Propagation of elastic waves in semiconductor superlattices under the action of a laser field

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Within the framework of the dispersion-equation treatment, the characteristics of the propagation of elastic waves along one-dimensional superlattices radiated by an infrared laser field are investigated. The Kubo formalism is generalized to evaluate the conductivity tensor in this system. When the condition $s_0/\Delta d|J_0(eFd/\Omega)|\simeq 1$ is satisfied (2Δ and d are the miniband width and period of superlattice, F and Ω are amplitude and frequency of the laser field, respectively, and s_0 is the sound velocity in the absence of interaction), a strong renormalization of the sound velocity is predicted, along with intense absorption peaks indicating the localization of waves. The spectrum of plasma oscillations propagating along the superlattice axis is also calculated. The characteristic feature of these plasmons is the absence of Landau damping. In the dynamic localization regime the plasma oscillations along the superlattice axis disappear.

I. INTRODUCTION

Recently, there has been much activity in the area of the electronic behavior of quantum semiconductor structures (quantum wells and superlattices) interacting with strong far-infrared laser radiation (see, for example, Refs. 1–11). The most interesting effect in these systems is that, if the laser power and frequency are chosen appropriately, all the quasienergies of the system can be crossed. Particularly, in the superlattices (SL) all quasienergies group in a miniband with a finite width, and when these eigenstates cross, the miniband width reduces to zero (collapse of minibands).[7,8] In this situation, if the electron is initially localized in one of the wells (or sites), it can be found there again after many times. This phenomenon is called dynamic or laser-induced localization of electrons.[1–6] In the regime of dynamic localization (or collapsing of minibands), a number of interesting effects are predicted (see, for example, Refs. 9 and 10, and references therein). Recently we have shown that in the regime of dynamic localization, electrons in superlattices can be considered as internal resonances, which efficiently localize acoustic waves at the resonance frequency.[12] Note that the idea of attaining strong localization of acoustic waves by internal resonances (gas bubbles in a liquid or Helmholtz resonators in air) was suggested by Sornette and Souillard[13] (see also Refs. 14 and 15).

In this paper we present a theory of propagation of elastic waves in superlattices under far-infrared laser radiation. We will focus attention on the regime of the collapsing of minibands, when the localization of waves may be observed. We shall also discuss plasma oscillations and related phenomena in that system. Note that the propagation of elastic waves in superlattices (in the absence of a laser field) is investigated in various aspects in a larger number of works.[16–25]

The plan of this paper is as follows. In Sec. II the dispersion equation of the propagation of elastic waves is reviewed. It is shown that the attenuation length, sound velocity, and plasma frequency are determined by the complex dielectric function or complex conductivity tensor. In Sec. III we develop the Kubo formula to calculate the complex conductivity of superlattices under the laser field. In Sec. IV we analyze the conditions for the localization of acoustic waves in this system. In Sec. V we calculate the spectrum of plasma oscillations, and in Sec. VI we discuss these results and make some concluding remarks.

II. DISPERSION EQUATION OF THE PROPAGATION OF ACOUSTIC WAVES IN SUPERLATTICES

In this section we present a model that we have used to obtain the dispersion equation for longitudinal elastic waves in SL.[26,27] We assume that there is no piezoelectric effect, so that the interaction of the acoustic waves with the electrons can be realized only via the deformation potential. Furthermore, for simplicity the electron-phonon interaction in superlattices is considered the same as in the bulk semiconductors. As it is known an accurate evaluation of the influence of the superlattice potential leads to renormalization of the electron-phonon interaction constant by a factor of the order of unity. The basic system of equations describing the propagation of acoustic waves in solids consists of (i) the equation of motion

$$\rho\frac{\partial^2 u_i}{\partial t^2}-\tilde{\lambda}_{iklm}\frac{\partial u_{lm}}{\partial x_k}=-\Lambda_{ik}\frac{\partial n_1(\mathbf{x},t)}{\partial x_k}\ , \tag{2.1}$$

(ii) the Maxwell's equations

$$\mathrm{rot\,rot}\mathbf{E}_1+\frac{1}{c^2}\frac{\partial^2\mathbf{D}}{\partial t^2}=-\frac{4\pi}{c^2}\frac{\partial \mathbf{j}(\mathbf{x},t)}{\partial t}\ ,\qquad D_i=\varepsilon_{ij}E_{1j} \tag{2.2}$$

and (iii) the continuity equation

$$\mathrm{div}\mathbf{j}(\mathbf{x},t)+e\frac{\partial n_1(\mathbf{x},t)}{\partial t}=0\ , \tag{2.3}$$

where ρ is the density of the lattice, $\tilde{\lambda}_{iklm}$ is the modulus of elasticity tensor, $\mathbf{u}$ is the displacement vector, u_{ik} is the deformation tensor, Λ_{ik} is the tensor of the electron-phonon interaction constant, ε_{ij} is the tensor of dielectric constant of the lattice, $\mathbf{D}$ is the induction vector, and $n_1(\mathbf{x},t)$ is the deviation of electron concentration from the equilibrium value caused by the sound wave. The electric current density can be expressed in the form

$$j_i(\mathbf{x},t)=\sigma_{ij}\left[E_{1j}-\frac{1}{e}\Lambda_{lk}\frac{\partial u_{lk}}{\partial x_j}\right]\equiv\sigma_{ij}E_j\ , \tag{2.4}$$

where the first term in Eq. (2.4) is due to electric field of the space charge $\mathbf{E}_1$, and the second term is due to the deformation potential. Note that in the presence of the additional laser field $\mathbf{F}(t)=\mathbf{F}\sin\Omega t$, the linear response of the system to the electric field induced by acoustic wave $\mathbf{E}(t)=\mathbf{E}\exp(i\omega t-i\mathbf{qr})$ consists of all harmonics of type $\exp[i(\omega+n\Omega)t]$ [see Eq. (3.13)]. It should be emphasized that the current density $\mathbf{j}$ in Eq. (2.4) is only the harmonic with the frequency ω.

Assuming that the quantities $\mathbf{u}$, $\mathbf{E}_1$, and n_1 depend on the coordinates and on the time in accordance with the plane-wave law $\mathbf{u}$, $\mathbf{E}_1$, $n_1\sim\exp(i\omega t-i\mathbf{qx})$, we obtain the Fourier components of the nonequilibrium values of the electron density,

$$n_1(\mathbf{q},\omega)=A_l u_l\ , \tag{2.5}$$

where

$$A_l=-\frac{1}{e\omega}q_i\sigma_{ij}\left\{\frac{4i\pi\omega}{c^2q^2e}\Gamma_{jt}^{-1}\sigma_{ts}\Lambda_{lk}q_sq_k-\frac{1}{3}\Lambda_{lk}q_jq_k\right\}, \tag{2.6}$$

$$\Gamma_{ij}=\delta_{ij}-\frac{q_iq_j}{q^2}-\frac{\omega^2}{c^2q^2}\left[\varepsilon_{ij}-\frac{4\pi i}{\omega}\sigma_{ij}\right], \tag{2.7}$$

and ω and $\mathbf{q}$ are the frequency and wave vector of the acoustic wave, respectively.

Now substituting (2.5) into (2.1), we obtain a dispersion equation for the propagation of acoustic waves in a solid as follows:

$$\mathrm{Det}|-\rho\omega^2\delta_{ij}+\tilde{\lambda}_{iklj}q_kq_l+i\Lambda_{ik}q_kA_j|=0\ . \tag{2.8}$$

We shall consider the idealized situation where there is an absolute matching of the lattice parameters of adjacent layers forming superlattices; thus, all quantities become scalars with only one component $\tilde{\lambda}$, Λ, and ε. Then, it follows from the dispersion equation (2.8) that

$$\omega^2-s^2q^2=\frac{i\Lambda^4q^4\sigma\varepsilon}{\rho e^2\omega\left[\varepsilon-\frac{4i\pi\sigma}{\omega}\right]}\ , \tag{2.9}$$

where $\sigma(\mathbf{q},\omega)$ is the longitudinal conductivity and $s=(\tilde{\lambda}/\rho)^{1/2}$ is the sound velocity. If we suppose that $q=q_1+iq_2$, $s=s_0+\delta s$, $\sigma=\sigma_1+i\sigma_2$, and $q_2/q_1\ll1$, $\delta s/s_0\ll1$, the dispersion equation may be solved analytically. In this case, for sound velocity s and absorption coefficient $\Gamma=-q_2$ one can get

$$s=s_0\left[1+\frac{\Lambda^2q^2\varepsilon}{2\rho s_0^2e^2\omega}\frac{\left[\varepsilon\sigma_2+\frac{4\pi}{\omega}(\sigma_1^2+\sigma_2^2)\right]}{\left[\varepsilon+\frac{4\pi\sigma_2}{\omega}\right]^2+\left[\frac{4\pi\sigma_1}{\omega}\right]^2}\right], \tag{2.10}$$

$$\Gamma=\frac{\Lambda^2q^3}{2\rho e^2s_0^2\omega}\frac{\varepsilon^2\sigma_1}{\left[\varepsilon+\frac{4\pi\sigma_2}{\omega}\right]^2+\left[\frac{4\pi\sigma_1}{\omega}\right]^2}\ . \tag{2.11}$$

We put $\hbar=1$ throughout this paper.

The dispersion equation of plasma oscillations can be obtained as follows.[28] For sufficiently weak applied potential $V_0(\mathbf{x},t)=V_0\exp(i\omega t-i\mathbf{qx})$, the deviation of electron density $n_1(\mathbf{q},\omega)$ can be written as follows:

$$n_1(\mathbf{q},\omega)=F(\mathbf{q},\omega)V_0\ . \tag{2.12}$$

The deviation of electron density causes an additional screening potential $V_1(\mathbf{x},t)=V_1(\mathbf{q},\omega)\exp(i\omega t-i\mathbf{q}\cdot\mathbf{x})$, which is governed by Poisson's equation

$$\nabla^2V_1(\mathbf{x},t)=-4\pi e^2n_1(\mathbf{x},t)\ . \tag{2.13}$$

On the other hand, the deviation of electron density can be expressed in the form of a linear response to the total potential:

$$n_1(\mathbf{q},\omega)=X(\mathbf{q},\omega)V,\quad V=V_0+V_1\ . \tag{2.14}$$

From Eqs. (2.12)–(2.14) one gets

$$V=\frac{V_0}{1-\frac{4\pi e^2}{q^2\varepsilon}X(\mathbf{q},\omega)}\ , \tag{2.15}$$

and the spectrum of plasma oscillations is defined by the poles of the total potential V,

$$1-\frac{4\pi e^2}{q^2\varepsilon}\,\mathrm{Re}X(\mathbf{q},\omega)=0\ . \tag{2.16}$$

Note that the relation between conductivity $\sigma(\mathbf{q},\omega)$ and the function $X(\mathbf{q},\omega)$ can be obtained from Eqs. (2.3) and (2.4), and from the well-known relation $e\mathbf{E}=-\nabla V$:

$$\sigma(\mathbf{q},\omega)=\frac{e^2(\omega-i\nu)}{iq^2}X(\mathbf{q},\omega)\ . \tag{2.17}$$

In Eq. (2.17), ν is an adiabatic parameter and is taken to be very small. The relation (2.17) between $\sigma(\mathbf{q},\omega)$ and $X(\mathbf{q},\omega)$ is obtained here phenomenologically. It will be shown below that this relation can also be obtained from microscopic calculations (see Sec. III). It is seen from these equations that the character of the propagation of the elastic waves is determined completely by the complex conductivity tensor $\sigma(\mathbf{q},\omega)$. There are some different ways to calculate this fundamental quantity. The semiclassical approach to the calculation of the conductivity tensor uses the Boltzmann equation for the distribution function of electrons in the presence of acoustic waves. However, the Boltzmann-equation treatment is only valid when the wavelength of the acoustic waves is much less than the de Broglie wavelength of electrons or the wave vector **q** is very small. In Sec. III, we present a quantum approach to the calculation of the conductivity tensor based on the generalized Kubo formula for the conductivity in the far-infrared laser field.

III. GENERALIZED KUBO FORMULA FOR THE CONDUCTIVITY TENSOR

Let the system be acted upon by a weak electric field (induced by acoustic wave) $\mathbf{E}(z,t)=\mathbf{E}\exp(i\omega t-iqz)$ ($\mathbf{E}\|\mathbf{q}\|$ superlattice axis OZ), and by a strong laser field $\mathbf{F}(t)=\mathbf{F}\sin\Omega t$ (the laser field is described here in the dipole approximation, $\mathbf{F}$ and Ω are amplitude and frequency of the laser field, respectively and $\mathbf{F}\|OZ$). We shall restrict our investigation to the region where the wavelength of the acoustic wave λ is larger than the superlattice period d. In this case the system can be considered to be homogeneous and the conductivity tensor is connected with the current density $\mathbf{j}(\mathbf{q},t)$ by the relation $\mathbf{j}(\mathbf{q},t)=\sigma(\mathbf{q},t)\mathbf{E}(t)$. We shall also limit our investigation to the region of frequencies $\omega,\tau^{-1}<\langle\varepsilon_\perp\rangle,2\Delta<\Omega<\varepsilon_g$, where τ is the relation time, 2Δ is the ground miniband width, $\langle\varepsilon_\perp\rangle$ is the in-plane characteristic energy of electrons, and ε_g is the band gap between the ground and first excited minibands. These conditions at first show the validity of the Bloch picture of electron motion along the superlattice axis. It also means that both acoustic and electromagnetic waves do not cause interminiband transitions. Furthermore, the laser field cannot cause the intraminiband transitions (the real absorption of light is absent). These limitations are quite reasonable for experimental observation. For example, for GaAs/AlAs superlattices with corresponding 14/12 (13/13) monolayers of well and barrier width ($d=73.58$ Å) from a Kronig-Penny model for an electron ($m_{\mathrm{GaAs}}=0.067m_0$, $m_{\mathrm{AlAs}}=0.0895m_0$, band offset of 1 eV) one can get $2\Delta=3.1(2.7)$ meV and $\varepsilon_g=489.2$ (526.2) meV. So the above conditions can be satisfied for acoustic waves with frequency $\omega\simeq(2-5)\times10^{11}$ s^{-1}, temperature $T\le100$ K, and laser frequency in the range of $\Omega\simeq(1.5\times10^{13}-4.5\times10^{14}\,\mathrm{s}^{-1})$.

The quantum approach to the calculation of the current density makes use of the Liouville equation to determine the density matrix $\rho(t)$,

$$i\frac{\partial\rho(t)}{\partial t}=[H_0(t)+H_1^t,\rho(t)]\ . \tag{3.1}$$

Here $H_0(t)$ is a sum of two terms:

$$H_0(t)=H_e(t)+H_{\mathrm{sc}}=\sum_{\mathbf{p}}\varepsilon\left[\mathbf{p}-\frac{e}{c}\mathbf{A}(t)\right]a_{\mathbf{p}}^\dagger a_{\mathbf{p}}+H_{\mathrm{sc}}\ . \tag{3.2}$$

The first term in Eq. (3.2) is the Hamiltonian of electrons in the laser field. The second term includes the Hamiltonian of scatterers and their interaction with electrons. Let us suppose that electrons are scattered by "internal" phonons and charged impurities; in this case the Hamiltonian H_{sc} has a form

$$H_{\mathrm{sc}}=\sum_{\mathbf{k}}\omega_{\mathbf{k}}b_{\mathbf{k}}^\dagger b_{\mathbf{k}}+\sum_{\mathbf{pk}}C_{\mathbf{k}}a_{\mathbf{p}+\mathbf{k}}^\dagger a_{\mathbf{p}}(b_{\mathbf{k}}+b_{-\mathbf{k}}^\dagger)+\sum_{\mathbf{k}}\pi_{\mathbf{k}}a_{\mathbf{p}+\mathbf{k}}^\dagger a_{\mathbf{p}}\ , \tag{3.3}$$

where $\pi_{\mathbf{k}}=4\pi Ze^2/\varepsilon(k^2+r_D^{-2})$, $C_{\mathbf{k}}$ is the matrix element of electron-phonon scattering, $a_{\mathbf{p}}^\dagger$ and $a_{\mathbf{p}}$ are electron creation and annihilation operators, $b_{\mathbf{k}}^\dagger$ and $b_{\mathbf{k}}$ are creation and annihilation operators of phonons, and $\omega_{\mathbf{k}}$ is the phonon frequency with wave vector **k**. The vector potential of the laser field is defined in the dipole approximation by $-(1/c)\partial\mathbf{A}(t)/\partial t=\mathbf{F}\sin\Omega t$. For simplicity in the Hamiltonian (3.3), umklapp processes are neglected.

In the tight-binding single miniband approximation, the electron spectrum has the form

$$\varepsilon(\mathbf{p})=\varepsilon_\perp+\varepsilon_z=\frac{p_\perp^2}{2m}-\Delta\cos p_z d+\varepsilon_1\ , \tag{3.4}$$

where 2Δ is the miniband width of the superlattice, m is the in-plane electron effective mass, and ε_1 determines the position of the lowest miniband.

The interaction of electrons with the acoustic wave can be treated as the interaction with the self-consistent field $\mathbf{E}(z,t)$ and is regarded as an external perturbation:

$$H_1^t=\frac{eE}{iq}e^{i\omega t}\sum_{\mathbf{p}}a_{\mathbf{p}-\mathbf{q}}^\dagger a_{\mathbf{p}}\ . \tag{3.5}$$

The solution of Eq. (3.1) is split as usual in the form $\rho(t)=\rho_0(t)+\rho_1(t)$, and working in the linear response approximation with respect to H_1^t we have

$$i\frac{\partial\rho_0(t)}{\partial t}=[H_0(t),\rho_0(t)]\ , \tag{3.6}$$

$$i\frac{\partial\rho_1(t)}{\partial t}=[H_e(t)+\rho_1(t)]+[H_1^t,\rho_0(t)]-\frac{i\rho_1(t)}{\tau}\ . \tag{3.7}$$

In Eq. (3.7), the scattering effect (connected with H_{sc}) is treated by a simple relaxation-time approximation:[28,29] $[H_{\mathrm{sc}},\rho_1(t)]\to-i\rho_1(t)/\tau$. Note that in the system of Eqs. (3.6) and (3.7), the electric field $\mathbf{E}(t)$ is taken into account in the linear approximation. However, the laser field is included in $H_0(t)$ and $\rho_0(t)$ exactly. In Eq. (3.6), $\rho_0(t)$ is the nonequilibrium density matrix.

The solution of Eq. (3.7) has a form

$$\rho_1(t)=-i\int_{-\infty}^{t}dt'e^{(t'-t)/\tau}S(t,t')[H_1^{t'},\rho_0(t')]S(t',t)\ , \tag{3.8}$$

where

$$S(t,t')=T\exp\left[-i\int_{t'}^{t}H_e(x)dx\right]\ , \tag{3.9}$$

and T is the time-ordering operator.

The current density now can be calculated according to the formula

$$\mathbf{j}(\mathbf{q},t)=\mathrm{Tr}[\mathbf{j}(\mathbf{q},t)\rho_1(t)]\ , \qquad (3.10)$$

where the current density operator is

$$\mathbf{j}(\mathbf{q},t)=\tfrac{1}{2}\sum_{\mathbf{p}}[\mathbf{v}(\mathbf{p}+\mathbf{q},t)+\mathbf{v}(\mathbf{p})]a^\dagger_{\mathbf{p}+\mathbf{q}}a_{\mathbf{p}}\ ,$$
$$\mathbf{v}(\mathbf{p},t)=\frac{\partial\varepsilon\left[\mathbf{p}-\frac{e}{c}\mathbf{A}(t)\right]}{\partial\mathbf{p}}\ . \qquad (3.11)$$

Substituting (3.8) and (3.11) into (3.10), one gets

$$\mathbf{j}(\mathbf{q},t)=-\frac{e^{-t/\tau}}{2q}Ee^2\int_{-\infty}^{t}dt'e^{i(\omega-i\nu)t'}\sum_{\mathbf{p}}[\mathbf{v}(\mathbf{p}+\mathbf{q},t)+\mathbf{v}(\mathbf{p},t)][f_0(\mathbf{p}+\mathbf{q},t')-f_0(\mathbf{p},t')]\exp\left[-i\int_t^{t'}[\varepsilon_{\mathbf{p}+\mathbf{q}}(\tau)-\varepsilon_{\mathbf{p}}(\tau)]d\tau\right]\ , \qquad (3.12)$$

where $\nu=1/\tau$, $f_0(\mathbf{p},t)=\mathrm{Tr}[a^\dagger_{\mathbf{p}}a_{\mathbf{p}}\rho_0(t)]$, and $\varepsilon_{\mathbf{p}}(t)=\varepsilon[\mathbf{p}-(e/c)\mathbf{A}(t)]$.

In general the current density $\mathbf{j}(\mathbf{q},t)$ consists of all harmonics,

$$\mathbf{j}(\mathbf{q},t)=\sum_{n=-\infty}^{\infty}\sigma(\mathbf{q},\omega+n\Omega)e^{i(\omega+n\Omega)t}\mathbf{E}\ . \qquad (3.13)$$

Here we are interested only in the linear response of the system to the perturbation (3.5) with the frequency ω. (Note that it is consistent with the dispersion-equation treatment described in Sec. II.) The limitations on frequencies made above imply that we are far from resonance ($|\omega+n\Omega|\neq\omega$ for any n). From Eq. (3.12) one can get an expression for the zero-order Fourier component ($n=0$) of the conductivity tensor as follows:

$$\sigma(\mathbf{q},\omega)=\frac{\Omega}{2\pi}\int_{-\pi/\Omega}^{\pi/\Omega}dt\int_{-\infty}^{t}dt'e^{i(\omega-i\nu)(t'-t)}\left[-\frac{e^2}{2q}\right]$$
$$\times\sum_{\mathbf{p}}[v_z(\mathbf{p}+\mathbf{q},t)+v_z(\mathbf{p},t)][f_0(\mathbf{p}+\mathbf{q},t')-f_0(\mathbf{p},t')]\exp\left[-i\int_t^{t'}[\varepsilon_{\mathbf{p}+\mathbf{q}}(\tau)-\varepsilon_{\mathbf{p}}(\tau)]d\tau\right]\ . \qquad (3.14)$$

Equation (3.14) is an exact expression for the conductivity tensor in the time-dependent electric field $\mathbf{F}(t)$ [except for the relaxation-time approximation made in Eq. (3.7)]. To evaluate the conductivity tensor $\sigma(\mathbf{q},\omega)$ one should know the explicit form of the nonequilibrium distribution function $f_0(\mathbf{p},t)$. Generally it is quite a difficult problem. One of the ways to treat it is to replace the distribution function $f_0(\mathbf{p},t)$ by the equilibrium distribution function $f_0(\mathbf{p})$. This approximation is equivalent to neglecting the influence of the incident radiation on the energy distribution of electrons (heating effect), and it is valid only under the condition $e^2F^2/m\Omega^3\ll 1$ (high frequency and weak electric field) (see, for example, Refs. 30 and 31, and references therein).

The other, more accurate way to solve this problem is as follows. The quantum transport equation for the distribution function $f_0(\mathbf{p},t)$ in superlattices can be derived from Eq. (3.6) by the method of Refs. 31–33. Generally the distribution function $f_0(\mathbf{p},t)$ consists of all harmonics of type

$$f_0(\mathbf{p},t)=\sum_{n=-\infty}^{\infty}f_0^n(\mathbf{p})e^{in\Omega t}\ . \qquad (3.15)$$

The limitations on laser frequencies made above mean that the real absorption of light is absent. In this case, all higher harmonics of the distribution function (3.15) vanish, except the steady-state part $f_0^0(\mathbf{p})$ [below we denote it by $\tilde{f}_0(\mathbf{p})$]. The equation for $\tilde{f}_0(\mathbf{p})$ was given in Ref. 32 as follows:

$$\sum_{\mathbf{k}}|C_{\mathbf{k}}|^2P_1\delta(\beta_1)\{(N_{\mathbf{k}}+1)\tilde{f}_0(\mathbf{p}+\mathbf{k})[1-\tilde{f}_0(\mathbf{p})]-N_{\mathbf{k}}\tilde{f}_0(\mathbf{p})[1-\tilde{f}_0(\mathbf{p}+\mathbf{k})]\}$$
$$+\sum_{\mathbf{k}}|C_{\mathbf{k}}|^2P_2\delta(\beta_2)\{N_{\mathbf{k}}\tilde{f}_0(\mathbf{p}-\mathbf{k})[1-\tilde{f}_0(\mathbf{p})]-(N_{\mathbf{k}}+1)\tilde{f}_0(\mathbf{p})[1-\tilde{f}_0(\mathbf{p}-\mathbf{k})]\}$$
$$+\sum_{\mathbf{k}}|\pi_{\mathbf{k}}|^2P_1\delta(\beta_3)\{\tilde{f}_0(\mathbf{p}+\mathbf{k})[1-\tilde{f}_0(\mathbf{p})]-\tilde{f}_0(\mathbf{p})[1-\tilde{f}_0(\mathbf{p}+\mathbf{k})]\}=0\ , \qquad (3.16)$$

where $Z_{1,2n}=(1/n\Omega)[a_n(\mathbf{p}\pm\mathbf{k},\mathbf{F})-a_n(\mathbf{p},\mathbf{F})]$, $\beta_{1,2}=\frac{1}{2}[a_0(\mathbf{p}+\mathbf{k},\mathbf{F})-a_0(\mathbf{p},\mathbf{F})]\mp\omega_{\mathbf{k}}$, β_3 is identical with β_1 when $\omega_{\mathbf{k}}=0$, and

$$P_{1,2}=\prod_{n=1}^{\infty} J_0^2(Z_{1,2n}) ,$$

$$a_n(\mathbf{p},\mathbf{F})=\frac{2}{\pi}\int_0^{\pi}dx\,\cos nx\,\varepsilon[\mathbf{p}+(e/\Omega)\mathbf{F}\cos x] .$$

As mentioned in Ref. 32, in our case the absence of any real light absorption indicates that the phonons are strictly in equilibrium. In this case, the quantum transport equation (3.16) can be solved exactly. Direct substitution shows that the distribution function $\tilde{f}_0(\mathbf{p})$ has the exact form of the Fermi function with a change: $\varepsilon_z\to\tilde{\varepsilon}_z=-\tilde{\Delta}\cos p_z d$, $\tilde{\Delta}=\Delta J_0(eFd/\Omega)$. Thus, the distribution function $f_0(\mathbf{p},t)$ in Eq. (3.14) can be replaced by $\tilde{f}_0(\mathbf{p})$. The other quantities in (3.14) can be expressed in terms of the small parameter Δ/Ω, and in zero order in Δ/Ω one can get

$$\sigma(\mathbf{q},\omega)=\frac{ie^2}{2q}\sum_{\mathbf{p}}[v_z(\mathbf{p}+\mathbf{q})+v_z(\mathbf{p})]\frac{\tilde{f}_0(\mathbf{p}+\mathbf{q})-\tilde{f}_0(\mathbf{p})}{\omega-\tilde{\varepsilon}_{\mathbf{p}+\mathbf{q}}+\tilde{\varepsilon}_{\mathbf{p}}-i\nu} . \tag{3.17}$$

In the limit $\mathbf{q}\to 0$ and when the laser field is absent, from Eq. (3.17) one can get the result for the longitudinal conductivity $\sigma(0,\omega)$ obtained by semiclassical Boltzmann-equation treatment,[10]

$$\sigma(0,\omega)=-iAe^2N_e\Delta d^2\frac{1}{\omega-i\nu} , \tag{3.18}$$

where $A=I_1(\Delta/T)/I_0(\Delta/T)$ in the case of nondegenerated electron gas, and $A=\Delta/2\varepsilon_F$ in the case of strong degenerated electron gas ($\varepsilon_F=\pi dN_e/m>2\Delta$); $I_n(z)$ are Bessel functions with an imaginary argument, ε_F is the Fermi energy, N_e is the electron density, and T is the temperature of the crystal in the energy units. In the presence of the laser field, the conductivity tensor has the exact form of Eq. (3.18) with the change $\Delta\to\tilde{\Delta}$; it can easily be seen that at the roots of the Bessel function $J_0(eFd/\Omega)$ the conductivity tensor vanishes. This shows that dynamic localization of electrons takes place, in agreement with the results obtained for the mean-square displacement in Refs. 4–6.

Note that for the electron spectrum in a superlattice [see Eq. (3.4)],

$$[v_z(\mathbf{p}+\mathbf{q})+v_z(\mathbf{p})]=d\cot\frac{qd}{2}[\tilde{\varepsilon}_{\mathbf{p}+\mathbf{q}}-\tilde{\varepsilon}_{\mathbf{p}}] , \tag{3.19}$$

so the conductivity tensor (3.17) can be written in the other form

$$\sigma(\mathbf{q},\omega)=\frac{e^2d}{2qi}\cot\frac{qd}{2}(\omega-i\nu)\sum_{\mathbf{p}}\frac{\tilde{f}_0(\mathbf{p})-\tilde{f}_0(\mathbf{p}+\mathbf{q})}{\omega-\tilde{\varepsilon}_{\mathbf{p}+\mathbf{q}}+\tilde{\varepsilon}_{\mathbf{p}}-i\nu} . \tag{3.20}$$

The density matrix $\rho_1(t)$ (3.8) can be used to evaluate other quantities. For example, the deviation of the electron density can be calculated according to the formula

$$n_1(\mathbf{q},t)=\mathrm{Tr}[a_{\mathbf{p}}^{\dagger}a_{\mathbf{p}}\rho_1(t)] . \tag{3.21}$$

Substituting (3.8) into (3.21) and following all procedures described above for calculating the current density, one gets

$$n_1(\mathbf{q},\omega)=\frac{eE}{iq}X(\mathbf{q},\omega) , \tag{3.22}$$

where

$$X(\mathbf{q},\omega)=\sum_{\mathbf{p}}\frac{\tilde{f}_0(\mathbf{p})-\tilde{f}_0(\mathbf{p}+\mathbf{q})}{\omega-\tilde{\varepsilon}_{\mathbf{p}+\mathbf{q}}+\tilde{\varepsilon}_{\mathbf{p}}-i\nu} \tag{3.23}$$

is the polarization operator.

By comparison with Eq. (3.20) and taking into account the condition of homogeneity $qd<1$, we obtain again the phenomenological relation (2.17), but this time through a microscopic calculation.

IV. LOCALIZATION OF ACOUSTIC WAVES IN SUPERLATTICES

We proceed to the analysis of the characteristics of the propagation of acoustic waves in superlattices. It will be shown below that for a certain range of the parameter $x=s_0/d\tilde{\Delta}\simeq 1$ the localization of waves may occur. This condition can be reached for narrow miniband (in the range of a few meV) superlattices or by varying the parameters of the laser field.

It is commonly accepted that the localization of the wave occurs when the diffusion constant ($D\sim sl_e$, l_e is the elastic mean free path) of the wave reduces to zero,[34–39] while the wave absorption characterized by an attenuation length l_a increases. The apparent contradictory results presented by John[34] and Anderson[35] on the renormalization of absorption by localization were reconciled and shown to apply to different situations by Sornette.[36] In a recent experimental work,[40] evidence for localization of acoustic waves in a three-dimensional system was reported. According to Graham, Piche, and Grant, the localization of acoustic waves is demonstrated by a strong renormalization of wave velocity (around 5%) together with intense peaks in attenuation. The similar effect (strong renormalization of wave velocity associated with intense attenuation peaks) can happen when an acoustic wave propagates along the superlattice axis.

To analyze the sound velocity s and absorption coefficient Γ according to Eqs. (2.10) and (2.11), we should first evaluate the complex conductivity tensor $\sigma(\mathbf{q},\omega)$ by using Eq. (3.20). Passing from summation to integration in Eq. (3.23), one gets

$$X(\mathbf{q},\omega)=\frac{N_eA}{\pi\tilde{\Delta}}\int_0^{2\pi}\frac{\cos\varphi\,d\varphi}{b-\cos\varphi} , \tag{4.1}$$

where $b=(\omega-i\nu)/qd\tilde{\Delta}$. In the case of nondegenerated electron gas, we assumed that $\Delta/T<1$. The integral in Eq. (4.1) can be calculated by residues; the result is

$$X(\mathbf{q},\omega)=-\frac{2N_eA}{\tilde{\Delta}}\left\{1-\frac{\rho\sqrt{\mathrm{sgn}\zeta}}{\sqrt{|\zeta|-i\,\mathrm{sgn}\zeta\sin 2\varphi}}\right\} , \tag{4.2}$$

where ρ and φ are modulus and argument of the complex parameter $b=\rho e^{-i\varphi}$, $\zeta=x^2(1+a^2)-(1-a^2)(1+a^2)^{-1}$, $a=(\omega\tau)^{-1}$, $\mathrm{sgn}x=x/|x|$. Substituting (4.2) into (3.20) we obtain the analytical expression for the real and imaginary parts of σ as follows: $\sigma_{1,2}=\sigma_0 f_{1,2}(z,a)$, where

$$f_1(z,a)=\begin{cases} a\left[1-\frac{\alpha}{\sqrt{r}}\cos\frac{\psi}{2}\right]-\frac{\alpha}{\sqrt{r}}\sin\frac{\psi}{2}, & \zeta>0 \\ a\left[1-\frac{\alpha}{\sqrt{r}}\sin\frac{\psi}{2}\right]+\frac{\alpha}{\sqrt{r}}\cos\frac{\psi}{2}, & \zeta<0 \end{cases} \tag{4.3}$$

$$f_2(z,a)=\begin{cases} \left[1-\frac{\alpha}{\sqrt{r}}\cos\frac{\psi}{2}\right]+a\frac{\alpha}{\sqrt{r}}\sin\frac{\psi}{2}, & \zeta>0 \\ \left[1-\frac{\alpha}{\sqrt{r}}\sin\frac{\psi}{2}\right]-a\frac{\alpha}{\sqrt{r}}\cos\frac{\psi}{2}, & \zeta<0 \end{cases} \tag{4.4}$$

$z=eFd/\Omega$, $\alpha=x\sqrt{1+a^2}$, $r=\sqrt{\zeta^2+4a^2/(1+a^2)^2}$, $\cos\psi/2=\sqrt{(r+|\zeta|)/2r}$, $x=\omega/qd\tilde{\Delta}$, $\sigma_0=e^2N_e\omega/q^2\langle\varepsilon\rangle$, and $\langle\varepsilon\rangle=T$ in the case of nondegenerated electron gas; and $\langle\varepsilon\rangle=\varepsilon_F$ in the case of strong degenerated electron gas. Note that in superlattices a maximum value of electron velocity is $\tilde{\Delta}d$, so the maximum value of the electron mean free path is $l_m=\tilde{\Delta}d\tau$. Our results (4.3) and (4.4) indeed contain the traditional parameters ql_m and $a=(\omega\tau)^{-1}$ of the theory of acoustic phenomena [via the parameter $x=(aql_m)^{-1}$] and are valid for a wide range of these parameters.

When screening effects can be neglected, $\omega>4\pi\sigma_0/\varepsilon$ from (2.10) and (2.11), and (4.3) and (4.4) we have

$$s=s_0[1+\beta f_2(z,a)]\ , \tag{4.5}$$

$$\Gamma=q\beta f_1(z,a)\ , \tag{4.6}$$

where $\beta=\Lambda^2N_e/2\rho s_0^2\langle\varepsilon\rangle$.

In the high-frequency (or collisionless) limit $(a\to 0)$, Eqs. (4.3) and (4.4) are simplified to

$$f_1(z,0)=\Theta(1-x^2)\frac{x}{\sqrt{1-x^2}}\ , \tag{4.7}$$

$$f_2(z,0)=1-\Theta(x^2-1)\frac{x}{\sqrt{x^2-1}}\ , \tag{4.8}$$

where $\Theta(x)$ is the step function. Equations (4.6) and (4.7) recover the result of Ref. 41 for the absorption coefficient in the collisionless and short wavelength $(ql_m\gg 1)$ limits. From Eqs. (4.7) and (4.8) it is easy to see that f_1 and f_2 diverge at the points $|x|=s_0/d\Delta|J_0(z)|=1$. This divergence leads simultaneously to strong absorption peaks and sharp renormalization of sound velocity indicating the localization of waves. Since this localization of waves can be achieved by varying the parameters of the laser field $(z=eFd/\Omega)$, it is called dynamic localization of waves. [Note that the dynamic localization of electrons occurs when $J_0(z)=0$.] It is well known that important parameters in the localization theory (renormalized sound velocity, absorption length, mean free path of waves, etc.) are related to the scattering cross section. Localization can occur if the scattering cross section resonantly increases.[13,37,38] From this point of view, the phenomenon described here can be explained as follows. The process of phonon scattering by electrons is governed by the momentum and energy conservation laws,

$$\tilde{\varepsilon}_{\mathbf{p}+\mathbf{q}}-\tilde{\varepsilon}_{\mathbf{p}}=2\tilde{\Delta}\sin\left[p_z+\frac{q}{2}\right]d\sin\frac{qd}{2}=\omega\ . \tag{4.9}$$

Since $qd<1$ and $|\sin(p_z+q/2)d|\le 1$, the maximum energy change of electrons is $qd|\tilde{\Delta}|$. If $\omega>qd|\tilde{\Delta}|$ $[s_0/d\Delta>|J_0(z)|]$, the momentum and energy conservation laws (4.9) cannot be satisfied. In this case, electrons cannot interact with the wave. When $\omega=qd|\tilde{\Delta}|$ $[s_0/d\Delta=|J_0(z)|]$, this interaction occurs resonantly and it tends to localize the wave. This situation is very similar to the localization of waves by internal resonances suggested by Sornette and co-workers in Refs. 13–15. The internal resonances in our system are bound electrons in a superlattice with a resonant frequency $\omega_0=qd\Delta$ and the localization occurs over a narrow range of frequencies $\omega\simeq\omega_0$. For typical values of superlattice parameters d and Δ, the resonant frequency ω_0 is often larger than the sound frequency ω $(\omega_0>\omega)$, so in order to localize, that is, to reduce the resonant frequency $\omega_0\to\tilde{\omega}_0\simeq\omega$, the presence of a laser field is required.

When the scattering frequency ν is finite, the peaks of f_1 and f_2 have finite heights and weights. From Eqs. (4.3) and (4.4), we can see that these peaks occur at the points

$$x=\frac{\sqrt{1-a^2}}{1+a^2}\quad (a<1)\ . \tag{4.10}$$

If $a\ge 1$, f_1 and f_2 are monotonic functions and localization of waves is almost destroyed. The situation is very similar to the acoustic Anderson localization problem where the dissipation of the acoustic energy hampers the observation of localization (see, for example, Ref. 36).

The behavior of f_1 and f_2 as functions of the dimensionless parameter x for the case $z=0$ and for several values of $a=(\omega\tau)^{-1}$ is depicted in Figs. 1(a) and 1(b). In Figs. 2(a) and 2(b) we present the behavior of f_1 and f_2 as functions of the parameter z for the case $s_0/d\Delta=0.25$ ($d=7$ nm, $2\Delta=2.6$ meV, $s_0=3.5\times10^5$ cm/s). We can see a set of intense and narrow peaks showing a strong absorption (function f_1) and a sudden change of sound velocity (function f_2). By increasing the scattering, the height and distinction of the peaks decrease, and the localization is suppressed. A similar situation is well known in the theory of dynamic localization of electrons.[6]

For nondegenerated electron gas with $N_e=10^{15}$ cm^{-3}, $T=100$ K, and for typical values of parameters (of GaAs): $\Lambda=10$ eV, $\varepsilon=13.1$, $s_0=3.5\times10^5$ cm/s, $\rho=5.3$ g cm^{-3} one gets $\beta\simeq 0.003$. So we can see that in the regime of dynamic localization, the absorption coefficient increases sharply 2–10 times and the sound velocity reduces ~ 1–$5\,\%$. In the case of strong degenerated electron gas for acoustic-wave frequencies up to 10^{12} s^{-1}, the screening effect cannot be neglected. In this case the ab-

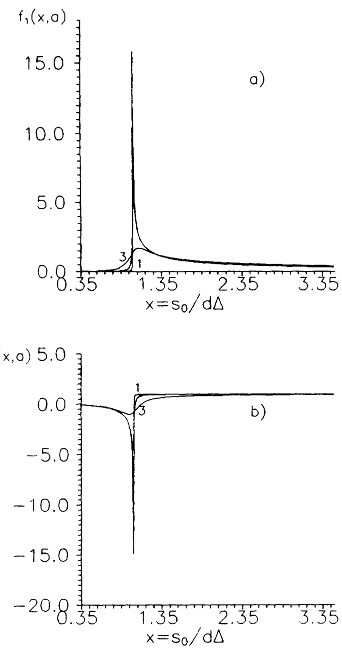

FIG. 1. (a) The function $f_1(x,a)$ for the following parameters: $z=eFd/\Omega=0$; curve 1, $a=0.001$; curve 2, $a=0.01$; curve 3, $a=0.1$. (b) The function $f_2(x,a)$ with the same parameters as (a).

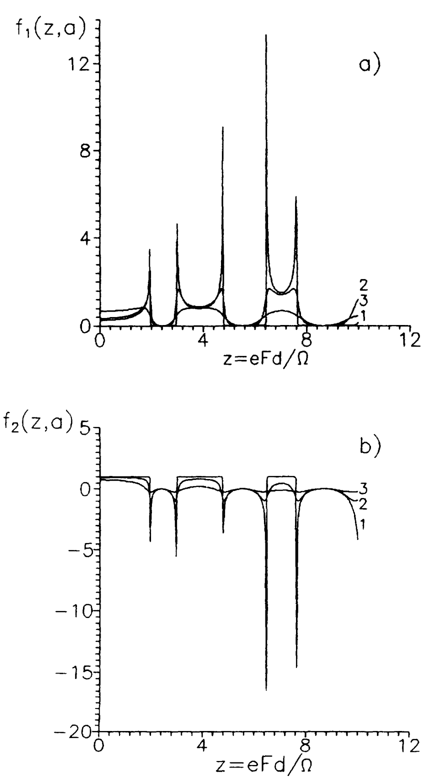

FIG. 2. (a) The function $f_1(z,a)$ for the following parameters: $s_0/\Delta d=0.25$; curve 1, $a=0.001$; curve 2, $a=0.1$; curve 3, $a=0.5$. (b) The function $f_2(z,a)$ with the same parameters as Fig. 1(a).

sorption coefficient and sound velocity are defined by the complete set of equations (2.10), (2.11), (4.3), and (4.4), and localization of the wave is not observed.

Let us estimate the magnitude of the scattering effect and wave frequency required to observe the effect described in this section. Two conditions $d<\lambda$ and $\omega>4\pi\sigma_0/\varepsilon$ limit the wave frequency within a region $s_0/d>\omega>4\pi\sigma_0/\varepsilon$. For typical numerical values given above and $d=7$ nm, both conditions can be satisfied for $\omega\simeq(2-5)\times10^{11}\ \mathrm{s}^{-1}$. On the other hand, to avoid the scattering effect, the condition $\omega\tau>1$ should be satisfied, thus, the scattering time must be of order $\tau\geq10^{-10}$ s. So, it is expected to observe the dynamic localization of high-frequency acoustic waves in highly pure superlattices at low temperature. The magnitude of the amplitude F and frequency Ω of the laser field can be estimated as follows. The dynamic localization occurs near the points $z\simeq2.4, 5.5\ldots$ [see Figs. 2(a) and 2(b)]. So if $\hbar\Omega=10$ meV ($\Omega\simeq1.5\times10^{13}\ \mathrm{s}^{-1}$), the localization will be observed for a laser field of the order of few hundreds kV/cm.

V. PLASMA OSCILLATIONS IN SUPERLATTICES

In this section we briefly consider the plasma oscillations in superlattices under the laser field by using results for the conductivity tensor $\sigma(\mathbf{q},\omega)$ obtained in the preceding section. According to Eqs. (2.16) and (2.18), the plasma frequency $\omega_{\mathrm{pl}}(\mathbf{q})$ and damping constant $\gamma(\mathbf{q})$ are defined as follows:

$$1+\frac{4\pi}{\varepsilon\omega_{\mathrm{pl}}}\sigma_2(\mathbf{q},\omega_{\mathrm{pl}})=0\ , \tag{5.1}$$

$$\gamma(\mathbf{q})=\left|\frac{\sigma_1(\mathbf{q},\omega_{\mathrm{pl}})}{\partial\sigma_2(\mathbf{q},\omega_{\mathrm{pl}})/\partial\omega_{\mathrm{pl}}}\right|\ . \tag{5.2}$$

When deriving Eqs. (5.1) and (5.2), we assume as usual that $\omega_{\mathrm{pl}}\gg\gamma$.

Let us consider the simple case when the collisions in plasma can be neglected ($\omega_{\mathrm{pl}}\gg\nu$). In this case the real and imaginary parts of the conductivity tensor are given by Eqs. (4.7) and (4.8). Substituting (4.7) into Eq. (5.1) one can see that it has a solution only when $\omega_{\mathrm{pl}}>|\tilde{\Delta}|qd$. In this case, one gets

$$\omega_{\mathrm{pl}}(\mathbf{q})=|\tilde{\Delta}|qd\left[\frac{[1+(r_Dq)^2]^2}{[1+(r_Dq)^2]^2-1}\right]^{1/2}\ ,\quad \omega_{\mathrm{pl}}>|\tilde{\Delta}|qd\ , \tag{5.3}$$

where r_D is the Debye screening radius: $r_D^2=\varepsilon\langle\varepsilon\rangle/4\pi e^2N_e$. It is interesting to note that in the case of strong degenerated electron gas, the Debye screening radius become independent on the carrier density: $r_D^2=\varepsilon d/4me^2$.

In the case of strong screening when $r_Dq\ll1$ we have

$$\omega_{\mathrm{pl}}=\frac{|\tilde{\Delta}|d}{\sqrt{2}r_D}\ . \tag{5.4}$$

On the other hand, when the screening is weak $r_Dq\gg1$

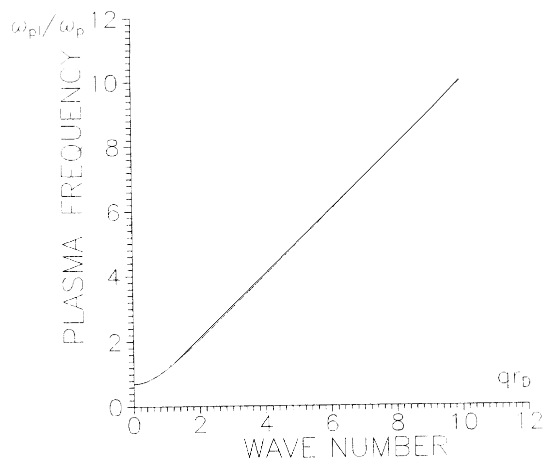

FIG. 3. The plasma frequency ω_{pl}/ω_p as the function of dimensionless wave number qr_D.

formula (5.3) gives

$$\omega_{pl}=|\tilde{\Delta}|qd \ . \tag{5.5}$$

For this range of wave numbers, the plasma oscillations follow the acoustic law of dispersion. The velocity of such an "electron sound" $s=|\tilde{\Delta}|d$ has the same order of magnitude as the electron velocity along the superlattice axis. Formulas (5.3)–(5.5) recover the result of Ref. 16 when the laser field is absent: $\tilde{\Delta}\rightarrow\Delta$. The spectrum of plasma oscillations in units of $\omega_p=|\tilde{\Delta}|d/r_D$ as a function of the dimensionless wave number qr_D is presented in Fig. 3.

For the plasma oscillations with the spectrum (5.3), one can see that the damping constant (5.2) equals zero. As mentioned already in Ref. 16, this phenomenon is explained as follows. The damping of plasma oscillations is governed by the momentum and energy conservation laws (4.9). For the plasma oscillations with the frequency (5.3), the conservation laws cannot be satisfied, which in turn allows the plasmons to propagate along the superlattice axis without decay. Note that in the dynamic localization regime when $J_0(eFd/\Omega)=0$ the frequency (5.3) turns to zero and plasma oscillations along the superlattice axis disappear.

VI. DISCUSSION

Let us summarize the results presented in this paper. As reviewed in Sec. II, within the framework of the dispersion-equation treatment, all characteristics of the propagation of the elastic wave are connected with the conductivity tensor. To obtain this fundamental quantity, we have generalized the Kubo formalism, which allows us to evaluate the current and electron deviation densities characterized by their response to the longitudinal self-consistent electric field $\mathbf{E}(\mathbf{x},t)$ driven by the far-infrared laser radiation. The generalized Kubo formula for these physical quantities together with the quantum transport equation for nonequilibrium electron distribution in a laser field completely describe the behavior of the electron system in the presence of a laser field. For infrared laser radiation, whose frequency exceeds the miniband width but does not cause interminiband transitions, the distribution function of electrons can be found, which has the exact form of the Fermi function with the renormalized electron spectrum. In this case, the analytical expression for the conductivity is obtained. We have shown that for a certain range of the parameter $s_0/d|\tilde{\Delta}|\simeq 1$, a strong renormalization of sound velocity and intense absorption peaks indicating the localization of wave are observed. The conditions for this kind of localization are discussed. The spectrum of plasma oscillations propagating along the superlattice axis is also calculated. The characteristic feature of these plasmons is the absence of Landau damping. In the dynamic localization regime, the plasma oscillations along the superlattice axis disappear.

We would like to comment on the approximations made in this work.

(1) The absorption coefficient $\Gamma=-q_2$ and the change in sound velocity given by the dispersion-equation treatment are correct only under the conditions $q_2/q_1\ll 1$ and $\delta s/s_0\ll 1$. That means the attenuation length $l_a=\Gamma^{-1}$ must be larger than the wavelength λ (weak absorption), and the change in the sound velocity must be less than the sound velocity s_0 in the absence of the interaction. However, for complete localization we should have $s\rightarrow 0$ or $\delta s/s_0\sim 1$. But, because of the limitation of our theory, we were not able to come to this regime.

(2) The dependence of the response to the action is local in space and time, that is, $\mathbf{j}(\mathbf{x},t)=\sigma(\mathbf{q},\omega)\mathbf{E}(\mathbf{x},t)$. This approximation is good only for homogeneous systems and restricts our investigations only in the region where the wavelength λ is larger than the superlattice period.

(3) The constant relaxation-time approximation is adequate when the condition $\omega>\nu$ is fulfilled. Note that these restrictions might be eliminated by the Green's-function treatment of the problem (see, for example, Refs. 37, 42, and 43).

Beside these main approximations in evaluating the conductivity tensor, we limited ourselves to zero order in the parameter Δ/Ω. However, as was shown by additional calculations, the corrections to our results appear only in second order of this parameter, thus, for the case of $\Delta/\Omega<1$, we consider that the inclusion of these corrections are not relevant.

Note that here we consider the localization and propagation of waves along the superlattice axis. However, a more interesting case should be when surface acoustic waves propagate parallel to the layers. In this situation the localization of acoustic waves is experimentally observed.[19,20,25,44] The theoretical investigation of this problem will be addressed in the future.

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