Collective excitations of a two-component one-dimensional quantum plasma confined in semiconductor quantum wires

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(Received 11 May 1994)

The collective plasmon excitation spectrum of a one-dimensional electron gas confined in a GaAs quantum wire is calculated within the two-subband random-phase approximation. We calculate the collective-charge-density-excitation (CDE) dispersion assuming that both the ground and the first excited subband states are occupied by electrons. We find four different branches of CDE modes, two of which are intrasubband CDE's and the other two intersubband CDE's. The mode-coupling effect between them is investigated by introducing a parity-breaking asymmetry in the confinement potential. Our calculated mode dispersion is in excellent quantitative agreement with a recent inelastic-light-scattering experiment.

In an important recent experiment Goñi *et al.* investigated the one-dimensional plasmon dispersion in GaAs quantum wire structures by using resonant inelastic-light-scattering spectroscopy.[1] In particular, the wave vector dispersion of the collective-charge-density excitation (CDE) modes in the one-dimensional (1D) electron gas in GaAs quantum wires was obtained both for intrasubband and intersubband transitions. While the reported experimental results for the collective mode dispersion are in reasonable agreement with the predictions of an earlier strictly 1D theoretical calculation[2] by Li and Das Sarma, there were a number of significant discrepancies and the experiment typically found three or four collective CDE branches while the theory[2] at most predicts two (one for the intrasubband plasmon and one for the intersubband plasmon). The purpose of this paper is to report results of a theoretical calculation of the elementary excitation dispersion in the quantum wire samples of Ref. 1 by going beyond the extreme 1D limit of single-subband occupancy and considering a two-subband model, i.e., by assuming that the quantum wire samples of Ref. 1 were, in fact, not in the extreme single channel 1D quantum limit, but had two 1D subbands occupied by carriers. The known experimental sample parameters (electron density, wire width, etc.) of Ref. 1 are consistent with the 1D Fermi level E_F being in the second confined subband, leading to two-subband occupancy in the system. The important result of this paper is to show that the two-subband model gives an excellent quantitative description of the elementary CDE spectra obtained in Ref. 1.

We use the random-phase approximation (RPA) for our calculation of collective mode dispersion. As emphasized elsewhere,[3] RPA is a very good approximation for the collective mode dispersion in 1D quantum wire structures by virtue of the essential vanishing of all vertex corrections to the 1D polarizability function.[4] The RPA, therefore, gives essentially an exact result for the collective mode dispersion in quantum wires.[3] The collective CDE is given within the RPA by the determinantal equation[2,5]

$$\det|\epsilon_{ijlm}| = 0, \tag{1}$$

where

$$\epsilon_{ijlm}(q,\omega) = \delta_{il}\delta_{jm} - v_{ijlm}(q)\Pi_{lm}(q,\omega). \tag{2}$$

In Eqs. (1) and (2), $\epsilon_{ijlm}(q,\omega)$ is the wave vector (q) and frequency (ω) dependent dielectric function for the 1D subband quantized system with i, j, l, and m as the 1D subband indices and q as the 1D wave vector, $v_{ijlm}(q)$ is the interelectron Coulomb interaction in the 1D subband representation,[2] and $\Pi_{lm}(q,\omega)$ is the 1D noninteracting irreducible electron polarizability function (the so-called "bare bubble").[2] Equations (1) and (2) provide the collective CDE dispersion for a multicomponent quantum plasma with each subband index (i,j,l,m, etc.) signifying a different 1D channel or, equivalently, a different quantized 1D plasma component.

If we restrict to a two-subband model, then each of the subband indices (i,j,l,m) in Eqs. (1) and (2) can be either 1 or 2 where 1 and 2, respectively, signify the ground and the first excited quantum 1D subbands. For the two-subband model, therefore, there are sixteen components of the dielectric matrix ϵ_{ijlm}, and Eq. (1) reduces to a 4×4 determinantal equation which, when solved for ω as a function of q, yields the CDE dispersion $\omega(q)$. Since the determinantal equation is of the fourth rank, there can, in general, be four different branches of CDE modes, each with its own dispersion $\omega(q)$. In the most general case of two-subband occupancy this is precisely what happens, whereas in the one-subband occupancy case one finds only two solutions for Eq. (1). We are, therefore, solving for the collective CDE mode spectra of a two-component (i.e., two-subband) 1D quantum plasma. If the one electron wave functions for 1D confinement are parity eigenstates (as they would be for symmetric confining potentials such as square well or parabolic confinement), then the off-diagonal elements of the Coulomb matrix elements (e.g., v_{1112}, v_{2111}, etc.) vanish by sym-

0163-1829/94/50(23)/17267(4)/$06.00

metry, leading to an exact separation between intasubband and intersubband elementary excitations. Thus, there is no mode-coupling effect between intrasubband and intersubband CDE plasmons for symmetric confinement potentials. For parity violating asymmetric confinement potentials there would be mode-coupling effects, particularly at finite wave vectors where the intrasubband and intersubband CDE energies are approximately equal (the resonant coupling effect). In the long wavelength limit ($q \to 0$), the mode-coupling effect disappears due to the orthogonality of subband wave functions, and a strict separation of the collective modes into intrasubband and intersubband CDE modes becomes possible even for nonsymmetric confinement. In the following we consider both symmetric and asymmetric 1D confinement.

We first consider the symmetric confinement situation when the collective mode condition [i.e., Eqs. (1) and (2)] is exactly separable and reduces to two separate equations for intrasubband and intersubband plasmons:

$$\epsilon_{\text{intra}}(q,\omega) = (1 - v_{1111}\Pi_{11})(1 - v_{2222}\Pi_{22}) + v_{1122}^2\Pi_{11}\Pi_{22} = 0 \tag{3}$$

and

$$\epsilon_{\text{inter}}(q,\omega) = 1 - v_{1212}(\Pi_{12} + \Pi_{21}) = 0, \tag{4}$$

where we suppress the (q,ω) dependence on the right-hand side for the sake of brevity. Using the known[2] expression for 1D polarizabilities it is easy to show that the long wavelength behavior of the two intrasubband CDE modes $\omega_\pm^{\text{intra}}$ implied by Eq. (3) are

$$\omega_+^{\text{intra}}(q \to 0) = 2q\sqrt{r_s(1+\eta)}\,|\ln(aq)|^{1/2}, \tag{5}$$

$$\omega_-^{\text{intra}}(q \to 0) = 2q\sqrt{\frac{1+\alpha^3}{1+\alpha}}, \tag{6}$$

where $\eta = \sqrt{1-\alpha}$, $\alpha = E_{12}/E_F < 1$, and $r_s = 4m^*e^2/\pi\kappa k_F$. In Eqs. (5) and (6), the wave vector q is measured in units of the 1D Fermi wave vector k_F and the frequencies ω are measured in units of $E_F/\hbar$ (i.e., $q \to q/k_F$ and $\hbar\omega \to \hbar\omega/E_F$), a is the effective linear width of the 1D quantum wire (i.e., the confinement size), E_{12} is the subband bottom energy difference between the ground and the excited subbands, α is the subband occupancy factor (with $\alpha < 1$ signifying two-subband occupancy), and m^* and κ are the electron effective mass and the background lattice dielectric constant, respectively. In the extreme quantum limit, when α approaches unity and $\eta \to 0$, we get the following results in the one subband occupancy case:

$$\omega_+^{\text{intra}}(q \to 0)|_{\alpha=1} = 2q\sqrt{r_s}\,|\ln(aq)|^{1/2}, \tag{7a}$$

$$\omega_-^{\text{intra}}(q \to 0)|_{\alpha=1} = 2q. \tag{7b}$$

Note that in this limit of one-subband occupancy we recover the known results; ω_+^{intra} becomes the usual 1D plasmon mode[2,3] and ω_-^{intra} is the single-particle excitation (SPE) energy which, being degenerate with the electron-hole Landau continuum, is damped and carries no spectral weight in the $q \to 0$ limit. When both subbands are occupied ($\alpha < 1$), we conclude from Eq. (6) that ω_+^{intra} is peaked above the 1D plasma dispersion [Eq. 7(a)] and ω_-^{intra} at long wavelength is a purely linear acoustic plasmon mode that lies below the 1D plasma dispersion.

Similarly, using the analytic forms for Π_{12} and Π_{21} (Ref. 2) we can solve Eq. (4) and obtain the long wavelength intersubband CDE dispersion relation. The result for the two-subband occupancy ($\alpha < 1$) case again produces two uncoupled modes $\omega_\pm^{\text{inter}}(q)$,

$$\omega_+^{\text{inter}}(q \to 0) = E_{12} + N_w(1-\eta)v_{1212}(q \to 0), \tag{8}$$

$$\omega_-^{\text{inter}}(q \to 0) = E_{12}, \tag{9}$$

where N_w is the 1D electron density in the quantum wire. Here ω_+^{inter} is the usual depolarization shifted intersubband CDE,[2] and ω_-^{inter} appears as a new intersubband collective mode only when the first excited subband gets occupied by carriers. Since the occupation of the second subband opens up a nondamping gap region inside the SPE intersubband continuum (see Fig. 1) ω_-^{inter} is an observable CDE collective mode in this region. When the second subband is empty (i.e., $\alpha > 1$), this gap closes, and consequently the ω_-^{inter} mode becomes a completely damped mode within the intersubband SPE continuum. Thus, while ω_+^{inter} exists both in one- and two-subband occupancy situations, ω_-^{inter} exists as a true collective mode only when both subbands are occupied. In this sense, ω_-^{intra} and ω_-^{inter} are both true undamped CDE collective modes only when both subbands are occupied; in the one-subband occupancy case ω_-^{inter} is a completely damped zero of the dielectric function which resides entirely inside the SPE electron-hole continua and carries little spectral weight. We note that by definition $(\omega_+^{\text{inter}} - \omega_-^{\text{inter}})$ is the depolarization shift for the 1D intersubband transition.[2] We emphasize that the existence of ω_-^{inter} as a true intersubband CDE mode is a feature peculiar to 1D systems only; there is no such intersubband collective mode in the corresponding two-dimensional systems in the two-subband occupancy situation, where one only finds the depolarization shifted ω_+^{inter} intersubband CDE mode.[5]

We show in Fig. 1 our calculated results for intrasubband and intersubband CDE mode dispersion obtained by direct numerical solutions of Eqs. (3) and (4). We use an infinite square well confinement potential of width a, employing parameters corresponding to experimental samples of Ref. 1 in Fig. 1: $a = 300$ Å, $N_w = 8.5$ (a), 8.6 (b) and (c), and 6.7 (d) $\times 10^5$ cm^{-1} corresponding to four different values of the subband occupancy factor $\alpha = \frac{E_{12}}{E_F} = 0.76$ (a), 0.90 (b), 0.73 (c), and 0.91 (d). The calculational parameters are taken from the various experimental samples of Ref. 1 in Fig. 1 with E_{12} (meV)/E_F (meV)= 3.2/4.2 (a), 5.2/5.8 (b), 3.2/4.4 (c), 3.1/3.4 (d). In each figure we show the calculated CDE mode dispersion (solid lines) along with the corresponding boundaries of the Landau damping electron-hole SPE continua regions (dotted lines). Each set of results has four branches of collective CDE dispersion: the lower (upper) two corresponding to the $\omega_{\pm}^{\rm intra}(\omega_{\pm}^{\rm inter})$ modes. (There are four sets of electron-hole Landau continua in each of Fig. 1, corresponding to two each of intrasubband and intersubband SPE, for the two occupied subbands; the lower two Landau continua dotted lines define the allowed regions of intrasubband SPE modes and the upper two define the intersubband SPE regions.) We also show in Fig. 1 the available experimental points for CDE dispersion as taken from Ref. 1. The agreement between theory and experiment in Fig. 1 is, in general, excellent. Our theoretical results account very well the four observed branches of CDE modes in Fig. 1(a), explaining the two lower ones as $\omega_{\pm}^{\rm intra}$ and the two upper ones as $\omega_{\pm}^{\rm inter}$. In the samples of Figs. 1(b), (c), and (d) our theory provides quantitative explanation for the observed CDE modes [three for Fig. 1(b) and two for Figs. 1(c) and 1(d)]. In these samples [Figs. 1(b)–1(d)] only the lower energy branches were looked for in the experiments of Ref. 1. We predict the existence of additional higher energy branches ($\omega_{\pm}^{\rm inter}$) for these samples [Figs. 1(b)–1(d)] whose verification in future experiments would be strong support for our theoretical calculations. We have also shown as an inset of Fig. 1(b) the details of our calculated results for the intrasubband CDE dispersion ($\omega_{\pm}^{\rm intra}$) with $\omega_0(q)$ being the strictly 1D CDE plasma dispersion in the one-subband occupancy limit. As $\alpha \to 1$, $\omega_{+}^{\rm intra}(q) \to \omega_0(q)$, and $\omega_{-}^{\rm intra}(q)$ merges with the SPE continuum becoming overdamped and experimentally unobservable.

In Fig. 2 we show our calculated two-subband model inelastic-light-scattering intensity (obtained by calculating the dynamical structure factor[2,5] of the two-component 1D system) for the sample corresponding to Fig. 1(b). The calculations are done at a finite temperature $T = 1.8$ K, with a small collisional broadening $\gamma = 0.05 E_F$ (which corresponds to experimental mobility values), for $q = 0.1 k_F$, and at a transverse wave vector transfer of $2\pi/a$. In the inset of Fig. 2 we show the light-scattering spectra (same sample) for a transverse wave vector transfer of $3\pi/a$. In general, we find the lower peaks to be sharper than the higher peaks at higher values of transverse wave vectors. Comparison with the experimental line shape[1] gives very good qualitative ageement; an exact quantitative comparison is not possible because the experimental values of the transverse wave

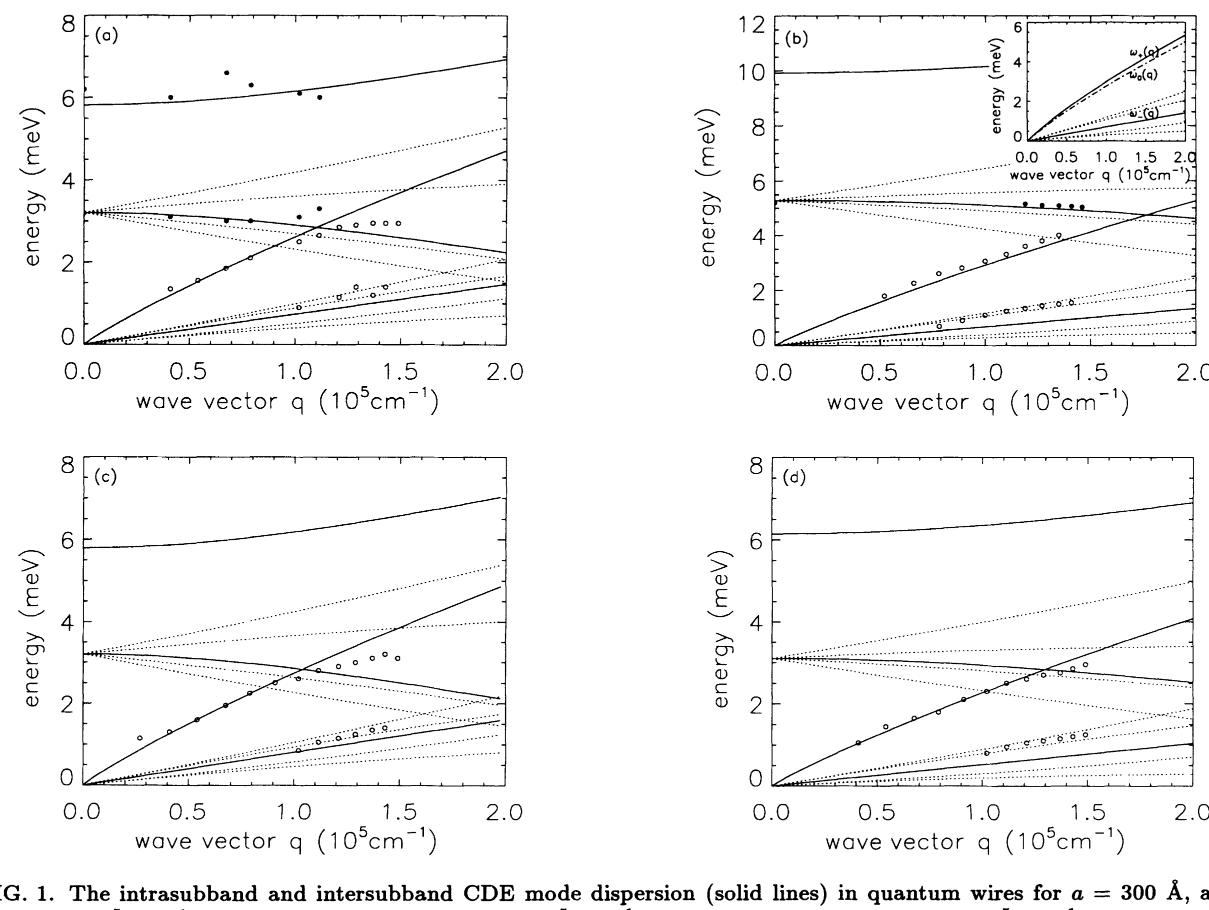

FIG. 1. The intrasubband and intersubband CDE mode dispersion (solid lines) in quantum wires for $a = 300$ Å, and (a) $N_w = 8.5 \times 10^5$ cm^{-1} and $\alpha = 0.76$; (b) $N_w = 8.6 \times 10^5$ cm^{-1} and $\alpha = 0.90$; (c) $N_w = 8.6 \times 10^5$ cm^{-1} and $\alpha = 0.73$; (d) $N_w = 6.7 \times 10^5$ cm^{-1} and $\alpha = 0.91$. Inset in (b) shows the comparison of the intrasubband CDE modes in the two-subband model [$\omega_{\pm}(q)$] with the corresponding one-subband result [$\omega_0(q)$]. The electron-hole SPE Landau continua regions lie inside the boundaries shown as dotted lines. The experimental points from Ref. 1 are also shown.

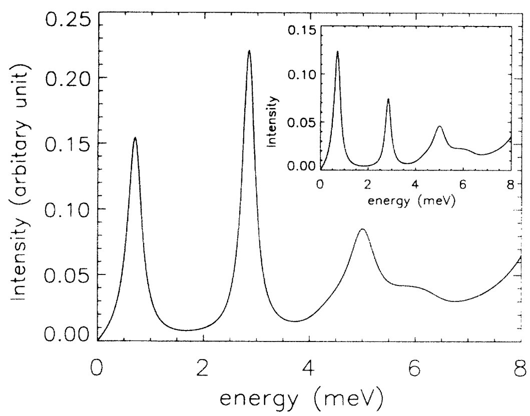

FIG. 2. The dynamical structure factor of the CDE modes corresponding to the results of Fig. 1(b) are shown for $q = 0.1k_F$ and for a transverse wave vector of $2\pi/a$. The inset shows the same at a transverse wave vector of $3\pi/a$.

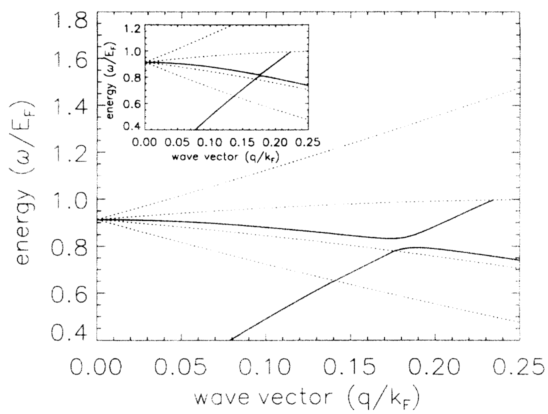

FIG. 3. The mode-coupling effect of intrasubband and intersubband CDE for the results shown in Fig. 1(a) but with an asymmetry of 2 meV in the confinement potential. The inset shows the same with a weaker asymmetry of 1 meV. Only the mode-coupled $\omega_+^{\rm intra}$ and $\omega_-^{\rm inter}$ are shown.

vector and the background intensity are not known.

Finally, in Fig. 3 we show the mode-coupling effect on our calculated CDE dispersion by including in our theory an asymmetry in the square well potential. This asymmetry destroys parity conservation of the problem and causes intrasubband-intersubband mode-coupling effect at finite wave vectors, as can be clearly seen in Fig. 3. For small asymmetry the mode-coupling effect is weak. In general, the mode-coupling effect is significant only at level crossings, i.e., only when the uncoupled intrasubband and intersubband CDE energies are equal. We believe that most of the small (but systematic) discrepancies in Fig. 1 between theory and experiment are caused by intrasubband-intersubband mode-coupling effects, as is obvious from a comparison between Fig. 1 and Fig. 3. Since the exact confinement potential is not known, it is futile to try a quantitative comparison; the strength of the mode-coupling effect obviously depends on the details of the asymmetry. We have established, however, that the resonant mode-coupling effect is operational in the experimental results of Ref. 1.

We conclude by emphasizing that our RPA-based two-subband collective mode calculation gives excellent qualitative and quantitative agreement with the experimental CDE dispersion in GaAs based 1D quantum wire structures. The success of this detailed quantitative comparison establishes that the collective electronic CDE modes in 1D quantum wires can be well described by RPA-type linear response theories provided the details of the 1D confinement potential are approximately known. It may be of interest to remark in this context that while the RPA plasmon dispersion in the strict 1D limit (i.e., the one-subband occupancy case) is the same as the collective CDE dispersion of the 1D Luttinger liquid,[3] a two-channel (i.e., the two-subband occupancy case) generalization of the Luttinger liquid theory including interchannel excitations is not yet available and, therefore, the applicability of Luttinger liquid ideas to explain the four observed CDE branches in Ref. 1 remains unknown.

This work was supported by the U.S. ARO and U.S. ONR.

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