PHYSICAL REVIEW B VOLUME 50, NUMBER 20 15 NOVEMBER 1994-II

Temperature variation of the time of inelastic electron relaxation in disordered bismuth films

Yu. F. Komnik, V. Yu. Kashirin, B. I. Belevtsev, and E. Yu. Beliaev
B. Verkin Institute for Low Temperature Physics and Engineering of Ukrainian Academy of Sciences, 47 Lenin Avenue,
310164 Kharkov, Ukraine

(Received 5 April 1994)

The behavior of quantum corrections to magnetoconductivity associated with the effect of weak electron localization is studied on thin Bi films ($\simeq 100$ Å thick) in a temperature range of 0.3–10 K with the effect of electron overheating being realized. The temperature dependences of the time $\tau_{e\text{-ph}}$ of electron-phonon energy relaxation are determined. The weakening of the electron-phonon interaction predicted for the case of the inequality $q_{\rm ph}l<1$ that yields a so-called "dirty limit" ($q_{\rm ph}$ is the thermal phonon wave vector, l is the electron mean free path) is observed. Two ways are used to realize the dirty limit: an artificial decrease in the mean free path l and a lowering of the temperature below the characteristic one, $T_2=\hbar s/kl$ (s is the phonon velocity).

I. INTRODUCTION

At present the scientific literature numbers many reports of experimental investigations of the quantum interference effects in disordered metal systems, i.e., weak electron localization (WL) and enhancement of electron-electron interaction (EEI). These effects arouse interest because they provide information on the times τ_φ of phase relaxation of the electron wave function (in essence the inelastic relaxation time) as well as on the electron spin relaxation times for spin-orbital ($\tau_{\rm so}$) and spin-spin (τ_s) scattering and on the electron-electron interaction constants. The main results of these investigations are presented in reviews[1–5] where thorough analysis of experimental data is given and the relationship to the current theoretical models is demonstrated.

Analysis of results for quantum corrections to the conductivity of thin metal films showed that in the low-temperature range ($T\approx 1$ K) the temperature variation of τ_φ is characterized by the law $\tau_\varphi^{-1}\propto T$ and at higher temperatures ($T\approx 10$ K), as a rule, by the law $\tau_\varphi^{-1}\propto T^2$.

In the former case, τ_φ was unambiguously identified as the time $\tau_{e\text{-}e}$ of electron-electron interaction dominated by collisions with small transfer of momenta. For two-dimensional disordered systems it is given by the expression[5]

$$\tau_{e\text{-}e}^{-1}=\frac{\pi kT}{\hbar}\frac{e^2R_\square}{2\pi^2\hbar}\ln\frac{\pi\hbar}{e^2R_\square}\ , \tag{1}$$

where $R_\square$ is the resistance of a square area element of the film. It was assumed that in this range of temperatures, $\tau_{e\text{-}e}\ll\tau_{e\text{-ph}}$.

At higher temperatures, when $\tau_{e\text{-}e}\gg\tau_{e\text{-ph}}$, the effect of electron-phonon relaxation is predominant. Therefore it becomes essentially possible to find $\tau_{e\text{-ph}}$, though identity of τ_φ and $\tau_{e\text{-ph}}$ is not evident in this temperature range.

For electron-phonon scattering in disordered metals some ideas were theoretically considered which suggest that the temperature dependence of $\tau_{e\text{-ph}}$ is different from that in the pure case. In a pure metal, as is known, $\tau_{e\text{-ph}}$ is at low temperature inversely proportional to the number of thermal phonons, and therefore $\tau_{e\text{-ph}}^{-1}\propto T^3$.

A situation can occur in a disordered metal, where, because of the small mean free path of electrons at low temperature, the inequality $q_{\rm ph}l<1$ is true, where $q_{\rm ph}$ is the wave vector of a thermal phonon and l is the electron mean free path. This inequality means smallness of l as against the phonon wavelength, and the electron-phonon interaction should therefore decrease. According to the common terminology, we shall refer to the case of $q_{\rm ph}l<1$ as the "dirty limit," in contrast to the case of $q_{\rm ph}l>1$, i.e., the "pure limit." Schmid[6] was the first to obtain the dependence $\tau_{e\text{-ph}}^{-1}\propto lT^4$ for the dirty limit. During the same period the dependence $\tau_{e\text{-ph}}^{-1}\propto l^{-1}T^2$ was obtained[7] for the noncoherent inelastic electron scattering by the excitation regions in the disordered metal (by vibrating impurities). (Note that the exponent 2 in T is also possible for the processes of electron interaction during the exchange of virtual phonons.[8]) Schmid's result $\tau_{e\text{-ph}}^{-1}\propto lT^4$ was later obtained for transverse and longitudinal phonons also in Refs. 9 and 10. In the pure limit ($q_{\rm ph}l>1$), Ref. 11 obtained for longitudinal phonons the dependence $\tau_{e\text{-ph}}^{-1}\propto T^3$ which is known for pure metals, but for transverse phonons, according to Ref. 9, inelastic scattering of electrons by vibrating impurities is important, and $\tau_{e\text{-ph}}^{-1}\propto l^{-1}T^2$. The contribution of this mechanism, as was shown in Ref. 10, decreases in the dirty limit. In the pure limit, according to Reizer,[11] it is absent; for the longitudinal and two transverse phonon modes at low temperatures the dependence $\tau_{e\text{-ph}}^{-1}\propto T^3$ was obtained. Thus, as found in Refs. 10 and 11, when the transition from pure to dirty limit occurs in isotropic disordered metals, the dependence $\tau_{e\text{-ph}}^{-1}\propto T^3$ changes to $\tau_{e\text{-ph}}^{-1}\propto lT^4$, and the dependence $\tau_{e\text{-ph}}^{-1}\propto l^{-1}T^2$ predicted in Ref. 7 is actually not seen anywhere.

The condition $q_{\rm ph}l\approx 1$ determines a certain temperature $T_2=\hbar s_{l,t}/kl$ ($s_{l,t}$ is the phonon velocity) below which the dirty limit situation arises. Thus in disordered metals the transition from pure to dirty limit can be achieved by lowering temperature.

The electron relaxation time calculated in Refs. 9–11 reflects the electron-phonon interaction contribution to relaxation of the electron wave function phase. To distinguish it from the electron-phonon energy relaxation time $\tau_{e\text{-ph}}$,[12] we shall denote it, as in Refs. 13 and 14, by $\tau^i_{e\text{-ph}}$. These characteristics times are both governed by the Eliashberg function $\alpha^2F(\omega)$ and are in a certain temperature-independent ratio for the pure and dirty limits taking into account the dimension of the electron-phonon interaction. In Ref. 13 the low-frequency part of the Eliashberg function, which is most important in the processes of electron-phonon scattering at low temperatures, was described as the exponential function $\alpha^2F(\omega)=C_\zeta\omega^\zeta$ and a suitable expression was obtained to calculate the ratio of the times $\tau_{e\text{-ph}}$ and $\tau^i_{e\text{-ph}}$. The ratio is determined by the exponent ζ or, in other words, by the exponent p in the temperature dependence of the electron-phonon interaction time $\tau_{e\text{-ph}} \propto T^{-p}$ because $p=\zeta+1$ in the low-temperature region. According to Ref. 13, in the relation $\tau^{-1}_{e\text{-ph}}=A^*(\tau^i_{e\text{-ph}})^{-1}$ the coefficient A^* is 2.5 and 4.285 for $p=3$ and 4, respectively.

Here are the complete expressions for $\tau^i_{e\text{-ph}}$ for the model of one longitudinal and two transverse phonon modes which were obtained in Refs. 10 and 11.

Pure limit:[11]

$$(\tau^i_{e\text{-ph}})^{-1}=\frac{7\pi\zeta(3)\beta_l}{(p_Fs_l)^2}\left[1+\frac{16}{\pi^2}\left[\frac{s_l}{s_t}\right]^4\right](kT)^3 . \tag{2}$$

This relation was obtained for the limitation on the temperature $T_2<T<T_1$, where $T_1=(\hbar s_t/kc)(V_Fk_D^2s_t)^{1/2}$, $k_D^2=4\pi e^2\nu$, and $\nu=mp_F/\hbar^3\pi^2$.

A result similar to Eq. (2) is presented in Ref. 9 for longitudinal phonons, for the case of $\hbar s_l/kl<T<\hbar s_lk_D/k$.

Dirty limit,[10] $T<T_2$:

$$(\tau^i_{e\text{-ph}})^{-1}=\frac{\pi^2\beta_l(p_Fl)}{(p_Fs_l)^3}\left[1+\frac{3}{2}\left[\frac{s_l}{s_t}\right]^5\right](kT)^4 . \tag{3}$$

This relation coincides with the results of Refs. 8 and 9. In Eqs. (2) and (3), p_F is the Fermi electron momentum, s_l and s_t are the velocities of longitudinal and transverse phonons, respectively, $\beta_l=(\frac{2}{3}E_F)^2\nu/2MNs_l^2$ is a dimensionless constant characterizing the interaction between electrons and longitudinal phonons, ν is the electron density of states, M is the mass of an ion, and N is the number of unit cells per unit volume.

In the case of two-dimensional electron-phonon interaction in a thin film, the form of the temperature dependence of $\tau^i_{e\text{-ph}}$ in the dirty limit is different from Eq. (3). Reference 14 presents the following formula for this case:

$$(\tau^i_{e\text{-ph}})^{-1}=\frac{56\zeta(3)\beta_ll}{5\pi^2(p_Fs_l)^2L}\left[1+\frac{3}{2}\left[\frac{s_l}{s_t}\right]^5\right](kT)^3 , \tag{4}$$

where L is the film thickness.

A similar result was earlier obtained in the general form in Ref. 15.

Besides the quantum corrections to the conductivity of thin films, information on the electron-phonon interaction time $\tau_{e\text{-ph}}$ may be provided by experiments on the electron overheating in thin films. A pure effect of electron overheating occurs when there is a good acoustic coupling of the film and substrate, which permits the nonequilibrium phonons emitted by the electrons on overheating to escape from the film. In this case the lattice temperature remains practically invariable, and a certain temperature T_e may be assigned to the electrons, which described the nonequilibrium distribution function of the electrons. Data on the time $\tau_{e\text{-ph}}$ can be obtained in such experiments because transfer of the excess energy from the electron to the phonon system, even with strong elastic scattering, is determined by the time $\tau_{e\text{-ph}}$. Experimentally, the task amounts to determining the electron gas overheating $\Delta T_e=T_e-T_{\text{ph}}$ (against the phonon temperature T_{ph}) in the case of passage of a high current or the effect of other heating factors.

In recent years, several experimental works have been carried out which obtain data on the time $\tau_{e\text{-ph}}$ from the electron overheating effect.[13,14,16–19] These experiments gave widely varying functional dependences $\tau_{e\text{-ph}}(T)$ in the low-temperature range. Thus Ref. 16 gave the exponent p in the dependence $\tau^{-1}_{e\text{-ph}} \propto T^p$ as 3 for copper film on sapphire; Ref. 17 for a gold film on glass and a bismuth film on gold with SiO intercalation gave p equal to 2.2 and 3, respectively; Ref. 18 gave $p=1.3$–1.5 for antimony on glass; Ref. 19 gave $p=2$ for niobium on sapphire; Ref. 13 gave $p=2$ for gold on quartz; and Ref. 14 found $p=3$ for gold on glass and silicon.

The exponent $p<3$ appearing in some cases may be accounted for by the fact that a two-dimensional electron-phonon interaction is possible in thin films: in a free film or in the film with poor film-substrate (or environment) acoustic coupling quantization of the phonon spectrum is observed at low temperatures.[13] The quantization-induced modification of the phonon spectrum lowers the exponent p from 4 to 3 in the dirty limit [see Eq. (4)] and, supposedly, from 3 to 2 in the pure limit. There is no rigorous theory for the latter case. In the meantime, a visual physical interpretation of $p=2$ for thin films was proposed in Refs. 1 and 13: the modified phonon spectrum of films must be characterized by a linear variation of the Eliashberg function in the low-frequency region ($\zeta=1$), which gives the dependence $\tau^{-1}_{e\text{-ph}} \propto T^2$. Note in addition, that in the case of poor acoustic coupling between the film and the substrate the Joule heating can cause errors in $\tau_{e\text{-ph}}$ found from the equation of thermal balance [see Eq. (7) below] and may thus be another reason for $p<3$ (e.g., 1.5 or 2).

In earlier experiments on electron overheating in thin bismuth films[20] analysis of a large number of measurements resulted in the conclusion that the exponent p for films of a thickness more than 100 Å is usually close to 3 in the range 2–3 K and the dependence $\tau_\varphi^{-1} \propto T^2$ observed at higher temperatures results from summation of the frequencies of electron-electron and electron-phonon relaxations:

$$\tau_\varphi^{-1}=\tau^{-1}_{e\text{-}e}+\tau^{-1}_{e\text{-ph}}=AT+A_pT^p . \tag{5}$$

The result $p=3$ is reasonably treated in Ref. 20 as a manifestation of electron-phonon scattering in the three-dimensional case, where $q_{\mathrm{ph}} l > 1$. Indeed, with the mean free path of charge carriers l roughly equal to the film thickness $L \geq 100$ Å, the temperature T_2 for bismuth is 1–1.5 K, below the temperature range of the measurements. The two-dimensional character of the electron-phonon interaction is improbable because of the nature of the experiments involving electron overheating which requires good acoustic coupling between the film and the substrate.

(3) The present work was intended to search for the conditions of observation of the temperature dependence $\tau_{e\text{-ph}}^{-1} \propto T^4$ expected for the dirty limit ($q_{\mathrm{ph}} l < 1$) on thin bismuth films. This can be done in two ways: (i) artificial decrease of the mean free path l by decreasing the film thickness towards the percolation limit, so that the temperature T_2 is raised, or (ii) making measurements on films of thickness of about 100 Å, as earlier, but at ultralow temperatures, i.e., below T_2. We have used both.

II. EXPERIMENT

Thin bismuth films (of thicknesses about 100 Å) were produced by condensation of the molecular bismuth beam, in vacuum of roughly 10^{-6} Torr, onto a room-temperature substrate. The samples were so shaped that the resistance could be measured by the four-probe method. The film width was 60–600 μm.

Two types of substrates were used: mica and calcite, providing bad and good acoustic coupling with bismuth, respectively. The criterion of the latter was comparison of the acoustic impedance $Z=\rho^* s/\cos\vartheta$ (ρ^* is the density, s the sound velocity, and ϑ the angle of incidence or passage of a sound wave) of the two media. In the case of a film on mica, electron overheating can be provided, with small energy evolution, at temperatures below that of the helium transition to the superfluid state. It is known, if the heat flow through the film surface to liquid helium does not exceed 0.6–0.8 W cm^{-2}, which corresponds to the convective or nucleate boiling regime of helium I boiling and the crisis of helium I boiling does not begin yet, that the surface overheating of the metal in which heat is released is not more than 0.8 K. For helium II such a regime corresponds to the film-free boiling, and the overheating of the metal surface is several times lower than in helium I. This occurs due to the extremely high thermal conductivity of superfluid helium despite the Kapitza jump. Thus below T_λ the temperature of the film may be considered practically invariable on passing moderate currents, and the situation may be interpreted as the electron overheating regime.

The measurements were made at temperatures 0.38–20 K (with the sample in vacuum or in liquid helium) and magnetic fields up to 45 kG. The current through samples was varied between 1 and 500 μA, the maximum current density was 5×10^4 A cm^{-2}; the electric field in this case was up to 33 V cm^{-1}.

The value of τ_φ was calculated by the relation[5]

$$\Delta\sigma_H^{\mathrm{loc}} = \frac{e^2}{2\pi^2\hbar}\left[\frac{3}{2} f_2\left[\frac{4eDH}{\hbar c}\tau_\varphi^*\right] - \frac{1}{2} f_2\left[\frac{4eDH}{\hbar c}\tau_\varphi\right]\right], \tag{6}$$

giving the localization correction to the conductivity of thin film in a perpendicular magnetic field. In Eq. (6), $\tau_\varphi^{*\,-1} = \tau_{\varphi 0}^{-1} + \frac{4}{3}\tau_{\mathrm{so}}^{-1} + \frac{2}{3}\tau_s^{-1}$; $\tau_\varphi^{-1} = \tau_{\varphi 0}^{-1} + 2\tau_s^{-1}$ (in a sample without magnetic impurities the time τ_s may be neglected); $f_2(x)=\ln(x)+\psi(\frac{1}{2}+1/x)$, ψ being the logarithmic derivative of the Γ function and D the electron diffusion coefficient. This one was calculated by the relation $D=v_F^2\tau/3$, where τ is the time of elastic electron relaxation. In the experiment and calculations we took into account the results of Ref. 21 and used the conditions at which there is no contribution to the magnetoconductivity of the quantum corrections besides the localization one. This governed the values of the currents and the magnetic fields.

In an electric field E electrons gain an additional energy $\Delta\varepsilon = eE(D\tau_{e\text{-ph}})^{1/2}$,[22] where $(D\tau_{e\text{-ph}})^{1/2}$ is the diffusion length. Energy transfer from the electron system to the phonon system is controlled by the time $\tau_{e\text{-ph}}$, and therefore the electron temperature T_e can be higher than the phonon temperature T_{ph}. These two temperatures are related as follows:[13,22]

$$(kT_e)^2 = (kT_{\mathrm{ph}})^2 + 6(eE)^2 D\tau_{e\text{-ph}}/\pi^2 . \tag{7}$$

In the electron overheating effect taking place in the case of good acoustic coupling with the substrate the temperature T_{ph} is identical to the initial temperature of the film and coincides with the substrate temperature. Thus calculation of $\tau_{e\text{-ph}}$ by Eq. (7) requires knowing the effective electron temperature T_e.

In the present study, as in Ref. 20, information on the electron temperature T_e in the case of electron overheating was obtained using the dependence $\tau_\varphi(T)$ for various current values. The magnitudes of τ_φ were determined from the magnetic-field-related variation of the quantum correction associated with weak localization of electrons, so that the contribution of the electron-electron interaction is canceled, which is sensitive, as was found in Ref. 21, in bismuth films to the electron energies and therefore to the electric field. It follows from the results of Ref. 20 that, though, rigorously, weak localization of electrons is determined by both the phonon temperature and the electron temperature, the predominant effect on τ_φ belongs to the latter. The temperature T_e can be found from comparison of τ_φ for passage of the heating current with that in the "equilibrium" curve of $\tau_\varphi(T)$, for a low current. After this, comparison by formula (7) yields the value of $\tau_{e\text{-ph}}$.

III. EXPERIMENTAL RESULTS AND DISCUSSION

The dirty limit of bismuth films at liquid-helium temperatures can be provided by decrease of the mean free path so that the temperature T_2 should be raised above the measurement range. We could do so by making the film thickness less than 100 Å, which is close the percolation threshold, i.e., 40–50 Å for bismuth films prepared at room temperature.

Figure 1 shows the results for quantum interference and overheating for a 74-Å-thick bismuth sample on mica with the mean free path sufficiently small: $l \simeq 10$ Å; this value of l corresponds to the characteristic temperature T_2 above 10 K. The alteration of quantum correction in magnetic field for this sample is very well described by Eq. (6), suggesting a considerable distance from the percolation threshold. Values of τ_φ (solid symbols) were found for various temperatures and currents, and then, by the above method, values of $\tau_{e\text{-ph}}$ (open symbols) were calculated by Eq. (7). The calculation of τ_φ was made using only five experimental points in two regimes at temperatures below T_λ of the helium transition to the superfluid state. It is noted above that under this condition the effect of electron overheating may occur in films on mica. Since superfluid helium effectively removes the heat from the film, it is hardly possible under this condition to expect any influence of the modified phonon spectrum in the Bi film. The dependence $\tau_{e\text{-ph}}(T)$ constructed in the range 1.8–2.7 K is well described by the law $\tau_{e\text{-ph}}^{-1} \propto T^4$. Since the relation between the times $\tau_{e\text{-ph}}$ and $\tau_{e\text{-ph}}^{i}$ does not depend on temperature,[11,13] the result conforms to the theoretical prediction for $\tau_{e\text{-ph}}^{i}$ in the case of dirty limit.

In order to improve the reliability of these values, we extended the temperature range towards ultralow temperatures. We used a cryostat with a ^{3}He pumping system providing, besides production and stabilization of temperature between 0.3 and 350 K, application of the magnetic field in the range 0–45 kG generated by a superconducting solenoid. The sample represented a thin bismuth film (a 60-μm-wide strip) on a calcite crystal. It was in vacuum; the calcite crystal had a good thermal contact with a copper block cooled with ^{3}He.

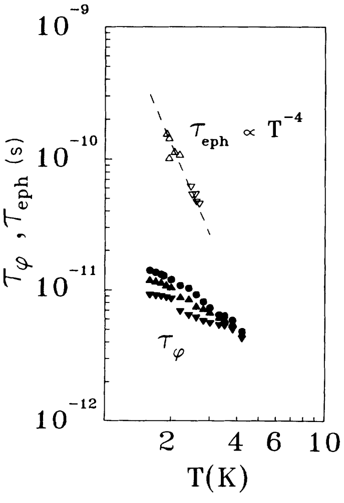

FIG. 1. Temperature dependence of τ_φ (solid symbols) and $\tau_{e\text{-ph}}$ (open symbols) for a thin Bi film on mica 74 Å in thickness for the following current values (μA): ●, 50; ▲, △, 100; ▼, ▽, 200.

Figure 2 shows the values of τ_φ and $\tau_{e\text{-ph}}$ for a 92-Å-thick sample. The mean free path of charge carriers was about 25 Å, which corresponds to the characteristic temperatures $T_2 = 6$ and 3 K of longitudinal and transverse phonons, respectively. It should be noted that the magnitude of $\tau_{e\text{-ph}}$ in electron-phonon relaxation at low temperatures is most considerably affected by the interaction with transverse phonons.[10] Values of τ_φ for various currents were obtained in the temperature range 0.38–5.1 K; this enabled us to obtain $\tau_{e\text{-ph}}$ values in the range 0.6–2.3 K from the electron overheating effect. The minimum current value for this sample was 1 μA, which did not cause heating at and above approximately 1 K. In the range 0.7–2.3 K the dependence $\tau_\varphi^{-1} \propto T$ was clearly seen which had always been observed for $\tau_\varphi(T)$ in the low-temperature range.[21,23,24] However, below 0.7 K such a small current causes heating, as is indicated by the departure of experimental points downwards from the curve of $\tau_\varphi^{-1} \propto T$ extrapolated to this temperature range. This permitted using all current values, including 1 μA, for calculation of $\tau_{e\text{-ph}}$.

The $\tau_{e\text{-ph}}$ values found represent a set of points which in the range 0.6–1.2 K is well describable by the dependence $\tau_{e\text{-ph}}^{-1} \propto T^4$; at higher temperature there is some deviation of the points from this law.

We analyzed the "equilibrium" dependence $\tau_\varphi(T)$ obtained with a small current, considering the τ_φ values in the range 0.7–5 K and representing this function, according to Eq. (5), as the sum $\tau_\varphi^{-1} = AT + A_p T^p$. We used the program of search for A, A_p, and p by the least-squares technique. The result is shown in Fig. 2 by

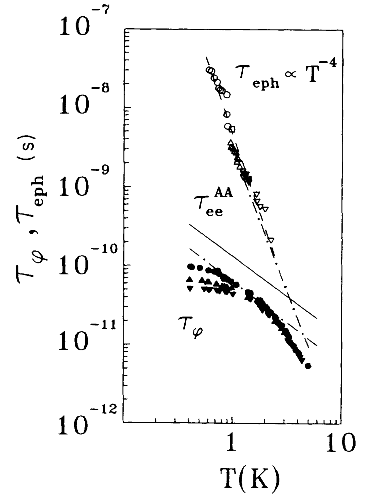

FIG. 2. Temperature dependence of τ_φ (solid symbols) and $\tau_{e\text{-ph}}$ (open symbols) for a thin Bi film on calcite 92 Å in thickness for the following current values (μA): ●, ○, 1; ▲, △, 5; ▼, ▽, 10; □, 20. The dot-dashed lines show the components $\tau_{e\text{-}e}(T)$ and $\tau_{e\text{-ph}}(T)$ obtained by representing the curve of $\tau_\varphi(T)$ for the current of 1 μA by expression (5) in the temperature range 0.7–6 K. The solid line shows the time $\tau_{e\text{-}e}$ calculated by formula (1).

dot-dashed lines. It was found that $p=3.6$, while a similar analysis of bismuth films of thickness over 100 Å always gave the exponent $p=3$ or slightly less. In this case p was slightly smaller than 4, since the analysis also included points outside the dirty limit (above 3 K).

Thus the results obtained support the predictions of the theory on the dependence $\tau_{e\text{-ph}}^{-1} \propto T^4$ in the dirty limit in the three-dimensional case fairly well. The next task is to find the functional dependence of $\tau_{e\text{-ph}}$ on the mean free path of charge carriers.

Quantitative comparison of the $\tau_{e\text{-ph}}$ values obtained for bismuth films with formula (3) is complicated by the following facts ignored by the theory: the existence of the two types of charge carriers in bismuth, the large anisotropy of the spectrum, and a certain deviation from the square law of electron dispersion. Yet, it is more important that in thin bismuth films the charge carrier concentration is essentially higher than that in a bulk crystal,[23] which naturally causes different spectrum parameters (E_F, p_F, v_F, etc.). The growth of the charge carrier concentration which is not attended by any marked breaking of balance of charge carrier concentration is due to the quantum size effect[25] and the effect of the subsurface bending of the film potential.[26] According to Refs. 25 and 26, the change of the charge carrier concentration n with the bismuth film thickness in the range 450–4000 Å is given by the expression

$$n=n_0\left[1+\frac{L_0}{L}\right], \tag{8}$$

$n_0=3.02\times10^{17}$ cm^{-3} and $L_0 \sim 1000$ Å.

Extrapolation of this dependence into the range of thickness $L \sim 100$ Å results in the value $n \sim 3\times10^{18}$ cm^{-3}.

In order to know the n values of our bismuth films more precisely, we made a combined measurement, on 100–400-Å-thick films, of the three kinetic properties: the resistance ρ, the magnetoresistance coefficient $\Delta\rho/(\rho H^2)$ (with the quantum localization correction to the magnetoresistance previously separated), and the Hall constant R_H. Each of these can be represented in the two-band model as a function of the concentrations and mobilities of electrons and holes, respectively. These characteristics can be found, if it is assumed that in films compensation is the case[25,26] or that mobilities of electrons and holes are similar. Both the calculations yielded practically identical results. It was found that in the range of thicknesses studied the dependence $n(L)$ is stronger than in expression (8), and for the thickness ≈ 100 Å the concentrations of electrons and holes are $\approx 10^{20}$ cm^{-3} and the mobilities are ≈ 20 cm^2 V^{-1} s^{-1}. Hence bismuth films have electronic characteristics quite different from those of bulk crystals. As regards the effect of the spectrum anisotropy, the following can be said. The structure of our films has a small grain size (the size in the film plane is slightly larger than the film thickness) and an axial texture, i.e., the C_3 axis is directed by the normal to the film plane. Grains are oriented in the film plane at random, which should result in isotropy of kinetic properties. Therefore $\tau_{e\text{-ph}}$ and D can be calculated from the characteristics averaged in the C_1C_2 plane; to do so, it seems suitable to take the cyclotron masses and velocities of electrons and holes, which correspond to the magnetic-field orientation along the C_3 axis.

To calculate $\tau_{e\text{-ph}}$ we transformed formula (3) so that it contained the charge carrier concentration n instead of v, E_F, and p_F:

$$(\tau_{e\text{-ph}}^{i})^{-1}=\frac{10n^{1/3}(\hbar/e^2)}{\hbar^2 m^*\rho^* R_\Box L s_l^5}\left[1+\frac{3}{2}\left[\frac{s_l}{s_t}\right]^5\right](kT)^4\,, \tag{9}$$

where ρ^* is the density. To take account of both types of charge carriers determining $R_\Box$, we used as a value of m^* the average values of the cyclotron masses of electrons and holes for $H\parallel C_3$.[27]

The dependence $\tau_{e\text{-ph}}(T)$ that we obtained (Fig. 2) may be represented as $\tau_{e\text{-ph}}^{-1}=A_4T^4$, where $A_4^{\text{expt}}=2.25\times10^8$ s^{-1} K^{-4}. Calculation by formula (9) gave $A_4^{\text{theor}}=4.01\times10^7$ s^{-1} K^{-4}. Note that formula (9) is for the phase relaxation time $\tau_{e\text{-ph}}^{i}$. For the dependence $\tau_{e\text{-ph}}^{-1}\propto T^p$, where $p=4$, the following relation is true: $\tau_{e\text{-ph}}^{-1}=4.285(\tau_{e\text{-ph}}^{i})^{-1}$. Hence we have $A_4^{\text{theor}}=1.72\times10^8$ s^{-1} K^{-4}, in fairly good agreement with the experimental value.

The solid line in Fig. 2 shows the temperature dependence of the electron-electron interaction time $\tau_{e\text{-}e}$ as calculated by formula (1). This dependence is fairly similar to the dependence $\tau_{e\text{-}e}(T)$ separated from $\tau_\varphi(T)$ by means of the separation (5) (the dot-dashed line). If $\tau_{e\text{-}e}^{-1}=AT$, then $A^{\text{expt}}=1.56\times10^{10}$ s^{-1} K^{-1}, while calculation by (1) gives $A^{\text{theor}}=7.81\times10^9$ s^{-1} K^{-1}, the difference being less than by a factor of 2. Such agreement between theoretical and experimental values, both for $\tau_{e\text{-ph}}$ and for $\tau_{e\text{-}e}$, is really very good.

ACKNOWLEDGMENT

This work was supported, in part, by a Soros Foundation Grant awarded by the American Physical Society.

---

[1] G. Bergmann, Phys. Rep. **107**, 1 (1984).

[2] B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interaction in Disordered Systems*, edited by A. L. Efros and M. P. Pollak, Modern Problems in Condensed Matter Science Vol. 10 (Elsevier, Amsterdam, 1985), p. 1.

[3] H. Fukuyama, in *Electron-Electron Interaction in Disordered Systems* (Ref. 2), p. 155.

[4] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).

[5] B. L. Altshuler, A. G. Aronov, M. E. Gershenzon, and Yu. V. Sharvin, Sov. Sci. Rev. Sect. A **9**, 223 (1987).

[6] A. Schmid, Z. Phys. **259**, 421 (1973).

[7] H. Takayama, Z. Phys. **263**, 323 (1973).

[8] D. Belitz, Phys. Rev. B **36**, 2513 (1987).

[9] J. Rammer and A. Schmid, Phys. Rev. B **34**, 1352 (1986).

[10] M. Yu. Reizer, and A. V. Sergeev, Zh. Eksp. Teor. Fiz. **90**, 1056 (1986) [Sov. Phys. JETP **63**, 616 (1986)].

[11] M. Yu. Reizer, Phys. Rev. B **40**, 5411 (1989).

[12] P. V. Allen, Phys. Rev. Lett. **59**, 1460 (1987).

[13] G. Bergmann, Wei Wei, Yao Zou, and R. M. Mueller, Phys. Rev. B **41**, 7386 (1990).

[14] P. M. Echternach, M. R. Thoman, C. M. Gould, and H. M. Bozler, Phys. Rev. B **46**, 10 339 (1992).

[15] D. Belitz and S. Das Sarma, Phys. Rev. B **36**, 7701 (1987).

[16] M. L. Roukes *et al.*, Phys. Rev. Lett. **55**, 422 (1985).

[17] S. I. Dorozhkin, F. Lell, and W. Schoepe, Solid State Commun. **60**, 245 (1986).

[18] J. Liu and N. Giordano, Phys. Rev. B **43**, 3928 (1991).

[19] E. M. Gershenzon *et al.*, Zh. Eksp. Teor. Fiz. **97**, 901 (1990) [Sov. Phys. JETP **70**, 505 (1990)].

[20] V. Yu. Kashirin and Yu. F. Komnik, Fiz. Nizk. Temp. **19**, 410 (1993) [Low Temp. Phys. **19**, 288 (1993)]; Yu. F. Komnik and V. Yu. Kashirin, *ibid.* **19**, 908 (1993) [*ibid.* **19**, 647 (1993)].

[21] V. Yu. Kashirin and Yu. F. Komnik, Fiz. Nizk. Temp. **19**, 165 (1993) [Low Temp. Phys. **19**, 117 (1993)].

[22] P. W. Anderson, E. Abrahams, and N. V. Ramakrishnan, Phys. Rev. Lett. **43**, 718 (1979).

[23] F. Komori, S. Kobayashi, and W. Sasaki, J. Phys. Soc. Jpn. **52**, 317 (1983).

[24] V. Yu. Kashirin and Yu. F. Komnik, Fiz. Nizk. Temp. **18**, 1246 (1992) [Sov. J. Low Temp. Phys. **11**, 872 (1992)].

[25] Yu. F. Komnik, E. I. Bukhshtab, Yu. V. Nikitin, and V. V. Andrievsky, Zh. Eksp. Teor. Fiz. **60**, 669 (1971) [Sov. Phys. JETP **33**, 364 (1971)].

[26] Yu. F. Komnik and V. V. Andrievsky, Fiz. Nizk. Temp. **1**, 104 (1975) [Sov. J. Low Temp. Phys. **1**, 51 (1975)].

[27] V. S. Edel'man, Usp. Fiz. Nauk. **123**, 257 (1977) [Sov. Phys. Usp. **20**, 819 (1977)].