Wigner crystal versus Hall insulator

H. C. Manoharan and M. Shayegan
Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544
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In a very low-disorder two-dimensional hole system we observe a *direct* transition from a well-developed *high-order* fractional quantum Hall state at $\nu = 2/5$ to an insulating state at $1/3 < \nu < 2/5$. As $T \to 0$, $\rho_{xx} \to \infty$ for the insulating phase, while $\rho_{xx} \to 0$ for the fractional quantum Hall states at $\nu = 2/5$ and $1/3$, yielding quasiparticle excitation gaps of 0.60 and 3.6 K, respectively. The topology of the theoretical disorder versus magnetic field global phase diagram for this system expressly forbids the direct quantum liquid-to-insulator transition we observe, ruling out the disorder-induced "Hall insulator" as the explanation.

The quest for the two-dimensional (2D) quantum Wigner crystal has focused considerable attention on the nature of the insulating phases (IP's) observed in various 2D systems subjected to a strong perpendicular magnetic field. Insulating phases reentrant around well developed integral or fractional quantum Hall (IQH or FQH) states have been reported for a number of different 2D systems.[1–9] In the less disordered samples these IP's have generally been interpreted as magnetic-field-induced pinned Wigner solid states of the 2D system. This interpretation, however, has been disputed as the relative roles of disorder and electron-electron interactions are still unknown, and it is precisely the competition between these two elements that uniquely determines the ground state of the system. In particular, based on their proposed disorder vs magnetic field global phase diagram for the quantum Hall effect, Kivelson, Lee, and Zhang (KLZ) have argued that IP's arising primarily from disorder can exist around integral as well as *principal* FQH states, characterized by Landau level filling factor $\nu = 1/(2p+1)$, where p is an integer.[10] These IP's were named the "Hall insulator" since they are expected to have a normal (nondiverging) Hall resistivity as temperature $T \to 0$.[10,11] The fact that IP's reported so far have indeed been observed around either IQH states or a principal FQH state (such as 1/3 or 1/5), as well as the observation of normal Hall coefficients in the IP of a number of 2D systems[1,8,9] appear to lend support to, or at least be consistent with, the Hall insulator picture.

In this paper, we report the observation of a *direct* transition from a well-developed *high-order* FQH state at $\nu = 2/5$ to an IP at $1/3 < \nu < 2/5$ in a very high-quality 2D hole system (2DHS) confined in a GaAs quantum well. By significantly improving the intrinsic quality of our 2DHS's, we are able for the first time to demonstrate the incontrovertible hallmarks of a FQH liquid, namely a quantized Hall resistance *and* activated temperature dependence signifying the presence of an incompressibility gap, at $\nu = 2/5$ when this state is immediately followed by an IP at lower ν. A direct transition from a high-order FQH state to a disorder-induced IP (the Hall insulator) is explicitly forbidden in the global phase diagram of Ref. 10. Thus, our data rules out the association of the IP observed in this system with a disorder-induced Hall insulator state, and furthermore provides unique and compelling evidence that the global quantum Hall phase diagram must be modified to correctly account for the quantum liquid-to-insulator transitions observed at low filling in near-ideal 2D systems.

The 2DHS's discussed in this work were fabricated by molecular beam epitaxy on undoped (311)A substrates. Samples consist of 200 Å GaAs quantum wells sandwiched between thick (700–2500 Å) $Al_{0.30}Ga_{0.70}As$ spacer layers above and below, followed by Si-doped regions. Doping from both sides allows us to obtain higher hole densities and mobilities than possible with single-interface heterojunction structures. Ohmic contacts were made to the 2DHS's by alloying eutectic In:Zn in an H_2:N_2 ambient. Transport measurements on samples with standard van der Pauw and Hall bar geometries were performed in a dilution refrigerator system and a superconducting magnet (16 T maximum field). Adjacent to the samples were a calibrated RuO_2 resistor with known magnetoresistance for T measurement, and a light-emitting diode used to lower the density of the 2DHS's through a persistent photoconductivity effect. A lock-in technique was used to measure transport coefficients, with excitation currents of 0.1–10 nA at frequencies < 10 Hz. When cooled in the dark, the samples had hole densities and mobilities of $p \simeq 1\text{–}2 \times 10^{11}$ cm^{-2} and $\mu \simeq 1 \times 10^6$ cm^2/V s.

First, we establish the unprecedentedly high quality of these samples. Figure 1 shows the diagonal and Hall resistivities (ρ_{xx} and ρ_{xy}), as a function of applied magnetic field B, for a 2DHS at $p = 1.48 \times 10^{11}$ cm^{-2}. The exceptionally high quality of this 2DHS is evidenced by the set of FQH states at $\nu = p/(2p \pm 1)$, converging to $\nu = 1/2$. Minima in ρ_{xx} up to $\nu = 5/11$ and 6/11 are observed. Indeed, the data shown in this figure are quite similar to high-quality GaAs 2D *electron* systems (2DES's).[12,13] This is particularly remarkable because, in GaAs, FQH states are in general expected to be weaker in 2DHS's than in 2DES's: the much larger hole effective mass ($\simeq 0.38 m_0$)[14] compared to electrons ($0.067 m_0$) leads to significant Landau level mixing, which is known to reduce the strength of the FQH states.[15] A further affirmation of the low disorder in our 2DHS's is shown in the inset of Fig. 1, where we plot ρ_{xx} and ρ_{xy} for the same

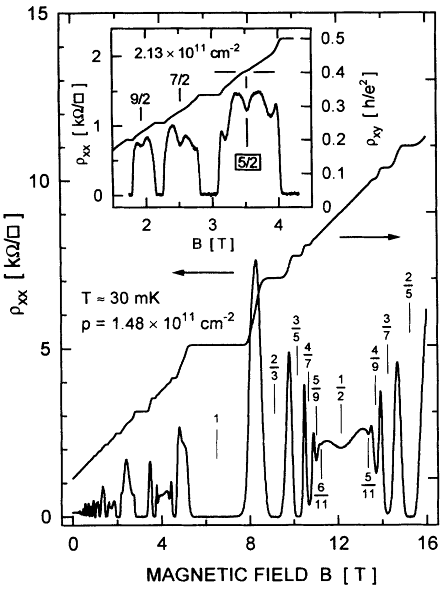

FIG. 1. Main figure: ρ_{xx} and ρ_{xy} vs B for a 2DHS. Filling factors ν are marked with vertical lines. Inset: data from the same sample at higher density, where a developing $\nu = 5/2$ FQH state is observed.

sample at higher density ($p = 2.13\times10^{11}$ cm^{-2}). Here, we see the first evidence of a $\nu = 5/2$ FQH state in a 2DHS. We observe a developing plateau at $\rho_{xy} = (2/5)(h/e^2)$ and a strong dip in ρ_{xx} at $\nu = 5/2$. We also observe similar ρ_{xx} minima at $\nu = 7/2$ and 9/2, although corresponding plateaus are not detectable.

In Fig. 2, we show data from a second 2DHS whose transport data (not shown) at its dark density ($p = 1.32\times 10^{11}$ cm^{-2}) is virtually identical to that shown in Fig. 1. Density was reduced via illumination to $p = 1.06 \times 10^{11}$ cm^{-2} to move the IP near $\nu = 1/3$ into our accessible field range. The data in Fig. 2 is highlighted by reentrant IP's characterized by large and, as we shall show, strongly T-dependent resistivities for $1/3 < \nu < 2/5$, $2/7 < \nu < 1/3$, and $\nu < 2/7$. At the same time, the data show well-developed FQH states at $\nu = 2/5$ and 1/3, and a pronounced ρ_{xx} minimum at $\nu = 2/7$. ρ_{xy} was taken for both field directions with T held reasonably constant and then averaged to remove the ρ_{xx} admixture resulting from any misalignment in the contacts.[8,9] This averaged trace is also plotted in Fig. 2, and shows quantization at $\rho_{xy} = (5/2)(h/e^2)$ and $3h/e^2$ for the $\nu = 2/5$ and 1/3 FQH states, respectively.[16] Despite the deep ρ_{xx} minimum at $\nu = 2/7$, there is no corresponding ρ_{xy} plateau, probably due to remaining admixture from the large ρ_{xx}. The IP is thus reentrant around the $\nu = 1/3$ FQH state *and* around the $\nu = 2/7$ minimum, exhibiting peaks at $\nu = 0.37$ and $\nu = 0.30$. Notably, ρ_{xy} appears to remain close to its classical value $(1/\nu)(h/e^2)$ in these IP regions.

To illustrate the FQH liquid-to-insulator transitions observed in this system, we plot the T dependence of ρ_{xx} at four separate ν in Fig. 3. The two IP peaks at $\nu = 0.37$ and 0.30 diverge as T is lowered, while the ρ_{xx} minima at $\nu = 2/5$ and 1/3 tend to vanish in a way typical of FQH liquid states. From fits to the activated regions of ρ_{xx} minima vs $1/T$, we obtain quasiparticle excitation gaps $\Delta_{2/5} = 0.60$ K and $\Delta_{1/3} = 3.6$ K, using $\rho_{xx} \propto \exp(\Delta_\nu/2T)$.

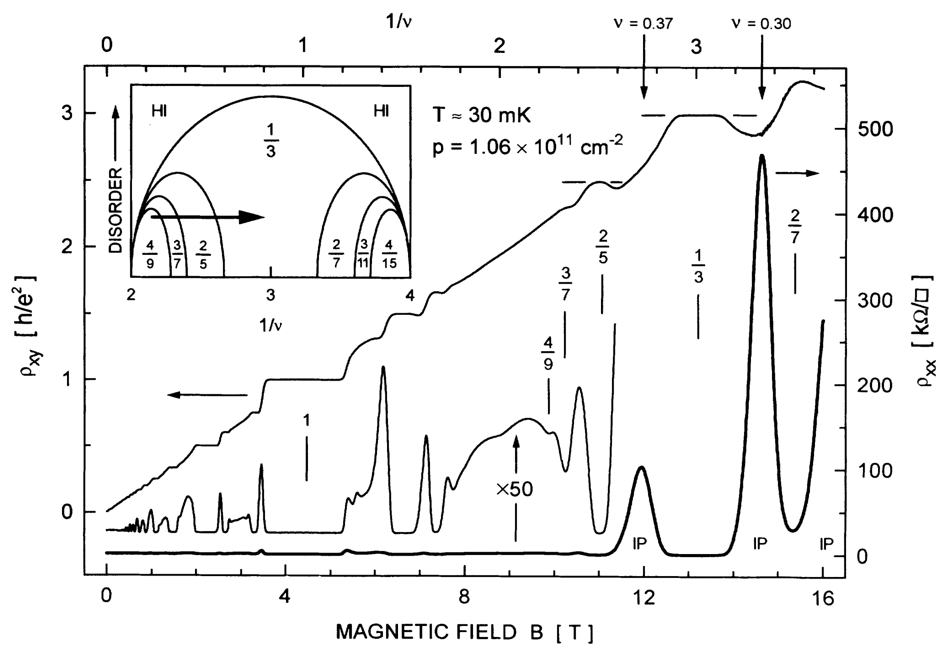

FIG. 2. Main figure: ρ_{xy} averaged over both field directions and ρ_{xx} vs B for a second 2DHS. The insulating phase regions are marked with "IP." A magnified trace (×50) is shown for lower fields, shifted up slightly for clarity. Expected plateaus for the $\nu = 2/5$ and 1/3 FQH states are marked with horizontal lines. Inset: the theoretical global phase diagram according to Ref. 10 for ν centered around 1/3. Fractional quantum Hall liquids are marked by fractional quantum number ν, and the Hall insulator state is denoted "HI." A bold arrow marks the direction and region traversed in the phase diagram as we increase B.

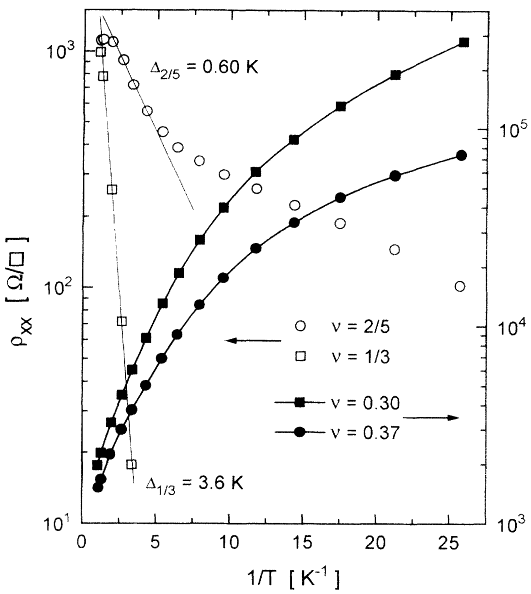

FIG. 3. Temperature dependence of ρ_{xx} for the reentrant IP at $\nu = 0.37$ and 0.30 (right axis), and for the FQH states at $\nu = 2/5$ and 1/3 (left axis). Lines through the FQH data represent fits to the activated regions, from which the quasiparticle excitation gaps Δ_ν were determined.

The relevant part of the global phase diagram, derived from Ref. 10, is shown in the inset of Fig. 2. In general, the topology of the entire diagram can be reproduced from the depicted phase by iteratively applying various transformations: $\nu \leftrightarrow \nu + 1$ (Landau level addition transformation), $\nu \leftrightarrow 1 - \nu$ (electron-hole transformation), and $(1/\nu) \leftrightarrow (1/\nu) + 2$ (flux attachment transformation).[10,17] The law of corresponding states, derived via a Chern-Simons Ginzburg-Landau formulation,[10,17,18] asserts that states linked through these transformations should exhibit similar behavior. We have chosen the root $\nu = 1/3$ phase to compare directly to our data. The actual phase boundaries are computed using results from two-parameter scaling theory, namely, that there exists a critical point between states $\nu = n$ and $\nu = n + 1$, where σ_{xy} remains fixed under renormalization-group transformations[19] and obeys $\sigma_{xy} = (e^2/h)(n + 1/2)$. Using relations between transport coefficients, this yields

$$\beta = \left[\frac{1}{\nu} - \left(n + \frac{1}{2}\right)\frac{1}{\nu^2}\right]^{1/2} \tag{1}$$

at the phase boundary, where β is a dimensionless measure of microscopic disorder. We have plotted β vs $1/\nu$ in the figure. Kivelson, Lee, and Zhang have proposed that surrounding the quantum Hall liquid states is a disorder-induced insulating phase (the "Hall insulator" state), in which, as $T \to 0$, $\rho_{xx} \to \infty$, $\sigma_{xx} = \rho_{xx}/(\rho_{xx}^2 + \rho_{xy}^2) \to 0$, and $\sigma_{xy} = \rho_{xy}/(\rho_{xx}^2 + \rho_{xy}^2) \to 0$ in such a way that $\rho_{xy} \sim (1/\nu)(h/e^2)$. Thus, the Hall insulator (HI) is characterized by a roughly classical Hall resistivity. Note that the law of corresponding states links the IP in the FQH regime to the IP in the $\nu \to \infty$ ($B \to 0$) limit. Since these same transformations map electrons and holes to quasielectrons and quasiholes, the Hall insulator we refer to in the rest of the paper is a quasiparticle insulator.

We now compare our data to the salient features of the theory.[20] In the global phase diagram, the *only* quantum liquid-to-insulator phase transitions occur between states $\nu = 1/(2p + 1)$ and the HI state. Specifically, a $\nu = 1/3$ FQH liquid $\leftrightarrow$ HI transition is the only such transition allowed in the section of phase space we are examining, while high-order FQH states are separated from the HI by at least one other phase boundary. From our data we can confidently assert the following as $T \to 0$: (a) For the FQH states at $\nu = 2/5$ and 1/3, ρ_{xx} and $\sigma_{xx} \to 0$, while ρ_{xy} and σ_{xy} are quantized at $h/\nu e^2$ and $\nu e^2/h$, respectively; (b) For the reentrant IP peaks at $\nu = 0.37$ and 0.30, $\rho_{xx} \to \infty$ while ρ_{xy} appears to remain finite and close to its classical value, implying σ_{xx} and $\sigma_{xy} \to 0$. Thus, we observe a direct transition from a FQH liquid at $\nu = 2/5$ to an IP at $1/3 < \nu < 2/5$, followed by another transition from the IP to a $\nu = 1/3$ FQH liquid, followed by a third transition to an IP at $\nu < 1/3$. It is possible yet another transition occurs to a FQH liquid at $\nu = 2/7$. In the global phase diagram, there exists no HI phase between the $\nu = 2/5$ and 1/3 FQH states; thus, while the low-temperature behavior of ρ_{xy} is consistent with the HI state, our observation of a direct transition from the $\nu = 2/5$ FQH state to the $1/3 < \nu < 2/5$ IP dissociates this IP from any disorder-induced HI state. The observation of reentrance around a possible $\nu = 2/7$ FQH state is further evidence against a HI explanation.[21]

Our observed IP's are consistent, on the other hand, with pinned Wigner crystal (WC) states. As was pointed out in Ref. 6, the larger effective mass of holes in GaAs 2DHS's[14] results in significant Landau level mixing, which can reduce the difference between the ground state energies of the $\nu = 1/3$ FQH liquid and the WC. A crossing of the liquid and solid energies near $\nu = 1/3$ is therefore possible. This conjecture has recently received theoretical justification.[22–24] Furthermore, if it is assumed that conduction in this state occurs via thermally activated dislocation pairs, recent theory also predicts an approximately classical $[\sim(1/\nu)(h/e^2)]$ Hall coefficient *at finite temperatures*,[25] consistent with our experiment.

To further contrast WC and HI states in this system, a brief discussion of the role of disorder is appropriate. In the case of zero disorder, the phase transition at small ν from an incompressible quantum liquid to a compressible Wigner solid is inescapable and is expected to be first order. For finite disorder, the nature of the transition is less clear. In this case, we can define a finite WC correlation length $\xi_{\rm wc}$ for the 2D hole solid. Casting these remarks into the framework of the KLZ theory, the first order WC transitions occur along the horizontal axis of the phase diagram, whereas all of the phase boundaries [Eq. (1)] are lines of second-order (continuous) phase transitions. Imagine starting at a zero-disorder (localization length $\xi_0 \to \infty$) WC state and then increasing disorder slightly. We maintain that long-range WC order still exists as long as $\xi_0 > \xi_{\rm wc} \gg a$, where $a \equiv (\pi p)^{-1/2}$ is the mean interhole spacing. Such WC phases are absent from the global phase diagram but consistent with our observations.

It is important to note that KLZ have specifically used the observation of a reentrant IP around $\nu = 1/3$ in a 2DHS (Ref. 6) as evidence in favor of the HI state and the consistency of the global phase diagram. They argue that the reduced mobility of the 2DHS, compared to 2DES's in which reentrance around $\nu = 1/5$ is observed, implies increased disorder and therefore easier resolution of the HI phase surrounding the $\nu = 1/3$ FQH liquid. Our samples, however, are comparable in quality to low-disorder 2DES's (as evidenced by comparable mobilities and FQH features), *yet they still exhibit an IP near* $\nu = 1/3$. Furthermore, the direct transition to this insulator from a carefully demonstrated quantum liquid provides the smoking gun that the IP observed in this 2DHS is not a HI.

Finally, we make the following observation. Clearly the reentrant IP around $\nu = 1/3$ in 2DHS's is quite similar to the IP around $\nu = 1/5$ in low-disorder 2DES's. Since we observe a direct transition from a $\nu = 2/5$ FQH state to a $1/3 < \nu < 2/5$ IP that precludes a HI explanation, we expect a similar violation of the HI selection rules near $\nu = 1/5$ for electrons: there should exist a direct transition from a $\nu = 2/9$ FQH liquid to a $1/5 < \nu < 2/9$ IP in 2DES's with low enough disorder.

In conclusion, we report the observation of a direct transition from a FQH liquid at $\nu = 2/5$ to an IP in a very low-disorder 2DHS. The IP we observe is reentrant around the $\nu = 1/3$ FQH state and around a pronounced $\nu = 2/7$ minimum. It is evident from our transport and T-dependent measurements that the states at $\nu = 2/5$ and 1/3 are true FQH liquids, while there exists a clear IP between them with diverging longitudinal resistivity as $T \to 0$. This observation is inconsistent with the position of the disorder-induced Hall insulator phase in the global phase diagram describing this system. Our experiments indicate that another explanation is required, and suggest that the reentrant IP we observe is characterized by substantial Wigner crystalline order.

Note added. Since the submission of this manuscript, we have learned that R. R. Du and co-workers,[26] examining the temperature dependence of ρ_{xx} near the IP reentrant around $\nu = 1/5$ in a 2DES, have recently seen evidence of a direct $\nu = 2/9$ FQH liquid to IP transition. This observation supports our conclusions.

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[1] M. D'Iorio, V. M. Pudalov, and S. G. Semenchinsky, Phys. Lett. A **150**, 422 (1990).
[2] V. J. Goldman *et al.*, Phys. Rev. Lett. **65**, 2189 (1990).
[3] H. W. Jiang *et al.*, Phys. Rev. Lett. **65**, 633 (1990).
[4] F. I. B. Williams *et al.*, Phys. Rev. Lett. **66**, 3285 (1991).
[5] Y. P. Li *et al.*, Phys. Rev. Lett. **67**, 1630 (1991).
[6] M. B. Santos *et al.*, Phys. Rev. Lett. **68**, 1188 (1992); Phys. Rev. B **46**, 13 639 (1992).
[7] Y. W. Suen, M. B. Santos, and M. Shayegan, Phys. Rev. Lett. **69**, 3551 (1992).
[8] V. J. Goldman *et al.*, Phys. Rev. Lett. **70**, 647 (1993); Phys. Rev. B **47**, 10 548 (1993).
[9] T. Sajoto *et al.*, Phys. Rev. Lett. **70**, 2321 (1993).
[10] S. Kivelson, D.-H. Lee, and S.-C. Zhang, Phys. Rev. B **46**, 2223 (1992).
[11] O. Viehweger and K. B. Efetov, J. Phys. Condens. Matter **2**, 7049 (1990).
[12] R. Willett *et al.*, Phys. Rev. Lett. **59**, 1776 (1987).
[13] T. Sajoto *et al.*, Phys. Rev. B **41**, 8449 (1990).
[14] Cyclotron resonance measurements on a sample from the same wafer as the sample characterized in Fig. 2 reveal a hole effective mass $m^*/m_0 = 0.38$ at $B \simeq 15$ T [C. Engelhardt *et al.* (private communication)].
[15] D. Yoshioka, J. Phys. Soc. Jpn. **53**, 3740 (1984); **55**, 885 (1986).
[16] Note that the separate ρ_{xy} traces for each field direction (not shown here) both have quantized Hall plateaus at these two states and significantly deviate from each other only in the IP regions where ρ_{xx} is large.
[17] J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989); J. K. Jain, S. A. Kivelson, and N. Trivedi, *ibid.* **64**, 1297 (1990); **64**, 1993 (1990).
[18] S.-C. Zhang, T. H. Hansson, and S. Kivelson, Phys. Rev. Lett. **62**, 82 (1989).
[19] D. E. Khmel'nitskiĭ, Pisma Zh. Eksp. Teor. Fiz. **38**, 454 (1983) [JETP Lett. **38**, 552 (1983)]; R. B. Laughlin, Phys. Rev. Lett. **52**, 2304 (1984); A. M. M. Pruisken, Phys. Rev. B **32**, 2636 (1985).
[20] Note that we are comparing experiment to the *topology* of the theoretical phase diagram. As stated in Ref. 10, the actual shape of the phase boundaries may be distorted by disorder and interaction.
[21] Further experiments indicate that at higher density, the reentrant IP is shifted to lower ν and then $\nu = 2/7$ plays an analogous role to $\nu = 2/5$ in this paper. This $\nu = 2/7$ FQH liquid to insulator transition correspondingly violates the Hall insulator selection rules.
[22] S. T. Chui and K. Esfarjani, Europhys. Lett. **14**, 361 (1991).
[23] X. Zhu and S. G. Louie, Phys. Rev. Lett. **70**, 335 (1993).
[24] R. Price, P. M. Platzman, and S. He, Phys. Rev. Lett. **70**, 339 (1993).
[25] S. T. Chui, Solid State Commun. **86**, 605 (1993).
[26] R. R. Du *et al.* (unpublished).