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Nonlinear carrier-plasmon interaction in a one-dimensional quantum plasma

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Periodic growth and decay of linearly unstable plasmon modes are predicted by numerical solution of the one-dimensional collisionless quantum Boltzmann equation. The nonlinear mode stabilization is accompanied by the generation of higher harmonics. The results are analyzed using quantum generalized quasilinear theory.

With the fabrication of quasi-one-dimensional (1D) semiconductor quantum wires, collective plasma waves in 1D quantum systems have become an issue of considerable current interest.[1-5] Plasmons in these systems are well established experimentally,[3] and have been studied theoretically.[1,2] Besides quasiequilibrium situations, nonequilibrium configurations have also been analyzed, leading to the prediction of carrier acoustic plasmon instabilities in quantum wires.[4,5] Special conditions have been proposed which should make instabilities easier to observe in a stationary regime (e.g., density modulation[6]). Even though there is not yet direct experimental evidence of plasma instabilities in quantum-confined structures, several circumstances are in favor of their occurrence. First, due to density-of-states effects, the 1D carrier-carrier- and carrier-phonon-scattering rates and, hence, the dephasing rates, are reduced in comparison to higher dimensions. More importantly, for quantum wires, the corresponding relaxation times of the carrier distributions τ_R are increased significantly. Time-resolved luminescence measurements[7] as well as Monte Carlo simulations[8] yield values for τ_R on the order of picoseconds.

So far most theoretical investigations have been restricted to the random-phase approximation (RPA), i.e., to linear-response theory. Within RPA the dispersion of longitudinal plasmons follows from the zeros of the analytic continuation of the dielectric function. However, as in classical plasmas,[9,10] nonlinear phenomena are also of considerable interest. In this paper we therefore investigate the nonlinear evolution of unstable modes in a 1D quantum plasma. In particular we study the temporal evolution of linearly unstable initial distributions well into the nonlinear regime. Our results reveal a nonlinear mechanism governing the evolution of weak short-wavelength fluctuations in quantum plasmas: carrier-plasmon interaction gives rise to additional maxima of the homogeneous part of the carrier distribution at positions $\pm nk_0$ away from the original ones (k_0 is the wave number of the externally excited unstable mode, $n=1,2,\ldots$). This reshaping not only removes the instability of the basic mode but even leads to mode damping. This is of principal interest, and allows us to compare the behavior of quantum plasmas to that of classical plasmas. Our calculations refer primarily to an idealized model 1D plasma. In order to visualize the main features more clearly, we study the collisionless relaxation over a relatively long time. The applicability of our results to realistic quasi-1D, in particular to electron-hole plasmas in quantum wires, is of course confined to times significantly less than the previously mentioned relaxation time τ_R.[11] With the fabrication of cleaner samples, this time interval is expected to increase further.

We study the nonlinear mode dynamics by direct solution of the 1D collisionless Boltzmann equation (also known as the time-dependent Hartree-Vlasov equation). In Fourier representation, this equation can be written as

$$\left\{\frac{\partial}{\partial t}+i\frac{pk}{m}\right\}f(p,k,t) = i\sum_q \left[f\left[p-\frac{q}{2},k-q\right]-f\left[p+\frac{q}{2},k-q\right]\right] \times\left[U_q+2V_q\sum_{p'}f(p',q)\right], \quad (1)$$

where $f(p,q,t)$, U_q, and V_q are the spatial Fourier transforms of the Wigner function, the external potential, and the Coulomb potential, respectively. In the numerical calculations we use $V(q)=2e^2K_0(qd)/\epsilon_b$ for a one-band quantum wire. Here K_0 denotes the modified Bessel function of the second kind, d is the wire width, and ϵ_b is the background dielectric constant.[1]

Our numerical results are summarized in Figs. 1 and 2. In both cases we start from a distribution function which is spatially homogeneous and has a main peak at $p=0$ (a Fermi function with chemical potential $0.4E_R$ and temperature $T=5$ K) plus a second peak (a Gaussian, $Ae^{-(p-p_{\max})^2/\sigma^2}$, with $a_B\sigma=0.25$, $A=0.3$) with a pronounced minimum between both maxima [solid lines in Figs. 1(c) and 2(c)]. One mode is excited externally using a small-amplitude potential $U_q = U\delta_{k_0,q} + U^*\delta_{k_0,-q}$. Figure 1(a) shows the temporal density evolution $n(R,t) = \int f(p,R,t)dp/\pi$ at the fixed space point $R=0$. After the strong initial amplitude increase, which is predicted by linear theory, we see a significant decrease. Instead of regular harmonic oscillations, which are characteristic of the linear regime (there is a transient damped mode mixed with the unstable mode at early times), we clearly observe nonharmonic behavior. This temporal variation is caused by the superposition of several frequencies $\omega(k_0)$, $\omega(k_1)$, etc. Correspondingly, in the spectrum [Fig. 1(b)], $n(k,t) = \int f(p,k,t)dp/\pi$, we see the fundamental mode at k_0 plus a second spatial harmonic at $k_1 = 2k_0$ and even a very weak third harmonic. Since the mode dispersion $\omega(k)$ is nearly linear, the presence of spatial harmonics causes the corresponding frequency harmonics in the density.

Shifting the second maximum of the initial distribution to slightly higher moments (from $|p_{\max}|a_B = 1.5$ in Fig. 1 to $|p_{\max}|a_B = 2.0$ in Fig. 2), we change the density evolution drastically [Fig. 2(a)]. We see irregular behavior, with the mode-beating period shortened and substantial enhancement of higher harmonics [up to the fourth harmonic, Fig. 2(b)]. Analyzing the numerical results, we note that the beating of the harmonics is in phase and the higher harmonics start after and disappear before the lower ones.

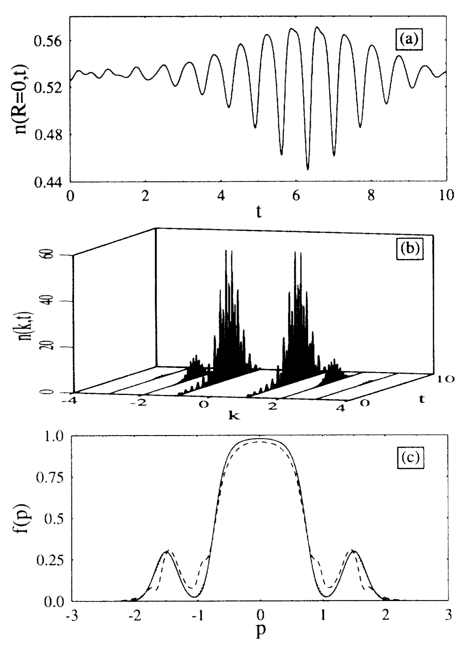

FIG. 1. Nonlinear evolution of a linearly unstable plasma [solution of Eq. (1) for $k_0a_B = 0.7013$, $p_{\max}a_B = 1.5$, $U = 0.05$, and $d/a_B = 0.25$]. Shown is (a) the time evolution of the carrier density at $R=0$, (b) the time evolution of the spatial Fourier transform of the carrier density, and (c) the space-averaged Wigner function for different times (solid line is $t=0$, dashed line is $t=6.83$ ps, and dotted line is $t=10.0$ ps). The units for the plots are as follows: n is in carriers/a_B, t is in ps, k is in k_0, and p is in a_B^{-1}. For these calculations we used $E_R = 4.2$ meV as the exciton Rydberg energy, $m = 0.067m_0$ (m_0 is the free-electron mass) as the electron, and $a_B = 140.0$ Å as the exciton Bohr radius for GaAs.

To gain further information we plot the spatially averaged Wigner function $f(p,t) = \int f(p,R,t)dR$ as a function of p for different times in Figs. 1(c) and 2(c). With increasing time, we observe a decrease of the original second maximum and the simultaneous occurrence of additional peaks. These additional maxima appear at a distance of roughly k_0 on both sides of the original sidepeak. As we will verify in our subsequent theoretical analysis, the initial reshaping of the Wigner function is responsible for the undamping of the higher harmonics. At the next stage, the growing peak between the original two maxima in $f(p,t)$ leads to the removal of the original instability condition, causing damping of the density oscillations.

To analyze the results, we now consider the necessary condition for the occurrence of an instability in linear theory, $\mathrm{Im}\epsilon < 0$. For a one-component plasma, in the case of small damping or growth, this criterion can be written as

$$\mathrm{Im}\epsilon(\Omega,q) = \frac{m}{q} V(q)[f_0(p^-) - f_0(p^+)] < 0 \ , \qquad (2)$$

with $p^{\pm} = (m/q)\Omega \pm (q/2)$, where Ω is the real part of the complex mode frequency $\hat{\Omega} = \Omega - i\Gamma$, and f_0 is the zeroth-order 1D carrier distribution, which is the solution of the stationary, homogeneous, field-free Boltzmann equation. Γ is the linear damping/growth rate ($\Gamma \sim \mathrm{Im}\epsilon$). Note that condition (2) for quantum plasmas depends on the details of the distribution on a *macroscopic* scale given

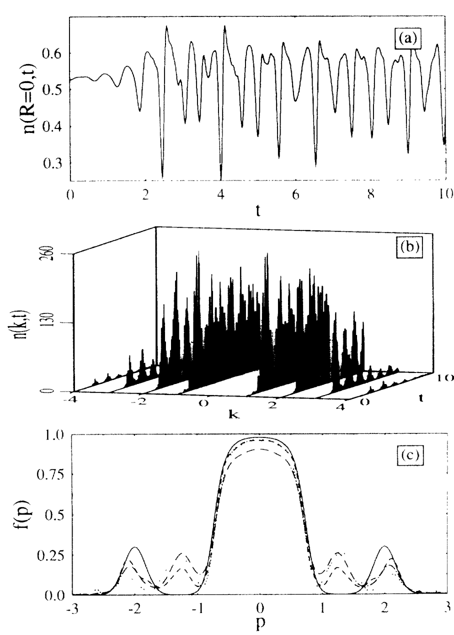

FIG. 2. Same as Fig. 1, but for a shifted peak position of the initial distribution function ($|p_{\max}|a_B = 2.0$). In (c) the times plotted are solid line $t=0$, dash-dotted line $t=2.24$ ps, dashed line $t=3.60$ ps, and dotted line $t=4.27$ ps.

by the basic wave number $k_0=q$. There always exists a maximum value k_{max} beyond which all waves are stable.[5]

The numerical results suggest that the evolution of the plasma and the excited modes is governed by the evolution of the homogeneous part of the distribution f_0. One therefore can try to derive an equation for the time evolution of f_0 under the influence of the plasma oscillations. Using the general ansatz

$$f(p,k)=f_0(p)\delta_{k,0}+\sum_{l=1}^{\infty}[f_l(p)\delta_{k,lk_0}+f_l^*(p)\delta_{k,-lk_0}] \quad (3)$$

and the abbreviations

$$\Delta f\left[p,\frac{q}{2}\right]=f\left[p-\frac{q}{2}\right]-f\left[p+\frac{q}{2}\right], \quad (4)$$

$n_i=\int f_i dp/\pi$, $U_1=U+2V_{k_0}n_{k_0}$, and $U_i=2V_{ik_0}n_{ik_0}$, $i>1$, the equation for f_0 reads

$$\frac{\partial}{\partial t}f_0(p)=2\sum_{l=1}^{\infty}\mathrm{Im}\left[f_l\left[p,\frac{lk_0}{2}\right]U_l^*\right]. \quad (5)$$

Together with the corresponding equations for $f_1,f_2,\ldots$, Eq. (5) is equivalent to the original kinetic equation and just as complicated. However, Eq. (5) already reveals the basic nonlinear mechanism in collisionless quantum plasmas: the homogeneous part of the distribution is changed (slowly, on times much longer than the oscillation period) due to interaction of carriers with all harmonics. The system of equations for the harmonics is particularly useful if the number of excited harmonics is small, as in our numerical examples. The simplest approximation is obtained assuming that f_0 is only changed due to the influence of f_1, which is determined using linear theory. This corresponds to the level of quasilinear theory.[12]

A very interesting result follows if we use for f_1 and U_1 in Eq. (5) a single-pole approximation: $f_1(p,t)=f_1(p)\exp[-i\hat{\Omega}_1 t]$ and $U_1(t)\approx U_1\exp[-i\hat{\Omega}_1 t]$. Here $\hat{\Omega}_1(k_0)=\Omega_1(k_0)-i\Gamma_1(k_0)$, where $\Omega_1(k_0)$ and $\Gamma_1(k_0)$ are the dispersion and growth rates $[\Gamma_1(k_0)<0]$ of the unstable mode determined within the RPA. Then

$$f_1(p)=\frac{\Delta f_0\left[p,\frac{k_0}{2}\right]}{pk_0/m-\hat{\Omega}_1}U_1. \quad (6)$$

With this expression inserted into Eq. (5), we obtain

$$\frac{\partial}{\partial t}f_0(p)=2|U_1|^2(-\Gamma_1)\exp[-2\Gamma_1 t]\left[\frac{f_0(p-k_0)-f_0(p)}{\left[\left[p-\frac{k_0}{2}\right]\frac{k_0}{m}-\Omega_1\right]^2+\Gamma_1^2}+\frac{f_0(p+k_0)-f_0(p)}{\left[\left[p+\frac{k_0}{2}\right]\frac{k_0}{m}-\Omega_1\right]^2+\Gamma_1^2}\right]. \quad (7)$$

Along with f_0, the dispersion and growth rates become slowly time dependent. From Eq. (7) one can see that the generation of an unstable mode causes a deformation of the homogeneous distribution in such a way that the instability is weakened. This becomes obvious if one considers $\partial f(p_{max})/\partial t$ for the momentum corresponding to the original side maximum. This derivative is negative, causing a lowering of the maximum. Furthermore, additional maxima start growing at $p_{max}\pm k_0$. While the lowering of the maximum is evident in Fig. 2(c), it is not seen in Fig. 1(c). This is because only carriers on the high momentum side of the maximum have a less-occupied momentum state to move to; these carriers contribute energy to the growing mode. The occupation of states on the low momentum side of the maximum actually increases as some carriers scatter into them from below the Fermi momentum (absorbing some energy from the mode).

The appearance of additional maxima weakens the growth rate of the mode. The side maximum at $p_{max}+k_0$ and the interaction with the higher harmonics lead to a continuation of this process so that no stationary state is reached. After damping sets in, all harmonics gradually vanish and the original maximum of the distribution grows again. Now the complete evolution process starts all over again. From Eq. (7) one can see that the cycle of this evolution is defined mainly by the linear growth rate; stronger linear growth leads to a shorter cycle period. The number of generated harmonics can be controlled by the ratio between the wave number k_0 and the distance between the Fermi momentum and the peak position p_{max}. In situations where the relative density change is small [Fig. 1(a)], the result of Eq. (7) can be generalized to include also the contributions of the higher harmonics to the change of f_0. Then one has to replace the right-hand side of Eq. (7) by a sum over the harmonics s, substituting s for the index 1 and sk_0 for k_0.

The present theory can be generalized to systems with more than one component. Formally, an application to higher-dimensional systems is possible; however, the respective time scale for the validity of the collisionless kinetic equation may be more restricted. We want to emphasize that our model describes a truly reversible evolution. This is in contrast to the classical quasilinear theory,[13] where a turbulent superposition of a large number of modes leads to irreversible loss of phase memory and the formation of a plateau in the distribution function.

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[11]The standard notion in plasma physics is that the effect of collisions on collective modes is negligible if $\Omega(k) \gg 1/\tau_c$ (τ_c is the two-particle correlation time). This means that there will be many periods of the oscillation of a carrier until it enters a scattering process, destroying the correlated behavior. However, even if $\Omega(k) < 1/\tau_c$, oscillations might be observable in certain situations. This is the case if there exists an effective (e.g., external) excitation mechanism. Then it depends on the ratio of the amplitude of the oscillation, the average energy transfer in a single-carrier collision process, and how long it takes to damp out the mode. In this situation, a collisionless approximation should be applicable as long as $\tau_R \gg 1/|\Gamma(k)|$ (τ_R denotes the relaxation time of the one-particle distribution), and the damping and growth of the oscillations are only slightly modified by collisional damping. The exact range of validity of the collionless equation depends on numerous system parameters, and has to be determined from the solution of the Boltzmann equation above for an inhomogeneous plasma including a microscopic scattering term.

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