Miniband transport in semiconductor superlattices in a quantized magnetic field

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Miniband transport of electrons in GaAs-based superlattices under quantized magnetic fields normal to the layers is investigated based on nonparabolic balance equations recently developed. We calculate the drift velocity and electron temperature in both low and high electric fields considering impurity, acoustic, and polar-optic-phonon scatterings. As many as 25 Landau levels are included in the calculation of the frictional accelerations and electron-energy-loss rate. Strong oscillation of linear mobility and drastically different behavior of nonlinear drift velocity versus electric field are exhibited in superlattices with narrow miniband width when polar-optic-phonon scatterings are resonantly enhanced or forbidden due to Landau quantization. The results are in good qualitative agreement with recent experiments.

I. INTRODUCTION

Recent experimental demonstrations of the Esaki-Tsu negative differential velocity[1-4] in superlattice vertical transport have stimulated widespread studies on the various aspects of this exciting phenomenon.[5-17] In particular, a magnetic field, parallel or perpendicular to the growth axis of the semiconductor superlattice, has been observed experimentally[5,7,8,17] to have profound influences on the linear and nonlinear miniband conduction of the system. The effect of a transverse magnetic field (perpendicular to the direction of the applied electric field, or the growth axis of the superlattice), which stems from the Lorentz force on the drifting carriers and the interplay between the magnetic field and the periodical potential of the superlattice, has been investigated in several recent publications.[5,7,8,11,17] A longitudinal (parallel) magnetic field, which leads to the Landau quantization in the superlattice plane and the electron confinement within a distance of the order of the cyclotron radius, plays a completely different role in miniband transport. Besides the normal positive magnetoresistance,[5,17] which could appear even in ordinary wide-band systems, strong oscillations of magnetophonon resonance have been observed,[8] reflecting the distribution of the density of states of the superlattice miniband under quantized magnetic fields. Although the main physics in the longitudinal magnetic-field configuration is generally clear,[5,8] many details have not yet been explored and a systematic theoretical treatment is still lacking.

The purpose of the present paper is to report a systematic theoretical analysis of miniband transport of electrons in a GaAs-based superlattice under the influence of a quantized magnetic field normal to the layer plane. The investigations are based on the recently developed nonparabolic balance equations for superlattice miniband transport,[10,18-20] which have been extended to include the effect of a quantized longitudinal magnetic field. Drift velocity and electron temperature in both low and high electric fields are calculated for superlattices having miniband widths $\Delta = 8.5$, 21.3, and 60 meV. Effects of realistic impurity, acoustic-phonon and polar-optic-phonon scatterings are taken into account. As many as 25 Landau levels are included in the calculation of the frictional accelerations and electron energy-loss rate. At low lattice temperature ($T = 50$ K) when impurity or acoustic-phonon scattering dominates, the linear mobility decreases monotonically with increasing magnetic field. At higher temperature ($T > 150$ K) when optic-phonon scattering dominates, strong oscillations of the linear mobility appear due to resonant scatterings of electrons between Landau levels by the longitudinal-optic (LO) phonons. Calculated nonlinear drift velocity as a function of the electric field exhibits drastically different behavior for a narrow miniband superlattice when LO-phonon scatterings are prohibited due to Landau quantization. These predictions are in good qualitative agreement with recent experiments.

II. BALANCE EQUATIONS IN A LONGITUDINAL MAGNETIC FIELD

We consider a system that is composed of N electrons moving in a model superlattice with periodical potential wells of period d along the z direction under the influence of a longitudinal magnetic field B (in the z direction). The electron energy spectrum of the superlattice still forms minibands in the longitudinal direction due to the superlattice periodic potential. In the superlattice layer (x-y plane), however, it is quantized into Landau levels due to the magnetic field. Considering only the lowest miniband the electron state can be described, in the Landau representation, by the quantum number n of the Landau level, the wave vectors k_x and k_z ($-\pi/d < k_z \leq \pi/d$), and the spin index σ. The electron energy can be written as (we neglect the spin-related energy for simplicity)

$$\varepsilon_n(k_z) = (n + \tfrac{1}{2})\omega_c + \varepsilon(k_z), \quad n = 0, 1, 2, \ldots . \quad (1)$$

Here $\omega_c = |e|B/m$ is the cyclotron frequency and m is electron band effective mass in the x-y plane, and $\varepsilon(k_z)$ represents the energy dispersion of the lowest superlattice miniband, which, under the tight-binding approximation, is given by

$$\varepsilon(k_z) = \frac{\Delta}{2}(1 - \cos k_z d), \quad (2)$$

Δ being the miniband width. The electron space wave function can be expressed as

$$\psi_{n,k_x,k_z}(x,y,z) = e^{ik_x x}\chi_{n,k_x}(y)\xi_{k_z}(z), \quad (3)$$

where

$$\chi_{n,k_x}(y) = \left(\frac{1}{\pi\alpha 2^n n!}\right)^{1/2} \exp\left[-\frac{(y-y_0)^2}{2\alpha^2}\right] \times H_n\left(\frac{y-y_0}{\alpha}\right) \quad (4)$$

with

$$\alpha \equiv \sqrt{\frac{1}{|e|B}}, \quad (5)$$

$y_0 \equiv -\mathrm{sgn}(e)k_x/\alpha^2$ and $H_n(x)$ the harmonic function, and $\xi_{k_z}(z)$ stands for the tight-binding Bloch function in the z direction.

When a uniform electric field E is applied parallel to the superlattice axis, the electrons are accelerated by the field and scattered by impurities and by phonons, resulting in an overall drift motion in the z direction and a possible heating or cooling of the electron system. Since the longitudinal magnetic field has no direct effect on the electron drift motion in the z direction, the transport state of this electron system can still be described by the center-of-mass momentum $P_d \equiv Np_d$ and the electron temperature T_e. And the effective force and energy balance equations has the same form as those without a magnetic field,[10,18]

$$eE/m_z^* + A_i + A_p = 0, \quad (6)$$

$$eEv_d - W = 0. \quad (7)$$

The effect of the magnetic field shows up via the quantized electron energy spectrum (1) and the electron wave function (3) which is confined in the y direction. The average drift velocity v_d and the ensemble-averaged inverse effective mass along the z direction, $1/m_z^*$, are given by

$$v_d = \frac{1}{2\pi^2\alpha^2 N_s n_z}\sum_{n,k_z}\frac{d\varepsilon(k_z)}{dk_z} f(\varepsilon_n(k_z - p_d), T_e) \quad (8)$$

and

$$1/m_z^* = \frac{1}{2\pi^2\alpha^2 N_s n_z}\sum_{n,k_z}\frac{d^2\varepsilon(k_z)}{dk_z^2} f(\varepsilon_n(k_z - p_d), T_e). \quad (9)$$

Here $N_s = N/n_z S$ is the electron sheet density per period, S denotes the area of the superlattice layer, and n_z is the total number of the periods of the whole superlattice. $f(\varepsilon, T_e) = (\exp[(\varepsilon - \mu)/T_e] + 1)^{-1}$ is the Fermi distribution function at the electron temperature T_e, and μ is the chemical potential determined by the sheet density of electrons

$$1 = \frac{1}{2\pi^2\alpha^2 N_s n_z}\sum_{n,k_z} f(\varepsilon_n(k_z), T_e). \quad (10)$$

The localized transverse wave function of the Landau quantized state gives rise to an additional form factor related to the x-y plane motion in the expressions of the frictional accelerations A_i (due to impurities) and A_p (due to phonons), and the electron energy-loss rate W to phonons:

$$A_i = \frac{n_i}{2\pi\alpha^2 N_s n_z}\sum_{n,n',k_z,\mathbf{q}} |u(\mathbf{q})|^2|g(q_z)|^2 \times C_{n,n'}(\alpha^2 q_\parallel^2/2)[v(k_z+q_z+p_d) - v(k_z+p_d)] \times \frac{f(\varepsilon_n(k_z),T_e) - f(\varepsilon_{n'}(k_z+q_z),T_e)}{|\epsilon(\mathbf{q},\varepsilon_n(k_z) - \varepsilon_{n'}(k_z+q_z))|^2} \times \delta(\varepsilon_{n'}(k_z+q_z+p_d) - \varepsilon_n(k_z+p_d)) , \quad (11)$$

$$A_p = \frac{1}{\pi\alpha^2 N_s n_z}\sum_{n,n',k_z,\mathbf{q}} |M(\mathbf{q},\lambda)|^2|g(q_z)|^2 \times C_{n,n'}(\alpha^2 q_\parallel^2/2)[v(k_z+q_z+p_d) - v(k_z+p_d)] \times \frac{f(\varepsilon_n(k_z),T_e) - f(\varepsilon_{n'}(k_z+q_z),T_e)}{|\epsilon(\mathbf{q},\varepsilon_n(k_z) - \varepsilon_{n'}(k_z+q_z))|^2} \times \delta(\varepsilon_{n'}(k_z+q_z+p_d) - \varepsilon_n(k_z+p_d) + \Omega_{\mathbf{q},\lambda}) , \quad (12)$$

$$W = \frac{1}{\pi\alpha^2 N_s n_z}\sum_{n,n',k_z,\mathbf{q}} |M(\mathbf{q},\lambda)|^2|g(q_z)|^2 \times C_{n,n'}(\alpha^2 q_\parallel^2/2)\,\Omega_{\mathbf{q},\lambda} \times \frac{f(\varepsilon_n(k_z),T_e) - f(\varepsilon_{n'}(k_z+q_z),T_e)}{|\epsilon(\mathbf{q},\varepsilon_n(k_z) - \varepsilon_{n'}(k_z+q_z))|^2} \times \delta(\varepsilon_{n'}(k_z+q_z+p_d) - \varepsilon_n(k_z+p_d) + \Omega_{\mathbf{q},\lambda}) . \quad (13)$$

In these expressions, $u(\mathbf{q})$ is the impurity potential, $M(\mathbf{q},\lambda)$ is the electron-phonon matrix element for phonons of wave vector $\mathbf{q} = (q_x, q_y, q_z)$ in branch λ, having frequency $\Omega_{\mathbf{q},\lambda}$, $n(x) = (e^x - 1)^{-1}$ is the Bose function, $\epsilon(\mathbf{q},\omega)$ is the dielectric function within random-phase approximation. $g(q_z)$ and $C_{n,n'}(\alpha^2 q_\parallel^2/2)$ are form factors due to the longitudinal miniband wave function $[q_\parallel \equiv (q_x^2 + q_y^2)]$ and the transverse Landau quantized wave function. The former is expressed as [$\phi(z)$ is the single-well function and a is the well width]

$$g(q_z) = \frac{2}{d}\int_0^{a/2} dz|\phi(z)|^2 \cos q_z z. \quad (14)$$

The latter is given by

$$C_{n,n+l}(x) = \frac{n!}{(n+l)!}x^l e^{-x}[L_n^l(x)]^2, \quad (15)$$

$L_n^l(x)$ being the associate Laguerre polynomials,

$$L_n^l(x) = \sum_m^n (-1)^m \frac{(n+l)!}{(l+m)!(n-m)!} \frac{x^m}{m!} \quad (16)$$

in which the summation over m begins from $m = -l$ if $l < 0$, otherwise from $m = 0$.

III. LINEAR AND NONLINEAR TRANSPORT

Based on balance equations (6) and (7), we have calculated the linear and nonlinear transport properties under the influence of a strong longitudinal magnetic field ranging from 6 T to 30 T for three GaAs-based quantum well superlattices with the same period $d = 9.0$ nm, well width $a = 6.0$ nm, and carrier (electron) sheet density $N_s = 2.0 \times 10^{14}$ m^{-2} per period, but having miniband width $\Delta = 8.5, 21.3$, and 60 meV, respectively. Since effects of different types of elastic scatterings (remote or background impurities, interface roughness, and other types of defects) on superlattice miniband transport are essentially the same, we use only randomly distributed background impurities to mimic the overall elastic scattering for simplicity and assume that the impurity density $n_i = 8.6 \times 10^{21}\,\mathrm{m}^{-3}$, which yields a low-temperature (4.2 K) mobility $\mu_i(0) \sim 0.17$ m^2/V s for a $\Delta = 8.5$ meV system, $\mu_i(0) \sim 0.78$ m^2/V s for a $\Delta = 21.3$ meV system, and $\mu_i(0) \sim 2.8$ m^2/V s for a $\Delta = 60$ meV system. Both longitudinal and transverse acoustic phonons (through the deformation potential and piezoelectric couplings with electrons) and longitudinal (polar) optic phonons (through the Fröhlich coupling with electrons) are taken into account in the calculation. All the material and electron-phonon coupling parameters are taken to be typical values of GaAs and are the same as those listed in Ref. 20.

To obtain the correct result for relatively low magnetic field, a sufficient number of Landau levels has to be included in the calculation of the frictional accelerations A_i and A_p and the electron energy-loss rate W. In the program, we perform the summation over the Landau levels up to the index n for each case according to such a criterion that at the highest electron temperature reached more than 99% of the total electrons is accommodated in the levels of indices from 1 to n. As many as 25 Landau levels are included in the calculation. This enables us to go through a magnetic field as low as 6 T in the worst case.

In Fig. 1, we plot the calculated linear (low electric field) mobility as a function of the longitudinal magnetic field B for the superlattice of miniband width $\Delta = 8.5$ meV at three lattice temperatures $T = 50, 150$, and 300 K. Since the longitudinal-optic-phonon energy $\Omega_{\rm LO} \simeq 35.6$ meV, intraminiband LO-phonon scatterings are always excluded for this narrow miniband system. At low lattice temperature ($T = 50$ K) impurities and acoustic phonons (spread energy spectrum) are main scatterers and the mobility decreases monotonically with increasing magnetic field. At higher lattice temperatures when the optic-phonon scatterings are greatly enhanced ($T = 150$ K) or become dominant ($T = 300$ K), it appears as a strong mobility oscillation superposed on the decreasing background component. There clearly exhibit three minima at $B \simeq 7, 11$, and 22 T. They can easily be explained by the maxima in the density of states at the bottom and top of each separate Landau miniband under a quantizing magnetic field, as pointed out by Noguchi *et al.*[8] When the magnetophonon resonance condition

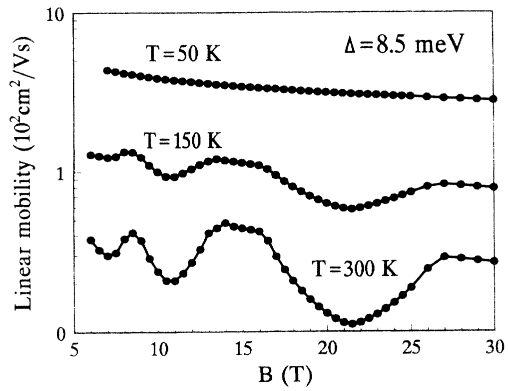

FIG. 1. Linear (low electric field) mobility is shown as a function of the longitudinal magnetic field B for a superlattice of miniband width $\Delta = 8.5$ meV at three lattice temperatures $T = 50, 150$, and 300 K.

$$n\,\omega_c = \Omega_{\rm LO} \quad (17)$$

(n = integer) is satisfied, the final states exist for all electrons in the miniband to scatter to if they absorb an optic phonon, resulting in a larger magnetoresistance. On the contrary, deviation from the above resonance condition greatly reduces the available final states for electron scattering by optic phonons, leading to a smaller resistance. The three minima just correspond to $n = 1, 2$, and 3 of the resonance condition. The $n = 1$ valley, which corresponds to the highest magnetic field to satisfy resonance condition (17), is the deepest since the density of states in each Landau level is proportional to B. The oscillation amplitudes are larger at $T = 300$ K than at $T = 150$ K, simply because that LO-phonon scatterings are much stronger at higher temperature.

For fixed d the total number of electron states (unit volume) in each miniband is the same. In wider miniband these electron states spread over a wider energy range and the singularity of their density of states reduces. The magnetophonon oscillation in the linear mobility is anticipated to weaken, as is clearly seen from Fig. 2, in which we plot the calculated linear mobility as a function of the longitudinal magnetic field B for superlattices having $\Delta = 8.5$, 21.3, and 60 meV at lattice temperature $T = 300$ K (the strongest resonance shows up). For the case of $\Delta = 21.3$ meV the $n = 2$ and $n = 3$ minima almost disappear and only $n = 1$ minimum retains. It further weakens in the $\Delta = 60$ meV system. At lower lattice temperatures, these structures are further smoothed out, exhibiting a monotonic decrease of the mobility with increasing magnetic field. All these features are in good qualitative agreement with the experimental results of Sibille *et al.*,[5] Noguchi *et al.*,[8] and Hutchinson *et al.*[17]

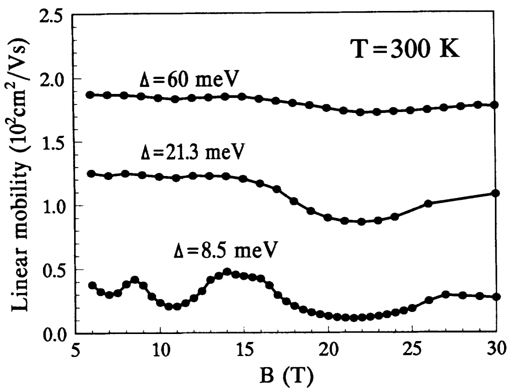

FIG. 2. Linear mobility is shown as a function of the longitudinal magnetic field B for three superlattices having miniband widths $\Delta = 8.5, 21.3$, and 60 meV at lattice temperature $T = 300$ K.

Strong oscillation also appears in the nonlinear transport in narrow miniband superlattices. In Fig. 3 we plot the calculated nonlinear drift velocity v_d as a function of the electric field E for the superlattice of $\Delta = 8.5$ meV at lattice temperature $T = 50$ K, under the influence of a longitudinal magnetic field having strength $B = 0, 11, 15, 22, 26$, or 30 T, respectively. Two curves, $B = 30$ T and $B = 15$ T, are distinctively different from the other four. That is caused by the prohibition of the LO-phonon scattering. There are two cases in which the optic-phonon scattering is forbidden. One is when the magnetic field is large enough that electrons can never jump to the higher Landau miniband even if they absorb an optic phonon: $\omega_c > \Omega_{\rm LO} + \Delta$. The other is when the magnetic field satisfies the condition $n\omega_c + \Delta < \Omega_{\rm LO} < (n+1)\omega_c - \Delta$, such that the whole range of the electron energy in a miniband, after absorbing an optic phonon, will completely fall inside the gap between two neighboring Landau minibands. Since acoustic phonons are far less efficient in dissipating energy from the carrier system, the electron temperature increases rapidly with increasing electric field. This quick rise of the electron temperature drastically limits the maximum average drift velocity of the carriers in the miniband and thus the peak drift velocity v_p is reached at a small critical electric field E_c. This case is very similar to that of a laterally confined superlattice.[12] $B = 30$ T and $B = 15$ T correspond to the first and the second cases of the prohibition of the LO-phonon scattering. The electron temperatures in these two cases grow very quickly, as can be seen in the inset of the figure, where we show the electron temperature T_e as a function of the applied electric field for all five magnetic fields. In Fig. 3 the $B = 22$ T curve and $B = 11$ T curve correspond to cases when the magnetophonon resonance condition (17) is satisfied and the optic-phonon scatterings are strong. Since the LO-phonon scattering is very efficient in transferring energy from the carrier system, the increase of the electron temperature with increasing electric field slows down greatly and the peak drift velocity reaches at a much higher electric field. The $B = 26$ T curve is an intermediate case when only a small part of electrons in the miniband can absorb an optic phonon and jump to a higher Landau level. Although this part of electrons is only about 20% of all electrons in this special case, the allowed LO-phonon scattering has already shown the ability to dissipate the electron energy and to keep the carrier system much cooler than at the $B = 15$ T and 30 T.

Figure 4 shows the calculated drift velocity v_d and the electron temperature T_e as functions of the applied electric field E for the same $\Delta = 8.5$ meV system at lattice temperature $T = 300$ K. At this temperature a sufficient number of LO phonons is excited and available to be absorbed by the carrier if the electron–LO-phonon scattering is energetically allowed. In the case of reso-

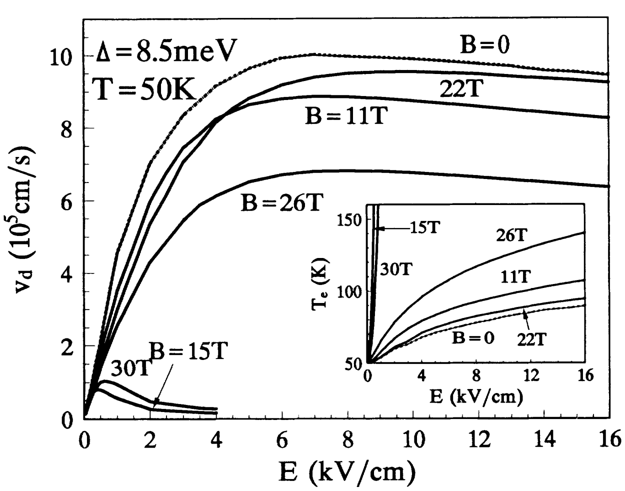

FIG. 3. Nonlinear drift velocity v_d and electron temperature T_e are plotted as functions of the applied electric field E for a superlattice of $\Delta = 8.5$ meV at lattice temperature $T = 50$ K, under the influence of longitudinal magnetic fields having strengths $B = 0, 11, 15, 22, 26$, and 30 T, respectively.

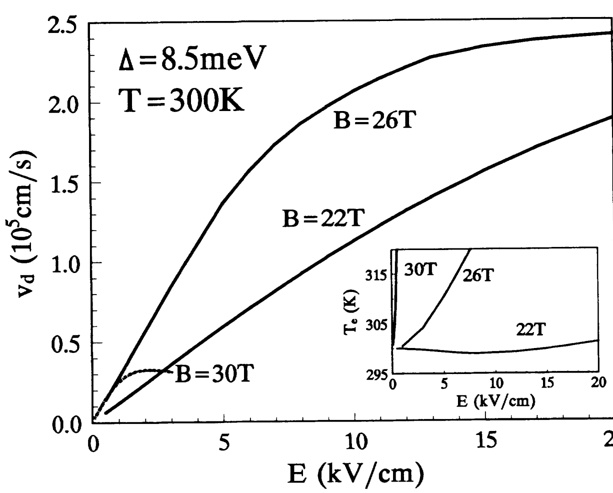

FIG. 4. Drift velocity v_d and electron temperature T_e are shown as functions of the applied electric field E at lattice temperature $T = 300$ K, for the same $\Delta = 8.5$ meV superlattice as describe in Fig. 3.

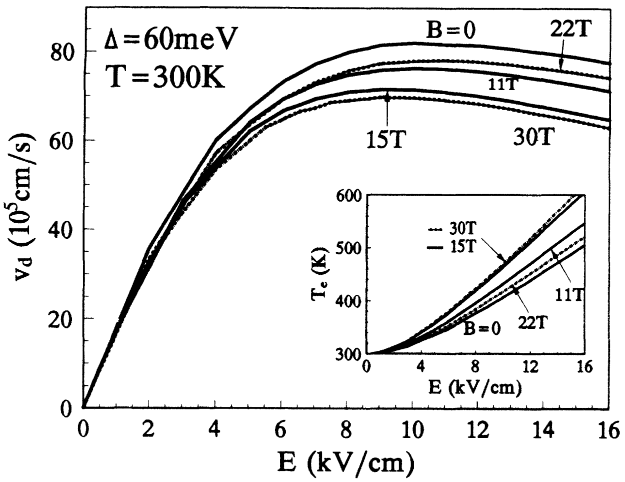

FIG. 5. Drift velocity v_d and electron temperature T_e are shown as functions of the applied electric field E at lattice temperature $T = 300$ K, for a superlattice having miniband width $\Delta = 60$ meV, under the influence of longitudinal magnetic fields having strengths $B = 0$, 11, 15, 22, and 30 T, respectively.

nance, $B = 22$ T, LO-phonon scattering is so strong that the electron temperature experiences almost no change with increasing the electric field, even becomes lower than the lattice temperature within a wide range of the field strength between $E = 0$ and $E = 15$ kV/cm, as is seen in the inset of the figure. In the absence of a magnetic field this type of electron cooling was predicted for systems where the impurity scattering is very weak.[19,21] The present result indicates that a magnetic field may promote the appearance of the electron cooling in a system with less weak impurity scattering due to the enhancement of the LO-phonon scattering by the magnetophonon resonance. It is expected that a stronger cooling will show up if the impurity scattering is weaker.

Figure 5 shows the calculated nonlinear drift velocity and the electron temperature for a wider miniband, $\Delta = 60$ meV, at lattice temperature $T = 300$ K. For this miniband width intraband LO-phonon scattering is always allowed and in no case can a v_d-vs-E behavior similar to that of $B = 15$ T or $B = 30$ T curve in Fig. 3 appear.

IV. SUMMARY

We have analyzed linear and nonlinear miniband transport of electrons in GaAs-based superlattices under a quantized magnetic field normal to the layer plane. The investigations are based on the nonparabolic balance equations extended to include the effect of Landau quantization by the magnetic field. Strong oscillation of linear mobility and drastically different behavior of nonlinear velocity versus electric field are found in superlattices of narrow miniband width when LO-phonon scatterings are resonantly enhanced or forbidden. A longitudinal magnetic field, which gives rise to the Landau quantization in the superlattice plane and the electron confinement within a distance of the order of the cyclotron radius, is able to strongly tune the vertical transport properties of a superlattice and thus to serve as a useful tool for the investigation of miniband conduction.

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