

Experimental investigation of the damping of low-frequency edge magnetoplasmons in GaAs-Al_xGa_{1-x}As heterostructures

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A detailed experimental study of damping and velocity of low-frequency edge magnetoplasmons in GaAs-Al_xGa_{1-x}As heterostructures is presented. The damping is observed to be frequency dependent at filling factors close to integer values. The magnitude of the damping increases with frequency, the dependence being somewhere between linear and quadratic. This finding indicates that the damping of low-frequency edge magnetoplasmons cannot be described by the effective relaxation time. The experimental results are discussed in terms of existing models of low-frequency edge magnetoplasmons.

INTRODUCTION

Recently there has been an interest in the low-frequency edge magnetoplasmons (EMP's) in two-dimensional electron gas (2DEG) in GaAs/Al_xGa_{1-x}As heterostructures¹⁻¹⁴ and on liquid-helium surface.¹⁵⁻¹⁷ Low frequency in this case refers to the situation where $\omega\tau \ll 1$, τ being the momentum relaxation time at zero magnetic field. In a bounded 2DEG these low-frequency plasma oscillations associated with the edge of the sample are observed as sharp resonances in the frequency domain and can be thought to be a rotation of edge charge, electric field, and Hall current distributions around the external magnetic-field direction. Around integer filling factors (ν), the field dependence of the velocity and damping of the low-frequency EMP's has been shown to be non-monotonic. Indeed, the velocity response exhibits plateaus while the damping forms sharp minima. This behavior distinguishes the low-frequency EMP response to that observed for high-frequency EMP's (Refs. 18 and 19), where no oscillatory behavior of the damping is observed. Below we will use the abbreviation EMP to refer to low-frequency EMP only. The behavior of the velocity of the EMP's has been discussed in detail⁴⁻¹² but little is known about the reasons causing EMP damping. This problem is of interest because it is closely related to the main problem of EMP physics, the edge charge spatial distribution. The EMP damping is much more sensitive to details of the edge charge distribution than that of EMP frequency.¹

Several theories exist as to the nature of the edge charge distribution in GaAs/Al_xGa_{1-x}As 2DEG structures. Volkov and Mikhailov¹ (VM) proposed a model in which the 2DEG is described by local conductivities σ_{xx} and σ_{xy} and where the edge of 2DEG is assumed to be infinitely sharp. In this theory, the edge charge distribution is a result of the compromise between the Hall current taking charge to the boundary and the Ohmic current trying to distribute charge over the bulk of the 2DEG. They defined a strip of width l along the 2DEG boundary, which accommodates the main part of the edge charge given by

$$l = \frac{2\pi i \sigma_{xx}(\omega)}{\epsilon \omega}, \quad (1)$$

where ω is the EMP frequency and ϵ is the dielectric permittivity. For the propagation of EMP's along the boundary of a 2DEG half plane, the frequency $\omega'(k) = \text{Re}\omega(k)$ and the damping $\omega''(k) = \text{Im}\omega(k)$ are given in Ref. 1 by

$$\omega'(k) + \omega''(k) = \frac{2k\sigma_{xy}}{\epsilon} \ln \left(\frac{2}{kl+1} \right). \quad (2)$$

In a bounded 2DEG, the EMP resonant frequencies can be determined from Eq. (2) and the condition for the wave vector k ,

$$k = \frac{2\pi n}{P}, \quad (3)$$

where P is the length of the perimeter of the 2DEG and n is an integer. The resonant linewidth is determined by $2\omega''(k)$. In this theory, the value of the damping $\omega''(k)$ is described in terms of the effective relaxation time τ^* , as shown here:

$$\omega'' = \frac{1}{\tau^*} \left[\ln \frac{1}{kl} \right]^{-1}. \quad (4)$$

Equation (4) is only valid when $\omega\tau^* \gg 1$. In the frequency region $\omega\tau^* \ll 1$, EMP damping is predicted¹ to depend on σ_{xy} rather than on $\sigma_{xx}(\omega)$. As this contradicts the experimental results presented below, we shall discuss only the $\omega\tau^* \gg 1$ domain of the VM theory. Usually $kl \ll 1$ so that the weak dependence of ω'' on k in Eq. (4) can be neglected. Thus, in a bounded 2DEG the fundamental and spatial-harmonics EMP modes are predicted to have the same linewidth. This theory has subsequently been applied to the interpretation of experimental results published by Ashoori *et al.*,¹¹ where the magnetic-field dependence of τ^* was presented. The applicability of the VM theory in the quantum Hall effect (QHE) regime was, however, questioned in this paper. Other theories exist in which the EMP charge distribution is assumed to be controlled by the Larmor radius.^{9,20} This assumption is

reasonable for a 2DEG with an infinitely sharp edge potential. At small enough $\sigma_{xx}(\omega)$, the value of l from Eq. (1) can be smaller than the Larmor radius, such that the EMP edge charge distribution is no longer determined by l , but by the Larmor radius.

We have previously reported a different model for the EMP problem.¹² This approach is based on the assumption that in the QHE regime the EMP charge is accommodated in edge channels (EC's). The concept of EC's has been used extensively to explain many interesting effects in direct current (dc) magnetotransport experiments and a detailed theoretical discussion was given by Buttiker in Ref. 21. In Ref. 12, the edge excitations in the 2DEG are considered as interactive charge density waves propagating in the EC's. The EMP's are considered to be the simplest type of the edge excitations corresponding to the in-phase vibrations of the charge in all the EC's. This model will be referred to as the EC model of EMP's. The model provides a link with dc measurements of EC's and helps reduce the problem of EMP charge distribution to the problem of the spatial structure of the edge channels.

The EC model of EMP is applicable if l from Eq. (1) is much smaller than the strip width occupied by the EC's. In reality, the edge potential of a 2DEG is believed not to be infinitely sharp but varies with distance into the sample. This smooth edge potential results in wide edge channels. By treating experimental EMP spectra in terms of the EC model of EMP, we previously estimated the edge channel width to be $\approx 3000 \text{ \AA}$.¹² More recent measurements of edge channel width by an independent method have given rise to values of $\approx 4000 \text{ \AA}$.²² The large width of the EC's allows one to propose that EMP damping must mainly occur inside EC's. This assumption leads to the conclusion that EMP damping should decrease with a decrease in frequency and, thus, it is impossible to introduce an effective relaxation time. A simple consideration results in a $\omega'' \propto \omega^2$ dependence, however, this contradicts views adopted in the literature.^{1,11} As a result, detailed measurements of EMP damping have been carried out with the conclusion that the damping is indeed frequency dependent. We will show that the dependence of $\omega''(\omega)$ is observed to be somewhere between linear and quadratic. A microscopic theory of the EC structure is needed for a qualitative description of EMP damping.

EXPERIMENTAL TECHNIQUE AND RESULTS

The method that we used for the investigation of EMP's is to treat the 2DEG as a transmission resonator.⁸ The EMP's are excited and detected with the help of two open-ended coaxial cables coupling the sample with an rf generator and sensitive receiver, see Fig. 1. The source frequency was swept and the power transmitted through the sample was measured with the aid of a spectrum analyzer preceded by a low-noise rf amplifier. Our system utilized the tracking generator of the spectrum analyzer as an rf source. The experimental setup allowed us to make measurements in the frequency range up to 400 MHz, in a magnetic field up to 8 T, and at tempera-

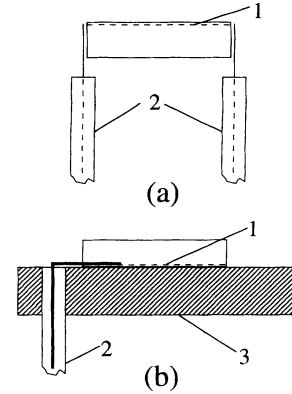


FIG. 1. Schematic representation of a sample with coaxial cables. (a) 2DEG is far away from a metal surface. (b) 2DEG is close to a metal surface. Only one coaxial cable is shown. (1) 2DEG, (2) coaxial cable, (3) the metal plate.

tures down to 0.3 K. For some measurements the sample was put face down on a polished metal plate, see Fig. 1(b). In this situation the distance between the 2DEG and the metal is controlled by the surface roughness of the metal and is estimated to be $\approx 1-2 \mu\text{m}$. Four different high-mobility GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$ heterostructures were analyzed, all samples were cleaved into squares of length 7 mm except for sample 2, which was rectangular ($3 \times 6 \text{ mm}^2$). Carrier densities and mobilities of the samples measured at 1.7 K in the dark are as fol-

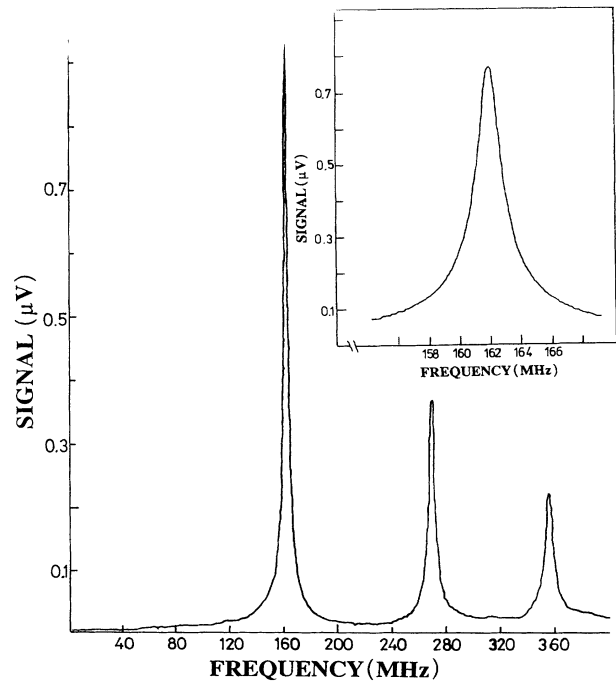


FIG. 2. Transmitted signal vs frequency measured on sample 1 at filling factor $\nu=2$ and rf source power -70 dBm ($70.7 \mu\text{V}$). Inset: the fundamental EMP mode. To resolve the narrow line, the data were taken in small frequency domain. The source power is -73 dBm .

lows: sample 1, $3.4 \times 10^{11} \text{ cm}^{-2}$ and $1.3 \times 10^6 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$; sample 2, $1.4 \times 10^{11} \text{ cm}^{-2}$ and $6.7 \times 10^5 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$; sample 3, $1.2 \times 10^{11} \text{ cm}^{-2}$ and $2.0 \times 10^6 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$; and finally for sample 4, $1.2 \times 10^{11} \text{ cm}^{-2}$ and $2.1 \times 10^6 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$.

The transmitted signal versus frequency of sample 1 at filling factor $\nu=2$ is shown in Fig. 2. The peaks in the data correspond to the excitation of the fundamental mode and several spatial EMP harmonics. The fundamental mode in this figure demonstrates the EMP vibration with highest value of quality factor $Q \approx 110$ that has so far been reported ($Q = \omega' / 2\omega''$ is the ratio of the resonant frequency to the linewidth). Figure 3(a) presents a similar trace for sample 2 except that the 2DEG is close to the metal surface as described in Fig. 1(b). For the problem of concern here, the measurements with the metal plate are important because they help to demonstrate that the EMP charge distribution does not depend on frequency. It can be seen from Fig. 3(a) that the peaks in the EMP spectrum are equidistant, the frequencies of the fundamental and subsequent harmonics being 17.5, 35.0, 51.8, and 67.8 MHz, respectively. The reason for this is that the metal plate restricts the electric field of the EMP's and the Hall current to a strip of width $d=1-2 \mu\text{m}$ near the boundary, making the EMP a perimetric ex-

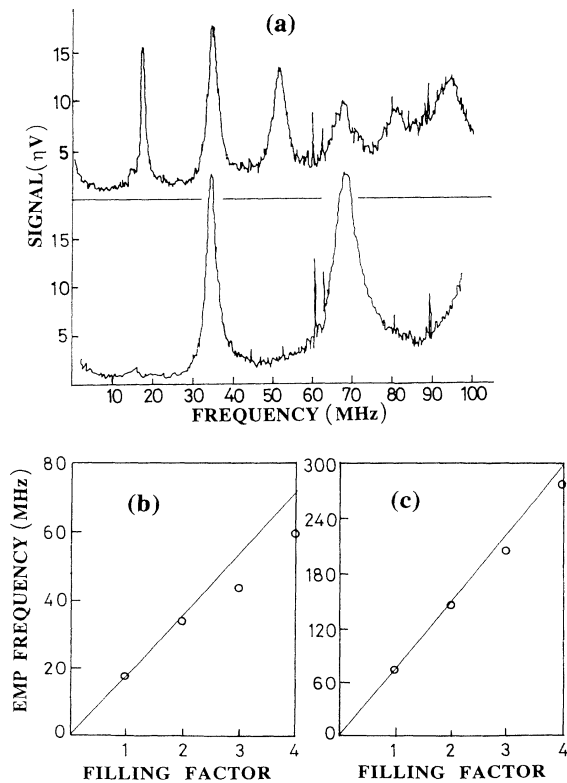


FIG. 3. (a) Transmitted signal vs frequency measured on sample 2 near the metal plate at filling factor $\nu=1$ (the top panel) and $\nu=2$ (bottom panel). rf source power is -65 dBm (the top panel) and -68 dBm (bottom panel). (b) The fundamental EMP mode resonant frequency vs integer filling factor values. (c) The same as in (b) but for a sample far away from the metal plate.

citation. For this situation, Eq. (2) is still applicable provided that $\ln|k/l|$ is substituted by $\ln d/l$.¹ Similar expressions appear in the EC model of EMP.¹² For a perimetric excitation, wave vector k can be found from Eq. (3). The equidistance of the EMP resonances shown in Fig. 3(a), therefore, indicates that l is not frequency dependent. For samples without the metal plate, the assumptions made in Eq. (3) are rather approximate and can lead to significant error.

The resonant frequency of the fundamental EMP mode of sample 2 with the metal plate for different integer filling factors is shown in Fig. 3(b). The corresponding data for the same sample but without the metal plate are shown in Fig. 3(c). It can be seen that for both cases there is a deviation from linearity of ω' as a function of filling factor, but this is more clearly seen for the sample with the metal plate close to the surface. This deviation supports the EC model for EMP and has been discussed in detail in Ref. 12.

The position and linewidths of the EMP peaks for samples 3 and 1 were measured in order to determine the resonant frequencies and damping as a function of magnetic field, and are shown in Figs. 4(a) and 4(b), respectively. The very narrow EMP resonances observed lead to the measurement of the plateaulike behavior of $\omega(\nu)$ near integer filling factors more precisely than has been done before, see Fig. 5 (sample 1). The linewidths of the fundamental and spatial harmonics were also measured in order to examine the frequency dependence of the damping. The normalized EMP resonant frequencies versus nor-

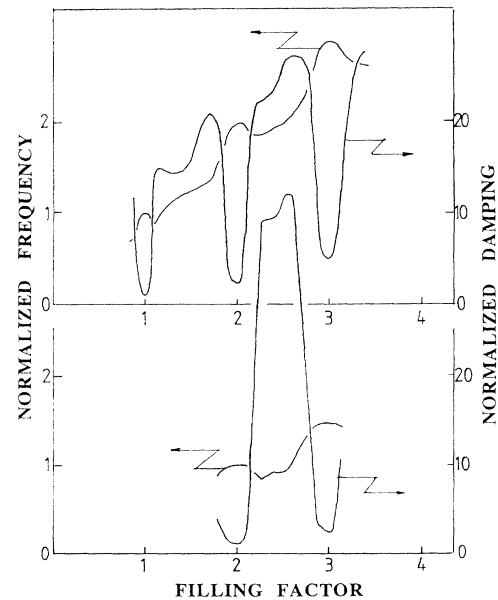


FIG. 4. Resonant frequency and damping (linewidth) of the fundamental EMP mode vs filling factor. The top panel shows the data taken on sample 3. The resonant frequency and damping are normalized to those at filling factor $\nu=1$. The resonant frequency and the linewidth at $\nu=1$ are 77.5 and 2.13 MHz, respectively. The bottom panel shows the data for sample 1. The resonant frequency and damping are normalized to those at filling factor $\nu=2$, which are 157.84 and 1.5 MHz, respectively.

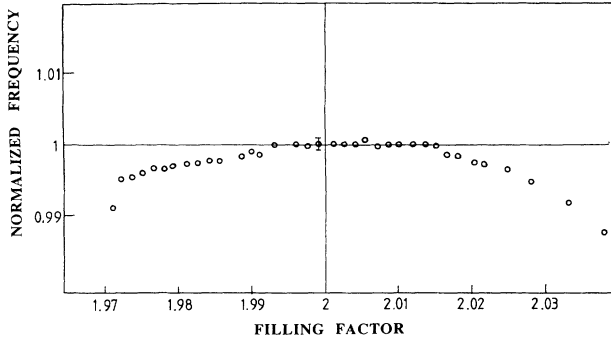


FIG. 5. Normalized resonant frequency of the fundamental EMP mode for sample 1 as a function of filling factor values close to $\nu=2$.

malized damping for samples 2 (with a metal plate), 3, and 4 at the filling factor $\nu=1$ are presented in Fig. 6. The corresponding actual frequencies for the sample close to the metal plate are smaller [from 17 to 68 MHz, Fig. 3(a)] than those for the other samples. It can be seen that for sample 2, close to the metal plate, the damping has a linear dependence with frequency, whereas for the samples with no metal plate, this dependence is somewhere between linear and quadratic. A similar frequency dependence of the damping was observed at $\nu=2$. The damping of the fundamental, second, and third spatial-harmonic EMP modes near $\nu=2$ for sample 1 is shown in Figs. 7(a), 7(b), and 7(c), respectively. It can be seen that the frequency dependence of the damping remains essentially unchanged at small intervals of ν .

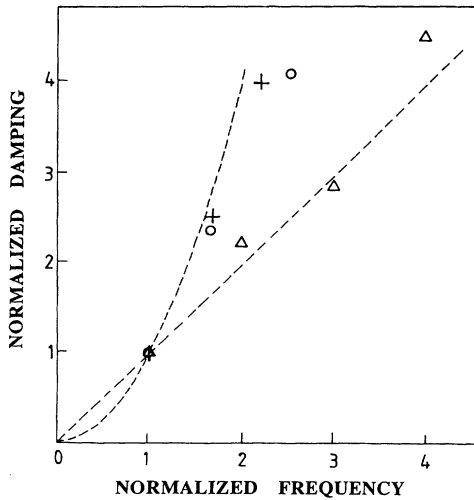


FIG. 6. EMP damping vs EMP frequency at filling factor $\nu=1$. Resonant frequencies and the linewidth of EMP spatial harmonics are normalized to those of the EMP fundamental mode. Triangles, circles, and crosses represent data for samples 2, 3, and 4, respectively. The data on sample 2 were taken for the sample closest to the metal surface [Fig. 3(a)]. Resonant frequency and damping of EMP fundamental mode are the following: sample 3, 77.5 and 2.1 MHz; sample 4, 58.4 and 1.75 MHz; sample 2, 17.5 and 1 MHz.

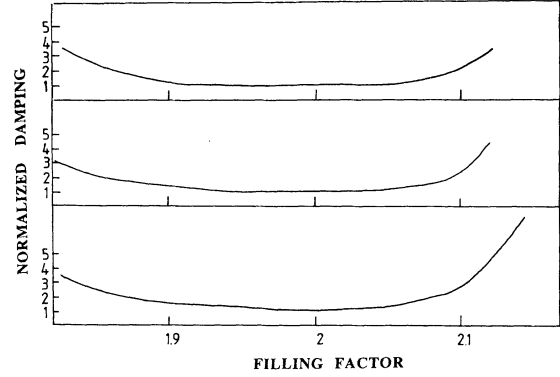


FIG. 7. The damping of EMP modes near $\nu=2$ for sample 1. Panels from bottom to top correspond to fundamental, second, and third spatial EMP harmonics, respectively.

All the above data have been obtained on cleaved samples. The same measurements have been performed on a 2DEG mesa and on a 2DEG confined electrostatically by means of gate-induced depletion. In all cases similar results were obtained to those described above.

DISCUSSION

From our experimental data it can be seen that the EMP damping cannot accurately be described by an effective relaxation time. This distinguishes EMP's from 2D bulk plasmons, where detailed investigations in the IR region have revealed that the spatial harmonics all have the same linewidth.²³

We now consider a possible explanation for the results obtained in terms of both the VM model and the EC model of EMP's. In the VM model in order to obtain a minimum of the damping at integer filling factors, one has to assume that in the QHE regime

$$\text{Im}\sigma_{xx}(\omega) \gg \text{Re}\sigma_{xx}(\omega). \quad (5)$$

Thus, from Eqs. (1) and (2), it follows that $\text{Im}\sigma_{xx}(\omega)$ controls the width of the edge charge distribution l and $\text{Re}\sigma_{xx}(\omega)$ controls the magnitude of the EMP damping. Further, in Ref. 1, it is assumed that

$$\text{Re}\sigma_{xx}(\omega) = \sigma_{xx}(0), \quad (6)$$

$$\text{Im}\sigma_{xx}(\omega) = -\text{Re}\sigma_{xx}(0)\omega\tau^*. \quad (7)$$

Equation (7) defines the effective relaxation time τ^* . The EMP damping can also be expressed¹ as $\omega'' \propto \text{Re}\sigma_{xx}/l$, which is equivalent to expression (4). The value of l was shown to be independent of ω ; therefore, in order to describe our results in terms of the VM model, one has to discard assumption (6) and treat $\text{Re}\sigma_{xx}$ as a frequency-dependent adjustable parameter in the QHE regime. This, therefore, implies that τ^* obtained from Eq. (7) can no longer describe EMP damping. Independent checks that Eqs. (5) and (6) are valid have not been carried out, since the quantity $\text{Im}\sigma_{xx}$ in the quantum Hall regime has

not been measured. It is not clear what the conductivity mechanism in the 2DEG should be in order for Eq. (5) to be valid. There exist contradictory data about the frequency dependence of $\text{Re}\sigma_{xx}$ in the QHE regime. Lee *et al.*²⁴ and Batov *et al.*²⁵ have observed a frequency dependence but measurements of the surface acoustic-wave damping²⁶ are described well by the assumption that $\text{Re}\sigma_{xx}$ is frequency independent. This discrepancy requires further investigation.

In the EC model of EMP, the edge charge distribution is controlled by the spatial structure of the EC's. A theoretical description of the EC structure was recently presented by Chklovskii, Shklovskii, and Glazman (CSG).²⁷ In this model a steplike behavior of electrostatic potential was found with plateaus corresponding to strips of compressible liquids and steps corresponding to strips of incompressible liquids. The compressible strips in this picture are the EC's. The density of states was adopted to be a sum of delta-function-like distributions, each delta function corresponding to one Landau level. Disorder results in the localization of states at the edges of EC's and in hopping conductivity across the EC's. The broadening of the Landau levels due to this disorder results in a varying potential inside the compressible strips, i.e., the edge channels. In order to get a rough idea of what the potential behavior inside the edge channels is like, one can take the density of states of the broadened Landau level to be a sum of the delta-function-like distributions and then apply the CSG consideration to that Landau level. Nevertheless, the idea of the EC as a wide strip with a high density of localized states at Fermi level seems to be valid. Recent experimental estimations of the EC width^{12,22} confirm this point of view.

We now discuss our understanding of the propagation and damping of EMP's. Let us consider for simplicity the 2DEG at $\nu=1$, see Fig. 8. The electric field of the edge charge drives the Hall current, which transfers charge from one part of the edge to another. In addition, the edge charge drifts due to the confining potential of the 2DEG. Both these processes determine the velocity of the edge-charge movement or EMP velocity. Edge charge entering a new part of the boundary redistributes across the edge channel by either filling or emptying adjacent localized states. For wide edge channels, this can lead to considerable damping. The time taken to reestablish the local equilibrium across the EC is estimated to be

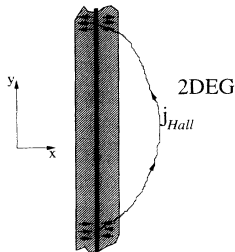


FIG. 8. An illustration of a sample edge. The strip represents an edge channel. There are delocalized states under Fermi level in the right side of the solid line.

$t_0 = l/\sigma$ where l is the EC width and σ is effective conductivity across the EC. For $l = 1 \mu\text{m}$ and $\sigma = e^2/h$, this estimation gives $t_0 = 10^{-12}$ s, such that $\omega t_0 \ll 1$ in the frequency range concerned. This allows us to estimate the current per unit length of the EC, I_x arising from equilibration processes to be $I_x = \omega\rho$, where ρ is the EMP charge per unit length of the EC and is given by $\rho = \rho_0 \exp[-i(ky - \omega t)]$. The EMP power dissipation W inside an EC can be estimated as

$$W \approx l\sigma(E_x^2 + E_y^2). \quad (8)$$

The components of the electric field E_x and E_y in the EC can be estimated as $E_x \approx I/\sigma = \omega\rho/\sigma$ and $E_y \approx k\rho \ln l/k l$. The EMP linewidth ($2\omega''$) is proportional to the ratio W/E where $E = \rho^2 \ln l/k l$ is the electrostatic energy of the EMP's per unit length. By combining these equations and assuming $\ln l/k l = 1$, the EMP linewidth can be estimated as

$$\omega'' \approx l \left[\frac{\omega^2}{\sigma} + \sigma k^2 \right]. \quad (9)$$

Thus, we have the conclusion that $\omega'' \propto \omega^2$. This estimation shows that the frequency dependence of the damping is unavoidable in the EC model of EMP's, but it does not, however, quantitatively agree with our experimental data. In our data, the damping as a function of frequency is somewhere between a linear and quadratic dependency, as shown in Fig. 6. It is clear that a more detailed calculation is needed to accurately describe the damping in the EC model of EMP.

While the EMP resonant frequencies reported here (taking into account differences in sample size) are in qualitative agreement with values observed by other authors,^{4,5,7,8,10,11} there is a disagreement in respect to EMP damping between our data and that published in Ref. 11. A much stronger decrease of the damping on approaching integer filling factors was observed in our data. In order to understand this discrepancy, it is important to consider that in Ref. 11 damping of an EMP wave packet was studied as compared with an EMP sine wave in our samples. The packet consists of several spatial EMP harmonics, which can have slightly different phase velocities and different damping. In this case, the EMP pulse would "wash out" showing not only a decrease in amplitude but a change in the shape of the response as well. The broadening of EMP pulse was indeed observed by Ashoori *et al.*¹¹ If this "washing out" process dominates the pulse amplitude (which is very probable in the QHE regime where the damping is small), then the damping of the EMP wave packet can differ significantly from the damping of an EMP sine wave.

In conclusion, we present a detailed experimental study of the EMP damping and velocity. The main experimental finding is a strong dependence of EMP damping on frequency in the QHE regime. It means that the description of the EMP damping in terms of effective relaxation time, which is often found in the literature, is not adequate. While the edge-channel model of EMP gives rise

to a frequency dependence of the damping, we have not yet determined a detailed quantitative description of the experimental data. Such a description is believed to require a microscopic theory of EC structure.

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- ¹V. A. Volkov and S. A. Mikhailov, *Zh. Eksp. Teor. Fiz.* **94**, 217 (1988) [*Sov. Phys. JETP* **67**, 1639 (1988)]; *Modern Problems in Condensed Matter Sciences*, edited by V. M. Agranovich and A. A. Maradudin (North-Holland, Amsterdam, 1991), Vol. 27.2, Chap. 15, p. 855.
- ²X. G. Wen, *Phys. Rev. Lett.* **64**, 2206 (1990); *Phys. Rev. B* **41**, 12 838 (1990); **43**, 11 025 (1991).
- ³A. A. Andreev, Ya. M. Blanter, and Yu. E. Lozovik, *Solid State Commun.* **88**, 231 (1993).
- ⁴S. A. Govorkov, M. I. Reznikov, A. P. Senichkin, and V. I. Talyanskii, *Pis'ma Zh. Eksp. Teor. Fiz.* **44**, 380 (1986) [*JETP Lett.* **44**, 187 (1986)].
- ⁵V. A. Volkov, D. V. Galchenkov, L. A. Galchenkov, I. M. Grodnenskii, O. R. Matov, and S. A. Mikhailov, *Pis'ma Zh. Eksp. Teor. Fiz.* **44**, 510 (1986) [*JETP Lett.* **44**, 655 (1986)].
- ⁶S. A. Govorkov, M. I. Reznikov, M. I. Medvedev, V. G. Mokerov, A. P. Senichkin, and V. I. Talyanskii, *Pis'ma Zh. Eksp. Teor. Fiz.* **45**, 252 (1987) [*JETP Lett.* **45**, 316 (1987)].
- ⁷E. I. Andrei, D. C. Glattli, F. I. B. Williams, and M. Heiblum, *Surf. Sci.* **196**, 5051 (1988).
- ⁸V. I. Talyanskii, I. E. Batov, B. K. Medvedev, J. P. Kotthaus, M. Wassermeier, A. Wixfoth, J. Weimann, W. Schlapp, and H. Nickel, *Pis'ma Zh. Eksp. Teor. Fiz.* **50**, 196 (1989) [*JETP Lett.* **50**, 221 (1989)].
- ⁹M. Wassermeier, J. Oshinovo, J. P. Kotthaus, A. H. MacDonald, C. T. Foxon, and J. J. Harris, *Phys. Rev. B* **41**, 10 287 (1990).
- ¹⁰I. Grodnensky, D. Heitmann, and K. von Klitzing, *Phys. Rev. Lett.* **67**, 1019 (1991).
- ¹¹R. C. Ashoori, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. West, *Phys. Rev. B* **45**, 3894 (1992).
- ¹²V. I. Talyanskii, A. V. Polisski, D. D. Arnone, M. Pepper, C. G. Smith, D. A. Ritchie, J. E. F. Frost, and G. A. C. Jones, *Phys. Rev. B* **46**, 12 427 (1992).
- ¹³V. I. Talyanskii, J. E. F. Frost, M. Pepper, D. A. Ritchie, M. Grimshaw, and G. A. C. Jones, *J. Phys. Condens. Matter* **5**, 7643 (1993).
- ¹⁴N. B. Zhitenev, R. J. Haug, K. v. Klitzing, and K. Eberl, *Phys. Rev. Lett.* **71**, 2292 (1993).
- ¹⁵D. B. Mast, A. J. Dahm, and A. L. Fetter, *Phys. Rev. Lett.* **54**, 1706 (1985).
- ¹⁶D. C. Glattli, E. Y. Andrei, G. Deville, J. Poitrenaud, and F. I. B. Williams, *Phys. Rev. Lett.* **54**, 1710 (1985).
- ¹⁷P. J. M. Peters, M. J. Lea, A. M. Jansen, A. O. Stone, W. P. N. M. Jacobs, P. Fozzoni, and R. W. van der Heijden, *Phys. Rev. Lett.* **67**, 2199 (1991).
- ¹⁸S. I. Allen, H. L. Stormer, and J. C. M. Hwang, *Phys. Rev. B* **28**, 4875 (1983).
- ¹⁹T. Demel, D. Heitmann, P. Grambow, and K. Ploog, *Phys. Rev. Lett.* **64**, 788 (1990).
- ²⁰V. B. Shikin, *Pis'ma Zh. Eksp. Teor. Fiz.* **47**, 471 (1988) [*JETP Lett.* **47**, 555 (1988)].
- ²¹M. Buttiker, *Phys. Rev. B* **38**, 9375 (1988).
- ²²S. W. Hwang, D. C. Tsui, and M. Shayegan, *Phys. Rev. B* **48**, 8161 (1993).
- ²³E. Batke, D. Heitman, and C. W. Tu, *Phys. Rev. B* **34**, 6951 (1986).
- ²⁴J. I. Lee, B. B. Goldberg, M. Heiblum, and P. J. Stiles, *Solid State Commun.* **64**, 447 (1987).
- ²⁵I. E. Batov, A. V. Polisskii, M. I. Reznikov, and V. I. Talyanskii, *Solid State Commun.* **76**, 25 (1990).
- ²⁶A. Wixforth, J. P. Kotthaus, and G. Weimann, *Phys. Rev. Lett.* **56**, 2104 (1986).
- ²⁷D. B. Chklovskii, B. I. Shklovskii, and L. I. Galzhan, *Phys. Rev. B* **46**, 4026 (1992).

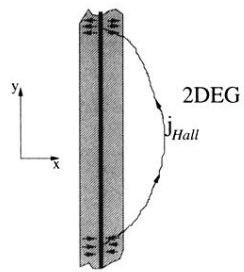


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