + \frac{9u^2(3-u)(3+2u+u^2)}{(1-u)^2(1+u)(3+u^2)^2}\delta^2 \quad \text{(A1g)}$$

and

$$A_1^a|_{\rm I} = -u^2 + u^2\delta^2(3-u)\frac{3+2u+u^2}{(1+u)(1-u)^2}. \quad \text{(A1h)}$$

From the above values of the Landau parameters, we can obtain the expressions for the following observables in the first regime:

$$\left.\frac{m^*}{m}\right|_{\rm I} = \frac{1}{1-u^2} + u^2\delta^2\frac{-1-2u+u^2}{(1+u)^2(1-u)^4}, \quad \text{(A1i)}$$

$$\left.\frac{\chi_s}{\chi_s^0}\right|_{\rm I} = \frac{1+u}{1-u}\frac{2u^2\delta^2}{(1-u)^4}, \quad \text{(A1j)}$$

$$K/K_0|_{\rm I} = \frac{1-u}{1+u} + \frac{6u^2\delta^2}{(1-u^2)^2}, \quad \text{(A1k)}$$

$$W_r/W_r^0|_{\rm I} = (1+u)^2 - \frac{u^2(1+u)\delta^2}{(1-u)}, \quad \text{(A1l)}$$

and

$$c_1/(c_1)_0|_{\rm I} = \left[\frac{1+u}{1-u}\right]^{1/2} \frac{3u^2\delta}{(1-u)^3\sqrt{1-u^2}}. \quad \text{(A1m)}$$

As we mentioned earlier, when u approaches 1, the Fermi liquid approaches the Brinkman-Rice transition and it tends to localize.

Case II: $u>1,\ |\delta| \ll 1$

There is no correspondence between case II and the Brinkman-Rice theory. One has to be careful in doing the expansion in this case, since the filling δ may appear in the denominator. A tedious calculation leads to the following Landau parameters:

$$F_0^s|_{\rm II} = \frac{(2u-1)}{2\delta\sqrt{u(u-1)}} - \frac{(1+u^2)}{4u(1-u)^2}, \quad \text{(A2a)}$$

$$F_0^a|_{\rm II} = -1 + \frac{1}{4u} + \frac{\delta\sqrt{1-1/u}}{4u}, \quad \text{(A2b)}$$

$$F_1^s|_{\rm II} = \frac{3\sqrt{1-1/u}}{2\delta} - \frac{3(2u^2-2u-1)}{4u(u-1)}, \quad \text{(A2c)}$$

and

$$F_1^a|_{\rm II} = -\frac{3(4u-5)}{4(u-2)} - \frac{9(6-13u+4u^2)\delta}{8\sqrt{1-1/u}\,(u-2)^2(2u-1)}. \quad \text{(A2d)}$$

The corresponding scattering amplitudes are

$$A_0^s|_{\rm II} = 1 + \frac{2u\delta\sqrt{1-1/u}}{1-2u}, \quad \text{(A2e)}$$

$$A_0^a|_{\rm II} = 1 - 4u + 4\delta\sqrt{u(u-1)}, \quad \text{(A2f)}$$

$$A_1^s|_{\rm II} = 3 - \frac{6\delta}{\sqrt{1-1/u}}, \quad \text{(A2g)}$$

and

$$A_1^a|_{\rm II} = 4u - 5 - \frac{2\delta(6-13u+4u^2)}{(2u-1)\sqrt{1-1/u}}. \quad \text{(A2h)}$$

The normalized observables are given by the following expressions:

$$\left.\frac{m^*}{m}\right|_{\rm II} = \frac{\sqrt{1-1/u}}{2\delta} + \frac{u-1}{4u}, \quad \text{(A2i)}$$

$$\left.\frac{\chi_s}{\chi_s^0}\right|_{\rm II} = \frac{2\sqrt{u(u-1)}}{\delta} + \frac{2u-1}{u-1}, \quad \text{(A2j)}$$

$$K/K_0|_{\rm II} = \frac{u-1}{2u-1} + \frac{3u\delta}{2(2u-1)^2\sqrt{1-1/u}}, \quad \text{(A2k)}$$

$$W_r/W_r^0|_{\rm II} = 4u - 4\delta\sqrt{u(u-1)}, \quad \text{(A2l)}$$

and

$$c_1/(c_1)_0|_{\rm II} = \left[\frac{2u-1}{u-1}\right]^{1/2} - \frac{3u\delta}{4(u-1)^2}\left[\frac{u}{2u-1}\right]^{1/2}. \quad \text{(A2m)}$$

Case III: $u \ll 1$

Similarly, in the weak-coupling limit, we obtain the Landau parameters

$$F_0^s|_{\rm III}=2u+(3-7\delta^2)u^2\ , \quad \text{(A3a)}$$

$$F_0^a|_{\rm III}=-2u+(3+\delta^2)u^2\ , \quad \text{A3b)}$$

$$F_1^s|_{\rm III}=3(1-\delta^2)u^2\ , \quad \text{(A3c)}$$

and

$$F_1^a|_{\rm III}=(-1+9\delta^2)u^2\ . \quad \text{(A3d)}$$

The corresponding scattering amplitudes are

$$A_0^s|_{\rm III}=2u-(1+7\delta^2)u^2\ , \quad \text{(A3e)}$$

$$A_0^a|_{\rm III}=-2u-(1-\delta^2)u^2\ , \quad \text{(A3f)}$$

$$A_1^s|_{\rm III}=3(1-\delta^2)u^2\ , \quad \text{(A3g)}$$

and

$$A_1^a|_{\rm III}=(-1+9\delta^2)u^2\ . \quad \text{(A3h)}$$

The normalized observables are

$$\left.\frac{m^*}{m}\right|_{\rm III}=1+u^2(1-\delta^2)\ , \quad \text{(A3i)}$$

$$\left.\frac{\chi_s}{\chi_s^0}\right|_{\rm III}=1+2u+2u^2(1-\delta^2)\ , \quad \text{(A3j)}$$

$$K/K_0|_{\rm III}=1-2u+2(1+3\delta^2)u^2\ , \quad \text{(A3k)}$$

$$W_r/W_r^0|_{\rm III}=1+2u+(1-\delta^2)u^2\ , \quad \text{(A3l)}$$

and

$$c_1/(c_1)_0|_{\rm III}=1+u+\frac{(1-6\delta)u^2}{2}\ . \quad \text{(A3m)}$$

Case IV: $u \gg 1$

Finally, in the strong-coupling regime, taking $1/u$ as an expansion parameter, tedious calculations lead to the following expressions:

$$F_0^s|_{\rm IV}=\frac{1}{\delta}-\frac{1+\delta}{4u}\ , \quad \text{(A4a)}$$

$$F_0^a|_{\rm IV}=-1+\frac{1+\delta}{4u}\ , \quad \text{(A4b)}$$

$$F_1^s|_{\rm IV}=\frac{3(1-\delta)}{2\delta}-\frac{3(1-\delta^2)}{4u\delta}\ , \quad \text{(A4c)}$$

$$F_1^a|_{\rm IV}=-3-\frac{9(1+\delta)}{4u}\ , \quad \text{(A4d)}$$

$$A_0^s|_{\rm IV}=\frac{1}{1+\delta}\frac{\delta^2}{4(1+\delta)u}\ , \quad \text{(A4e)}$$

$$A_0^a|_{\rm IV}=\frac{-4u}{1+\delta}-\frac{2(1-2\delta)}{(1+\delta)}\ , \quad \text{(A4f)}$$

$$A_1^s|_{\rm IV}=\frac{3(1-\delta)}{1+\delta}-\frac{3(1-\delta)\delta}{u(1+\delta)}\ , \quad \text{(A4g)}$$

$$A_1^a|_{\rm IV}=\frac{4u}{(1+\delta)}-\frac{2+\delta}{1+\delta}+\frac{1+6\delta-6\delta^2}{4u(1+\delta)}\ , \quad \text{(A4h)}$$

$$\left.\frac{m^*}{m}\right|_{\rm IV}=\frac{1+\delta}{2\delta}-\frac{1-\delta^2}{4u\delta}\ , \quad \text{(A4i)}$$

$$\left.\frac{\chi_s}{\chi_s^0}\right|_{\rm IV}=\frac{2u}{\delta}+\frac{1-\delta}{2\delta}-\frac{1-\delta^2}{8u\delta}\ , \quad \text{(A4j)}$$

$$K/K_0|_{\rm IV}=\tfrac{1}{2}-\frac{2-3\delta}{8}\ , \quad \text{(A4k)}$$

$$W_r/W_r^0|_{\rm IV}=\frac{4u}{1+\delta}+\frac{3(1-\delta)}{(1+\delta)}+\frac{(1-\delta)(1-5\delta)}{4u(1+\delta)}\ , \quad \text{(A4l)}$$

and

$$c_1/(c_1)_0|_{\rm IV}=\sqrt{2}+\frac{2-3\delta}{2^{5/2}}u\ . \quad \text{(A4m)}$$

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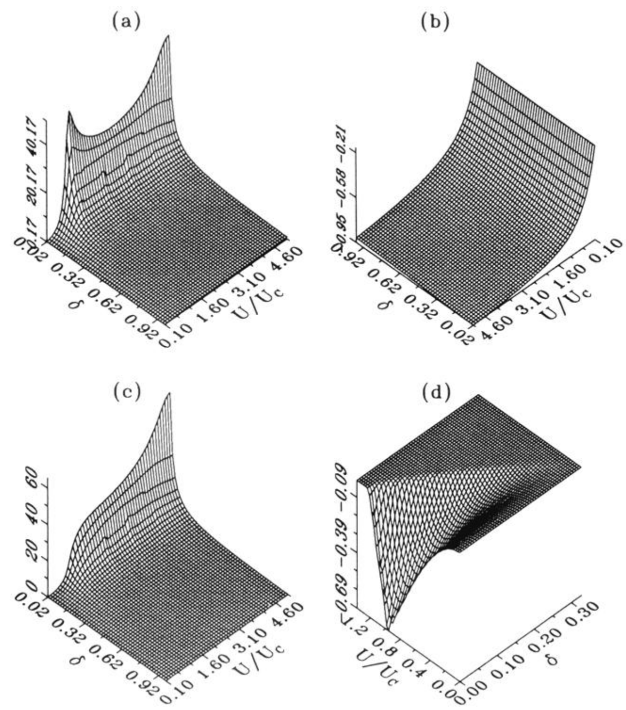

FIG. 4. The 3D diagram of the four leading Landau parameters for the Hubbard model in the SB approach. The flatband assumption is used. The Landau parameters are plotted as a function of the scaled interaction U/U_c and the doping factor δ. F_1^a was obtained with the *s*-*p* approximation. When the values of F_1^a became unstable (<-3) or unphysical, they were set to zero. Thus from this figure one can find the regime of validity of the SB calculation of F_1^a on the $(U/U_c,\delta)$ manifold.