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Fracture surfaces: Apparent roughness, relevant length scales, and fracture toughness

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The Griffith criterion is rewritten to account for the propagation of a self-affine crack. It is shown that the stress-field singularity in the close vicinity of the crack tip involves an exponent $-(d_F-1)/2$ instead of $-\frac{1}{2}$. Furthermore, the fracture toughness is shown to be related to the ratio of the maximum height $z_{\max}$ to the self-affine correlation length ξ.

It is now well established experimentally that fracture surfaces are self-affine objects.[1–9] The universality of the roughness exponent, however, is still controversial. In the pioneering work of Mandelbrot, Passoja, and Paullay,[1] it was concluded that the fracture toughness was a monotonic decreasing function of the fractal dimension. This surprising result—since one would at least expect a rougher surface to be the result of a higher energy spending—gave rise to many experimental investigations that, however, were unable to confirm it. Some of these experiments, in fact, report a correlation that relates higher energies to rougher surfaces, but the actual meaning of these results is not particularly clear since nothing is known about the experimental precision. On the other hand, more recent results concerning very different materials, either ductile (aluminum alloys,[5] steels[3,7]) or brittle (six brittle materials,[6] rocks,[8] intermetallics,[9]...), studied with very different experimental techniques (scanning electron microscopy and image analysis,[5,9] mechanical profile determination,[6,8] electrochemistry,[7]...), show that the roughness exponent is in all cases very close to the value $\zeta=0.8$. These results strongly support the conjecture made in Ref. 5 that this exponent could well be *universal*, i.e., independent of the material and of the fracture mode. One can note, however, that values of ζ closer to the roughness exponent of the *minimal energy surface*,[10–13] $\zeta\simeq0.5$ (Refs. 14 and 15), have been reported in cases that can be considered as very different. One of them concerns very small (nanometric) length scales,[16] and the other one analyzes fatigue-fracture surfaces of steel[17] in the micrometric domain.[18] Fatigue might be considered as a quasistatic process for which the *minimal energy surface* hypothesis could hold (i.e., the fact that the fracture surface globally minimizes the fracture energy). Very locally, up to a typical kinetic length scale, this assumption might be true as well. Recent experiments on a Ti_3Al-based alloy indeed show the evolution of this length scale with the local stress-intensity factor.[19] But whenever dynamical effects are relevant (at higher length scales for rapid crack-propagation modes, as reported in all the other experiments[1–9,19]), there seems to be only one *universality class*, for which $\zeta\simeq0.8$.

In order to explain why the fracture toughness should increase with the fractal dimension, Mosolov[20] recently proposed a rewriting of the Griffith criterion in the case of self-affine cracks. Although we disagree with his calculations and conclusions, we have investigated this idea and shown that the revisited criterion in fact leads to a correlation of the fracture toughness K_{Ic} with $z_{\max}/\xi$ of the feature surfaces, where ξ is the self-affine correlation length and $z_{\max}$ is the typical height on the fracture surface outside the fractal regime. The quantity $z_{\max}/\xi$ measures how spiky the surface is.

Let us first briefly recall the content of the classical Griffith criterion[21] for a crack propagating in mode I (see Fig. 1) in a sample of width W submitted to a uniaxial tension. The critical value of the stress at which propagation actually takes place is determined by equating the elastic energy ΔU_{el} due to crack propagation to the energy ΔU_s required to create two free surfaces in the material. Furthermore, it is assumed that the stress field is singular in the vicinity of the crack tip:

$$\sigma(r)\simeq Kr^{-\alpha} \tag{1}$$

at a distance r ahead of the crack front. K is the stress-intensity factor.

When the fracture path is regular, the surface energy is proportional to

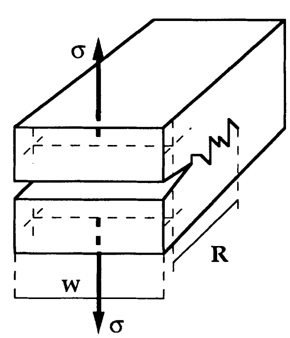

FIG. 1. Propagation of a mode-I crack in a sample of width W.

$$\Delta U_s = 2\gamma W R, \qquad (2)$$

where γ is the surface tension and R is the crack length increment.

On the other hand, $\sigma^2/2E$ being the elastic energy per unit volume, the released elastic energy can be estimated by noting that the stress field is essentially relaxed on scales $r<R$ and unperturbed on larger scales:

$$\Delta U_{\rm el} \simeq \frac{K^2}{2E}\int_a^R r^{-2\alpha} W r\, dr \simeq \frac{K^2}{2(-2\alpha+2)E}\, W R^{(-2\alpha+2)}, \qquad (3)$$

where a is a microscopic cutoff (plastic zone size, for example) below which the stress field saturates. At the onset of fracture, the two energies should be of the same order of magnitude. This implies first that $\alpha=\frac{1}{2}$, which is the classical result in elasticity theory.[21] Second, one obtains the critical value K_{Ic} of the stress-intensity factor below which the crack is unable to progress because the elastic energy is not high enough to compensate for the creation of two free surfaces. One finds a material-dependent fracture toughness, given by $K_{Ic} \propto \sqrt{\gamma E}$.

Let us assume now that the fracture surface is a self-affine characterized both by a roughness exponent ζ and a correlation length ξ. This means that if z is the height, and r the coordinate along the horizontal axis (in two dimensions) or in the horizontal plane (in three dimensions), one can write

$$z = \eta(r) z_{\rm max}\left(\frac{r}{\xi}\right)^{\zeta} \quad (r \ll \xi), \qquad (4a)$$

$$z = \eta(r) z_{\rm max} \quad (r \gg \xi), \qquad (4b)$$

where $\eta(r)$ is a random variable of order 1, which varies on the scale of r itself.

The surface energy corresponding to the opening of the crack along a distance $R \ll \xi$ can be expressed in the following way:

$$\Delta U_s \simeq 2\gamma W \int_a^R \left[1+\left(\frac{dz}{dr}\right)^2\right]^{1/2} dr, \qquad (5)$$

with $dz/dr \simeq z/r$ for $r \ll \xi$, since $d\eta(r)/dr$ is also of order z/r.

From Eq. (5), one can see that a new length scale r^* appears. r^* is the distance where the local slope of the self-affine relief is of order 1: $(dz/dr)(r=r^*) \simeq 1$, while for $r \ll r^*$, $dz/dr \gg 1$.

It can be shown that

$$\frac{r^*}{\xi} \simeq \left(\frac{z_{\rm max}}{\xi}\right)^{1/(1-\zeta)}. \qquad (6)$$

Since ζ is an exponent lying between 0 and 1, two cases may occur.

(i) $z_{max} \ll \xi$ or $r^* \ll \xi$: "*Shallow*" *surface*

In this case, r^* is smaller than ξ [Eq. (6)]. Such a surface will be called "shallow." An estimation of the surface energy then provides

$$\Delta U_s \propto 2\gamma W \xi \left(\frac{z_{\rm max}}{\xi}\right)\left(\frac{R}{\xi}\right)^{\zeta} \qquad (7)$$

as long as $R<r^*$. Note that this result is different from Mosolov's[20] expression for the surface contribution, which he estimated as $R^{(d-\zeta)}$, d being the dimension of space.

On the other hand,

$$\Delta U_s \simeq 2\gamma W(\lambda R) \qquad (8)$$

as soon as $R>r^*$ (even if $R<\xi$); λ is a number of order 1 describing the effective dilation of length brought about by the small scale roughness (see below).

In this case, even at length scales smaller than the correlation length ξ, provided that these length scales are larger than r^*, the surface energy is similar (up to a factor λ) to the one necessary to create a flat surface, although the surface is actually rough up to distances of the order of ξ. This property even holds if ξ is infinite. One consequence of this is that the stress-field singularity is the usual one at length scales greater than r^*: $\sigma \simeq K r^{-1/2}$ ($r^*<r<\xi$). On the contrary, Eq. (7) implies the existence of a nontrivial singularity closer to the crack tip, i.e., at distances from it smaller than r^*: $\sigma \simeq K' r^{-\alpha}$ ($r<r^*$), which gives rise to an elastic-energy term $\Delta U_{\rm el}$ the expression of which is given in Eq. (3). Equating $\Delta U_{\rm el}$ and ΔU_s [Eq. (7)] leads to the relation $-2\alpha+2=\zeta$, which is valid in two as well as in three dimensions. When the surface is flat, i.e., when ζ is equal to unity, the value $\alpha=\frac{1}{2}$ is recovered. In three dimensions, for which $3-\zeta$ is the fractal dimension d_F of the surface, the previous relation can also be written $\alpha=(d_F-1)/2 =(2-\zeta)/2$. As expected intuitively, "rougher" cracks (large d_F) are associated to more singular stress fields.

The crossover between this regime and the $-\frac{1}{2}$ singularity regime takes place at $r=r^*$, yielding: $K' r^{*(\zeta-2)/2} = K r^{*-1/2}$. On the other hand, the equality $\Delta U_s = \Delta U_{\rm el}$ applied in the regime $R>r^*$ leads to a critical value of K, corresponding to the *measured* fracture toughness K_{Ic}, which is independent of the characteristic length scales of the problem in the regime $K_{Ic} \simeq \sqrt{\gamma^* E}$, where $\gamma^* = \lambda\gamma$ is the effective surface energy at large length scales. It is now shown that this conclusion is not valid when surfaces with $z_{\rm max}/\xi \gg 1$, which we define as "spiky," are considered.

(ii) $z_{max} \gg \xi$ or $\xi \ll r^*$: "*Spiky*" *surface*

From the definition of r^*, one realizes that this case corresponds to $\xi<r^*$. In this case, whenever R is smaller than ξ, ΔU_s can be written as in Eq. (7): in the whole fractal domain, the local slope of the surface is much larger than one. This also implies that, close to the crack tip ($r<\xi$), the stress-field singularity is characterized by an exponent $\alpha=(2-\zeta)/2$. The crossover between the $(2-\zeta)/2$ and the $\frac{1}{2}$ singularities now takes place for $r=\xi$: r^* is no longer a relevant length of the problem. Thus, one can write: $K' \xi^{(\zeta-2)/2} = K \xi^{-1/2}$ (stress-field continuity, which allows one to find K') and, for $R>\xi$,

$$\Delta U_s \simeq 2\gamma W\left(\frac{z_{\rm max}}{\xi}\right) R, \qquad (9)$$

which leads to an interesting expression for the fracture toughness:

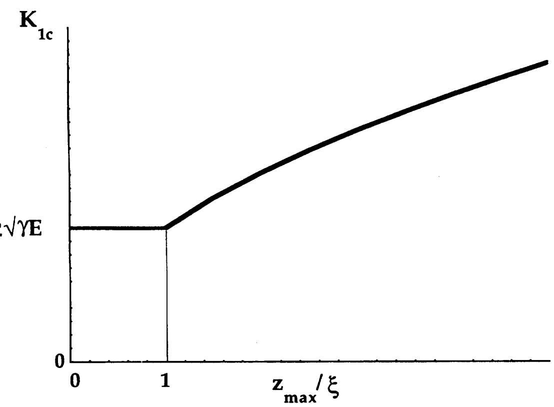

FIG. 2. Evolution of the fracture toughness K_{Ic} as a function of z_{max}/ξ: for *shallow* surfaces, i.e., when $z_{max}/\xi<1$, K_{Ic} is a constant for a given material, while it evolves as $\sqrt{z_{max}/\xi}$ when $z_{max}/\xi>1$.

$$K_{Ic} \simeq 2\sqrt{\gamma E}\left(\frac{z_{max}}{\xi}\right)^{1/2}. \quad (10)$$

The evolution of K_{Ic} as a function of z_{max}/ξ of the surface is shown in Fig. 2.

Note that the results pertaining to the case of a very flat cylindrical crack are similar to those reported above since in that case the sample width W is replaced by the crack radius R. Indeed, in the short length-scale regime ($r<r^*$ when $z_{max}\ll\xi$ and $r<\xi$ when $z_{max}\gg\xi$), the surface energy scales as $\Delta U_s \simeq R^{1+\zeta}$ while the elastic energy is proportional to $\Delta U_{el} \simeq R^{(-2\alpha+3)}$.

Let us emphasize that the small-scale fractal nature of the cracks has the consequence that on large scales, the total length traveled by the crack is larger by a factor λ (when $z_{max}\ll\xi$) or z_{max}/ξ (when $z_{max}\gg\xi$) than the horizontal length R. This means correlatively that if the crack propagates at its maximal speed (i.e., the Rayleigh wave velocity) along its tortuous path, its *apparent* velocity, measured by the time needed to cross a certain distance larger than $\min(r^*,\xi)$, is reduced by the corresponding factor. This fact could explain the well-known observation that the maximal measured crack velocity is always found to be a certain *nonuniversal* fraction of the expected Rayleigh velocity.[22,23] It would be extremely interesting to relate this velocity reduction factor to z_{max}/ξ, and hence, through Eq. (10), to the fracture toughness.

We have thus shown that although the roughness index of fracture surfaces is universal, the morphology of the fracture surface and the effective fracture toughness K_{Ic} are closely related. On fracture surfaces for which the maximum height z_{max} is smaller than the self-affine correlation length ξ, the relevant length is the distance r^* smaller than ξ for which the local slope becomes of the order of unity. In this case, K_{Ic} does not depend on the various length scales, and although ξ may be infinite, the energy spending is the same as that for fracturing a regular flat surface.

On the contrary, in the case for which the ratio z_{max}/ξ is greater than unity, ξ is the only relevant length scale in the horizontal plane, and K_{Ic} is proportional to the square root of this ratio.

This could well be an explanation for the difference in K_{Ic} measured for crack propagations in two orthogonal directions on anisotropic materials.[24] It might also explain the correlation between K_{Ic} and the fractal dimension d_F of the fracture surface, observed by Mandelbrot, Passoja, and Paullay.[1] Let us recall that the experimental method used was the "slit-island method": after the fracture surfaces were plated by nickel through vacuum evaporation, the samples were polished perpendicularly to the z axis, thus revealing "islands" of steel. The areas of these islands were then plotted against their perimeters. Because only the contour of these islands is fractal, two power-law regimes can be observed that way: at distances smaller than ξ, one measures an exponent $2/(d_F-1)$, *smaller* than 2; at larger distances, this exponent rises towards its classical value 2. Tougher samples might correspond to smaller correlation lengths ξ [Eq. (10), assuming z_{max} is fixed and $z_{max}\gg\xi$]. In this case, the apparent exponent measured in a fixed length-scale window is larger than the actual one, which leads to an apparent fractal dimension *smaller* than the real one. This could explain the trend observed in Ref. 1. Furthermore, let us note that vacuum evaporation leads to a very directional deposition. Even with no overhangs on the surface, one can assume that a slight inclination with respect to the incident direction might lead to leaving more zones unplated when the surface is more spiky. This means that the lower limit of the fractal domain might be overestimated, which also leads to an overestimation of the exponent, and to an underestimation of d_F.

In the case of our previous experiments on the 7475 aluminum alloy,[5] ξ could not be measured because it was larger than the limit length imposed by the microscope. Hence the measurements were not sensitive to the variation of ξ and an exponent closer to its actual value could be determined.

An experimental test of the model on a series of the same alloy should now be able to correlate the measured K_{Ic} and the relevant length scales determined through fracture-profiles analysis, or from crack-velocity measurements.

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[1] B. B. Mandelbrot, D. E. Passoja, and A. J. Paullay, Nature (London) **308**, 721 (1984).

[2] J. J. Mecholsky, D. E. Passoja, and K. S. Feinberg-Ringel, J. Am. Ceram. Soc. **72**, 60 (1989).

[3] Z. Q. Mu and C. W. Lung, J. Phys. D **21**, 848 (1988).

[4] R. H. Dauskardt, F. Haubensak, and R. O. Ritchie, Acta Metall. Mater. **38**, 143 (1990).

[5] E. Bouchaud, G. Lapasset, and J. Planès, Europhys. Lett. **13**, 73 (1990).

[6] K. J. Maloy, A. Hansen, E. L. Hinrichsen, and S. Roux, Phys.

Rev. Lett. **68**, 213 (1992).

[7] A. Imre, T. Pajkossy, and L. Nyikos, Acta Metall. Mater. **40**, 1819 (1992).

[8] J. Schmittbuhl, S. Gentier, and S. Roux, Geophys. Lett. **20**, 639 (1993).

[9] E. Bouchaud, G. Lapasset, J. Planès, and S. Navéos, Phys. Rev. B **48**, 2917 (1993); J.-P. Bouchaud, E. Bouchaud, G. Lapasset, and J. Planès, Phys. Rev. Lett. **71**, 2240 (1993).

[10] A. Chudnovsky and B. Kunin, J. Appl. Phys. **62**, 4124 (1987).

[11] M. Kardar, in *Disorder and Fracture*, edited by J. C. Charmet, S. Roux, and E. Guyon (Plenum, New York, 1990).

[12] S. Roux and D. François, Scr. Metall. **25**, 1092 (1991).

[13] A. Hansen, E. L. Hinrichsen, and S. Roux, Phys. Rev. Lett. **66**, 2476 (1991).

[14] M. Mézard and G. Parisi, J. Phys. France (I) **1**, 809 (1991).

[15] T. Halpin-Healy, Phys. Rev. A **42**, 711 (1990).

[16] P. McAnulty, L. V. Meisel, and P. J. Cote, Phys. Rev. A **46**, 3523 (1992).

[17] V. Y. Milman, R. Blumenfeld, N. A. Stelmashenko, and R. C. Ball, Phys. Rev. Lett. **71**, 204 (1993).

[18] A third case that should be mentioned is that of two-dimensional cracks, which seem to be well described by the statistics of *minimal energy paths* (or directed polymers): see A. Hansen, E. L. Hinrichsen, and S. Roux, Phys. Rev. Lett. **66**, 2476 (1991); J. Kertész, V. K. Horvath, and F. Weber, Fractals **1**, 67 (1993); C. Poirier, M. Ammi, D. Bideau, and J.-P. Troadec, Phys. Rev. Lett. **68**, 216 (1992).

[19] E. Bouchaud and S. Navéos (unpublished).

[20] A. B. Mosolov, Europhys. Lett. **24**, 673 (1993).

[21] B. R. Lawn, *Fracture of Brittle Solids*, 2nd ed. (Cambridge University Press, Cambridge, England, 1993).

[22] K. Ravi-Chandar and W. G. Knauss, Int. J. Fract. **26**, 65 (1984); **26**, 141 (1984).

[23] J. Fineberg, S. P. Gross, M. Marder, and H. L. Swinney, Phys. Rev. Lett. **67**, 141 (1984).

[24] G. Lapasset (private communication).