Magnetic field dependence of two-dimensional static shielding in the hydrodynamic model

Vassilios Fessatidis
Department of Physics, Fordham University, Bronx, New York 10458

Norman J. M. Horing and Hong-Liang Cui
Department of Physics and Engineering Physics, Stevens Institute of Technology, Hoboken, New Jersey 07030
(Received 14 July 1994)

The hydrodynamic description of two-dimensional plasma nonlocality in a magnetic field [as represented by the dynamic nonlocal longitudinal dielectric function of interacting two-dimensional (2D) electrons] is seen to be inappropriate for direct application in the determination of the magnetic-field dependence of shielding in the static limit. We find that this shortcoming is remedied by inversion of the full 2D hydrodynamic dielectric tensor in magnetic field and selection of its longitudinal element for static shielding.

I. INTRODUCTION

The hydrodynamic model of plasma nonlocality has served as a valuable guide in many complicated calculations relating to correlation phenomena in solids and the role of collective electron plasma modes. Moreover, a judicious choice of the nonlocal "sound velocity" parameter β also adequately represents the static Thomas-Fermi-Debye screening law (although this choice does not properly fit the nonlocal correction to the bulk plasmon, and depending on one's interest, the fitting choice could be reversed). The model has provided much physical insight within the limitations of its validity as a simplified representation of nonlocal three-dimensional (3D) plasma dynamics, surface correlation dynamics, and microstructure plasma dynamics.

The inclusion of a magnetic field in the hydrodynamic model follows simply from a Lorentz force, and it properly represents the local magnetoplasmon spectrum. This advantage has been successfully employed in a number of 2D theories, through the 2D longitudinal hydrodynamic dielectric function $\epsilon_{2\mathrm{D}}(\bar{\mathbf{p}},\omega)$ ($\bar{\mathbf{p}}$ is 2D wave vector, ω is frequency, ω_c is the cyclotron frequency, n_0 is the 2D sheet density, m is the effective mass, $-e$ is the effective charge moderated by the background dielectric constant, and δ is a positive infinitesimal):

$$\epsilon_{2\mathrm{D}}(\bar{\mathbf{p}},\omega)=1-\frac{2\pi e^2 n_0 p}{m\,[\omega(\omega+i\delta)-\omega_c^2-\beta^2 p^2]}\ , \tag{1}$$

with the local electrostatic magnetoplasmon for low wave number given by $\epsilon_{2\mathrm{D}}=0$,

$$\omega^2=\frac{2\pi e^2 n_0}{m}p+\omega_c^2=\omega_{p,2\mathrm{D}}^2+\omega_c^2\ , \tag{2}$$

where the 2D local plasmon in the absence of a magnetic field is given by $\omega_{p,2\mathrm{D}}^2=2\pi e^2 n_0 p/m$. (A more detailed discussion is given in the Appendix.) Furthermore, in the *absence* of a magnetic field the nonlocal structure of the longitudinal hydrodynamic dielectric function describes 2D static shielding in terms of

$$\epsilon_{2\mathrm{D}}^{-1}(\bar{\mathbf{p}},0)=\left[1+\frac{2\pi e^2 n_0}{m\beta^2 p}\right]^{-1}\ , \tag{3}$$

which is correct for screening at wave numbers $p \ll p_F$ (Fermi wave number) as a 2D counterpart of Thomas-Fermi-Debye static shielding, contingent, of course, on the choice of β as $\beta^2=\hbar^2 n_0/2m^2e^2$.[1] (The 2D counterpart of Thomas-Fermi-Debye shielding is not as dramatic as the 3D exponential decay in space, as shown in a detailed evaluation in Ref. 1.) Considering the hydrodynamic model of static shielding with the inclusion of a magnetic field $\mathbf{B}$, it is to be expected physically that there should be no change from Eq. (3), since the Lorentz force cannot do any of the work required for a rearrangement of the statically shielded charge distribution. However, a glance at Eq. (1) in the static limit seems to indicate otherwise,

$$\epsilon_{2\mathrm{D}}^{-1}(\bar{\mathbf{p}},0)=\left[1+\frac{2\pi e^2 n_0 p}{m\,[\omega_c^2+\beta^2 p^2]}\right]^{-1}\ , \tag{4}$$

because of the presence of ω_c^2 in the denominator of the 2D hydrodynamic polarizability. The resolution of this apparent dilemma is the object of this paper. An appropriate explanation of this seemingly unphysical feature of the hydrodynamic model is important to maintain confidence in its capability to provide reasonable physical insight in more complicated calculations.

II. INVERSION OF THE 2D HYDRODYNAMIC DIELECTRIC TENSOR IN MAGNETIC FIELD

In point of fact, it is insufficient to consider just the algebraic inverse of the 2D longitudinal dielectric function $\epsilon_{2\mathrm{D}}(\bar{\mathbf{p}},\omega)$ since response on the plane is tensorial. To construct the appropriate dielectric tensor $\overleftrightarrow{\epsilon}_{2\mathrm{D}}(\bar{\mathbf{p}},\omega)$, we employ its 2×2 matrix relation to the conductivity tensor $\overleftrightarrow{\sigma}_{2\mathrm{D}}(\bar{\mathbf{p}},\omega)$, on the plane[2-7]

$$\overleftrightarrow{\epsilon}_{2\mathrm{D}}(\bar{\mathbf{p}},\omega)=\overleftrightarrow{I}+i\frac{2\pi}{\omega}p\,\overleftrightarrow{\sigma}_{2\mathrm{D}}(\bar{\mathbf{p}},\omega)\ , \tag{5}$$

where $\bar{\mathbf{J}}=\overleftrightarrow{\sigma}_{2D}\cdot\bar{\mathbf{E}}$ with $\bar{\mathbf{J}}=-n_0 e\bar{\mathbf{v}}$ as sheet current density per unit normal length, n_0 is the sheet density per unit area, and $\bar{\mathbf{v}}$ is the drift velocity responding to the electric field $\bar{\mathbf{E}}$ parallel to the plane in a perpendicular magnetic field. In this circumstance, an alternative equivalent form may be written as[8]

$$\overleftrightarrow{\epsilon}_{2D}(\bar{\mathbf{p}},\omega)=\overleftrightarrow{I}+i\frac{4\pi}{\omega}\overleftrightarrow{\sigma}_{\text{block}}(\bar{\mathbf{p}},\omega)\ , \tag{6a}$$

where $\overleftrightarrow{\sigma}_{\text{block}}$ is the 2×2 part of the full block diagonal 3×3 conductivity tensor, and

$$\overleftrightarrow{\sigma}_{\text{block}}=\frac{p}{2}\overleftrightarrow{\sigma}_{2D}\ . \tag{6b}$$

The 2D current-field relation for the hydrodynamic model is based on the linearized continuity and Euler equations for the perturbed 2D density n and velocity $\bar{\mathbf{v}}$:

$$\frac{\partial n}{\partial t}+n_0\nabla\cdot\bar{\mathbf{v}}=0\ , \tag{7}$$

$$n_0\frac{\partial\bar{\mathbf{v}}}{\partial t}+\frac{n_0\bar{\mathbf{v}}}{\tau}=-\beta^2\nabla n-\frac{e}{m}n_0\bar{\mathbf{E}}-\frac{e}{m}n_0\bar{\mathbf{v}}\times\mathbf{B}\ , \tag{8}$$

where collisions are represented in terms of a constant phenomenological relaxation time τ. Fourier transforming ($\bar{\mathbf{r}}\to\bar{\mathbf{p}},t\to\omega$) Eqs. (7) and (8) yield the 2D conductivity tensor as

$$\overleftrightarrow{\sigma}_{2D}(\bar{\mathbf{p}},\omega)=\frac{in_0e^2\omega}{m\mathcal{D}}\begin{bmatrix}\Omega^2-\beta^2p_y^2 & \beta^2p_xp_y-i\omega\omega_c\\ \beta^2p_xp_y+i\omega\omega_c & \Omega^2-\beta^2p_x^2\end{bmatrix}\ , \tag{9}$$

where $\Omega^2=\omega(\omega+i/\tau)$ and $\mathcal{D}=\Omega^2(\Omega^2-\beta^2p^2)-\omega^2\omega_c^2$ and p_x,p_y are wave-vector components along two mutually orthogonal directions on the 2D x,y plane. This results in the 2D dielectric tensor (here, again, $\omega_{p,2D}^2=2\pi e^2n_0p/m$)

$$\overleftrightarrow{\epsilon}_{2D}(\bar{\mathbf{p}},\omega)=\begin{bmatrix}1-\frac{\omega_{p,2D}^2}{\mathcal{D}}(\Omega^2-\beta^2p_y^2) & -\frac{\omega_{p,2D}^2}{\mathcal{D}}(\beta^2p_xp_y-i\omega\omega_c)\\ -\frac{\omega_{p,2D}^2}{\mathcal{D}}(\beta^2p_xp_y+i\omega\omega_c) & 1-\frac{\omega_{p,2D}^2}{\mathcal{D}}(\Omega^2-\beta^2p_x^2)\end{bmatrix}\ , \tag{10}$$

which has the longitudinal component

$$\begin{aligned}\epsilon_L&=\frac{\bar{\mathbf{p}}\cdot\overleftrightarrow{\epsilon}_{2D}\cdot\bar{\mathbf{p}}}{p^2}\\ &=\frac{1}{p^2}[p_x^2\epsilon_{2D,xx}+p_y^2\epsilon_{2D,yy}+p_xp_y(\epsilon_{2D,xy}+\epsilon_{2D,yx})]=1-\frac{\omega_{p,2D}^2\Omega^2}{\Omega^2(\Omega^2-\beta^2p^2)-\omega^2\omega_c^2}\ ,\end{aligned} \tag{11}$$

corresponding to Eq. (1) with $1/\tau\to\delta$, a positive infinitesimal, which we take to be the case.

The matrix inverse of the dielectric tensor, $\overleftrightarrow{\kappa}_{2D}(\bar{\mathbf{p}},\omega)$ $[\overleftrightarrow{\kappa}_{2D}\cdot\overleftrightarrow{\epsilon}_{2D}=(\begin{smallmatrix}1&0\\0&1\end{smallmatrix})]$, is given by

$$\overleftrightarrow{\kappa}_{2D}(\bar{\mathbf{p}},\omega)=\frac{1}{\det|\overleftrightarrow{\epsilon}_{2D}|}\begin{bmatrix}1-\frac{\omega_{p,2D}^2}{\mathcal{D}}(\Omega^2-\beta^2p_x^2) & \frac{\omega_{p,2D}^2}{\mathcal{D}}(\beta^2p_xp_y-i\omega\omega_c)\\ \frac{\omega_{p,2D}^2}{\mathcal{D}}(\beta^2p_xp_y+i\omega\omega_c) & 1-\frac{\omega_{p,2D}^2}{\mathcal{D}}(\Omega^2-\beta^2p_y^2)\end{bmatrix}\ , \tag{12}$$

with

$$\det|\overleftrightarrow{\epsilon}_{2D}|=1-\frac{\omega_{p,2D}^2}{\mathcal{D}}(2\Omega^2-\beta^2p^2-\omega_{p,2D}^2)\ , \tag{13}$$

and its longitudinal part is

$$\begin{aligned}\kappa_L(\bar{\mathbf{p}},\omega)&=\frac{\bar{\mathbf{p}}\cdot\overleftrightarrow{\kappa}_{2D}\cdot\bar{\mathbf{p}}}{p^2}\\ &=\frac{1}{\det|\overleftrightarrow{\epsilon}_{2D}|}\left[1-\frac{\omega_{p,2D}^2}{\mathcal{D}}(\Omega^2-\beta^2p^2)\right]\ .\end{aligned} \tag{14}$$

III. CONCLUSIONS

In the static limit, $\omega\to 0$, Eq. (14) yields the longitudinal part of the inverse dielectric tensor as

$$\kappa_L(\bar{\mathbf{p}},0)=\frac{1}{1+(p_{\text{TF}}/p)}\ , \tag{15}$$

which has the proper form for the 2D counterpart of Thomas-Fermi-Debye shielding with the identification $p_{\text{TF}}=2\pi e^2n_0/m\beta^2$. As anticipated from physical arguments in the Introduction, the classical Lorentz force does not affect the static screening of the hydrodynamic model due to its classical dynamical character. Straightforward as these considerations are, they gain importance by way of contrast to the fact that $\epsilon_{2D}^{-1}(\bar{\mathbf{p}},0)$ as given by Eq. (4) fails to correctly represent static shielding in the 2D hydrodynamic model. Such a failure does *not* occur in the 2D random-phase approximation (RPA), and $\epsilon_{2D(\text{RPA})}^{-1}(\bar{\mathbf{p}},0)$ does indeed correctly represent static shielding in the presence of magnetic field, reproducing Eq. (15) as

$$\epsilon_{2D(RPA)}^{-1}(\bar{p},0)=\kappa_L(\bar{p},0)=\frac{1}{1+(p_{TF}/p)} , \quad (16)$$

for wave numbers well below the Fermi wave number, as shown in detailed analysis of the RPA in Ref. 1. In this light, the failure of $\epsilon_{2D}^{-1}(\bar{p},0)$ to correctly represent 2D static shielding in a magnetic field could be seen as a serious deficiency of the 2D hydrodynamic model, were it not for the fact that the correct static result is recovered by dielectric tensor inversion, in the form of $\kappa_L(\bar{p},0)$ given by Eq. (15). With this in mind, there should be no loss of confidence in the hydrodynamic model and its capability to provide physical insight into the role of nonlocal plasma dynamics in a complicated 2D probe of correlation and interaction in a magnetic field.

APPENDIX

In regard to the local 2D magnetoplasmon spectrum, it has long been known that the 2D coupled electromagnetic-plasma dispersion relation (retarded) has the form[2,7]

$$\det\left|\overleftrightarrow{I}+\frac{p^2}{p_\perp^2}\left[\overleftrightarrow{I}-\frac{\bar{p}_\parallel\bar{p}_\parallel}{p^2}\right]\left[\frac{\overleftrightarrow{\epsilon}_{2D}}{\epsilon_m}-\overleftrightarrow{I}\right]\right|=0 , \quad (A1)$$

where $p^2=p_\parallel^2+p_\perp^2=\epsilon_m\omega^2/c^2$ with $\bar{p}_\parallel$ and $\bar{p}_\perp$ being wave-vector components parallel and perpendicular to the 2D electron sheet and ϵ_m is introduced as the background dielectric constant. In the electrostatic limit $c\to\infty$, $p\to 0$, and $p_\perp=ip_\parallel$, and Eq. (17) reduces to

$$\det\left|\overleftrightarrow{I}+\frac{\bar{p}_\parallel\bar{p}_\parallel}{p^2}\left[\frac{\overleftrightarrow{\epsilon}_{2D}}{\epsilon_m}-\overleftrightarrow{I}\right]\right|=0 , \quad (A2)$$

or, alternatively

$$\frac{1}{\epsilon_m}\bar{p}_\parallel\cdot\overleftrightarrow{\epsilon}_{2D}\cdot\bar{p}_\parallel=\frac{\epsilon_L}{\epsilon_m}=0 , \quad (A3)$$

which reproduces the electrostatic magnetoplasmon given by Eq. (2). However, as shown above, it is necessary to employ the longitudinal component of the inverted dielectric tensor to correctly determine the static shielding of the hydrodynamic model.

[1] N. J. M. Horing and M. Yildiz, Ann. Phys. (N.Y.) **97**, 216 (1976).
[2] M. Orman, Ph.D. thesis, Stevens Institute of Technology, 1974.
[3] N. J. M. Horing, M. Orman, and M. Yildiz, Phys. Lett. **48A**, 7 (1974).
[4] M. Orman and N. J. M. Horing, Solid State Commun. **15**, 1381 (1974).
[5] M. Orman, N. J. M. Horing, and M. L. Glasser, in *Proceedings of the Seventh International Vacuum Congress and Third International Conference Solid Surfaces, Vienna, 1977*, edited by R. Dobrozemsky *et al.* (F. Berger & Söhne, Vienna, 1977).
[6] N. J. M. Horing, M. Orman, E. Kamen, and M. L. Glasser, Phys. Lett. **85A**, 378 (1981).
[7] K. W. Chiu and J. J. Quinn, Phys. Rev. B **9**, 4724 (1974).
[8] S. Das Sarma and J. J. Quinn, Phys. Rev. B **25**, 7603 (1982).