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Soliton-phonon interaction in anharmonic quasi-one-dimensional ferromagnetic crystals: Soliton-induced modification of the speed of sound

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The influence of the spin-phonon coupling on soliton characteristics and the soliton response on the lattice subsystem in a quasi-one-dimensional magnetic chain is examined. It was found that, in the case of the ferromagnetic coupling, these correlations may induce reduction of the effective exchange integral which, in the temperature range where one-dimensional ordering prevails, may be maximally about 1%. Consequently the soliton energy and width suffer the negligible modification, too. On the other hand, the magnetic subsystem may have a certain impact on elastic subsystem characteristics, the speed of sound in particular. The character of these changes depends strongly on the type of the excitations of the magnetic subsystem—solitons or magnons.

I. INTRODUCTION

The various phenomena in quasi-one-dimensional (quasi-1D) magnetic systems may be attributed to nonlinear dynamics and successfully explained in terms of the soliton solutions of the sine-Gordon (SG) equation.[1-7] Thus, for example, the appearance of the central peak in the energy spectrum of slow neutrons scattered on the quasi-1D ferromagnets $C_sN_iF_3$ and antiferromagnets $[(CH_3)_4N]M_nCl_3$ may be interpreted, on the basis of the SG model, in terms of an ideal gas of solitons.[2-7] However, the applicability of this idealized concept for the understanding of soliton properties is limited since the external perturbations, always present in realistic conditions, may affect soliton dynamics significantly.

In the present paper we shall examine the influence of the thermal fluctuations of crystal lattices on soliton properties and vice versa: the modification of phonon frequencies due to the soliton presence. The system we are considering consists of a compressible 1D magnetic chain placed in a magnetic field. The simplest Hamiltonian which may describe such a system includes the Heisenberg exchange interaction, anisotropy energy, and lattice Hamiltonian:

$$\mathcal{H}=-\frac{1}{2}\sum_{f,g}J(R_f-R_g)\mathbf{S}_f\cdot\mathbf{S}_g+\mathcal{H}_A-g\mu_B\sum_f \mathbf{H}\cdot\mathbf{S}_f+\mathcal{H}_l\ . \tag{1}$$

Here the spin-phonon coupling arises as a result of the exchange integral dependence on the instantaneous separation of magnetic ions which oscillate around their equilibrium positions. Thus the position of the magnetic ion in the lattice may be determined as $R_f=f+u_f$, where u_f denotes the ionic displacement from its equilibrium position f. Here $\mathcal{H}_A$ and

$$\mathcal{H}_l=\sum_f\frac{p_f^2}{2m}+\frac{1}{2}\sum_{f,g}\phi(R_f-R_g)$$

are the anisotropy energy (whose explicit form will be specified later) and lattice Hamiltonian, respectively, while H is the applied magnetic field.

The usual treatment of spin-phonon systems is based upon the application of the linear approximation for spin-phonon coupling [i.e., $J(R_f-R_g)\approx J_{f-g}+\chi(u_f-u_g)$] and the harmonic approximation for the lattice Hamiltonian. The dynamics of such a system is usually described by a system of coupled evolution equations for the magnetic and elastic degrees of freedom.[8-14] That approach, based upon the application of the classical approximation for the phonon subsystem, analogously to the large polaron problem,[15] leads to a physical picture which corresponds to the coexistence and simultaneous propagation of the coupled magnetic and lattice solitons. However, as a rule, in most of these papers favoring the classical picture, the discussion of the validity of the approximation involved is lacking. Therefore it is not clear to which realistic systems such a theoretical model corresponds. Following the analogy with polaron problem,[15] it could be expected that the above-described physical situation can be realized in systems with strong spin-phonon coupling. Namely, in the low-temperature limit, the degree of excitation of each phonon mode is influenced exclusively by spin-phonon coupling. Therefore the phonon modes become macroscopically (classically) occupied in the strong coupling limit. Unfortunately, the accessible experimental data for the typical quasi-1D magnetic systems[16-18] ($C_sN_iF_3$, for example) do not always support this condition. Therefore the theoretical treatment of soliton-phonon interactions should be completed by also taking into account the quantum nature of the lattice vibrations.

II. EFFECTIVE HAMILTONIAN AND VARIATIONAL PROCEDURE

In order to include the temperature effect in the magnetic subsystem characteristics, we shall take into account the anharmonic terms in the ionic displacement in both the exchange integral and lattice Hamiltonian. For

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that purpose we shall apply a pseudoharmonic approximation.[19–23] It consists of a transition from the model Hamiltonian (1) to an equivalent one with renormalized energy parameters, which should be determined self-consistently using the Bogoliubov theorem.[19–21] We first assume that the system is well described by the trial Hamiltonian

$$\mathcal{H}_0=-\frac{1}{2}\sum_{f,g}\tilde{J}_{f-g}(\theta)\mathbf{S}_f\cdot\mathbf{S}_g+\mathcal{H}_A-g\mu_B\sum_f \mathbf{H}\cdot\mathbf{S}_f+\mathcal{H}_l\ , \tag{2a}$$

where

$$\mathcal{H}_l=\frac{1}{2}\sum_q\left[\frac{p_q p_{-q}}{M}+M\Omega_q^2 u_q u_{-q}\right]\quad(\theta=k_BT) \tag{2b}$$

represents Hamiltonian of the so-called pseudoharmonic phonons and p_q and u_q are Fourier transforms of the momentum and displacement operators of the magnetic ion, respectively. $\tilde{J}_{f-g}$ and Ω_q are variational parameters which may be determined by minimizing the trial free energy of the system:

$$F_1=F_0+\langle\mathcal{H}-\mathcal{H}_0\rangle_0\ , \tag{3}$$

where $F_0=F_{\rm ph}+F_s$ represents the free energy of the fully decoupled spin-phonon model, while $\langle\cdots\rangle_0$ denotes averaging over the assembly of the noninteracting spin and vibrational excitations. Therefore we may write

$$\langle\mathcal{H}-\mathcal{H}_0\rangle_0=-\frac{1}{2}\sum_{f,g}\left[\langle J(R_f-R_g)\rangle_0-\tilde{J}_{f-g}(\theta)\right]\langle\mathbf{S}_f\cdot\mathbf{S}_g\rangle_0+\frac{1}{2}\left[\sum_{f,g}\langle(\phi(R_f-R_g)\rangle_0-\sum_q M\Omega_q^2\langle u_q u_{-q}\rangle_0\right]\ . \tag{4}$$

Following Mašković, Sajfert, and Marinković,[23] we may expand $J(R_f-R_g)$ and $\phi(R_f-R_g)$ into a Fourier series, and so the corresponding average values in (4) become

$$\langle Y(R_f-R_g)\rangle_0=\frac{1}{N}\sum_k Y(k)e^{ik(f-g)a}\langle e^{ik(u_f-u_g)}\rangle_0\ . \tag{5}$$

Here a denotes the lattice constant along the chain, while $Y(k)=\sum_p e^{ikp}Y(p)$, where Y may be either J or ϕ.

Calculation of the above average value is straightforward[19–23] and so we obtain

$$\langle Y(R_f-R_g)\rangle_0=\frac{1}{N}\sum_k Y(k)e^{ik(f-g)a}\exp\left[-\frac{2k^2}{N}\sum_q\langle u_q u_{-q}\rangle_0[1-\cos q(f-g)a]\right]\ . \tag{6}$$

Now we may determine the variational parameters $\tilde{J}_{f-g}(\theta)$ and Ω_q, demanding stationarity of the trial free energy over the $\tilde{J}_{f-g}(\theta)$ and Ω_q or equivalently over the correlation functions $\langle u_q u_{-q}\rangle_0$ and $\langle\mathbf{S}_f\cdot\mathbf{S}_g\rangle_0$:[22]

$$\frac{\partial F_1}{\partial\langle\mathbf{S}_f\cdot\mathbf{S}_g\rangle_0}=0\ ,\quad \tilde{J}_{f-g}(\theta)=\langle J(R_f-R_g)\rangle_0\ , \tag{7a}$$

$$\frac{\partial F_1}{\partial\langle u_q\cdot u_{-q}\rangle_0}=0\ ,\quad \Omega_q^2=-\frac{2}{MN^2}\left\{\sum_{f,g,k}k^2\left[\phi(k)-\frac{J(k)}{2}\langle\mathbf{S}_f\cdot\mathbf{S}_g\rangle_0\right]e^{-k^2a^2G(\theta)}e^{ik(f-g)a}\right\}[1-\cos q(f-g)a]\ , \tag{7b}$$

$$G(\theta)=\frac{2}{Na^2}\sum_q\langle u_q u_{-q}\rangle_0[1-\cos q(f-g)a]\ . \tag{7c}$$

In the nearest-neighbor approximation, $f-g=l$, where $l=\pm1$. Now the effective exchange integral attains the simplified form

$$\tilde{J}_{f-g}\equiv\tilde{J}(\theta)=\frac{1}{N}\sum_k e^{ika}e^{-k^2a^2G(\theta)}J(k)\ .$$

According to Bennett,[24] $J(k)$ is simply

$$J(k)\equiv\sum_{p=\pm1}J_p e^{ika}=2J\cos ka\ ,$$

where J is the nearest-neighbor exchange integral, which yields $\tilde{J}(\theta)=J\mathcal{F}(\theta)$, where

$$\mathcal{F}(\theta)=\frac{2}{N}\sum_k e^{-k^2a^2G(\theta)}\cos^2 ka\ . \tag{8}$$

It can be calculated in the continuum approximation

$$\frac{1}{N}\sum_k\cdots\to\frac{a}{2\pi}\int_{-\pi/a}^{\pi/a}\cdots dk\ .$$

Therefore the magnetic subsystem in the presence of the thermal motion of magnetic ions may be described by the effective Hamiltonian

$$\mathcal{H}_{\rm eff}=-\tilde{J}(\theta)\sum_f\mathbf{S}_f\cdot\mathbf{S}_{f+1}+\mathcal{H}_A-g\mu_B\sum_f\mathbf{H}\cdot\mathbf{S}_f\ . \tag{9}$$

Using an analogous procedure for the calculation of the $\phi(k)$, we find an expression for the frequency of "pseudoharmonic phonons":

$$\Omega_q^2=\frac{8\phi}{Ma^2}\left[1-\frac{J}{\phi}\frac{1}{N}\sum_f\langle \mathbf{S}_f\cdot\mathbf{S}_{f+1}\rangle_0\right]\times(1-\cos qa)\frac{\partial\mathcal{F}(\theta)}{\partial G(\theta)}\ , \tag{10}$$

where ϕ represents interaction potential, the nonmagnetic one, between neighboring magnetic ions.

From this equation one can find the speed of sound as modified by the spin-phonon interaction $c=\lim_{q\to 0}(\Omega_q/q)$,

$$c(\theta,H)=c_0(\theta)\left[1-\frac{J}{\phi}\frac{1}{N}\sum_f\langle \mathbf{S}_f\mathbf{S}_{f+1}\rangle_0\right]^{1/2}\ , \tag{11}$$

where $c_0(\theta)=2\sqrt{(\phi/M)[\partial\mathcal{F}(\theta)/\partial G(\theta)]}$ denotes the speed of sound in the absence of spin-phonon coupling. Here we assume that $\phi<0$ as a result of the stability condition of the lattice.[19]

III. CONSEQUENCE OF THE SPIN-LATTICE CORRELATIONS FOR THE SOLITON PROPERTIES

In what follows we shall restrict ourselves to an examination of the 1D ferromagnetic with planar (easy-plane) anisotropy $\mathcal{H}_A=A\sum_f(S_f^z)^2$, the magnetic field being perpendicular to the chain direction. If we take the chain direction to be along the z axis (x-y plane is the easy plane) and if the magnetic field is applied along the x axis, then in accordance with a well-known procedure,[2,6,7] the magnetic subsystem as modified by the phonon field may be described by the SG model:

$$\mathcal{H}_{\text{eff}}=S^2\tilde{J}(\theta)a^2\int\frac{dz}{a}\left\{\frac{1}{2}\left[\left[\frac{\partial\varphi}{\partial z}\right]^2+\frac{1}{v_0^2}\left[\frac{\partial\varphi}{\partial t}\right]^2\right]+k_0^2(1-\cos\varphi)\right\}\ , \tag{12}$$

where v_0 and k_0 are related to the parameters of the original Hamiltonian as $v_0^2=2A\tilde{J}(\theta)S^2a^2/\hbar^2$ and $k_0^2=g\mu_BH/S\tilde{J}(\theta)a^2$. Here $A>0$ is the easy-plane anisotropy constant, while φ is the polar angle of the magnetization vector.

Let us now calculate the contribution of SG solitons to the static spin correlation function in (11). Here we first apply the continuum approximation to obtain

$$\frac{1}{N}\sum_j\langle \mathbf{S}_f\cdot\mathbf{S}_{f+1}\rangle_0\approx S^2-\frac{S^2a^2}{2N}\int_{-\infty}^{\infty}\left\langle\left[\frac{\partial\varphi(z,t)}{\partial z}\right]^2\right\rangle_0\frac{dz}{a}\ . \tag{13}$$

Then, using the soliton solution of the SG equation,

$$\left[\frac{\partial\varphi}{\partial z}\right]^2=\frac{4k_0^2}{1-v^2/v_0^2}\text{sech}^2k_0\gamma(z-z_0) \tag{14}$$

$[\gamma^{-1}=\sqrt{1-(v/v_0)^2}]$, we finally obtain

$$J\frac{1}{N}\sum_f\langle \mathbf{S}_f\cdot\mathbf{S}_{f+1}\rangle_0\approx JS^2-\frac{J}{\tilde{J}(\theta)}\frac{1}{N}\langle E_{\text{sol}}(v)\rangle_0\ . \tag{15}$$

Here $E_{\text{sol}}(v)=8\tilde{J}(\theta)S^2k_0a\gamma=\varepsilon_s\gamma$ denotes the soliton energy (ε_s is the soliton rest energy).

The average value in (15) will be calculated within the ideal soliton gas approximation, which is applicable in the low-temperature limit $k_BT\ll\varepsilon_s$. The procedure of averaging is the standard one and may be carried out in two steps.[2,6,7,25] We first calculate $\langle\cdots\rangle_1$ assuming that the only one soliton is excited in the system. Here $\langle\cdots\rangle_1$ denotes the calculation of the following integral over the soliton momentum (p) and position (z_0):

$$\langle\cdots\rangle_1=\frac{1}{Z_1}\int\int\frac{dp\,dz_0}{h}(\cdots)e^{-E_{\text{sol}}(v)/\theta}\ . \tag{16}$$

It is easy to prove the relation

$$\langle E_{\text{sol}}(v)\rangle_1=-\frac{1}{Z_1}\frac{\partial Z_1}{\partial\beta}\quad\left[\beta=\frac{1}{\theta}\right]\ , \tag{17}$$

where

$$Z_1=\int\int\frac{dp\,dz_0}{h}e^{-E_{\text{sol}}(v)/\theta}$$

is the single-soliton partition function. In the low-temperature limit, it approaches[6]

$$Z_1=\frac{L}{hv_0}\left[\frac{\pi}{2}\right]^{1/2}\varepsilon_s(\beta\varepsilon_s)^{-1/2}e^{-\beta\varepsilon_s}\ , \tag{18}$$

where L is the length of the magnetic chain. From (17) and (18), we obtain $\langle E_{\text{sol}}(v)\rangle_1=\theta/2+\varepsilon_s$. Generalization to the system with small but finite soliton densities is straightforward,[2,6,7,25] and it simply demands the multiplication of $\langle\cdots\rangle_1$ by an average number of solitons: $N_s=4Lk_0(\beta\varepsilon_s/2\pi)^{1/2}e^{-\beta\varepsilon_s}$,[2,6,7] which finally results in

$$\frac{J}{N}\sum_f\langle \mathbf{S}_f\cdot\mathbf{S}_{f+1}\rangle_0\approx JS^2-\frac{J}{\tilde{J}(\theta)}\frac{n_sa}{4}(\theta+2\varepsilon_s)\ . \tag{19}$$

Here $n_s=2N_s/L$ is the soliton density. The factor of 2 is due to the presence of the equal number of solitons and antisolitons in the system.

With the help of this result, one can examine the character of the soliton impact on the phonon subsystem, which is determined through the explicit dependence of the speed of sound on the soliton parameters. However, before that, we first have to estimate the changes of the effective exchange integral due to spin-phonon coupling.

As a first step, we shall determine the explicit temperature dependence of the function $G(\theta)$, which, after the substitution of the phonon displacement correlation function $\langle u_qu_{-q}\rangle_0$ into (7c), attains the form

$$G(\theta)=\frac{2}{Na^2}\sum_q\frac{\hbar}{2M\Omega_q}\left[\bar{\nu}_q+\frac{1}{2}\right](1-\cos qa)\ , \tag{20}$$

where $\bar{\nu}_q=(e^{\hbar\Omega_q/\theta}-1)^{-1}$ is the phonon average number.

Substituting the summation over the phonon quasimomenta by an integration and using the explicit form of Ω_q, $G(\theta)$ becomes

$$G(\theta)=\frac{\pi\hbar}{8Mac(H,\theta)}\times\left[1+\frac{4a^2\theta^2}{\pi\hbar^2c^2(H,\theta)}\int_0^{\pi c(H,\theta)\hbar/a\theta}x(e^x-1)^{-1}dx\right]. \tag{21}$$

If we focus our attention on the archetypical example of a 1D ferromagnet $C_sN_iF_3$ where 1D ordering appears in the temperature range $3\le T\le 16$ K,[2,6,7] we may safely utilize the low-temperature limit for the calculation of the integral in (21). Namely, substituting the known values of the system parameters[2,6,7] $J/k_B=23.6$ K, $A/k_B\approx 5$ K, $S=1$, $g=2.4$, and $a=2.6$ Å, we may extend the integration boundary to infinity, while at the same time $(e^x-1)^{-1}$ may be approximated by e^{-x}, and the integral in (11) is practically equal to unity.

According to the explicit dependence of the renormalized speed of sound $c(H,\theta)$ on $G(\theta)$, relation (21) represents a rather complicated self-consistent equation for $G(\theta)$, and in the general case it cannot be found in a closed form. However, knowing the values of the basic physical parameters of the system, we may distinguish two limiting cases when $\mathcal{F}(\theta)$ and $G(\theta)$ have relatively simple forms. The first one corresponds to the high values of $G(\theta)$ $[G(\theta)\gg 1]$, while the other one is related to the opposite limit. For $G(\theta)\gg 1$ the Gaussian term in the expression for $\mathcal{F}(\theta)$ [Eq. (8)] rapidly decreases so that the integration boundaries may be extended to infinity yielding $\mathcal{F}(\theta)=[\pi G(\theta)]^{-1/2}(1+e^{-1/G})$.

Unfortunately, this result cannot be used for the analysis of the soliton properties in $C_sN_iF_3$ since it is relevant in the high-temperature limit only, but when 1D ordering is no longer preserved.[2,6,7] Namely, using the data for $C_sN_iF_3$ and according to Eq. (20), one can see that G may be greater than 1 for the highly populated $(\bar{\nu}_q\gg 1)$ phonon modes. Therefore, for this particular case, we should restrict ourselves to the calculation of $\mathcal{F}(\theta)$ assuming $G(\theta)\ll 1$. A preliminary estimate, neglecting the renormalization of the speed of sound due to spin-phonon coupling, confirms this statement. In that case we may expand the exponent in Eq. (8), and keeping just the two first terms, we easily obtain $\mathcal{F}(\theta)\approx 1-3.8G(\theta)$. Using this expression to obtain $\mathcal{F}(\theta)$, we find $G(\theta)\approx\alpha(1+\beta T^2)$, where $\alpha=1.4\times10^{-3}$ and $\beta=4.7\times10^{-4}K^{-2}$. Thus we may conclude that the spin-phonon correlation leads to a negligible reduction of the effective exchange integral. Using the data for $C_sN_iF_3$, we may estimate this reduction to be maximally about 1%. Consequently, the soliton width and energy, being both proportional to $\sqrt{\tilde{J}(\theta)}$, also decrease, while its effective mass increases. The degree of these changes with respect to their bare values is also very small. Since the soliton peak in neutron scattering in $C_sN_iF_3$ has considerable strength at low temperatures [$T\sim 6$ K (Ref. 2)], these temperature variations of $\tilde{J}(\theta)$ may not have a significant influence on the neutron-scattering data.

However, there arises an interesting possibility for an indirect experimental examination of solitons in accordance with the above predicted magnetic field dependence of the longitudinal speed of sound [Eq. (12)], which may be quite different for different types of the excitations of the magnetic subsystem:

$$c(H,\theta)=c_0(\theta)\sqrt{1-\psi(H,\theta)}, \tag{22}$$

where

$$\psi(H,\theta)=\frac{JS^2}{\phi}-\frac{Jn_sa}{4\tilde{J}(\theta)\phi}(\theta+2\varepsilon_s) \tag{23a}$$

for solitons and

$$\psi(H,\theta)=\frac{JS^2}{\phi}-\frac{JS^2\hbar v_0}{\tilde{J}(\theta)\phi}\frac{1}{N}\sum_k\frac{k^2}{\sqrt{k^2+k_0^2}}\left[\bar{\nu}_k+\frac{1}{2}\right] \tag{23b}$$

for magnons. Here $\bar{\nu}_k=(e^{\hbar\omega_k/\theta}-1)^{-1}$ is the magnon average number $(\omega_k=v_0\sqrt{k^2+k_0^2})$. Equation (23b) follows straightforwardly from a linearized version of the SG model $(\cos\varphi^2\approx 1-\frac{1}{2}\varphi^2)$, which may be quantized in the standard way.[26]

Summation over the magnon quasimomenta is rather complicated and cannot be performed exactly. However, in the low-temperature limit, which is the most interesting one, one can neglect the magnon occupation number $(\bar{\nu}_k\ll 1)$, so that the dominant contribution arises from the second term in (23b). Using the standard procedure, we may replace the summation over k by an integration. It gives $(S=1)$

$$\psi(H,\theta)=\frac{J}{\phi}-\frac{\hbar v_0}{4\phi}\left[\left[\frac{\pi^2}{a^2}+k_0^2\right]^{1/2}-\frac{a}{\pi}k_0^2\ln\frac{\pi/a+\sqrt{\pi^2/a^2+k_0^2}}{k_0}\right]. \tag{24}$$

Using the typical data for $C_sN_iF_3$ with H being of the order of a few kG, it follows that $\pi/a\gg k_0$, and so we finally may estimate the most important part of the soliton and magnon contribution to $\psi(\theta,H)$ as

$$\psi(H,\theta)\approx\begin{cases}\dfrac{J}{\phi}-D_1\dfrac{H^{5/4}}{\theta^{1/2}}e^{-D_2H^{1/2}/\theta} & \text{for solitons},\\[2ex] \dfrac{J}{\phi}-\dfrac{\hbar v_0\pi}{4\phi a}\left[1+\dfrac{g\mu_BH}{2\pi^2\tilde{J}(\theta)}\ln\dfrac{g\mu_BH}{4\pi^2\tilde{J}(\theta)}\right] & \text{for magnons},\end{cases} \tag{25}$$

$$D_1=\frac{64}{\sqrt{\pi}\phi}(g\mu_B)^{5/4}J^{1/4}\ ,\quad D_2=8\sqrt{g\mu_B\tilde{J}(\theta)}\ .$$

IV. CONCLUDING REMARKS

In the present paper we have examined the influence of spin-phonon coupling on the soliton properties in quasi-1D magnetic materials. It was shown that the thermal motion of magnetic ions may induce the reduction of the effective exchange integral, while at the same time the spin subsystem in response modifies the speed of sound. The degree of the changes of the effective exchange integral, within the utilized approximations, depends practically on the phonon subsystem characteristic only. Quite to the contrary, the character of the temperature and magnetic field dependence of the speed of sound depends on the type of magnetic coupling and on the type of excitation of the magnetic subsystem. On the basis of Eqs. (22) and (25), we obtain that the spin-lattice correlations for ferromagnetic coupling ($J>0$) increase the speed of sound. Increasing the magnetic field strength and temperature softens these modifications of the speed of sound if the system is exclusively populated by solitons. However, the magnon contribution shows a quite different behavior. Namely, using the typical data for $C_sN_iF_3$ and choosing H to be of the order of a few kG, we estimate $g\mu_B H/4\pi^2 J\sim\delta\times 10^{-2}$, where δ is of the order of unity. Consequently, the logarithmic term in (24) is negative, which implies an increase of the speed of sound with an increase of the strength of the magnetic field.

In the case of antiferromagnetic coupling ($J<0$), the above conclusions could be considerably modified. Namely, depending on the mutual ratio of J and ϕ, the renormalized speed of sound may be much smaller than in the absence of spin-phonon coupling. This, in response, influences a possibly significant increase of the parameter $G(\theta)$, which should lead to a quite different temperature behavior of the effective exchange integral. In particular, our preliminary investigations show that it can be highly reduced as compared to its bare value. Furthermore, the character of the temperature and magnetic field dependence of the renormalized speed of sound is also quite different. These details are very interesting in their own right and demand deeper investigations. The results of these examinations will be published separately.

The method applied here is the closest to that of Mašković, Sajfert, and Marinković[23] who have studied the temperature behavior of ferroelectric solitons utilizing the effective Hamiltonian approach, but without analysis of the response of soliton system on the frequency of lattice vibrations and speed of sound. Furthermore, we also must note that although the influence of the spin excitations on lattice properties in quasi-1D systems has been the subject of both experimental[16–18] and theoretical studies,[28–30] the soliton impact on phonon frequencies and speed of sound did not attract considerable attention.

The limitations of the applicability of the present approach for the understanding of soliton properties in a realistic system arise primarily as a result of the application of the idealized pure 1D model for both the magnetic and vibronic subsystems and as a result of the neglect of the remaining spin-phonon interaction. Although the magnetic subsystem may be considered as practically 1D, according to the smallness of the ratio of the interchain ($J_\perp$) to intrachain (J) magnetic interaction (10^{-2}–10^{-4}),[27] this is not necessarily the case for the phonon subsystem. Therefore examination of the influence of the three-dimensionality of the realistic phonon spectrum on the magnetic subsystem characteristics is of particular interest. However, according to the experimental examination of the elastic properties of $C_sN_iF_3$,[16–18] the phonon spectrum seems to possess considerable anisotropy, and so the approximation utilized above could be satisfactory.

The remaining spin-phonon interaction

$$V=\frac{1}{2}\sum_{f,g}[J(\mathbf{R}_f-\mathbf{R}_g)-\tilde{J}_{f-g}]\mathbf{S}_f\cdot\mathbf{S}_g$$

may have a certain impact on the soliton dynamics, and for a consistent treatment one should take into account also the consequences arising due to its presence. This term induces the energy exchange between soliton and phonons, which in turn causes the dynamic damping of soliton motion.[31] According to some previous examinations of the soliton-phonon system within other models,[31–33] we believe that a similar situation, as described there, may arise in this case too. Consequently, we expect that the soliton dynamics, as a result of the remaining interaction with the lattice should attain the character of the Brownian motion caused by a Cherenkov-like emission and absorption of real phonons.[31–33] This problem, its relevance for neutron-scattering experiments (i.e., the modification of the dynamical spin correlation function), and the corrections arising as a result of the difference in the dimensionalities of the constituent subsystems will be examined in a subsequent paper.

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