Multiphoton Wannier-Stark effect in semiconductor superlattices

B. S. Monozon
Physics Department, Marine Technical University, 3 Lotsmanskaya Strasse, St. Petersburg 190008, Russia

J. L. Dunn and C. A. Bates
Physics Department, University of Nottingham, University Park, Nottingham NG7 2RD, United Kingdom
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Interband optical transitions in a semiconductor superlattice induced by an intense optical wave in the presence of a uniform electric field are analyzed. Both the oscillating electric field of the optical wave and the uniform electric field are directed perpendicular to the heterolayers. The superlattice potential barriers are modeled by a periodical chain of δ functions. Quasienergetic time-dependent states are used. The explicit dependence of the coefficient of the multiphoton absorption on the frequency and magnitude of the oscillating electric field, the superlattice parameters, and on the magnitude of the uniform electric field is obtained. The importance of a sufficiently strong uniform electric field, which causes Wannier-Stark localization of the electrons and holes, is emphasized. It has been shown that this localization increases with the magnitude of both the uniform and oscillating electric fields. The main influence of the intense oscillating field is found to be in the narrowing of the energy minibands. Under localization conditions, the electroabsorption multiphoton spectrum consists of a sequence of intense steps such that the number of steps depends upon the number of photons and increases with this number. The effective red boundary of the spectrum shifts towards longer wavelengths as the magnitude of the uniform electric field increases. The form of the spectrum is shown to depend upon the parity of the number of photons involved. Estimates for the GaAs/$Ga_{1-x}Al_xAs$ superlattice are given.

I. INTRODUCTION

The study of the optical properties of multilayer periodic semiconductor structures is attracting the attention of both theoreticians and experimentalists. One important example is the heterostructure formed by alternating layers of GaAs and $Ga_{1-x}Al_xAs$ semiconductors which have similar properties but different forbidden gaps. In such superlattices, the electrons are under the influence of an additional periodic potential which affects the electron motion in the direction Oz (the superlattice direction) normal to the heterolayers. This potential consists of a periodic sequence of potential wells separated by barriers and causes changes in the three-dimensional energy-band spectrum. The energy spectrum associated with the z direction of the superlattice splits into an alternating series of allowed and forbidden minibands. This miniband spectrum is superimposed on the two-dimensional band energy spectrum associated with the transverse motion. The character of the superlattice energy-band spectrum reflects both the localized and extended carrier states of the potential wells due to tunneling through the barriers.

Optical experiments, including interband optical absorption, are the methods usually adopted for the investigation of these structures. These techniques are very effective in the presence of external fields. In particular, very interesting optical phenomena arise in the superlattice when it is subjected to a uniform electric field **E** parallel to Oz. It is well known[1,2] that the quasiclassical motion of the electron of charge e moving along Oz is finite. Its frequency Ω is given by $\Omega \sim eEa/\hbar$, where a is the relevant period of the structure under consideration. The localization length of the electron states is order of Δ/eE, where Δ is the energy-band width. This leads to a discrete electron energy spectrum consisting of equidistant Stark levels separated by an amount $\hbar\Omega$. However, in bulk semiconductors for which $\Delta \sim 1$ eV, $a \sim 1$ Å, and with available electric fields $E \sim 10^7$ V m^{-1}, the localization length ($\sim 10^3$ Å) is much greater than the crystal period, so that Stark quantization is not observed in such experiments. In contrast, in a superlattice for which $a \sim 50$ Å and $\Delta \sim 0.1$ eV, an electron can be localized within one period by the above-mentioned strong applied electric field E. We note that the influence of the localized electrons on the optical response of a superlattice with an electric field applied along the superlattice direction has been investigated previously both theoretically[3–5] and experimentally.[6–9]

It should be noted that most of the previous theoretical papers were based either on numerical- or variational-type calculations of one-photon effects induced by a weak optical wave. The numerical character of these calculations is a consequence of using a real superlattice potential which consists of a large number of rectangular wells separated by barriers with finite width and height.[6,10,11] Evidently such a potential cannot be studied by analytical methods. Since the optical wave is considered to be weak, these calculations involve the time-independent intraband carrier states and ignore the effects of the time-dependent electric field of the optical wave. The calculation of multiphoton transitions based on time-independent states is possible by using the formalism of high order of perturbation theory. However, if the num-

ber of photons is increased, these calculations rapidly become very cumbersome and thus only one- or two-photon absorption can be considered in practice. This approach has been illustrated by the example of two-photon magnetoabsorption in a superlattice induced by radiation polarized in the plane of the heterolayers.[12]

An analytical approach to the calculation of the interband optical transitions in a semiconductor superlattice has been developed previously[13–15] in which the superlattice potential barriers are modeled by carriers which are δ-type functions.[14,15] Explicit analytical expressions for the time-independent carrier states and for the coefficient of one-photon absorption in the presence of uniform electric[13,14] and magnetic[15] fields were obtained. The results were found to be in good agreement with both numerical calculations and with experimental data.[6,10] Explicit expressions for the time-dependent carrier states and for the coefficient of multiphoton magnetoabsorption[16] and exciton absorption[17] induced by the radiation polarized perpendicular to the heterolayers have been obtained. The theoretical results[16] are in agreement with experimental data.[18,19]

The aim of this paper is to extend the previous analytical approach[16,17] to calculate the interband multiphoton absorption in a semiconductor superlattice subjected to external uniform electric fields (the multiphoton Wannier-Stark effect). Both the oscillating electric field of the intense optical wave and the uniform electric field are parallel to Oz. The effective-mass approximation is used and the superlattice is modeled by a limiting form of the Kronig-Penney potential consisting of a periodic chain of δ-function-type barriers. This approach uses quasienergetic time-dependent intraband states. The influence of the dynamical Stark effect on the localization of electrons by the uniform electric field is studied. The explicit dependence of the coefficient of the multiphoton absorption, upon the frequency and magnitude of the oscillating electric field, upon the magnitude of the uniform electric field, and upon the superlattice parameters, is obtained.

It is shown that the dynamical Stark effect is favorable for the localization on account of the narrowing of the miniband width Δ by the strong oscillating electric field.[16,17] Also, when the conditions of localization are met, the electroabsorption multiphoton spectrum will be shown to consist of a sequence of intense steps such that the number of these steps depends upon the number l of photons and increases with l. As the uniform electric field increases in magnitude, it will be shown that the low-frequency edge of the spectrum will shift toward longer wavelengths. Finally, it is found that the form of the spectrum depends critically upon the parity of the number of photons. Detailed estimates for the $GaAs/Ga_{1-x}Al_xAs$ structure are also given.

II. THE QUASIENERGETIC STATES OF THE CARRIERS

Let us consider an electron of charge e in a semiconductor superlattice with a large number N of periods a in the presence of an oscillating electric field $\eta F_0\cos\omega t$ of frequency ω, magnitude F_0, polarization (unit) vector η, and a uniform electric field $\mathbf{E}$. Assuming that the usual effective mass approximation can be used, the equation for the envelope wave function Ψ describing a particle at coordinate $\mathbf{r}$ in a simple band with an effective mass m is given by

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t)+[V(\mathbf{r})-e(\mathbf{E}+\eta F_0\cos\omega t)\cdot\mathbf{r}]\Psi(\mathbf{r},t)=i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t}\ , \quad (2.1)$$

where

$$V(z)=\alpha\sum_s\delta(z-as),\quad V(z)=V(z+as),\quad \alpha>0 \text{ with } s \text{ integral} \quad (2.2)$$

is the periodic superlattice potential formed by the δ-function-type barriers of power α.

If both electric fields are directed along Oz, the solution to Eq. (2.1) is

$$\Psi(\mathbf{r},t)=e^{-i\mathcal{E}_\perp t/\hbar}\Phi_\perp(\boldsymbol{\rho})\phi(z,t)\ , \quad (2.3)$$

where $\Phi_\perp(\boldsymbol{\rho})$ is the transverse function of a particle with transverse energy $\mathcal{E}_\perp$, and $\phi(z,t)$ obeys the equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\phi(z,t)}{\partial z^2}+[V(z)-e(E+F_0\cos\omega t)z]\phi(z,t)=i\hbar\frac{\partial\phi(z,t)}{\partial t}\ . \quad (2.4)$$

The solutions to this equation with $E=F_0=0$ are

$$\phi_n^{(0)}(z,t;k)=e^{-i\varepsilon_n(k)t/\hbar}\psi_n(z;k)\ , \quad (2.5)$$

where

$$\varepsilon_n(k)=\varepsilon_n(k+2\pi/a),\quad \psi_n(z,k)=\psi_n(z,k+2\pi/a) \text{ for } n=1,2,3,\ldots . \quad (2.6)$$

ε_n are the energies of the allowed minibands, and ψ_n are the Bloch functions of the particle in the superlattice with the average momentum $\hbar k$. For weak barrier penetration for which

$$\lambda=\frac{\hbar^2}{2ma\alpha}\ll 1\ ,$$

where λ is the reciprocal dimensionless barrier power, the expressions for the Bloch functions ψ_n and energy minibands ε_n can be found in explicit form. The reciprocal power λ can be regarded as a parameter of the theory, but it can be calculated from the knowledge of the width and height of the barrier. Under the condition $\lambda\ll 1$, the expression for the energy spectrum is[14–17]

$$\varepsilon_n(k)=b_n+\tfrac{1}{2}\Delta_n(1-\cos ka)\ , \quad (2.7)$$

where

$$b_n=\frac{\hbar^2}{2m}\left[\frac{n\pi}{a}\right]^2[(1-2\lambda)^2+4(-1)^n\lambda]$$

and

$$\Delta_n = 8(-1)^{n+1}\frac{\hbar^2}{2m}\left[\frac{n\pi}{a}\right]^2\lambda \quad (n=1,2,3,\ldots) .$$

The lower boundary of each miniband is $\varepsilon_n(0)=b_n$, and the corresponding width is Δ_n. We shall consider the ground minibands only so that the $n=1$ label will henceforth be dropped.

Let us represent the solution to Eq. (2.4) with $E\neq 0$, $F_0\neq 0$ in the form

$$\phi(z,t)=C\int_{-\pi/a}^{+\pi/a} c(k,t)\psi[z,q(t,k)]e^{-i\mathcal{E}t/\hbar}dk , \tag{2.8}$$

where

$$q(t,k)=k+eF_0/\hbar\omega\sin\omega t ,$$

and C is a normalization factor. Substituting expression (2.8) into Eq. (2.4) and taking into account the relations[14]

$$\frac{\partial\psi}{\partial q}=iz[1+0(\lambda)] \quad \text{and} \quad z\psi=\left[i\frac{\partial}{\partial k}-\frac{a}{2}[1+0(\lambda)]\right]\psi$$

with $z\approx as$,

the equation for $c(k,t)$ is

$$[\varepsilon(q)-\mathcal{E}+\tfrac{1}{2}eEa]c(k,t)-ieE\frac{\partial c(k,t)}{\partial t}=i\hbar\frac{\partial c(k,t)}{\partial t} . \tag{2.9}$$

Let us represent the function $c(k,t)$ in the form

$$c(k,t)=c_0(k)c_1(k,t) , \tag{2.10}$$

where

$$c_0(k)=\exp\left\{-\frac{i}{eE}\int_0^k[\bar{\varepsilon}(k')-\mathcal{E}+\tfrac{1}{2}eEa]dk'\right\} \tag{2.11}$$

and

$$\bar{\varepsilon}(k)=\frac{1}{T}\int_0^T\varepsilon[q(\tau,k)]d\tau \quad \text{where } T=\frac{2\pi}{\omega} . \tag{2.12}$$

Substituting the function $c(k,t)$ from (2.10) into Eq. (2.9) and assuming that the condition

$$eE\frac{\partial c_1}{\partial k}\ll\hbar\frac{\partial c_1}{\partial t} \tag{2.13}$$

is valid, the term $\sim\partial c_1/\partial k$ in the equation for c_1 can be neglected. The equation for c_1 then becomes

$$c_1(k,t)\{\varepsilon[q(t,k)]-\bar{\varepsilon}(k)\}=i\hbar\frac{\partial c_1(k,t)}{\partial t} . \tag{2.14}$$

The solution to Eq. (2.14) can be found by direct integration and gives

$$c_1(k,t)=\exp\left\{-\frac{i}{\hbar}\int_0^t\{\varepsilon[q(\tau,k)]-\bar{\varepsilon}(k)\}d\tau\right\} . \tag{2.15}$$

Substituting the functions $c_0(k)$ from (2.11) and $c_1(k,t)$ from (2.15) into (2.10), and then into (2.8), the result for the function ϕ is

$$\phi(z,t)=C\int_{-\pi/a}^{+\pi/2}dk\,\psi[z,q(t,k)]\exp\left\{-\frac{i}{\hbar}\int_0^t\{\mathcal{E}-\bar{\varepsilon}(k)+\varepsilon[q(\tau,k)]\}d\tau-\frac{i}{eE}\int_0^k[\bar{\varepsilon}(k')-\mathcal{E}+\tfrac{1}{2}eEa]dk'\right\} . \tag{2.16}$$

Expression (2.16) is valid under condition (2.13). On taking into account expression (2.15), this condition can be written in an explicit form

$$\nu=\frac{eEa}{\hbar\omega}\ll 1 . \tag{2.17}$$

For example, if $\hbar\omega\sim\mathcal{E}_g\sim 1$ eV, $a\sim 50$ Å, and $E\sim 10^7$ V m^{-1}, then $\nu\sim 5\times 10^{-2}$.

It is easy to see that function (2.16) satisfies the conditions

$$\phi(z,t+T)=e^{-i\mathcal{E}T/\hbar}\phi(z,t) \tag{2.18}$$

$$\phi(z,t)=e^{-i\mathcal{E}t/\hbar}f(z,t) \quad \text{such that } f(z,t+T)=f(z,t) ,$$

with the explicit form of the periodic function $f(z,t)$ given below. Condition (2.18) means that expression (2.16) is a quasienergetic function[20] with quasienergy $\mathcal{E}$.

The requirement of periodicity of the function $c(k,t)\sim c_0(k)$ is given by

$$c(k,t)=c(k+2\pi/a,t) .$$

That is, the periodicity of the function $c_0(k)$ leads to the quantized Stark energy levels given by

$$\mathcal{E}_\sigma=eEa(\sigma+\tfrac{1}{2})+\frac{a}{2\pi}\int_0^{2\pi/a}\bar{\varepsilon}(k)dk$$

where $\sigma=0,\pm 1,\pm 2,\ldots .$ (2.19)

As expected, in the absence of a time-dependent electric field ($F_0=0$), function (2.16) tends to the function which describes the particle in the presence of the uniform electric field E given previously.[12,13] If $E=0$ but $F_0\neq 0$, the function $c_0(k)$ takes the form

$$c_0(k)=\delta(k-p) \quad \text{and} \quad \bar{\varepsilon}(p)=\mathcal{E} ,$$

and function (2.16) tends to that describing the particle subjected to the time-dependent electric field only.[16]

To take into account the periodicity of functions (2.6), the Bloch functions ψ are expanded in a Fourier series in the form

$$\psi(z,q)=\frac{1}{\sqrt{N}}\sum_\mu Q_\mu(z)e^{iqa\mu} , \tag{2.20}$$

where μ is summed over the superlattice indices, and where

$$Q_\mu(z)=-\frac{a\sqrt{N}}{2\pi}\int_{-\pi/a}^{+\pi/a}\psi(z,q)e^{-iqa\mu}dq \tag{2.21}$$

are the particle Wannier functions, such that

$$\langle Q_{\mu'}Q_\mu\rangle=\delta_{\mu\mu'} . \tag{2.22}$$

Using the explicit form of the Bloch functions in the cell with fixed index s, the explicit form of the Wannier functions can be obtained.[14] In the approximation $\lambda \ll 1$, the Wannier functions are not equal to zero in the superlattice cells with indices $s=\mu$, $\mu+1$, and $\mu-1$ with $Q_\mu \sim \lambda^{|s-\mu|}$. Formulas (2.3), (2.19), (2.20), (2.16), (2.7), and (2.12) define the quasienergetic miniband time-dependent states of a carrier in the superlattice in the presence of the total longitudinal electric field $E+F_0\cos\omega t$.

III. LOCALIZATION IN THE OSCILLATING ELECTRIC FIELD

To find the longitudinal function (2.16) and quasienergy (2.19) in an explicit form, we make the assumption that in the case of real superlattice and in the presence of a real oscillating electric field F_0, the parameter β is such that

$$\beta=\frac{eF_0a}{\hbar\omega}\ll 1 . \tag{3.1}$$

Thus if $\hbar\omega \sim \mathcal{E}_g \sim 1$ eV, $a\sim 50$ Å, and $F_0 \sim 5\times 10^7$ V m^{-1}, then $\beta=0.25$.

Under conditions (2.17) and (3.1), the expressions for functions (2.16) and (2.18) and the quasienergies (2.19) become

$$f_\sigma(z,t)=\frac{C}{a\sqrt{N}}\int_{-\pi}^{+\pi}\sum_\mu Q_\mu(z)e^{i(G+H+I)}d\theta , \tag{3.2}$$

$$\mathcal{E}_\sigma=eEa(\sigma+\tfrac{1}{2})+b+\tfrac{1}{2}\Delta . \tag{3.3}$$

where $G=\mu(\beta\sin\theta+\theta)$, $H=-\frac{1}{2}\zeta[\beta\sin\theta(1-\cos\omega t)-(\beta^2/8)\cos\theta\sin2\omega t]$, and $I=[\sigma\theta+(\zeta/2\nu)(1-\frac{1}{4}\beta^2)\sin\theta]$.

The other parameters are defined by $\zeta=\Delta/\hbar\omega$ and $C=a\sqrt{N}/2\pi$.

Using the expansions

$$e^{i\chi\cos\theta}=\sum_{n'=-\infty}^{+\infty} i^{n'}J_{n'}(\chi)e^{in'\theta} \quad \text{with } n'=0,1,2,\ldots$$

and

$$e^{i\Phi\sin\theta}=\sum_{n=-\infty}^{+\infty} J_n(\Phi)e^{in\theta} \quad \text{with } n=0,1,2,\ldots ,$$

where the J_n's are Bessel functions, we find that

$$f_\sigma(z,t)=\sum_{n,n'} Q_{-\sigma+n+n'}(z)e^{-i(\sigma-n-n')\beta\sin\omega t}(-1)^{n+n'}(i)^{n'}J_{n'}(\chi(t))J_n(\Phi(t)) , \tag{3.4}$$

where

$$\chi(t)=\frac{\beta^2}{16}\zeta\sin2\omega t$$

and

$$\Phi(t)=\tfrac{1}{2}(1-\tfrac{1}{4}\beta^2)\zeta/\nu-\tfrac{1}{2}\zeta\beta(1-\cos\omega t) .$$

Since $\Delta\sim 0.1$ eV and $\hbar\omega\sim 1$ eV, then $\zeta \ll 1$ and

$$\chi(t)\ll 1 ,$$
$$\Phi(t)\approx\Phi_0=\frac{\zeta}{2\nu}\left[1-\frac{\beta^2}{4}\right]=\frac{\Delta}{2eEa}\left[1-\frac{\beta^2}{4}\right] . \tag{3.5}$$

Therefore, only the item with index $n'=0$ should be retained in the sum (3.4), so that

$$f_\sigma(z,t)\approx\sum_n Q_{n-\sigma}(z)(-1)^nJ_n(\Phi_0) . \tag{3.6}$$

It follows from expression (3.6) that the degree of localization of the carriers is defined by the Bessel function $J_n(\Phi_0)$. If $\Phi_0 \ll 1$, then $J_0=1$ and $J_n \ll 1$ $(n\neq 0)$ and the carrier having the energy $\mathcal{E}_0$ is localized in the superlattice cell with index $s=-\sigma$. Expression (3.5) for Φ_0 shows that localization increases with an increase in the magnitudes of both the uniform E and oscillating F_0 electric fields. The uniform electric field E increases the slope of the chain of the potential wells and barriers describing the superlattice. The oscillating field F_0 reduces the miniband width to[16]

$$\Delta(F_0)=\Delta(1-\tfrac{1}{4}\beta^2) .$$

Both of these mechanisms are favorable for the concept of the localization. In contrast, the time dependence of the slope of the superlattice potential due to the oscillating field does not influence the localization, because this effect is suppressed by the high frequency of the oscillations.

IV. COEFFICIENT OF THE INTERBAND MULTIPHOTON ABSORPTION

The outline of the derivation of the multiphoton absorption coefficient will be given below; further details of the calculation can be found in a previous paper.[16] We consider the interband absorption as a transition of the electron-hole pair to an excited state in which an electron (e) is in the conduction miniband and a hole (h) is in the valence miniband. The ground state is then described[21] by the function $\Psi_0(\mathbf{r}_e,\mathbf{r}_h)=\delta(\mathbf{r}_e-\mathbf{r}_h)$, and the excited state by the function

$$\Psi(\mathbf{r}_e,\mathbf{r}_h,t)=\Psi_e(\mathbf{r}_e,t)\Psi_h(\mathbf{r}_h,t) . \tag{4.1}$$

The electron function $\Psi_e(\mathbf{r}_e,t)$ is defined by expressions (2.3), (2.18), (3.2) and (3.3), with the addition of subscripts e to all parameters $(\rho,z,\mathcal{E}_\perp,\zeta,\Delta,b,m)$. Similarly, the hole function $\Psi_h(\mathbf{r}_h,t)$ can be obtained from the electron function by replacing the subscript e by the subscript h, σ by σ', t by $-t$, e by $-e$, and taking complex conjugates. In expansion (3.3), the electron Wannier function $Q_\mu(z_e)$ should be replaced by the hole function $\bar{Q}_\mu'^*(z_h)$ with $\bar{Q}_\mu=Q_{-\mu}$.

The coefficient of the interband dipole transition under the oscillating field is defined by the matrix element of the operator

$$P(t)=P_0\cos\omega t \quad \text{where } P_0=\frac{i\hbar F_0 p_{ehz} e}{m_0\mathcal{E}_g} , \tag{4.2}$$

and where p_{ehz} is the matrix element of the momentum operator between the amplitudes of the Bloch functions of the electron and hole bands. This matrix element is given by

$$S(t)=\frac{1}{i\hbar}\int_0^t \delta(\mathbf{r}_e-\mathbf{r}_h)P(\tau)\Psi^*(\mathbf{r}_e,\mathbf{r}_h,\tau)d\mathbf{r}_e d\mathbf{r}_h d\tau . \tag{4.3}$$

The result connects the transition rate W with the coefficient of absorption α as

$$\alpha=\frac{n_0\hbar\omega}{cu\Omega}W \quad \text{and} \quad W=\frac{1}{t}\sum_{e,h}|S(t)|^2 , \tag{4.4}$$

where $n_0(\omega)$ is the refractive index, c is the speed of light, $u=\varepsilon_0 n_0^2 F_0^2$ is the optical energy density, $\Omega=L_xL_y\,Na$ is the volume of the crystal and $\sum_{e,h}$ a sum over band states. On substituting for the operator $P(\tau)$ from (4.2), and the function Ψ from (4.1) into (4.3), and using the relations

$$\begin{aligned} M(\tau)\cos\omega\tau &= \sum_{l=-\infty}^{+\infty} e^{-il\omega\tau}A_l(\omega) , \\ A_l(\omega) &= \frac{\omega}{2\pi}\int_{-\pi/\omega}^{+\pi/\omega} e^{il\omega t}\cos\omega t M(t)dt , \end{aligned} \tag{4.5}$$

where

$$\begin{aligned} M(\tau)=\int \delta(\mathbf{r}_e-\mathbf{r}_h)\Phi_{\perp e}^*(\boldsymbol{\rho}_e)\Phi_{\perp h}^*(\boldsymbol{\rho}_h) \\ \times f_e^*(z_e,\tau)f_h^*(z_h,\tau)d\mathbf{r}_e d\mathbf{r}_h , \end{aligned} \tag{4.6}$$

such that $M(\tau+2\pi/\omega)=M(\tau)$, the general form for the coefficient of absorption α is

$$\alpha=\sum_l \alpha_l \quad \text{where } \alpha_l(\omega)=\frac{2\pi\omega\hbar^2 e^2|p_{ehz}|^2}{\varepsilon_0 c\Omega m_0^2 n_0 \mathcal{E}_g^2}\sum_{e,h}|A_l(\omega)|^2\delta(l\hbar\omega-\mathcal{E}_g-\mathcal{E}_{\perp e}-\mathcal{E}_{\perp h}-\mathcal{E}_\sigma-\mathcal{E}_{\sigma'}) , \tag{4.7}$$

where α_l is the coefficient of l-photon interband absorption. A transformation of (4.6) into the relative coordinate $\boldsymbol{\rho}$ and the coordinate of the center of mass $\mathbf{R}_\perp$ is made using the relations

$$\boldsymbol{\rho}=\boldsymbol{\rho}_e-\boldsymbol{\rho}_h, \quad \mathbf{R}_\perp=\frac{m_e\boldsymbol{\rho}_e+m_h\boldsymbol{\rho}_h}{m_e+m_h} ,$$

and writing

$$\Phi_{\perp e}(\boldsymbol{\rho}_e)\Phi_{\perp h}(\boldsymbol{\rho}_h)=\frac{e^{i\mathbf{K}_\perp\cdot\mathbf{R}_\perp}}{\sqrt{L_xL_y}}\Phi_\perp(\rho) , \tag{4.8}$$

where $\Phi_\perp(\rho)$ is a function of the relative transverse motion and $\mathbf{K}_\perp$ is the total transverse momentum of the electron-hole pair, with $\mathbf{K}_\perp\simeq 0$ in the dipole approximation.

Substituting expressions (4.8), (3.2), and (3.3) into (4.6) and then into (4.5), the form for the coefficient $A_l(\omega)$ becomes

$$A_l(\omega)=\sqrt{L_xL_y}\Phi_\perp(0)\frac{1}{2\pi}\int_{-\pi}^{+\pi}\exp\left\{i\left[(\sigma-\sigma')\theta-\frac{\zeta_{eh}}{2\nu}\left[1-\frac{\beta^2}{4}\right]\sin\theta\right]\right\}B_l(\omega,\theta)d\theta , \tag{4.9}$$

where

$$B_l(\omega,\theta)=\frac{\omega}{2\pi}\int_{-\pi/\omega}^{+\pi/\omega}dt\,\cos\omega t\,\exp\left\{i\omega\left[\int_0^t d\tau[l-\tfrac{1}{2}\zeta_{eh}\{\beta\sin\theta\sin\omega\tau-\tfrac{1}{4}\beta^2\cos\theta(1-2\sin^2\omega\tau)\}]\right]\right\} \tag{4.10}$$

and where $\zeta_{eh}=\zeta_e+\zeta_h$.

It was shown previously[16] that the coefficient B_l can be represented in the simpler form

$$B_l(\omega,\theta)=\frac{1}{2\sqrt{l\pi}}e^{[-i\pi(l-1)+l]/2}\times\left[\frac{1}{2\gamma}\right]^{l-1}\sin[-2l\xi+\tfrac{1}{2}l\pi]\ , \tag{4.11}$$

where

$$2\xi(\theta)=\left[\frac{\zeta_{eh}}{l}\right]^{1/2}\frac{\sin\theta}{(\cos\theta)^{1/2}}$$

and

$$\frac{1}{\gamma^2(\theta)}=\frac{\zeta_{eh}}{4l}\beta^2\cos\theta\ ,$$

such that $\zeta_{eh}\ll 1$, $\gamma\gg 1$. Expressions (4.7), (4.9), and (4.11) define the coefficient of the l-photon interband electroabsorption in the superlattice. The above expressions have a common character but they are not related to any specific transverse states.

V. RESULTS AND DISCUSSION

A. General result

For the free transverse motion, the wave function and energy are

$$\Phi_\perp(\boldsymbol{\rho})=\frac{e^{i\mathbf{k}_\perp\cdot\boldsymbol{\rho}}}{2\pi}\quad\text{and}\quad \mathcal{E}_{\perp e}+\mathcal{E}_{\perp h}=\frac{\hbar^2k_\perp^2}{2\mu}$$

$$\text{where } \mu=\frac{m_em_h}{m_e+m_h}\ . \tag{5.1}$$

The sum in (4.7) is of the form

$$\sum_{e,h}=\sum_{\sigma,\sigma'}\int dk_\perp=N\sum_{\delta}\int dk_\perp\quad\text{with }\delta=\sigma-\sigma'\ . \tag{5.2}$$

On substituting expressions (4.11) into (4.9), the coefficient $A_l(\omega)$ can be expressed in terms of Bessel functions. Substituting $\Phi_\perp(0)$ and (5.1) into (4.9) and then substituting the explicit form of the coefficient $A_l(\omega)$ into (4.7), and using the summation rules (5.2), the result for α_l is

$$\alpha_l(\omega)=\overline{\alpha_l}\sum_{\delta=-\infty}^{+\infty}|M_\delta^{(l)}|^2\Theta(l\hbar\omega-\overline{\mathcal{E}}_g-eEa\delta)\ , \tag{5.3}$$

where

$$\overline{\alpha_l}=\alpha_0\frac{e^l}{4l\pi}\left[\frac{\zeta_{eh}\beta^2}{16l}\right]^{l-1}\ ,$$

and where

$$\overline{\mathcal{E}}_g=\mathcal{E}_g+b_e+b_h+\tfrac{1}{2}(\Delta_e+\Delta_h),\quad \Theta(x)=\begin{cases}1, & x>0\\ 0, & x<0\ ,\end{cases}$$

with

$$\alpha_0=\frac{\omega e^2|p_{ehz}|^2\mu}{ca\varepsilon_0n_0m_0^2\mathcal{E}_g^2}\ .$$

The matrix elements are as follows.

(a) For transitions involving an odd number of photons, such that $l=2s+1$ with $s=0,1,2,\ldots$, we have

$$M_\delta^{(l)}(\nu,\beta)=\frac{1}{2^s}\sum_{j=0}^{s}\binom{s}{j}J_{s-\delta-2j}(\Lambda)\ . \tag{5.4}$$

(b) For transitions involving an even number of photons, such that $l=2(s+1)$ with $s=0,1,2,\ldots$, we have

$$M_\delta^{(l)}(\nu,\beta)=(l\zeta_{eh})^{1/2}\frac{1}{2^{s+1}}\sum_{j=0}^{s}\binom{s}{j}[J_{s+1-\delta-2j}(\Lambda)-J_{s-1-\delta-2j}(\Lambda)]\ , \tag{5.5}$$

with

$$\Lambda=\frac{\zeta_{eh}}{2\nu}(1-\tfrac{1}{4}\beta^2),\quad \zeta_{eh},\nu,\beta\ll 1\ ,$$

and with

$$\binom{s}{j}=\frac{s!}{j!(s-j!)}\ .$$

[The latter are the binomial coefficients, and $J_n(\Lambda)$ are Bessel functions of integral order.]

Expressions (5.3)–(5.5) define the coefficients of the interband multiphoton electroabsorption in the superlattice. With a weak oscillating electric field $F_0(\beta\to 0)$, only the one-photon ($l=1$, $s=0$) transitions are large. In this case, the matrix element (5.4) is

$$M_\delta^{(l)}=J_{-\delta}(\Lambda)\ ,$$

and the coefficient $\alpha_l(\omega)$ from (5.3) tends to an analogous result for the one-photon absorption.[14] We can see from (5.3) that the l-photon spectrum consists of a sequence of rectangular steps. The boundary frequencies are given by

$$\omega_\delta^{(l)}=\frac{\overline{\mathcal{E}}_g+eEa\delta}{l\hbar}\quad\text{for }\delta=0,\pm1,\pm2,\pm3,\ldots\ . \tag{5.6}$$

The heights of the steps are governed by the relevant matrix elements $M_\delta^{(l)}$ and consist of Bessel functions. Their argument Λ depends upon the relationship

$$\frac{\zeta_{eh}}{\nu}=\frac{\Delta_e+\Delta_h}{eEa}\ ,$$

and on the parameter $\beta\ll 1$. For arbitrary values of $\zeta_{eh}/\nu\geq 1$, the steps with different δ contribute nearly equally in the absorption spectrum (5.3). Interband transitions between any Stark levels ($\sigma\to\sigma'$) and between any wells are possible. The intensity of the multiphoton absorption decreases with the number of photons l such that $\alpha_l\sim(\zeta_{eh}\beta^2)^{l-1}$ and $\zeta_{eh}\beta^2\ll 1$. These optical transitions between the extended states are the essence of the multiphoton Franz-Keldysch effect in the superlattice.[22]

Let us consider the most attractive case for the localization of carriers. The localization arises in the presence of a sufficiently strong uniform electric field E. With such fields, the separations between the Stark levels eEa become much greater than the sum of the widths of the electron and hole minibands $(\Delta_e+\Delta_h)$ such that $(\zeta_{eh}/\nu)\ll 1$. In this case, the magnitude of the argument Λ of the Bessel functions in (5.4) and (5.5) becomes $\ll 1$. Note also that an increase in the magnitude of the oscillating electric field $F_0\sim\beta$ causes Λ to decrease. Under the condition $\Lambda\ll 1$, the Bessel function $J_0(\Lambda)=1$ only contributes to the matrix elements (5.4) and (5.5) as $J_n(\Lambda)\sim\Lambda^n\ll 1$ for $n\neq 0$. Optical transitions occur between the localized states of the single quantum wells and cause the multiphoton Wannier-Stark effect.

B. The form of the spectrum

1. *For an odd number of photon transitions* $(l=2s+1, s=0,1,2,\ldots)$

It follows from (5.4) that only items with

$$j=\tfrac{1}{2}(s-\delta)\ , \tag{5.7}$$

with

$$0\le j\le s \tag{5.8}$$

contribute in the transition with the fixed number s. It follows from (5.7) that the quantum number δ should have the same parity as the integer s. The condition (5.8) limits the values of δ such that

$$-s\le\delta\le s\ . \tag{5.9}$$

For the one-photon absorption $(l=1, s=0)$ only the transition with $\delta=0$ is allowed. The absorption spectrum then consists of one step with a boundary frequency of $\omega_0^{(1)}$ and of height governed by the matrix element $M_0^{(1)}=1$. For three-photon absorption $(l=3, s=1)$, transitions with $\delta=\pm 1$ are possible. The absorption spectrum then consists of two steps with boundary frequencies of $\omega_{\pm 1}^{(3)}$ and heights governed by the matrix elements $M_1^{(3)}=M_{-1}^{(3)}=\frac{1}{2}$. For five-photon absorption $(l=5, s=2)$ the corresponding boundary frequencies are $\omega_{-2}^{(5)}$, $\omega_0^{(5)}$, and $\omega_{+2}^{(5)}$, with $M_0^{(5)}=\frac{1}{2}$ and $M_{+2}^{(5)}=M_{-2}^{(5)}=\frac{1}{4}$. The odd-photon electroabsorption spectrum then consists of $s+1$ pronounced steps of boundary frequencies $\omega_{-s}^{(l)}$, $\omega_{-s+2}^{(l)},\ldots,\omega_{s-2}^{(l)}$, $\omega_s^{(l)}$. The red edge of the spectrum is at $\omega_{-s}^{(l)}$. The form of the superlattice odd-photon electroabsorption spectrum is depicted in Fig. 1 for $l=1$, 3, and 5.

2. *For an even number of photon transitions* $(l=2(s+1), s=0,1,2,\ldots)$

It follows from (5.5) that, for $\Lambda\ll 1$, only items with

$$j=\tfrac{1}{2}(s-\delta)\pm\tfrac{1}{2} \tag{5.10}$$

contribute to the transition with s fixed. From (5.10), it follows that the quantum number δ should have a parity opposite to that of the integer s. Condition (5.8), which still applies, leads to the allowed values for δ in the range

$$-(s+1)\le\delta\le(s+1)\ . \tag{5.11}$$

For the two-photon absorption $(l=2, s=0)$, transitions with $\delta=\pm 1$ are possible with $(2\zeta)^{-1/2}M_{\pm 1}^{(2)}=\pm\frac{1}{2}$. If $l=4$, $s=1$, then $\delta=0$, ± 2 with $(2\zeta)^{-1/2}$ $M_{\pm 2}^{(4)}=\pm\frac{1}{4}$ and $M_0^{(4)}=0$. Thus the absorption spectrum for an even number of photons consists of $(s+2)$ steps having boundary frequencies $\omega_{-(s+1)}^{(l)},\omega_{-s+1}^{(l)},\ldots,\omega_{s-1}^{(l)},\omega_{s+1}^{(l)}$. The red edge of the spectrum is at $\omega_{-(s+1)}^{(l)}$. The form of the superlattice elctroabsorption spectrum for an even number of photons is also shown in Fig. 2 for $l=2$ and 4.

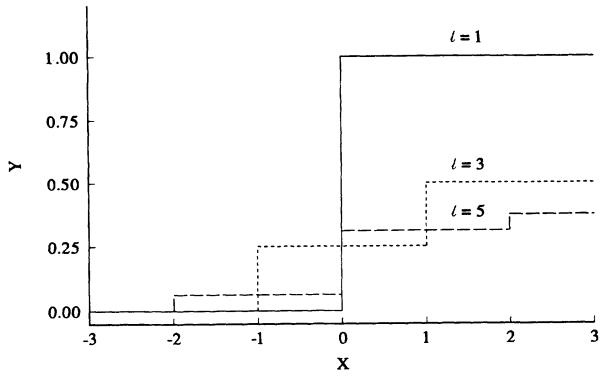

FIG. 1. The multiphoton electroabsorption spectrum represented by $Y=\alpha_l(\omega)/\bar{\alpha}_l$ for $l=1$, 3, and 5 as a function of $X=(l\hbar\omega-\bar{\mathcal{E}}_g)/(eEa)$.

C. Discussion of the results

In practice, in the presence of a strong electric field E, multiphoton transitions occur between isolated wells which are labeled by the quantum numbers σ and σ' and which satisfy conditions (5.9) and (5.11). Thus all frequencies are absorbed with the boundary at the red end of the spectrum shifted toward the lower frequencies. The absorption increases with an increase in the frequency ω of the light. In contrast to the increase in the absorption in bulk semiconductors (the Franz-Keldysh effect[22]), the multiphoton absorption in the superlattice

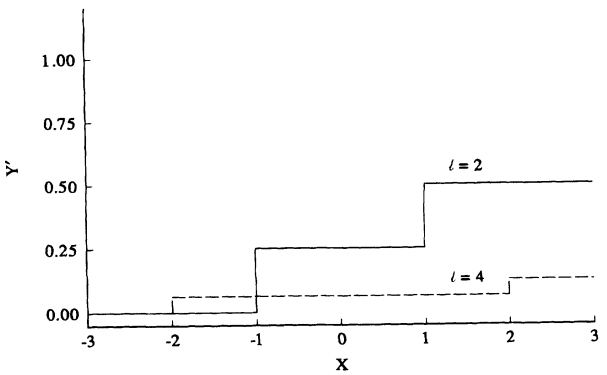

FIG. 2. The multiphoton electroabsorption spectrum represented by $Y'=\alpha_l(\omega)/(\bar{\alpha}_l l\zeta_{eh})$ as a function of X (defined in Fig. 1).

consists of equidistant intense steps caused by the Stark quantization (The Wannier-Stark effect[2]). Among these intense steps mentioned above, there are other steps of vanishingly small heights which are proportional to various powers of $\Lambda \ll 1$.

If the superlattice is formed with semiconductors with complex valence-band structures, then the light and heavy holes contribute to the electroabsorption spectrum, with the heavy holes localizing more rapidly than the light holes. Estimates are now made of the results using the parameters of $GaAs/Ga_{1-x}Al_xAs$ (with $x=0.35$) heterostructure for which $m_e=0.065m_0$, $m_{hh}=0.55m_0$, $m_{lh}=0.09m_0$, and $\mathcal{E}_g=1.53$ eV. We assume further that the superlattice has a typical period ($a=50$ Å) and that $\lambda_e=0.05$. We note that the selected quantity λ_e corresponds to the electron miniband width $\Delta_e=0.0926$ eV which is very close to the value of 0.0950 eV for Δ_e given previously[10] obtained by numerical calculation using a model of rectangular wells of width 40 Å separated by barriers of width 15 Å. Other parameters needed are $\lambda_{hh}=0.006$, $\lambda_{lh}=0.036$, $\Delta_{hh}=0.0013$ eV, and $\Delta_{lh}=0.048$ eV. The total width of the minibands is given by $\Delta_e+\Delta_{lh}=0.14$ eV and $\Delta_e+\Delta_{hh}=0.094$ eV. In an electric field $E=5\times10^6$ V m^{-1}, we find that the parameter Φ_0 in (3.5) is such that, for electrons, $\Phi_{0,e}=1.856$, for heavy holes $\Phi_{0,hh}=0.0258$ and, for light holes, $\Phi_{0,lh}=0.98$. If $E=10^7$ V m^{-1}, then $\Phi_{0,e}=0.928$, $\Phi_{0,hh}=0.0129$, and $\Phi_{0,lh}=0.49$. In such an electric field, the heavy hole is almost totally localized. By increasing the electric field E, first the light holes and then the electrons will become localized as the localization condition $\Lambda \ll 1$ is reached.

VI. CONCLUSION

In summary, we have developed an analytical approach to the problem of calculating the multiphoton interband absorption spectrum in the presence of a uniform electric field which is directed parallel to the oscillating electric field of the optical wave and the superlattice axis. It has been shown that as both the uniform and oscillating electric fields increase in magnitude, localization of the carriers increases. The influence of the intense oscillating field is to narrow the miniband width. If the localization is realized, the electroabsorption multiphoton spectrum or photocurrent consists of a sequence of intense steps. The number of steps depends upon the number of photons and increases as this number increases. The effective red boundary of the spectrum shifts toward longer wavelengths as the uniform electric field increases. The form of the spectrum depends upon the parity of the number of the absorbed photons.

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