Effect of a fluctuating magnetic field on weak localization in a two-dimensional disordered system

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We consider a charged quantum particle in a two-dimensional disordered system subject to a spatially and temporally fluctuating magnetic field. The fluctuations are assumed to be Gaussian, with correlations typical for a metal in the anomalous-skin-effect regime. We derive a scaling form for the quantum correction to the conductivity in terms of a scaled temperature, elastic mean free path, and magnetic field. The weak localization correction to the conductivity is calculated for the case of rapid magnetic-field fluctuations. We express the result in terms of a phenomenological phase relaxation rate $1/\tau_\phi^*$, which is found to scale with temperature as $T^{1/3}$, provided the potential disorder is sufficiently strong and the temperature is above a critical value. In all other cases, including the normal-skin-effect regime and the case of quasistatic field fluctuations, $1/\tau_\phi$ is found to be proportional to T, albeit with unusual prefactors.

I. INTRODUCTION

It has been known for some time that the transport of quantum particles in two-dimensional disordered systems is governed by quantum interference effects.[1–5] The constructive interference of time-reversed paths leads to a dramatic enhancement of the resistivity at low temperatures, and to the localization of particles of all energies at $T = 0$ in the infinite system. At finite T inelastic processes induced by the electron-electron interaction destroy the coherence beyond a characteristic length scale L_ϕ. The dominant scattering processes with small transferred energy have been shown to be equivalent to a fluctuating electric field acting on the charged particles.[3] The effect of this field is to give rise to a phase relaxation rate $1/\tau_\phi$, but it does not change the relation between diffusion length and diffusion time, $\tau_\phi \sim L_\phi^2$, characteristic of the motion of the particles in a random potential.

In this paper we discuss the effect of a fluctuating magnetic field on the quantum transport in a disordered system. One might expect that magnetic-field fluctuations will have a substantially different effect as compared to electric-field fluctuations. After all, a magnetic field couples to the velocity of a charged particle and in that way can lead to an effective change of the mass of the particle. This translates into a change of the diffusion constant, more precisely a slowing down of diffusion. It is clear that this effect can have a major influence on the form of the quantum correction to the transport coefficients. The well-known expression, e.g., for the conductivity quantum correction, $\delta\sigma \sim \ln(\tau/\tau_\phi)$, where τ is the elastic scattering time, will no longer be valid in this case.

We solve this problem within controlled approximation schemes in the limit of rapid fluctuations in time and in the quasistatic limit. In the former case the interaction may be shown to be local in time, and the problem can be reduced to that of a quantum particle with a diffusion coefficient depending on position and moving in an effective potential. As we will show, this requires a self-consistent treatment of the momentum cutoff necessary to guarantee the validity of the "instantaneous approximation." The small parameter of this approximation is the ratio $g = e^{*2}D/\chi_d$ where D is the diffusion coefficient in the static random potential, e^* is the charge of the particle in the gauge field, and χ_d is the diamagnetic susceptibility characterizing the dispersion of the gauge-field mode $\omega \propto (\chi_d/k_F)q^3$. We show that even for $g \sim 1$, the "instantaneous approximation" is valid in a wide temperature regime. Nonetheless, the approximation is only valid for sufficiently strong potential disorder. In the opposite, "quasistatic" case, we can show that the usual interpretation of phase breaking is correct. The phase-breaking rate $1/\tau_\phi$ turns out to be linear in T and proportional to the inverse resistance per square.

Our model calculation is applicable to high-T_c compounds doped close to the metal-insulator transition, such as Bi 2:2:0:1 compounds with superconducting transition temperatures well below 1 K. In this case one can hope to measure weak localization effects probing the inelastic scattering processes in the system. These effects have indeed been observed, so far, in a single study of the magnetoresistance of these compounds.[6] The results, when analyzed in terms of usual weak localization theory, lead to an apparent phase-breaking rate $1/\tau_\phi$ inconsistent with the conventional interpretation. The observed rate varies with temperature as $\tau_\phi^{-1} \sim T^{1/3}$, and at the lowest measuring temperature is about two orders of magnitude larger than the inelastic scattering rate $1/\tau_{\rm in} \sim T$, extrapolated down from higher temperatures. It is clear that the phase-breaking rate cannot exceed the inelastic scattering rate: $1/\tau_\phi \leq 1/\tau_{\rm in}$. This serious conflict with existing theory requires a major revision of the conventional interpretation.

As indicated above, a fluctuating magnetic field, as

opposed to an electric field, not only causes phase relaxation, but can lead to a nonlocal change of the diffusion coefficient, which causes a rapid decay of phase coherence. We will show within the instantaneous approximation that even though the relevant time scale for phase-breaking processes is still given by the inelastic relaxation time, the relevant length scale is no longer given by $L_\phi \sim (D\tau_{\rm in})^{1/2}$, but rather by $L_\phi \sim (\xi L_0)^{1/2}$, where ξ is a temperature-independent characteristic length and L_0 is proportional to the thermal wavelength of the gauge-field fluctuations, $L_0 \sim T^{-1/3}$. As a result, the quantum correction to the conductivity is found to vary with temperature in the same way as if the phase-breaking rate $1/\tau_\phi$ in the conventional theory obeyed the power law $T^{1/3}$. The agreement of our result with experimental observation[6] suggests the presence of a fluctuating gauge field of some kind, coupling to the charge carriers in these compounds. A brief report of this work has appeared.[7]

In the clean limit, or in the dirty limit at low temperatures, the "instantaneous approximation" is not valid. Rather, one expects the static approximation to be applicable. Thus the apparent phase-breaking rate is predicted to cross over from a $T^{1/3}$ law at intermediate T to a linear law at low T.

It may be shown that the limit of a static, or frozen, statistical distribution of magnetic fields leads to a logarithmically varying, highly retarded interaction. This case has been studied recently by several authors.[8–13] In Ref. 9 a particular gauge-invariant single-particle Green's function was considered in the limit of large particle energy, such that a quasiclassical approximation could be applied. The numerical studies of localization in a random magnetic field have been interpreted controversially. While the authors of Ref. 12 conclude that all states are localized in two dimensions, the authors of Refs. 11 and 13 argue in favor of the existence of extended states. We have recently been able to show[14] that the problem can be mapped on to a nonlinear σ model of matrices with unitary symmetry, for which all states are known to be localized. The single-particle properties have been shown to depend on the geometry considered and a single-particle relaxation time describing the broadening of Landau levels in an external uniform magnetic field has been calculated in quasiclassical approximation.[15]

II. GENERAL FORMULATION

A. Model

We consider a quantum particle of energy E in a static random potential $V(\mathbf{r})$ in two-dimensional (2D) space. The particle is subjected to a magnetic field fluctuating in space and time, described by the vector potential $\mathbf{a}(\mathbf{r},t)$. We will also consider a uniform static magnetic field $\mathbf{B}$ normal to the plane, with vector potential $\mathbf{A}(\mathbf{r}) = \frac{B}{2}(y,-x)$. The Hamiltonian of this system is given by

$$H = \int d^2r\psi^\dagger(\mathbf{r})\Big[\frac{1}{2m}[-i\nabla - e\mathbf{A}(\mathbf{r}) - e^*\mathbf{a}(\mathbf{r},t)]^2 + V(\mathbf{r})\Big]\psi(\mathbf{r}) , \quad (1)$$

where $\psi^\dagger(\mathbf{r})$ [$\psi(\mathbf{r})$] are field operators creating [annihilating] a quantum particle, m is the particle mass, and e is the electric charge, and we take the velocity of light $c = 1$. The coupling constant e^* is the effective charge of the quantum particle with respect to the fluctuating part of the magnetic field, which we allow to be different from e.

The random potential $V(\mathbf{r})$ will be assumed to be characterized by a δ-correlated Gaussian distribution with $\langle V(\mathbf{r})V(\mathbf{r}')\rangle = \langle V^2\rangle\delta(\mathbf{r}-\mathbf{r}')$, although the precise form of the distribution is not essential for the following. The elastic relaxation rate of particles of energy E is $\frac{1}{\tau} = 2\pi N(E)\langle V^2\rangle$, where $N(E)$ is the density of states for one spin, given by $N(E) = \frac{m}{2\pi}$ for noninteracting particles with quadratic dispersion $\epsilon_p = \frac{p^2}{2m}$ and $N(E) = \frac{E}{2\pi c^2}$ for a weakly interacting system of Bose particles with linear spectrum $\epsilon_p = cp$.

The fluctuating gauge field $\mathbf{a}(\mathbf{r},t)$ will be assumed to be Gaussian distributed as well, the variance being given by the correlation function in Fourier space,

$$\langle a_\alpha a_\beta\rangle_{\mathbf{k}\omega} = \delta^T_{\alpha\beta}(\hat{\mathbf{k}})\coth\frac{\omega}{2T}\mathrm{Im}\Big\{\frac{1}{-i\omega\sigma(k)+\chi_d k^2}\Big\} , \quad (2)$$

where $\delta^T_{\alpha\beta}(\hat{\mathbf{k}}) = \delta_{\alpha\beta} - \hat{\mathbf{k}}_\alpha\hat{\mathbf{k}}_\beta$ and $\hat{\mathbf{k}}_\alpha = k_\alpha/\mid\mathbf{k}\mid$. This form of the vector potential field propagator is reminiscent of the electromagnetic field in a metal. In applications to be discussed later the fluctuating field will be a fictitious internal gauge field, for example, associated with the separation in spin and charge degrees of freedom hypothesized in models of strongly correlated lattice electrons.[17–21] However, it is conceivable that fluctuations of the internal current distributions in these systems would give rise to the same type of magnetic-field fluctuations. The parameter χ_d in (2) may be interpreted as a diamagnetic susceptibility, whereas $\sigma(k)$ plays the role of the nonlocal conductivity. For small k, $k\ell_F \ll 1$, $\sigma(k) = (e^*)^2k_F\ell_F$ is k independent, whereas in the opposite limit $k\ell_F \gg 1$, $\sigma(k) = (e^*)^2k_F/\pi k$. In the case of the usual transport theory of metals k_F and ℓ_F are the Fermi wave number and the elastic mean free path of the electrons. Here we want to consider k_F and ℓ_F as parameters without an immediate physical interpretation. In particular, the elastic mean free path induced by the random potential $V(\mathbf{r})$, $\ell = v_E\tau$, where v_E is the velocity of the particles, will be assumed to be unrelated to ℓ_F.

In the following we will only consider the thermal fluctuations of $\mathbf{a}(\mathbf{r},t)$ for which the condition $\mid\omega\mid < T$ must be satisfied. Then,

$$\langle a_\alpha a_\beta\rangle_{\mathbf{k}\omega} = \delta^T_{\alpha\beta}(\hat{k})\frac{2T/\sigma(k)}{\omega^2+\Gamma^2(k)}, \qquad \mid\omega\mid < T, \quad (3)$$

where

$$\Gamma(k) = \chi_d k^2/\sigma(k) \quad (4)$$

characterizes the excitation spectrum of the gauge field. We will concentrate on the anomalous-skin-effect regime, $k\ell_F \gg 1$, where the most interesting behavior is expected. Then, $\Gamma(k) = [\pi\chi_d/k_F(e^*)^2]k^3$; i.e., the typical frequencies of the gauge field vary as the cube of the wave vector, leading to a very singular behavior of the gauge propagator at small ω, k.

B. Weak localization correction to conductivity

In this paper we calculate the quantum correction to the conductivity, $\delta\sigma$, for the system described above. It is well known that this can be expressed in terms of the so-called Cooperon amplitude,

$$C_{t_0}(\mathbf{r},t;\mathbf{r}',t') = \frac{i}{\pi N(E)\tau}\left\langle \psi\left(\mathbf{r},t+\frac{t_0}{2}\right)\psi\left(\mathbf{r},t-\frac{t_0}{2}\right)\psi^\dagger\left(\mathbf{r}',t'+\frac{t_0}{2}\right)\psi^\dagger\left(\mathbf{r}',t'-\frac{t_0}{2}\right)\right\rangle , \tag{5}$$

where the angular brackets denote averaging over the static random potential. One may interpret $C_{t_0}(\mathbf{r}t,\mathbf{r}'t')$ as the probability amplitude for two particles with energy E to move from point $\mathbf{r}'$ to point $\mathbf{r}$ within time intervals $t-t' \pm t_0$. The conductivity correction for a quantum particle with given energy E is found as

$$\delta\sigma = -\frac{2}{\pi}e^2 D\tau\int_{-\infty}^{\infty} dt_0 \langle C_{t_0}(0;0)\rangle_{\mathbf{a}} , \tag{6}$$

where $D = \frac{1}{2}v_E^2\tau$ is the diffusion constant, the angular brackets denote the average over the gauge field, $C_{t_0}(0,0) = C_{t_0}(\mathbf{r}=0,t=0;\mathbf{r}'=0,t'=0)$, and v_E is the velocity of the particle.

For a given space- and time-dependent gauge field the equation of motion for the Cooperon is obtained from the diffusion equation for C_{t_0} valid in the absence of magnetic fields by the gauge-invariant replacement of the momentum operator $-\frac{i}{2}\nabla$ by $(-\frac{i}{2}\nabla - e^*\mathbf{a} - e\mathbf{A})$, for both particles,

$$\left\{\frac{\partial}{\partial t_0} + D\left[-i\nabla - 2e\mathbf{A}(\mathbf{r}) - e^*\mathbf{a}(\mathbf{r},t+t_0/2) - e^*\mathbf{a}(\mathbf{r},t-t_0/2)\right]^2\right\}C = \frac{1}{\tau}\delta(\mathbf{r}-\mathbf{r}')\delta(2t_0) . \tag{7}$$

Both equations, (5) and (6), can be obtained from phenomenological considerations, without any microscopic model assumption.

The averaging over the gauge field is most conveniently done by representing the solution to (6) in the form of a path integral, and integrating over all gauge-field configurations with Gaussian weight,[3]

$$\langle C_{t_0}(0,0)\rangle = \frac{1}{\tau}\int_{\mathbf{r}(-t_0)=0}^{\mathbf{r}(t_0)=0}\mathcal{D}[\mathbf{r}(t)]\exp -(S_0+\Delta S), \tag{8}$$

where

$$S_0 = \int_{-t_0}^{t_0} dt_1\left[\frac{1}{4D}\dot{\mathbf{r}}^2(t_1) + 2e\dot{\mathbf{r}}(t_1)\cdot\mathbf{A}(\mathbf{r}(t_1))\right] \tag{9}$$

and

$$\Delta S = \int_{-t_0}^{t_0} dt_1\int_{-t_0}^{t_0} dt_2 \dot{r}_\alpha(t_1)\dot{r}_\beta(t_2)F_{\alpha\beta}(\mathbf{r}(t_1),t_1;\mathbf{r}(t_2),t_2), \tag{10}$$

with

$$F_{\alpha\beta}(\mathbf{r}_1t_1;\mathbf{r}_2t_2) = (e^*)^2\int\frac{d^2k\,d\omega}{(2\pi)^3}e^{i\mathbf{k}\cdot(\mathbf{r}_1-\mathbf{r}_2)} \times\left[\cos\frac{\omega}{2}(t_1+t_2) + \cos\frac{\omega}{2}(t_1-t_2)\right]\langle a_\alpha a_\beta\rangle_{\mathbf{k}\omega} . \tag{11}$$

The integration on $|\mathbf{k}|$ should be cut off at the inverse elastic mean free path, as the diffusion equation only holds for length scales larger than ℓ. Performing the ω integration yields

$$\Delta S = \frac{1}{2\pi}\frac{T}{\chi_d}(e^*)^2\int_{-t_0}^{t_0} dt_1 dt_2\int^{\ell^{-1}}\frac{d^2k}{2\pi}\frac{1}{k^2} \times e^{i\mathbf{k}\cdot(\mathbf{r}_1-\mathbf{r}_2)}\delta^T_{\alpha\beta}(\hat{\mathbf{k}})\dot{r}_\alpha(t_1)\dot{r}_\beta(t_2)\left[\Delta_k(t_1-t_2) + \Delta_k(t_1+t_2)\right] , \tag{12}$$

where

$$\Delta_k(t) = \exp\left(-\frac{1}{2}\Gamma(k)\,|\,t\,|\right) . \tag{13}$$

Since $\Delta_{\mathbf{k}}(t_1 \pm t_2)$ only depends on $|\mathbf{k}|$, the integration over angles of $\hat{\mathbf{k}}$ may also be done, leading to the final result

$$\Delta S = \frac{1}{2\pi}\frac{T}{\chi_d}(e^*)^2 \int_{-t_0}^{t_0} dt_1 dt_2 \int_0^{\ell^{-1}} \frac{dk}{k}\Big[(\mathbf{v}_1\cdot\mathbf{v}_2)F_1(kr) - (\mathbf{v}_1\cdot\hat{\mathbf{r}})(\mathbf{v}_2\cdot\hat{\mathbf{r}})F_2(kr)\Big]\Big[\Delta_k(t_1-t_2)+\Delta_k(t_1+t_2)\Big] , \quad (14)$$

where

$$F_n(x) = J_0(x) - nJ_1(x)/x, \qquad n = 1,2 , \quad (15)$$

and the $J_n(x)$ are Bessel functions. Here we have defined $\mathbf{v}_i = \dot{\mathbf{r}}(t_i)$, $\mathbf{r} = \mathbf{r}(t_1) - \mathbf{r}(t_2)$, $r = \mid \mathbf{r} \mid$, and $\hat{\mathbf{r}} = \mathbf{r}/r$.

The k integration in (14) appears to diverge logarithmically at the lower limit, since $\lim_{x\to 0} F_1(x) = \frac{1}{2}$. This divergence would be unphysical and one may expect it to be removed by taking gauge invariance into account: A uniform ($\mathbf{k} \to 0$), time-independent gauge field should drop out of (14). We may rewrite (14) by performing a partial integration and using the property

$$\int_{-t_0}^{t_0} dt_1 dt_2 \dot{\mathbf{r}}(t_1)\cdot\dot{\mathbf{r}}(t_2) = 0 ,$$

which holds for any closed loop path $\mathbf{r}(t)$. The term in question may be transformed as follows:

$$\begin{aligned}\lim_{k_1\to 0}\int dt_1 dt_2 (\mathbf{v}_1\cdot\mathbf{v}_2)\int_{k_1}^{\ell^{-1}}\frac{dk}{k}F_1(kr)\Big[\Delta_k(t_1-t_2)+\Delta_k(t_1+t_2)\Big] \\ = \lim_{k_1\to 0}\int dt_1 dt_2 (\mathbf{v}_1\cdot\mathbf{v}_2)\Big\{\Big[\ln(kl_0)F_1(kr)[\Delta_{k_1}(t_1-t_2)+\Delta_{k_1}(t_1+t_2)]\Big]_{k_1}^{\ell^{-1}} \\ - \int_{k_1}^{\ell^{-1}} dk \ln(kl_0)\frac{d}{dk}\{F_1(kr)[\Delta_k(t_1-t_2)+\Delta_k(t_1+t_2)]\}\Big]\Big\} , \end{aligned} \quad (16)$$

where l_0 is an integration constant. Note that (16) does not depend on l_0, and therefore l_0 may be chosen in the most convenient way. The first term in the latter expression vanishes for the choice $l_0 = \ell$, since

$$\lim_{k_1\to 0}\int dt_1 dt_2 (\mathbf{v}_1\cdot\mathbf{v}_2)\ln(k_1\ell)F_1(k_1 r)[\Delta_{k_1}(t_1-t_2)+\Delta_{k_1}(t_1+t_2)] = \lim_{k_1\to 0}\int dt_1 dt_2 (\mathbf{v}_1\cdot\mathbf{v}_2)\ln(k_1\ell) = 0$$

for any finite value of t_0 and any path satisfying $\mid \mathbf{r}_1(t) \mid < \infty$. Here we used $\lim_{k_1\to 0}\Delta_{k_1}(t) = 1$.

The fluctuation part of the action may therefore be written in the regularized form

$$\begin{aligned}\Delta S = -\frac{1}{2\pi}\frac{T}{\chi_d}(e^*)^2\int_{-t_0}^{t_0} dt_1 dt_2 \int_0^{\ell^{-1}} dk\Big\{(\mathbf{v}_1\cdot\mathbf{v}_2)\ln(k\ell)\frac{d}{dk}\{F_1(kr)[\Delta_k(t_1-t_2)+\Delta_k(t_1+t_2)]\} \\ +(\mathbf{v}_1\cdot\hat{\mathbf{r}})(\mathbf{v}_2\cdot\hat{\mathbf{r}})\frac{1}{k}F_2(kr)[\Delta_k(t_1-t_2)+\Delta_k(t_1+t_2)]\Big\} . \end{aligned} \quad (17)$$

We will now show that ΔS may be written in a scaled form, exhibiting the dependence on two external variables, the temperature T, and the elastic mean free path ℓ.

C. Scaling form of the quantum correction to the conductivity

It is useful to define a characteristic temperature

$$T_D = Dk_F^2 \quad (18)$$

and a dimensionless coupling constant

$$g = e^{*2}D/\chi_d . \quad (19)$$

The dimensional dependences may be scaled out of the action S by defining dimensionless variables τ and $\mathbf{x}$ as $t = \beta_0\tau$, $\mathbf{r} = L_0\mathbf{x}$. We determine β_0 and L_0 by requiring (i) $\frac{1}{2}\Gamma(k) \mid t_1 \pm t_2 \mid = \kappa^3 \mid \tau_\pm \mid$ where κ is the scaled momentum variable $\kappa = kL_0$ and $\tau_\pm = \tau_1 \pm \tau_2$ and (ii) that the prefactors of S_0 and ΔS be equal. Condition (i) leads to the relation $[\pi\chi_d/2k_F(e^*)^2]L_0^{-3}\beta_0 = 1$, whereas condition (ii) may be expressed as $\frac{L_0^2}{4D\beta_0} = \frac{TL_0^2(e^*)^2}{2\pi\chi_d}$. The two scales β_0 and L_0 are therefore determined as

$$\beta_0 = \frac{\pi}{2gT} , \quad L_0^3 = \frac{\pi^2 T_D}{4k_F^3 g^2 T} . \quad (20)$$

The total action takes the scaled form

$$S = \frac{1}{4}\left(\frac{T}{T_0}\right)^{1/3}\Big[s_0(\tau_0) + s_1(\tau_0, \ell/L_0)\Big] + s_B(\tau_0, r_B/L_0), \quad (21)$$

with

$$\begin{aligned} s_0(\tau_0) &= \int_{-\tau_0}^{\tau_0} d\tau \left(\frac{d\mathbf{x}}{d\tau}\right)^2 , \\ s_B(\tau_0, r_B/L_0) &= \frac{L_0^2}{r_B^2}\int_{-\tau_0}^{\tau_0} d\tau \frac{dx_i}{d\tau}\epsilon_{ij}x_j , \quad r_B^{-2} = eB , \end{aligned} \quad (22)$$

and

$$s_1(\tau_0,\ell/L_0)=-\int_{-\tau_0}^{\tau_0}d\tau_1 d\tau_2\int_0^{L_0/\ell}d\kappa\Big\{(\dot{\mathbf{x}}_1\cdot\dot{\mathbf{x}}_2)\ln(\kappa\ell/L_0)\frac{d}{d\kappa}[F_1(\kappa x)\tilde{\Delta}_\kappa(\tau_1,\tau_2)]+(\dot{\mathbf{x}}_1\cdot\hat{\mathbf{x}})(\dot{\mathbf{x}}_2\cdot\hat{\mathbf{x}})\frac{1}{\kappa}F_2(\kappa x)\tilde{\Delta}_\kappa(\tau_1,\tau_2)\Big\}, \tag{23}$$

where

$$\tilde{\Delta}_\kappa(\tau_1,\tau_2)=e^{-\kappa^3|\tau_+|}+e^{-\kappa^3|\tau_-|} \tag{24}$$

and $\mathbf{B}$ is a uniform, static magnetic field normal to the plane (ϵ_{ij} is the antisymmetric unit tensor). It is seen that in zero magnetic field S depends only on two parameters, (T/T_0) and (ℓ/L_0), and the scaled time $\tau_0=t_0/\beta_0$. The characteristic temperature T_0 is given by

$$T_0=\frac{2}{\pi}gT_D\ . \tag{25}$$

In order to determine the quantum correction to the conductivity, we have to scale out the dimensional dependence of $C_{t_0}(0,0)$. Apart from an explicit factor of $1/\tau$ [see the right-hand side (rhs) of (7)], $C_{t_0}(0,0)$ has dimension $1/(\text{length})^2$, so that one may define a dimensionless function $\tilde{C}_{\tau_0}(0,0;T/T_0,\ell/L_0)$ by

$$C_{t_0}(0,0)=\frac{1}{\tau}\frac{1}{L_0^2}\tilde{C}_{\tau_0}(0,0;T/T_0,\ell/L_0)\ . \tag{26}$$

The conductivity correction as given by (6) may be expressed as

$$\begin{aligned}\delta\sigma&=-\frac{2}{\pi}e^2\left(\frac{T_0}{T}\right)^{1/3}\int_{-\infty}^{\infty}d\tau_0\tilde{C}_{\tau_0}(0,0;T/T_0,\ell/L_0)\\&=-e^2f\left(\frac{T}{T_0},\frac{\ell}{L_0}\right)\ .\end{aligned} \tag{27}$$

Thus $\delta\sigma$ is found to scale as a function of temperature as T/T_0 and as a function of the elastic mean free path as ℓ/L_0.

In an external magnetic field B the conductivity correction $\delta\sigma$ is seen to depend on the magnetic field through the dimensionless variable $(r_B/L_0)^2$.

III. "INSTANTANEOUS" APPROXIMATION

A. Effective Hamiltonian

If the gauge fields fluctuate rapidly in time, such that $\Gamma(k)t_0\gg 1$ for typical times t_0, the exponentially time-dependent factor $\Delta_\kappa(t)$ in (14) restricts the relevant interaction times to $\mid t_1\pm t_2\mid\lesssim\Gamma^{-1}(k)$, $k>k_0$. This is assuming that the important interaction processes take place at not too long wavelengths, or $k>k_0$, where k_0 is a cutoff, chosen such that the relevant processes are included. Since these processes are not known *a priori*, we determine k_0 self-consistently. As we will show below, for given cutoff k_0 one can identify a characteristic time scale $\tau_c(k_0)$ for interaction processes. We require that for all wave numbers k of interest, i.e., for $k>k_0$, $\Gamma(k)\tau_c>1$. Hence $\Gamma(k_0)\tau_c(k_0)=1$ is the equation defining k_0. Later we will estimate the contribution of processes with small wave numbers $k<k_0$. We will show that their contribution is small in the regime where the instantaneous approximation is applicable. The rapid falloff of the exponential factors $\Delta_k(t_1\pm t_2)$ in the k regime $k>k_0$ allows us then to replace them by δ functions,

$$\Delta_k(t)\simeq\frac{4}{\Gamma(k)}\delta(t)\ . \tag{28}$$

As a consequence the fluctuation part of the action, ΔS, becomes local in time. The fluctuations change so rapidly in time that the coherent interaction of the "particle pair" described by the Cooperon can only take place instantaneously, i.e., if $t_1=+t_2$ for the particles propagating in the same direction and if $t_1=-t_2$ for the particles propagating in opposite directions around a closed loop. One may interpret the resulting problem as that of two particles with coordinates $\mathbf{r}(t)$ and $\mathbf{r}(-t)$ propagating in the time interval $0<t<t_0$, i.e., one-half of the original time interval. It is convenient to introduce relative and center of mass coordinates[3]

$$\boldsymbol{\rho}(t)=\mathbf{r}(t)-\mathbf{r}(-t)\quad,\quad\mathbf{R}(t)=\frac{1}{2}[\mathbf{r}(t)+\mathbf{r}(-t)]\ . \tag{29}$$

The total action then takes the form

$$\begin{aligned}S_{\text{inst}}[\mathbf{R}(t),\boldsymbol{\rho}(t)]=\int_0^{t_0}dt\Big\{&\frac{1}{2}\dot{R}_\alpha(D_s^{-1})_{\alpha\beta}\dot{R}_\beta\\&+\frac{1}{8}\dot{\rho}_\alpha(D_r^{-1})_{\alpha\beta}\dot{\rho}_\beta\\&+4e\dot{\mathbf{R}}\cdot\mathbf{A}(\mathbf{R})+e\dot{\boldsymbol{\rho}}\cdot\mathbf{A}(\boldsymbol{\rho})\Big\}\ ,\end{aligned} \tag{30}$$

where D_s and D_r are effective diffusion coefficient tensors

$$\begin{aligned}[D_{s,r}^{-1}(\rho)]_{\alpha\beta}&=\Big[\delta_{\alpha\beta}(I_0\pm 4I_1(\rho))\pm 4I_2(\rho)\hat{\boldsymbol{\rho}}_\alpha\hat{\boldsymbol{\rho}}_\beta\Big]D^{-1}\\&\equiv[D_{s,r}^{\parallel}(\rho)]^{-1}\hat{\boldsymbol{\rho}}_\alpha\hat{\boldsymbol{\rho}}_\beta+[D_{s,r}^{\perp}]^{-1}(\delta_{\alpha\beta}-\hat{\boldsymbol{\rho}}_\alpha\hat{\boldsymbol{\rho}}_\beta)\ ,\end{aligned} \tag{31}$$

which depend on the relative coordinate ρ through the functions $I_{1,2}(\rho)$, defined as

$$I_1(\rho)=\frac{1}{8}\int_0^{\ell^{-1}}dk\ln(k\ell)\frac{d}{dk}\Big[(kL_0)^{-3}F_1(k\rho)\Big]\Lambda(k/k_0)\ , \tag{32}$$

$$I_2(\rho)=\frac{1}{8}\int_0^{\ell^{-1}}\frac{dk}{k}(kL_0)^{-3}F_2(k\rho)\Lambda(k/k_0)\ , \tag{33}$$

where $\Lambda(x)$ is a cutoff function. For the numerical evaluation we have used $\Lambda(x)=[x^2/(1+x^2)]^3$. The constant

I_0 is given by

$$I_0 = 1 + \frac{1}{4}(k_0 L_0)^{-3} \ln(k_0 l) \ . \tag{34}$$

The Cooperon may be represented in the form of the path integral

$$\langle C_{t_0}(0,0)\rangle = \frac{1}{\tau}\int d^2x \int_{\mathbf{R}(0)=\mathbf{x}}^{\mathbf{R}(t_0)=0} \mathcal{D}[\mathbf{R}(t)] \times \int_{\boldsymbol{\rho}(0)=0}^{\boldsymbol{\rho}(t_0)=0} \mathcal{D}[\boldsymbol{\rho}(t)] e^{-S_{\text{inst}}} \ , \tag{35}$$

where $\mathbf{x}$ is the position vector $\mathbf{r}$ at time $t = 0$. The integral on $\mathbf{x}$ in (35) is the one in time slice $t = 0$ of the functional integral $\mathcal{D}[\mathbf{r}(t)]$ in (8). The integration in any other time slice $t \neq 0$ may be mapped onto the integrations on $\mathbf{R}(t)$ and $\boldsymbol{\rho}(t)$. Since the action S_{inst} is local in time, the path integral may be converted into a Schrödinger equation for the Green's function $\langle C_{t_0}(\boldsymbol{\rho}, \mathbf{R}; \boldsymbol{\rho}', \mathbf{R}')\rangle$ in imaginary time,

$$\left[\frac{\partial}{\partial t_0} + H(\boldsymbol{\rho}, \mathbf{R})\right]\langle C_{t_0}(\boldsymbol{\rho}, \mathbf{R}; \boldsymbol{\rho}', \mathbf{R}')\rangle = \frac{1}{\tau}\delta(\boldsymbol{\rho} - \boldsymbol{\rho}')\delta(\mathbf{R} - \mathbf{R}')\delta(t_0) \ . \tag{36}$$

The Hamiltonian H, as derived in Appendix A, is given by

$$H = -\frac{1}{2}D_{s,\alpha\beta}\nabla^R_\alpha \nabla^R_\beta - 2\nabla^\rho_\alpha D_{r,\alpha\beta}\nabla^\rho_\beta - \frac{1}{2}(\nabla^\rho_\alpha \nabla^\rho_\beta D_{r,\alpha\beta}) \ . \tag{37}$$

In (37) summation over repeated indices is understood. The gradient operators in (37) are defined as the gauge invariant combinations

$$\nabla^R_\alpha = \frac{\partial}{\partial R_\alpha} + i4eA_\alpha(\mathbf{R}) \ ,$$
$$\nabla^\rho_\alpha = \frac{\partial}{\partial \rho_\alpha} + ieA_\alpha(\boldsymbol{\rho}) \ . \tag{38}$$

The unusual values of the charge, $4e$ and e, are due to our choice of coordinates.

The diffusion constants $D_{r,s}$ in (37) depend on the relative variable $\boldsymbol{\rho}$, but not on the center of mass variable $\mathbf{R}$. This allows elimination of $\mathbf{R}$ from the Green's function in zero magnetic field,

$$\langle C_{t_0}(0,0)\rangle = \int d^2x \langle C_{t_0}(0,\mathbf{x};0,0)\rangle \ . \tag{39}$$

In the absence of an external magnetic field B, the eigenfunctions of H defined by

$$H\psi_{n\mathbf{k}}(\boldsymbol{\rho}, \mathbf{R}) = E_{n\mathbf{k}}\psi_{n\mathbf{k}}(\boldsymbol{\rho}, \mathbf{R}) \tag{40}$$

take the form

$$\psi_{n\mathbf{k}}(\boldsymbol{\rho}, \mathbf{R}) = e^{i\mathbf{k}\cdot\mathbf{R}}\phi_{n\mathbf{k}}(\boldsymbol{\rho}) \ . \tag{41}$$

Assuming the ϕ_n to be normalized,

$$\int d^2\rho \phi^*_{n\mathbf{k}}(\boldsymbol{\rho})\phi_{m\mathbf{k}}(\boldsymbol{\rho}) = \delta_{nm} \ , \tag{42}$$

we may express the Green's function in the form

$$\langle C_{\epsilon=0}(\boldsymbol{\rho}, \mathbf{R}; \boldsymbol{\rho}', \mathbf{R}')\rangle = \int_{-\infty}^{\infty} dt_0 \langle C_{t_0}(\boldsymbol{\rho}, \mathbf{R}; \boldsymbol{\rho}', \mathbf{R}')\rangle = \frac{1}{\tau}\int \frac{d^2k}{(2\pi)^2}\sum_n \frac{\psi^*_{n\mathbf{k}}(\boldsymbol{\rho}, \mathbf{R})\psi_{n\mathbf{k}}(\boldsymbol{\rho}', \mathbf{R}')}{E_{n\mathbf{k}}} \ . \tag{43}$$

It follows that only the eigenvalue $\mathbf{k} = 0$ enters the $\mathbf{x}$-integrated Green's function, as

$$\langle C_{\epsilon=0}(0,0)\rangle = \int d^2x \langle C_{\epsilon=0}(0,\mathbf{x};0,0)\rangle = \frac{1}{\tau}\sum_n \left.\frac{\phi^*_{n\mathbf{k}}(0)\phi_{n\mathbf{k}}(0)}{E_{n\mathbf{k}}}\right|_{\mathbf{k}=0} \ . \tag{44}$$

In other words, the kinetic energy operator involving ∇^R_α may be replaced by its zero eigenvalue. This is related to the fact that in zero external magnetic field the Cooperon amplitude is equal to the classical diffusion probability, which is conserved. The integral over the center of mass position $\mathbf{x}$ is therefore normalized to unity.

In the presence of an external magnetic field the equation of motion for the Cooperon is no longer the diffusion equation, but acquires nonconserving parts. Therefore the Cooperon amplitude is no longer conserved by itself. However, the absolute square of the Cooperon amplitude is of course conserved as it represents the quantum mechanical probability density. This property is unfortunately not sufficient to allow elimination of the center of mass variable $\mathbf{R}$. One is therefore dealing with a genuine two-particle problem in this case. In the following we will concentrate on the zero-magnetic-field case.

B. Cooperon in zero external magnetic field

As shown in the last section, the center of mass variable $\mathbf{R}$ drops out in zero magnetic field, leaving a one-body problem to be solved. The effective Hamiltonian for this is given by (37), omitting the first term $\propto \nabla^R_\alpha \nabla^R_\beta$. Fur-

thermore, it will be convenient to commute a factor of $(D_r^{\parallel})^{1/2}$ symmetrically to both sides of H. The ensuing equation is given by

$$\left[D\nabla^2 - U(\rho)\right](D_r^{\parallel})^{1/2}(\rho)\langle C_{\epsilon=0}(\boldsymbol{\rho},0)\rangle = -\frac{D^{1/2}}{2\tau}\delta(\boldsymbol{\rho}) \ . \tag{45}$$

Here

$$U(\boldsymbol{\rho}) = \frac{1}{4}D\nabla^2 \ln D_r^{\parallel}(\rho) - \frac{1}{4\rho}\frac{D}{D_r^{\parallel}}\frac{\partial}{\partial\rho}(D_r^{\parallel} - D_r^{\perp}) \tag{46}$$

plays the role of an effective potential for the relative motion of the Cooperon. Since $D_r^{\parallel,\perp}$ are functions of $\rho = \mid \boldsymbol{\rho} \mid$ only, the angular dependence may be separated out. In the limit $\boldsymbol{\rho} \to 0$ only the s-wave component will survive.

The additional term proportional to $(D_r^{\parallel} - D_r^{\perp})$ in (46) is small compared to the first term, because $D_r^{\parallel} - D_r^{\perp} \sim I_2$ and the integral I_2 defined in (33) is smaller than I_1 due to the logarithmic factor in (32). We will neglect this term later.

The solution of (45) can be found with the aid of Appendixes B and C as

$$\langle C_{\epsilon=0}(\rho,0)\rangle = -\frac{1}{4\pi D\tau}\left[\ln k_0\rho + \frac{1}{2\lambda}\right] , \tag{47}$$

where we have identified the range of the potential r_0 with the inverse momentum cutoff, $r_0 = k_0^{-1}$. The coupling constant λ is given by

$$\lambda = \frac{1}{32}\int_{\ell}^{\infty} d\rho\rho\left[\frac{d}{d\rho}\ln D_r^{\parallel}(\rho)\right]^2 + \frac{1}{4}\int_{\ell}^{\infty} d\rho \frac{1}{D_r^{\parallel}(\rho)}\frac{\partial}{\partial\rho}(D_r^{\parallel} - D_r^{\perp}) \ . \tag{48}$$

Note that λ is of second order in the perturbation $\frac{d}{d\rho}D_r^{\parallel}$. The first and dominant term is a positive definite quantity. The second term on the rhs of (48) is also a second-order contribution originating from the anisotropic part of the potential U, $\propto (D_r^{\parallel} - D_r^{\perp})$. We estimate it to be positive and at least one order of magnitude smaller than the main term.

A numerical evaluation of λ yields

$$\lambda = c_\lambda (16)^{-2}(k_0 L_0)^{-6}\ln^2(k_0\ell) \ , \tag{49}$$

where $c_\lambda \simeq 0.4$.

As shown in Appendix B, the effective potential $U(\rho)$ defined in (46) has a bound state of energy

$$E_b = -2Dk_0^2 \exp(-1/\lambda) \ . \tag{50}$$

We may identify the energy scale defined by $\mid E_b \mid$ as characteristic of this problem. Therefore the characteristic time scale τ_c introduced in the beginning of Sec. III A will be determined as

$$\tau_c = 1/\mid E_b \mid \ . \tag{51}$$

This provides the self-consistency equation from which the momentum cutoff k_0 may be determined:

$$\Gamma(k_0) = \mid E_b \mid \ . \tag{52}$$

Substituting E_b from (50) one finds

$$k_0 = \xi^{-1}\exp(-1/\lambda) \ , \tag{53}$$

where

$$\xi = (\pi/2g)k_F^{-1} \tag{54}$$

is a temperature-independent characteristic length. Inserting k_0 from (53) into (49) yields the following equation determining λ:

$$\frac{3}{\lambda} = \ln\left(\frac{T_0}{T}\right) - \frac{1}{2}\ln\left[\frac{1}{\lambda}\left(\frac{1}{\lambda} + \ln\frac{\xi}{\ell}\right)^2\right] + c_1 \ , \tag{55}$$

where T_0 is defined in (24) and $c_1 = \ln(16/\sqrt{c_\lambda}) \simeq 3$.

C. Correction to conductivity

We may now substitute the result (47) for the Cooperon into (6). The limit $\rho \to 0$ must be interpreted as $\rho = \ell$, the elastic mean free path. It is important to observe that the self-consistently determined cutoff k_0 depends on the coupling constant λ in an essential way.

The quantum correction to the conductivity thus is obtained using (6) as

$$\delta\sigma = -e^2\frac{(2S+1)}{4\pi^2}\left[\frac{1}{\lambda} + 2\ln\frac{\xi}{\ell}\right] , \tag{56}$$

where λ is a solution of (55) and the factor $(2S+1)$ accounts for the spin multiplicity of the carriers. It is interesting to note that in the limit $\frac{1}{\lambda} \gg \ln(\frac{\xi}{\ell})$, when the terms $\ln(\frac{\xi}{\ell})$ may be neglected in both (55) and (56), $\delta\sigma$ is only a function of (T/T_0). The dependence on the mean free path ℓ drops out in this limit, implying that the renormalization of the diffusion constant is so effective that the memory of the initial scale ℓ is completely lost. In other words, the relevant k modes in (19) satisfy the inequality $k \ll \ell^{-1}$.

More explicitly, the quantum correction to the conductivity is given by

$$\delta\sigma = -e^2\frac{(2S+1)}{4\pi^2}\ln\left(\frac{T^*}{T}\right)^{1/3} , \tag{57}$$

where

$$T^* = T_0\left(\frac{\xi}{\ell}\right)^6\left[\frac{1}{\sqrt{\lambda}}\left(\frac{1}{\lambda} + \ln\frac{\xi}{\ell}\right)\right]^{-1}$$

is a weakly temperature-dependent quantity.

A comparison with the usual expression for weak localization, $\delta\sigma = -\frac{e^2(2S+1)}{4\pi^2}\ln(\tau_\phi/\tau)$, shows that the effective phase-breaking time τ_ϕ varies as $\tau_\phi \propto T^{-1/3}$. It is unavoidable that at low temperatures the so-determined phase relaxation rate $\frac{1}{\tau_\phi}$ will be much larger than T. This

comparison, however, does not have any physical significance, as the energy scale $1/\tau_\phi$ does not appear in our derivation. Thus, one should in particular *not conclude* from this that the quasiparticle relaxation rate *exceeds* the temperature and that therefore *quasiparticle states are no longer well defined.* For later comparison we define the apparent phase-breaking rate $1/\tau_\phi^*$ by

$$\frac{1}{\tau_\phi^*} = \frac{1}{\tau}\left(\frac{T}{T^*}\right)^{1/3} . \tag{58}$$

It is worthwhile to return to the question how the self-consistently determined scales k_0 and $|E_b|$ relate to the scales L_0 and β_0 derived in Sec. II C on the basis of dimensional analysis. Using the definitions of L_0 and β_0 in (20), (50), and (53) we find

$$k_0 L_0 = \left[\frac{\sqrt{c_\lambda}}{16}\frac{1}{\sqrt{\lambda}}\left(\frac{1}{\lambda}+\ln\frac{\xi}{\ell}\right)\right]^{1/3} ,$$

$$|E_b|\beta_0 = \frac{\sqrt{c_\lambda}}{8}\frac{1}{\sqrt{\lambda}}\left(\frac{1}{\lambda}+\ln\frac{\xi}{\ell}\right) . \tag{59}$$

In other words, the length scale k_0^{-1} and the time scale $|E_b|^{-1}$ are identical to the formally defined scales L_0 and β_0 up to logarithmic corrections. This lends support to the expectation that the self-consistency argument employed in conjunction with the instantaneous approximation captures the principal contribution to $\delta\sigma$.

Equation (59) also serves to determine the regime of validity of the instantaneous approximation. The relevant condition follows from the requirement that the fluctuations may be treated classically, or $\Gamma(k_0) \simeq |E_b| \ll T$, which can be cast in the form

$$\frac{\sqrt{c_\lambda}}{4\pi} g \frac{1}{\sqrt{\lambda}}\left(\frac{1}{\lambda}+\ln\frac{\xi}{\ell}\right) \ll 1 . \tag{60}$$

On the other hand, the condition $\lambda \ll 1$ should be satisfied, which requires $T \ll T_0$. The instantaneous approximation is thus seen to be valid in the temperature interval

$$T_{\rm inst} \ll T \ll T_0 , \tag{61}$$

where

$$T_{\rm inst} = T_0 \exp -3\left(\frac{4\pi}{\sqrt{c_\lambda}}\frac{1}{g}\right)^{2/3} . \tag{62}$$

In Fig. 1 the regime of validity of the instantaneous approximation in the anomalous-skin-effect case is shown (area marked A). In addition to the boundaries 1 ($T = T_0$) and 2 ($T = T_{\rm inst}$), the boundary to the regime dominated by normal-skin-effect behavior, defined by $L_0(T) = \ell_F$, is shown. The crossover at the boundary $T = T_{\rm inst}$ may be obtained by imposing the restriction to the classical fluctuation regime by an upper cutoff k_T in the k integration, where k_T is defined through $\Gamma(k_T) = T$. When k_T approaches the lower cutoff k_0 from above, the contribution to $1/\tau_\phi$ from large k values, $k > k_0$, tends to zero as $k_T - k_0 \propto T - T_{\rm inst}$. The phase relaxation for

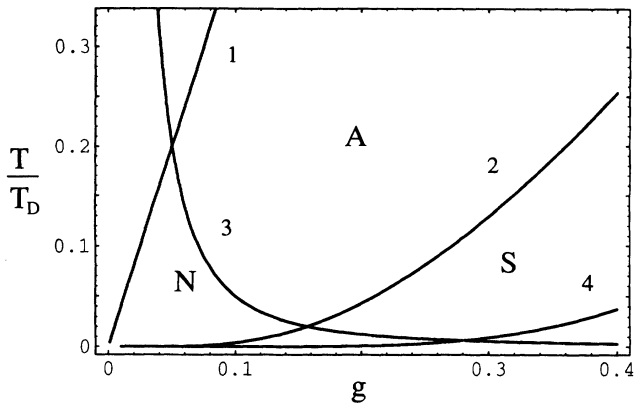

FIG. 1. Schematic overview of the regions of applicability of the instantaneous approximation in the anomalous- (region marked A) and normal- (N) skin-effect cases, and of the quasistatic approximation (S), in the temperature-disorder parameter plane [$T_D = Dk_F^2$, $g = D(e^*)^2/\chi_d$]. The curves shown are $T = T_0$ (1), $T = T_{\rm inst}$ (2), $L_0(T) = \ell_F$ (3), $T = T_{{\rm inst},n}$ (4), for typical values of the parameters.

$T < T_{\rm inst}$ is entirely due to small k processes, which will be considered in the quasistatic approximation below.

Finally, we note that using (55), the quantum correction to the conductivity may be expressed as

$$\delta\sigma = -e^2\frac{2S+1}{4\pi^2}\ln\left(\frac{\xi}{k_0 l^2}\right) = -e^2\frac{2S+1}{2\pi^2}\ln\frac{L_\phi}{l} , \tag{63}$$

where the phase-breaking length L_ϕ has been defined as $L_\phi = (\xi k_0^{-1})^{1/2} \simeq (\xi L_0)^{1/2}$. This result follows from the usual relation $L_\phi = (D\tau_\phi)^{1/2}$ if τ_ϕ is identified with $\beta_0 \sim \frac{k_F}{\chi_d} L_0^3$ and D is replaced by $D(\xi/L_0)^2$.

D. Normal-skin-effect regime

In this subsection we briefly consider the instantaneous approximation for the case $k_0\ell_F \ll 1$, when the gauge-field propagator has the normal-skin-effect form and

$$\Gamma(k) = \left(\frac{\chi_d}{e^{*2}\sigma_F}\right)k^2, \quad k < \ell_F^{-1} , \tag{64}$$

where $\sigma_F = k_F\ell_F$. We will assume here that the relevant processes take place in the regime of wave vectors $k < \ell_F^{-1}$.

The normal-skin-effect regime for a usual metal, i.e., for one species of particles, such that our parameters ℓ_F and ℓ are equal, has been considered before,[22,3] in particular in the so-called strong-skin-effect regime, where $k\delta(\omega) \gg 1$. Here k is the typical wavelength of electromagnetic fluctuations, and $\delta(\omega) = c^2/2\pi\sigma\omega$ is the skin depth for a $2d$ metal. Using $k \sim L_\phi^{-1}$ and $\omega \sim \tau_\phi^{-1}$ the condition for a strong skin effect can be written $(c/v_F)(\kappa\ell)^{-1} \gg 1$, where $\kappa = [4\pi e^2 N(0)]^{1/2}$ is the screening wave number. In this regime the longitudinal part of the electromagnetic fluctuations, i.e., the fluctuations of the electric field, dominates. As shown in Ref. 22,

the interaction is rigorously instantaneous in that case. The corresponding Hamiltonian is that of a particle (with constant mass) in a potential. The phase relaxation rate is found as

$$\frac{1}{\tau_\phi^{\mathrm{AAK}}} = \frac{e^2 T}{2\pi\sigma}\ln(\pi\sigma/e^2) \ . \tag{65}$$

It is worth noting that $e^2/\sigma = R_\Box e^2 \ll 1$ in a weakly disordered system ($R_\Box$ is the resistance per square). We will compare the result (65) in the following with the apparent $1/\tau_\phi$ induced by transverse magnetic-field fluctuations in the weak-skin-effect case.

We now return to the case of transverse magnetic-field fluctuations. The dimensional analysis analogous to the discussion in Sec. II C yields the same time scale β_0, given in (19), but a different length scale L_n defined by

$$L_n^2 = \frac{\pi T_D}{2g^2\sigma_F T} k_F^{-2} \ . \tag{66}$$

The instantaneous approximation can be applied to this case as well. The steps leading to (45)–(48) can be taken over unchanged. Only the explicit expression for λ changes:

$$\lambda = c_n (k_0 L_n)^{-4} \ln^2(k_0 \ell) \ , \tag{67}$$

where c_n is a constant. As in the case considered before, there is a single bound state in the weak coupling limit ($\lambda \ll 1$), with energy given by (50). The relation (52) now no longer determines k_0, as the factors of k_0^2 on both sides of the equation drop out. Rather, (52) serves to determine λ, which is now a parameter independent of T,

$$e^{-\frac{1}{\lambda}} = \frac{1}{2g\sigma_F} \ . \tag{68}$$

The condition $\lambda \ll 1$ is seen to carry over into $2g\sigma_F \gg 1$. The cutoff momentum k_0 is now determined from (67) as

$$k_0 \cong \left(\frac{c_n}{\lambda}\right)^{1/4} \left[\frac{1}{2}\ln\left(\frac{\lambda}{c_n}\right) + \ln\left(\frac{L_n}{\ell}\right)\right]^{1/2} L_n^{-1} \equiv c_k L_n^{-1} \ , \tag{69}$$

where c_k is a constant, apart from logarithmic corrections.

Inserting k_0 and λ into the expression (47) for the Cooperon, one finds the conductivity correction as

$$\delta\sigma = -\frac{e^2}{2\pi^2}(2S+1)\ln\left[\frac{L_n}{\ell}\frac{1}{c_k}\left(\frac{1}{2\sigma_F g}\right)^{1/2}\right] \ . \tag{70}$$

By equating the argument of the logarithm in (70) to $(\tau_\phi/\tau)^{1/2}$, we find the apparent phase relaxation rate as

$$\frac{1}{\tau_\phi^n} = \frac{8}{\pi} c_k^2 g^3 \sigma_F^2 T \ . \tag{71}$$

Thus τ_ϕ^{-1} is seen to be a linear function of T. For large σ_F, as required by the weak coupling condition, the prefactor of T is likely to be larger than unity.

Finally we have to consider the regime of validity of the instantaneous approximation in this case. It follows from $|E_b| \ll T$ that

$$\frac{1}{\pi}\left(\frac{c_n}{\lambda}\right)^{1/2} g \ln\left[\left(\frac{L_n}{\ell}\right)\left(\frac{\lambda}{c_n}\right)^{1/2}\right] \ll 1 \ . \tag{72}$$

The small parameter of the instantaneous approximation, $[(e^*)^2 D/\chi_d]$, guarantees a regime of temperatures, where (72) is satisfied. However, in the limit of low temperatures, the increase of $L_0 \propto T^{-1/2}$ will eventually lead to the violation of (72). Thus the validity of the instantaneous approximation in the normal-skin-effect regime is restricted to $T > T_{\mathrm{inst,n}}$, where

$$T_{\mathrm{inst,n}} = T_{0,n} \exp\left[-\pi\left(\frac{\lambda}{c_n}\right)^{1/2}\frac{1}{g}\right] \tag{73}$$

and $T_{0,n} = (\pi\lambda/c_n)(D/g^2 l^2)$ is a characteristic temperature.

On the other hand, the temperature regime is bounded from above by the requirement that $k_F \ell \ll 1$, which translates into $L_n \gg l_F$, or

$$T \ll T_{NA} \ , \tag{74}$$

where $T_{NA} = \pi T_D/\sigma_F^3 g^2$.

In order for $T_{\mathrm{inst},n}$ to be sufficiently low, the diffusion constant should be small, such that $g \ll 1$. This together with the condition $\lambda \ll 1$ confines g to

$$1 \ll g^{-1} \ll \sigma_F \ . \tag{75}$$

Comparing the result (71), with the phase relaxation rate $1/\tau_\phi^{\mathrm{AAK}}$ induced by electric-field fluctuations generalized to the two-component system considered in the present work, i.e., replacing σ by σ_F, we observe that the magnetic-field-fluctuation-induced rate (71) is dominant in the parameter range where

$$4c_k^2 (g\sigma_F)^3 \ln \pi\sigma_F \gg 1 \ .$$

For values of $\sigma_F \gg 1/g$, condition (76) is seen to be satisfied. The regime of validity of the instantaneous approximation in the normal-skin-effect case is shown in Fig. 1 (marked N). The boundary $T = T_{\mathrm{inst},n}$ is represented by the curve labeled 4.

It is instructive to rewrite the apparent phase-breaking rates $\frac{1}{\tau_\phi^*}$ and $\frac{1}{\tau_\phi^n}$ obtained within the instantaneous approximation in a form using the crossover temperature T_{NA} defined in (74),

$$\frac{1}{\tau_\phi^*} = \frac{g}{\sigma_F} T_D \left(\frac{T}{T_{NA}}\right)^{1/3} \text{ and } \frac{1}{\tau_\phi^n} = \frac{g}{\sigma_F} T_D \frac{T}{T_{NA}} \ , \tag{76}$$

up to logarithmic corrections. The $T^{1/3}$ behavior is seen to crossover smoothly into a linear T law.

IV. QUASISTATIC APPROXIMATION

In the limit where the magnetic-field fluctuations are very slow in time, the typical frequencies of the fluctuations are small compared to the inverse characteristic time of the problem, i.e., $\Gamma(k)t_0 \ll 1$. In the expression (19) for the action ΔS we may then replace $\Delta_k(t)$ by 1,

$$\Delta S = -\frac{1}{\pi}\frac{T(e^*)^2}{\chi_d}\int_{-t_0}^{t_0} dt_1 dt_2 \int_0^{l^{-1}} dk\Big\{(\mathbf{v}_1\cdot\mathbf{v}_2)\ln(k\ell)\frac{d}{dk}F_1(kr) + (\mathbf{v}_1\cdot\hat{\mathbf{r}})(\mathbf{v}_2\cdot\hat{\mathbf{r}})\frac{1}{k}F_2(kr)\Big\} . \tag{77}$$

The dependence on $\mathbf{r}_1$, $\mathbf{r}_2$ may be scaled out, to give

$$\Delta S = -\frac{1}{\pi}\frac{T(e^*)^2}{\chi_d}\int_{-t_0}^{t_0} dt_1 dt_2\Big\{(\mathbf{v}_1\cdot\mathbf{v}_2)\Big[Q(r/\ell)\ln\frac{r}{\ell} + P_1(r/\ell)\Big] + (\mathbf{v}_1\cdot\hat{\mathbf{r}})(\mathbf{v}_2\cdot\hat{\mathbf{r}})P_2(r/\ell)\Big\} , \tag{78}$$

where

$$\begin{aligned} Q(r/\ell) &= F_1(0) - F_1(r/\ell) , \\ P_1(r/\ell) &= \int_0^{r/\ell} dx \ln x \frac{dF_1(x)}{dx} , \\ P_2(r/\ell) &= \int_0^{r/\ell} dx \frac{1}{x}F_2(x) . \end{aligned} \tag{79}$$

In the diffusion approximation for the dynamics of the particle, which we have been using throughout, spatial distances less than the elastic mean free path ℓ are meaningless. On the contrary, we are assuming that typical distances are much larger than ℓ, and therefore we should consider (79) in the limit $r \gg \ell$. In this limit, the functions Q, P_1, and P_2 approach constant values, in particular $Q \to \frac{1}{2}$. The logarithmic term is seen to dominate, and we may write

$$\Delta S \simeq -\frac{1}{2\pi}\frac{T(e^*)^2}{\chi_d}\int_{-t_0}^{t_0} dt_1 dt_2\Big[\dot{\mathbf{r}}(t_1)\cdot\dot{\mathbf{r}}(t_2)\Big] \times\ln\frac{|\,\mathbf{r}(t_1)-\mathbf{r}(t_2)\,|}{\ell} . \tag{80}$$

This expression for the action is highly nonlocal in time. A similar problem was discussed in Ref. 8, in the context of single-particle properties without an additional scalar random potential. The authors of Ref. 8 approximated $\ln|\,\mathbf{r}(t_1)-\mathbf{r}(t_2)\,| \simeq f(t_1-t_2)$ and proposed to determine the function $f(t)$ self-consistently. However, it is not clear how to relate $f(t)$ self-consistently to the properties being calculated in this case.

The expression (80) for the action ΔS may be rewritten as

$$\begin{aligned} \Delta S &= -\frac{T(e^*)^2}{2\pi\chi_d}\oint d\mathbf{r}_1\cdot\oint d\mathbf{r}_2\ln\frac{|\,\mathbf{r}_1-\mathbf{r}_2\,|}{\ell} \\ &= 2\frac{T(e^*)^2}{\chi_d}\int dA_1\int dA_2\delta(\mathbf{r}_1-\mathbf{r}_2) , \end{aligned} \tag{81}$$

where $dA_{1,2}$ denotes integration over the area enclosed by the path.

The result (81) may be derived in a different way. In the quasistatic limit one can replace the arguments in the cosine function, $\frac{\omega}{2}(t_1\pm t_2)$, in (10) by zero. The remaining integral on $\langle a_\alpha a_\beta\rangle_{\mathbf{k}\omega}$ over ω yields

$$\int\frac{d\omega}{2\pi}\langle a_\alpha a_\beta\rangle_{\mathbf{k}\omega} = \delta^T_{\alpha\beta}(\hat{\mathbf{k}})\frac{T}{\chi_d k^2} . \tag{82}$$

The corresponding correlation function of the magnetic field $b(\mathbf{r}) = \nabla\times\mathbf{a}(\mathbf{r})$ is given by

$$\langle b(\mathbf{r})b(\mathbf{r}')\rangle = \frac{T}{\chi_d}\delta(\mathbf{r}-\mathbf{r}') . \tag{83}$$

The fluctuation contribution to the action (9) may be reformulated as

$$\begin{aligned} \Delta S &= 2e^{*2}\int_{-t_0}^{t_0} dt_1\int_{-t_0}^{t_0} dt_2\dot{r}_\alpha(t_1)\dot{r}_\beta(t_2)\langle a_\alpha(\mathbf{r}_1)a_\beta(\mathbf{r}_2)\rangle \\ &= 2e^{*2}\oint d\ell_{1\alpha}\oint d\ell_{2\beta}\langle a_\alpha(\mathbf{r}_1)a_\beta(\mathbf{r}_2)\rangle \\ &= 2\frac{e^{*2}T}{\chi_d}\int dA_1\int dA_2\delta(\mathbf{r}_1-\mathbf{r}_2) , \end{aligned} \tag{84}$$

where $d\ell_{1,2}$ denotes integration along the closed loop path $\mathbf{r}(t)$ and we have used Stokes' theorem to transform the line integral into integrals over the area enclosed by the path.

In the case of a self-intersecting path, it may happen that a certain area A is encircled more than once. In this case, the integral over the area A has to be performed a corresponding number of times (the winding number n) in both integrations (dA_1 and dA_2). If we now perform one of the integrations over the area with the aid of the δ function, we are left with a single integration over the area enclosed by the path, which in the absence of multiply winding trajectories just yields the area enclosed by the path, irrespective of the sense of rotation of the path. Any area A encircled n times by the path yields a contribution n^2A, because the δ function can be satisfied in n^2 different ways taking into account the n windings in integration dA_1 and the n windings in integration dA_2. The action ΔS can thus be expressed in terms of the areas A_i encircled by all closed loops formed by the (closed loop) path $\mathbf{r}(t)$ as

$$\Delta S = 2\frac{T(e^*)^2}{\chi_d}\sum_i n_i^2 A_i , \tag{85}$$

where n_i is the number of loops enclosing the area A_i.

In the limit of weak disorder where $E\tau \gg 1$ (E is the typical energy of the carriers), the probability of multiple loops is small, since the probability of return is proportional to $(E\tau)^{-1}$. One may obtain an estimate of $|\,\Delta S\,|$ by replacing the factor n_i^2 in (85) by 1,

$$\Delta S \simeq \frac{T(e^*)^2}{\chi_d}A , \tag{86}$$

where A is the total area enclosed by the path. In a quasiclassical approximation, A is given by the area covered

by a diffusing particle in time t_0: $A_{\rm cl} = Dt_0$. Substituting this into (86) and employing the definition (19) of the characteristic time β_0, we find

$$\Delta S \simeq \frac{t_0}{\beta_0} = \frac{t_0}{\tau_\phi^{\rm st}} \ . \tag{87}$$

A linear dependence of ΔS on time t_0 is equivalent to phase relaxation given by the rate

$$\frac{1}{\tau_\phi^{\rm st}} = \frac{(e^*)^2 D}{\chi_d} T = gT \ . \tag{88}$$

It is interesting to observe that the temperature dependence of $\frac{1}{\tau_\phi^{\rm st}}$ is linear, as is the case for $\frac{1}{\tau_\phi}$ induced by Coulomb interaction processes. However, in that case $\frac{1}{\tau_\phi} = \pi T(\frac{Re^2}{2\pi^2})\ln(\frac{\pi}{e^2 R})$ and the prefactor is seen to be proportional to the resistance per square R, whereas $\frac{1}{\tau_\phi^{\rm st}}$ in (88) is seen to be inversely proportional to R. This difference can be easily understood by observing that a random magnetic field induces fluctuations of the charge current. In the classical regime these fluctuations are proportional to the temperature and the "conductance" g, whereas the fluctuations of the electric field are proportional to the resistance.

A necessary, but not sufficient, condition for the validity of the quasistatic approximation is $\Gamma(k = L_\phi^{-1})\tau_\phi^{\rm st} \ll 1$ where $L_\phi^2 = D\tau_\phi$. Substituting $\Gamma(k)$ as given by (4) for the anomalous-skin-effect regime and $\tau_\phi^{\rm st}$, this condition takes the form $T \ll T_0$, with T_0 defined in (24). In the normal-skin-effect regime one finds the condition $\chi_d/\sigma D \ll 1$, which coincides with the weak coupling condition $\lambda \ll 1$ stated in the text following (68).

The contribution (88) to the phase-breaking rate is due to the quasistatic, or low-wave-vector processes, i.e., in the regime $0 < k < k_0$, which we neglected in the instantaneous approximation. As long as the conditions for the validity of the instantaneous approximation (61) hold, and $1/\tau_\phi^{\rm st}$ is less than the apparent phase-breaking rate $1/\tau_\phi^*$ derived from the correction to the conductivity (57), the quasistastic processes are negligible. This holds also in the normal-skin-effect regime, with the ratio of (71) and (88) given by $\tau_\phi^{\rm st}/\tau_\phi^n \sim \exp(1/\lambda) \gg 1$. However, in the low-temperature regime $T < T_{\rm inst}$, where $T_{\rm inst}$ is defined by (62), the quasistatic processes dominate. In this case the usual interpretation of the weak localization correction (see the discussion in the Introduction) holds: At the time scale $\tau_\phi^{\rm st}$ the coherent motion of particles is cut off. The regime of validity of the quasistatic approximation is indicated in Fig. 1 (area marked S).

It is interesting to note that the prefactor of T in (88) can be larger than 1 (the condition for the validity of the quasiclassical average over ΔS is $\frac{1}{\tau_\phi^{\rm st}} \ll \frac{1}{\tau}$). In other words, $\frac{1}{\tau_\phi^{\rm st}}$ appears to become larger than T. This may be thought to be in conflict with the inelastic transport relaxation rate $\frac{1}{\tau_{\rm tr}} \simeq T$.[19] However, whereas small momentum transfer processes do not contribute to $\tau_{\rm tr}$, they are shown in the above to give the main contribution here, so that $\frac{1}{\tau_\phi^{\rm st}}$ may well be larger than $\frac{1}{\tau_{\rm tr}}$.

A different approach would be to develop a perturbation theory scheme, which can deal with the severe infrared divergences appearing here. This has been done successfully for the calculation of the diffusion coefficient in the static random magnetic-field problem.[14] However, the calculation of the Cooperon requires a controlled description of the infrared properties of the single-particle Green's function first, which is a nontrivial task in itself.

V. APPLICATION TO HIGH-T_c COMPOUNDS

We will now discuss the possible relation of our results to the experimentally observed weak localization corrections for the magnetoconductance in Bi 2:2:0:1 compounds.[6] In these experiments the longitudinal and transverse magnetoconductance, i.e., for current oriented in the CuO plane and magnetic field parallel to the current or perpendicular to the CuO planes, was measured in the temperature range 0.5–20 K. The superconducting transition temperature in these samples was less than 0.3 K. The data showed evidence for both weak localization corrections (negative magnetoresistance) and interaction-induced quantum corrections (positive magnetoresistance). By subtracting the measured longitudinal magnetoresistance from the transverse one, assuming that the interaction-induced contribution is isotropic, one can isolate the weak localization contribution $\Delta\sigma_{\rm WL}$. The data for $\Delta\sigma_{\rm WL}$ obtained in this way were fitted to the usual expression[5]

$$\Delta\sigma_{\rm WL} = \frac{e^2}{2\pi^2}\left[\psi\left(\frac{1}{2}+\frac{r_B^2}{4\ell^2}\right) - \psi\left(\frac{1}{2}+\frac{r_B^2}{4L_\phi^2}\right) - \ln\left(\frac{\tau_\phi}{\tau}\right)\right], \tag{89}$$

where $r_B^2 = 1/(eB)$. Good fits to the data were obtained, except for temperatures $T < 2$ K in low magnetic fields, where an anomaly in the longitudinal magnetoconductivity was observed. It was found that $L_\phi^2 = 131 T^{-1/3}$ nm^2 (T in K), whereas the elastic mean free path ℓ obtained from the fit varied from sample to sample between 5.5 and 7 nm. It follows that the apparent phase relaxation rate, normalized to the elastic scattering rate, is given by $\tau/\tau_\phi = \ell^2/(2L_\phi^2) \cong (T/T^*)^{1/3}$, where $T^* \simeq 650$ or 150 K, for values of $\ell = 5.5$ or 7 mn.

In the gauge-field model of high-T_c compounds proposed in Refs. 17 and 18, the charge is transported mainly by the bosons (sometimes called holons), whereas the fermions (or spinons) govern the behavior of the gauge field. Unfortunately, the properties of the bosons, i.e., the energy spectrum, are not very well known yet. Elementary models of weakly interacting bosons are not sufficient to explain the observed behavior. For example, a quadratic spectrum $\epsilon_p = \frac{p^2}{2m}$ leads to an average (thermal) velocity of the bosons, $v_{\rm th} = \sqrt{\frac{2}{m}T}$. For the elastic relaxation rate one finds in Born approximation $\frac{1}{\tau} = 2\pi N(E)V^2$, and taking into account the constant density of states, $\frac{1}{\tau}$ is seen to be independent of temperature. The diffusion constant follows as $D = \frac{1}{2}v_{\rm th}^2\tau \propto T$. The conductivity is temperature independent in this case,

$\sigma = \frac{\partial n}{\partial \mu} D = \frac{e^2 n \tau}{m}$, since the compressibility $\frac{\partial n}{\partial \mu} \propto \frac{1}{T}$. A finite residual resistivity, obtained by extrapolating the linear T dependence observed over a wide temperature range, to zero temperature, is indeed observed in experiment. However, the corresponding weak localization correction, obtained by averaging the result for one particle of given energy E, over energy with weight factor $\frac{\partial n}{\partial E}$, where $n(E)$ is the Bose distribution function, yields a prefactor $\frac{\partial n}{\partial \mu}$ of the usual $\ln(\tau_\phi/\tau)$ term, which is much more strongly temperature dependent ($\propto \frac{1}{T}$) than $\ln(\tau_\phi/\tau)$. Such behavior is definitely not seen in experiment. Rather, the charge carriers appear to have the characteristics of fermions.

One possible way out of this dilemma is to assume that the bosons in low-energy states of the band are localized, up to a mobility edge E_c (the characteristic energy where strong localization crosses over to weak localization). Due to the mutual repulsion of the bosons, each localized state is only singly occupied. The number of bosons localized in states below E_c is thus temperature independent. These particles effectively correspond to a Fermi sea, and E_c plays the role of the Fermi energy. In the limit of low temperatures all the mobile bosons are within a narrow temperature interval of width T around the energy E_c. The weak localization contribution in the limit of low temperature, $T \ll E_c$, is thus given by (56), with the energy E given by E_c.

Another possibility would be that the charge carriers are actually fermions, in a different formulation of a slave particle representation, or else through an effective transmutation on account of the strong interaction of the bosons with fermions and the longitudinal gauge field. In principle, the distinction between bosonic charge carriers or fermions carrying spin is possible by measurement of the spin degeneracy factor $(2S+1)$ in (57).

In any case, a two-component description has the advantage of allowing the gauge field to be in the clean limit, while the charge carriers may be in the dirty limit. In the original formulation in terms of holons and spinons this is achieved by taking into account that the spinons couple less strongly to the charge distribution around an impurity than do the holons (see Ref. 20 for a discussion of the screening of the effective impurity potential seen by the spinons).

Consequently, the spinon mean free path ℓ_F can be much larger than the holon mean free path ℓ. This allows one to satisfy the condition for the validity of the instantaneous approximation, $g \lesssim 1$, in the anomalous-skin-effect regime of the gauge field, where $k_0 \ell_F > 1$ has to be satisfied. However, even if $\ell_F \gg \ell$, and for $g \ll 1$, the condition $k_0 \ell_F > 1$ will be violated below a characteristic temperature $T_{\rm an}$ given by $\ell_F = L_0(T_{\rm an})$, or $T_{\rm an} = (\frac{\chi_d^2}{(e^*)^2 k_F D}) \ell_F^{-3} \sim (\frac{\chi_d}{\ell_F^2})(\frac{\xi}{\ell_F})$. In the temperature regime $T_{\rm inst} < T < T_{\rm an}$ the normal-skin-effect form of the gauge-field propagator applies, and the phase relaxation rate, obeys a different temperature power law $\frac{1}{\tau_\phi} \sim T$, as given by (69).

A different source of fluctuating transverse gauge fields would be the magnetic-field fluctuations generated by current fluctuations in the system. In this case the mean free path ℓ_F entering the gauge-field propagator would be identical with the one of the charge carriers, $\ell_F = \ell$. It follows from the condition $k_0 \ell \ll 1$ that only the normal-skin-effect regime is accessible and therefore $\frac{1}{\tau_\phi} \sim T$ in this case.

The different temperature laws for $1/\tau_\phi$ predicted in the various regimes defined by the characteristic temperatures $T_{\rm inst}$ and $T_{\rm an}$ should be looked for in experiments. This requires experiments to cover a broader temperature range as well as a range of impurity concentrations, in particular extending to cleaner systems.

On the theoretical side, a calculation of the magnetoconductivity along the lines described in the present paper is necessary, in order to compare more directly with experiments. Work in this direction is in progress.

VI. CONCLUSION

We have shown that the weak localization correction to the electrical conductivity σ of charged quantum particles in two dimensions moving in a static random potential and subject to a fluctuating magnetic field leads to a variety of dependences on temperature, diffusion constant, and the parameters ℓ_F, k_F, χ_d of the gauge-field propagator in various regimes. As usual, we calculated the weak localization correction as an integral over all times of the Cooperon amplitude. The Cooperon in turn was obtained from its path integral representation. The quantum correction to σ is usually expressed in terms of the phase-breaking rate $1/\tau_\phi^*$. For the case of the electron-electron interaction, which may be modeled as a fluctuating electric field, it was found some time ago [3,22] that $1/\tau_\phi \propto (R_\Box \ln R_\Box) T$. As we are able to show, a linear T dependence is obtained for a large portion of parameter space also in the here considered case of a fluctuating transverse magnetic field, albeit with different prefactors. However, in not too clean systems, such that $(e^*)^2 D/\chi_d \ll 1$, where D is the diffusion coefficient, e^* is the charge of the particles in the (possibly fictitious) gauge field, and χ_d is the diamagnetic susceptibility, and in the special situation where the gauge field is in the clean limit (dispersion law $\omega \propto ik^3$), we find the highly unusual behavior $1/\tau_\phi \sim T^{1/3}$. In order for the charge carriers to be in the dirty limit and the gauge field to be in the clean limit it is necessary that the properties of the gauge field be governed by a different species of particles, which scatter only weakly off impurities.

This regime is characterized by rapid fluctuations of the magnetic field, such that the interaction processes become instantaneous, or local in time. The Cooperon amplitude, describing the interference of two quantum particles traversing a closed path in opposite direction and shifted in time by the amount t_0, has been shown to be influenced dramatically by the following effect: The diffusion of the two particles gets slowed down with increasing relative distance by the magnetic-field fluctuations. This in conjunction with phase relaxation leads to a much stronger correction to the conductivity, or expressed differently, to a much larger *apparent* phase-breaking rate $1/\tau_\phi^*$. A $T^{1/3}$ temperature law has been

observed experimentally in a Bi-cuprate compound, for which the existence of internal gauge fields has been conjectured, as discussed in Sec. V.

For systems still in the instantaneous regime, but in the normal-skin-effect domain ($\omega \propto ik^2$), we find a linear T law for $1/\tau_\phi$, with prefactor $\sim g^3\sigma_F^2$, larger than the one deriving from electric-field fluctuations in a wide domain of parameters.

Finally, we considered the opposite limit of a quasistatic magnetic field. In this case the usual situation of coherent quantum diffusion limited by phase relaxation is recovered. We were able to estimate the part of the action due to magnetic-field fluctuations in a quasiclassical approximation, leading to $1/\tau_\phi$ linear in T. Again, the prefactor $\sim g$ may be larger than the one found from electric-field fluctuations.

We found that the high-frequency, large-momentum processes featuring in the instantaneous approximation dominate the low-frequency processes taken into account in the quasistatic approximation at least at not too low temperatures.

Below a critical temperature $T_{\rm inst}$ the high-frequency processes are no longer available in the domain of classical fluctuations, $\omega \ll T$, and the quasistatic contribution takes over. The rich variety of behavior found here deserves further experimental study. On the theoretical side, work on various extensions including detailed studies of the quasiclassical approximation and the effect of an additional static uniform magnetic field are in progress.

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APPENDIX A: EFFECTIVE HAMILTONIAN

In this appendix we derive the Hamilton operator (37) corresponding to the action $S_{\rm inst}$ defined in (30). The action $S_{\rm inst}(t_0)$ defines the dynamics of a system of two quantum particles. We first consider the case of zero external magnetic field. The state of this system at time t is determined by the wave function $\psi(\mathbf{r},\mathbf{R},t)$ specifying the probability amplitude of the two particles at positions $\mathbf{r}$ and $\mathbf{R}$. The wave function at an infinitesimally later time $t+\epsilon$ can be expressed in terms of an integral over all positions the particles may have had at time t, weighted with the phase factor $\exp[iS_{\rm inst}(\epsilon)]$:

$$\psi(\mathbf{r},\mathbf{R},t+\epsilon) = \int d^2\xi d^2\eta \left[2\pi i\epsilon D_s\left(\mathbf{r}+\frac{\boldsymbol{\xi}}{2}\right)\right]^{-1}\left[8\pi i\epsilon D_r\left(\mathbf{r}+\frac{\boldsymbol{\xi}}{2}\right)\right]^{-1}\psi(\mathbf{r}+\boldsymbol{\xi},\mathbf{R}+\boldsymbol{\eta},t)$$
$$\times\exp\left\{\frac{i}{2\epsilon}D^{-1}_{s\alpha\beta}\left(\mathbf{r}+\frac{\boldsymbol{\xi}}{2}\right)\eta_\alpha\eta_\beta+\frac{i}{8\epsilon}D^{-1}_{r\alpha\beta}\left(\mathbf{r}+\frac{\boldsymbol{\xi}}{2}\right)\xi_\alpha\xi_\beta\right\}. \tag{A1}$$

Since the effective diffusion coefficients $D_{s\alpha\beta}$ and $D_{r\alpha\beta}$ are position dependent, the question arises as to which average position has to be used. As discussed by Feynman and Hibbs,[16] the correct procedure is to use the midpoint position $\mathbf{r}_{\rm MP} = \mathbf{r} + \boldsymbol{\xi}/2$. The normalization factors $[2\pi i\epsilon D_{s,r}(\mathbf{r}+\frac{\boldsymbol{\xi}}{2})]^{-1}$ depend on $\boldsymbol{\xi}$. They have been chosen such that probability is conserved, which dictates the choice of the midpoint position. In order words, only for this choice will the resulting Hamiltonian be Hermitian.

The rhs of (A1) is now expanded in ϵ to first order, resulting in

$$\psi(\mathbf{r},\mathbf{R},t+\epsilon) = \psi(\mathbf{r},\mathbf{R},t) - i\epsilon\hat{H}\psi(\mathbf{r},\mathbf{R},t). \tag{A2}$$

The operator $\hat{H}$ in (A2) may be identified as the Hamilton operator of the system. In the first step of this procedure, the rhs of (A1) is expanded to sixth order in the variables ξ_α and η_α, using the fact that they both scale as $\epsilon^{1/2}$:

$$\psi(\mathbf{r},\mathbf{R},t+\epsilon) = Z\int d^2\xi d^2\eta\exp\left\{\frac{i}{2\epsilon}D^{-1}_{s\alpha\beta}\eta_\alpha\eta_\beta+\frac{i}{8\epsilon}D^{-1}_{r\alpha\beta}\xi_\alpha\xi_\beta\right\}$$
$$\times\Big\{\Big[1+\frac{1}{4}D_rD_s(\boldsymbol{\xi}\cdot\boldsymbol{\nabla}D_s^{-1})(\boldsymbol{\xi}\cdot\boldsymbol{\nabla}D_r^{-1})+\frac{1}{8}D_s(\boldsymbol{\xi}\cdot\boldsymbol{\nabla})^2D_s^{-1}$$
$$+\frac{1}{8}D_r(\boldsymbol{\xi}\cdot\boldsymbol{\nabla})^2D_r^{-1}+\frac{i}{32\epsilon}\Big(D_s\boldsymbol{\xi}\cdot\boldsymbol{\nabla}D_s^{-1}+D_r\boldsymbol{\xi}\cdot\boldsymbol{\nabla}D_r^{-1}\Big)$$
$$\times\Big(\boldsymbol{\xi}\cdot\boldsymbol{\nabla}D^{-1}_{r\alpha\beta}\xi_\alpha\xi_\beta+4\boldsymbol{\xi}\cdot\boldsymbol{\nabla}D^{-1}_{s\alpha\beta}\eta_\alpha\eta_\beta\Big)-\frac{1}{2}\left(\frac{1}{16\epsilon}\right)^2(\boldsymbol{\xi}\cdot\boldsymbol{\nabla}D^{-1}_{r\alpha\beta}\xi_\alpha\xi_\beta+4\boldsymbol{\xi}\cdot\boldsymbol{\nabla}D^{-1}_{s\alpha\beta}\eta_\alpha\eta_\beta\Big)^2$$
$$+\frac{i}{64\epsilon}(\boldsymbol{\xi}\cdot\boldsymbol{\nabla})^2D^{-1}_{r\alpha\beta}\xi_\alpha\xi_\beta+\frac{i}{16\epsilon}(\boldsymbol{\xi}\cdot\boldsymbol{\nabla})^2D^{-1}_{s\alpha\beta}\eta_\alpha\eta_\beta\Big]\psi(\mathbf{r},\mathbf{R},t)$$
$$+\Big[\frac{1}{2}D_r\boldsymbol{\xi}\cdot\boldsymbol{\nabla}D_r^{-1}+\frac{1}{2}D_s\boldsymbol{\xi}\cdot\boldsymbol{\nabla}D_s^{-1}+\frac{i}{16\epsilon}\boldsymbol{\xi}\cdot\boldsymbol{\nabla}D^{-1}_{r\alpha\beta}\xi_\alpha\xi_\beta+\frac{i}{4\epsilon}\boldsymbol{\xi}\cdot\boldsymbol{\nabla}D^{-1}_{s\alpha\beta}\eta_\alpha\eta_\beta\Big]\boldsymbol{\xi}\cdot\boldsymbol{\nabla}\psi$$
$$+\frac{1}{2}(\boldsymbol{\xi}\cdot\boldsymbol{\nabla})^2\psi+\frac{1}{2}(\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R)^2\psi\Big\}, \tag{A3}$$

where $Z = [2\pi i\epsilon D_s(\mathbf{r})]^{-1}\,[8\pi i\epsilon D_r(\mathbf{r})]^{-1/2}$ and D_s, D_r are defined as $D_{s,r} = [\det D_{s,r}]^{1/2}$. The gradient operator ∇ is acting on the relative variable $\mathbf{r}$. After performing the various Gauss integrals on $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$, and rearranging terms, the resulting Hamilton operator may be written in the form

$$H = -\frac{1}{2}D_{s\alpha\beta}\nabla^R_\alpha\nabla^R_\beta - 2\nabla_\alpha D_{r\alpha\beta}\nabla_\beta - \frac{1}{2}(\nabla_\alpha\nabla_\beta D_{r\alpha\beta}) \ . \tag{A4}$$

Let us note again that this Hamiltonian is Hermitian. In terms of the eigenvalues of the diffusion tensors $D_s^{\parallel,\perp}$ and $D_r^{\parallel,\perp}$, defined by

$$D_{s,r\alpha\beta} = D^{\parallel}_{s,r}(r)\hat{\mathbf{r}}_\alpha\hat{\mathbf{r}}_\beta + D^{\perp}_{s,r}(\delta_{\alpha\beta} - \hat{\mathbf{r}}_\alpha\hat{\mathbf{r}}_\beta) \ , \tag{A5}$$

where $\hat{\mathbf{r}} = \mathbf{r}/r$ is the unit vector in the direction $\hat{\mathbf{r}}$, H is given by

$$H = -\frac{1}{2}(D^{\parallel}_s - D^{\perp}_s)(\hat{\mathbf{r}}\cdot\nabla_R)^2 - \frac{1}{2}D^{\perp}_s(\nabla_R)^2 - 2\Big[D^{\parallel}_r(\nabla)^2 + \frac{\partial D^{\parallel}_r}{\partial r}\frac{\partial}{\partial r} + \frac{1}{4}\Big((\nabla)^2 D^{\parallel}_r\Big) + \frac{1}{4r}\frac{\partial}{\partial r}(D^{\parallel}_r - D^{\perp}_r)\Big] \ . \tag{A6}$$

This result can also be obtained from the classical Hamilton function by properly quantizing the position and momentum variables. From the Lagrangian

$$L_{\text{inst}} = \frac{1}{2}D^{-1}_{s\alpha\beta}\dot{R}_\alpha\dot{R}_\beta + \frac{1}{8}D^{-1}_{r\alpha\beta}\dot{r}_\alpha\dot{r}_\beta \ , \tag{A7}$$

the generalized momenta are obtained as $p_\alpha = \frac{\partial L}{\partial \dot{r}_\alpha} = \frac{1}{4}D^{-1}_{r\alpha\beta}\dot{r}_\beta$ and $P_\alpha = \frac{\partial L}{\partial \dot{R}_\alpha} = D^{-1}_{s\alpha\beta}\dot{R}_\beta$.

The Hamilton function follows as

$$H_{\text{cl}} = \frac{1}{2}D_{s\alpha\beta}(r)P_\alpha P_\beta + 2D_{r\alpha\beta}(r)p_\alpha p_\beta \ . \tag{A8}$$

The corresponding Hamilton operator is derived by replacing $\mathbf{p}, \mathbf{P}$ and $\mathbf{r}, \mathbf{R}$ by their operator equivalents, after the Hamilton function has been properly symmetrized:

$$H = \frac{1}{2}D_{s\alpha\beta}(r)P_\alpha P_\beta + \frac{1}{2}[p_\alpha p_\beta D_{r\alpha\beta}(r) + 2p_\alpha D_{r\alpha\beta}p_\beta + D_{r\alpha\beta}p_\alpha p_\beta] \ . \tag{A9}$$

Using the identity $p_\alpha D_{r\alpha\beta} = D_{r\alpha\beta}p_\alpha + (p_\alpha D_{r\alpha\beta})$ and in position space representation, where $\mathbf{p} = -i\nabla$ and $\mathbf{P} = -i\nabla^R$, one finds (A9) to be identical to (A4). It is remarkable that this result depends on the choice of the full symmetrization procedure [with respect to the position of the operator $\mathbf{p}$ relative to the operator $D_{\alpha\beta}(r)$]. For example, the choice $Dp^2 \to \frac{1}{2}(Dp^2 + p^2D)$ leads to a result different from (A7).

In the presence of an external magnetic field, as included in (29), the additional terms obtained from the above procedure are exactly the ones obtained by the minimal coupling scheme, i.e., by replacing the gradient operators by their gauge-invariant counterparts:

$$-i\nabla \to -i\nabla + e\mathbf{A}(\mathbf{r}) \ ,$$
$$-i\nabla^R \to -i\nabla^R + 4e\cdot\mathbf{A}(\mathbf{R}) \ , \tag{A10}$$

where the charges e and $4e$ are taken from the action as given in (29).

APPENDIX B: BOUND STATE OF A QUANTUM PARTICLE WITH POSITION-DEPENDENT MASS

The Lagrangian of a particle with mass dependent on position is $L = \frac{1}{2}m(\mathbf{r})\dot{\mathbf{r}}^2$. The corresponding classical Hamiltonian $H_{\text{cl}} = \mathbf{p}^2/2m(\mathbf{r})$ can be quanitized as shown in Appendix A to give $H = \frac{1}{8}[\frac{1}{m}\mathbf{p}^2 + 2\mathbf{p}\frac{1}{m}\mathbf{p} + \mathbf{p}^2\frac{1}{m}]$. It is convenient to move factors of $[m_0/m(\mathbf{r})]^{1/2}$ to the left and to the right, so that the Hamiltonian takes the form

$$H = [m_0/m(\mathbf{r})]^{1/2}\Big[-\frac{1}{2m_0}\nabla^2 - \frac{1}{8m_0}[\nabla^2\ln m(\mathbf{r})]\Big] \times [m_0/m(\mathbf{r})]^{1/2} \ , \tag{B1}$$

where $m_0 = m(\mathbf{r}=0) > 0$. We also assume $\nabla m(\mathbf{r}) = 0$ and $m(\mathbf{r}) = m_\infty$ for $r \geq r_0$. The Hamiltonian (B1) is manifestly Hermitian. If $m(\mathbf{r})$ depends only on $r = |\,\mathbf{r}\,|$, which is the case we will discuss, H is rotation invariant and its eigenfunctions may be classified according to angular momentum.

We will show that the spectrum of H has at least one bound state (in 2D space), which is an s-wave state. The Schrödinger equation takes the form

$$\Big[-\frac{1}{2m_0}\nabla^2 + U(\mathbf{r})\Big]\tilde{\psi}(\mathbf{r}) = \frac{m(\mathbf{r})}{m_0}E\tilde{\psi}(\mathbf{r}) \ , \tag{B2}$$

where

$$U(\mathbf{r}) = -\frac{1}{8m_0}\nabla^2\ln m(\mathbf{r}) \quad r \leq r_0 \ , \tag{B3}$$

is a potential function, and $\tilde{\psi}(\mathbf{r}) = [m_0/m(\mathbf{r})]^{1/2}\psi(\mathbf{r})$.

In the regime outside the potential (region II), $r \geq r_0$, the s-wave solutions for negative energy E_b are given by

$$\tilde{\psi}_{\text{II}}(\mathbf{r}) = C_{\text{II}}K_0(\kappa r) \ , \tag{B4}$$

where $\kappa^2 = 2m_\infty \mid E_b \mid$ and $K_0(x)$ is a Bessel function.

Inside the potential region (region I), $r < r_0$, one can neglect the term $\frac{m(\mathbf{r})}{m_0}E\tilde{\psi}$ on the rhs of (B2) in comparison with the term $U(\mathbf{r})\tilde{\psi}(\mathbf{r})$ on the lhs. The remaining differential equation for the s-wave component is

$$\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}\tilde{\psi}_{\text{I}}(r) = -\frac{1}{4}\tilde{\psi}_{\text{I}}(r)\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}\ln m(r) \ . \tag{B5}$$

In the limit of weak interaction the energy of the bound state, E_b, will be small, and the extension of the wave function will be large, $\kappa^{-1} \gg r_0$. In this case $\tilde{\psi}(r)$ is approximately constant. We may integrate (B5) to obtain in first order in U

$$\frac{d}{dr}\tilde{\psi}_{\text{I}}(r) = -\frac{1}{4r}\tilde{\psi}_{\text{I}}(0)\Big[r\frac{d}{dr}\ln m(r)\Big]_0^r \ , \tag{B6}$$

which yields the logarithmic derivative at the point $r = r_0$, where the wave functions of the inner and outer regimes have to be matched,

$$\left.\frac{1}{\tilde{\psi}_{\rm I}}\frac{d}{dr}\tilde{\psi}_{\rm I}^{(1)}\right|_{r=r_0} = 0 \ . \tag{B7}$$

The logarithmic derivative of the wave function in regime II is

$$\left.\frac{1}{\tilde{\psi}_{\rm II}}\frac{d}{dr}\tilde{\psi}_{\rm II}\right|_{r=r_0} = \frac{1}{r_0 \ln \kappa r_0} \ . \tag{B8}$$

We conclude that in first order in U there is no bound state, since (B7) and (B8) are only equal for $\kappa = 0$.

Let us now consider the second-order correction. We integrate (B5) to get

$$\frac{d}{dr}\tilde{\psi}_{\rm I}(r) = -\frac{1}{4r}\int_0^r dr \tilde{\psi}_{\rm I}(r)\frac{d}{dr}r\frac{d}{dr}\ln m(r) \ . \tag{B9}$$

and perform a partial integration of the rhs,

$$\frac{d}{dr}\tilde{\psi}_{\rm I}(r) = -\frac{1}{4}\tilde{\psi}_{\rm I}(r)\frac{d}{dr}\ln m(r)\,|_0^r + \frac{1}{4r}\int_0^r dr r \frac{d\tilde{\psi}_{\rm I}(r)}{dr}\frac{d}{dr}\ln m(r) \tag{B10}$$

Next we substitute the first-order result (B6) for $\frac{d\tilde{\psi}_{\rm I}(r)}{dr}$ into the integral on the rhs to obtain

$$\left.\frac{1}{\tilde{\psi}_{\rm I}}\frac{d\tilde{\psi}_{\rm I}^{(2)}}{dr}\right|_{r=r_0} = -\frac{1}{16 r_0}\int_0^{r_0} dr r\left[\frac{d}{dr}\ln m(r)\right]^2 = -\frac{2\lambda}{r_0} \ . \tag{B11}$$

Matching this result with the one in regime II, Eq. (B8), yields the energy of a bound state,

$$E_B = -\frac{\kappa^2}{2m_\infty} = -\frac{1}{2m_\infty r_0^2}e^{-1/\lambda}. \tag{B12}$$

The coupling constant λ is defined by

$$\lambda = \frac{1}{32}\int_0^{r_0} dr r\left[\frac{d}{dr}\ln m(r)\right]^2 \ . \tag{B13}$$

Thus we find a bound state even though the potential U has repulsive components such that the integral over all space is zero; i.e., the coupling constant is zero in first order.

The second term in the effective potential (45), originating from the anisotropy of the diffusion tensor D_r, may be included in the above consideration. We note that its contribution to the coupling constant λ is zero in first order, since

$$\Delta\lambda^{(1)} = \frac{1}{4}\int_0^{r_0} d\rho \frac{1}{D}\frac{\partial}{\partial\rho}(D_r^{\parallel} - D_r^{\perp}) = 0 \ . \tag{B14}$$

This follows from the fact that $(D_r^{\parallel} - D_r^{\perp}) \to 0$ in the limits $\rho \to 0$ and $\rho \geq r_0$. However, in second order $\Delta\lambda$ contributes. The sign of $\Delta\lambda$ is not predetermined. We have calculated $\Delta\lambda$ numerically and find its contribution to be positive and about $\sim 10\%$ of λ for $k_0\ell = 0.01$.

We now turn to the calculation of the Green's funnction.

APPENDIX C: GREEN'S FUNCTION OF A QUANTUM PARTICLE WITH POSITION-DEPENDENT MASS IN 2D

In this appendix we derive the solution of the effective single-particle problem for the Cooperon, Eq. (44). We note that this problem is equivalent to the problem of a particle with mass $m(r)$ dependent on position $\mathbf{r}$, where the diffusion coefficient $D_r(r)$ has to be identified with $[4m(r)]^{-1}$. The single-particle Green's function for energy E satisfies the equation

$$(H - E)G(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') \ , \tag{C1}$$

where H is given by (B1). We are interested in calculating G for small distances $|\,\mathbf{r} - \mathbf{r}'\,|$. The source term on the rhs of (C1) may be replaced by the boundary condition

$$\lim_{r\to 0} G(\mathbf{r}, 0) = -\frac{1}{4\pi m_0}\ln r \ . \tag{C2}$$

In region II ($r \geq r_0$), the solution for finite energy $E = k^2/2m \geq 0$ is given by the outgoing wave solution

$$G_{\rm II}(r, 0) = C'_{\rm II}[J_0(kr) + iN_0(kr)] \ , \tag{C3}$$

where $J_0(x)$ and $N_0(x)$ are Bessel functions.

The solution in region I can be written in the form

$$G_{\rm I}(r, 0) = -\frac{1}{4\pi m_0}h(r)\ln r + C'_{\rm I}f(r) \ . \tag{C4}$$

Here $f(r)$ is a regular solution and $\frac{1}{2\pi}h(r)\ln r$ is a singular solution of (C1). The functions f and h are normalized at $r = 0$: $h(0) = f(0) = 1$. The derivative of the singular function is dominated by the $\ln r$ factor; so one may put $h(r) = 1$ for the following, but the derivative of the regular function has to be calculated along the lines of the discussion of the bound state in Appendix B, Eqs. (B2)–(B11). The result is

$$\left.\frac{df}{dr}\right|_{r_0} = -\frac{2\lambda}{r_0} \ , \tag{C5}$$

with λ defined by (B13).

Matching the Green's function and its derivative in the two regimes I and II gives the system of linear equations for the coefficients $C'_{\rm I}$ and $C'_{\rm II}$,

$$\begin{pmatrix} 1 & -i\frac{2}{\pi}\ln kr_0 \\ -\frac{2\lambda}{r_0} & -i\frac{2}{\pi r_0} \end{pmatrix}\begin{pmatrix} C'_{\rm I} \\ C'_{\rm II} \end{pmatrix} = -\frac{1}{4\pi m_0}\begin{pmatrix} -\ln r_0 & \\ \frac{2\lambda}{r_0}\ln r_0 & -\frac{1}{r_0} \end{pmatrix} \tag{C6}$$

In the limit of zero energy, $k \to 0$, one finds $C'_{\rm II} \to 0$ and

$$C'_{\rm I} = -\frac{1}{4\pi m_0}\left[-\ln r_0 + \frac{1}{2\lambda}\right] \ , \tag{C7}$$

and the Green's function for $r < r_0$ follows as

$$G_{\rm I}(r, 0) = -\frac{1}{4\pi m_0}\left[\ln\left(\frac{r}{r_0}\right) + \frac{1}{2\lambda}\right] \ . \tag{C8}$$

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