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Magnetoresistance of a two-dimensional electron gas in a random magnetic field

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We report magnetoresistance measurements on a two-dimensional electron gas made from a high-mobility $GaAs/Al_xGa_{1-x}As$ heterostructure, where the externally applied magnetic field was expelled from regions of the semiconductor by means of superconducting lead grains randomly distributed on the surface of the sample. A theoretical explanation in excellent agreement with the experiment is given within the framework of the semiclassical Boltzmann equation.

The response of a two-dimensional electron gas (2DEG) to a spatially inhomogeneous magnetic field is a subject of considerable interest both theoretically and experimentally.[1] One of the experimental techniques proposed is to deposit a pattern of small magnets on the semiconductor containing the 2DEG.[2] This technique, however, only gives rise to a very weak modulation. Another method is to grow the 2DEG on a substrate with a modulated thickness.[3] The applied magnetic field experienced by the electrons in the plane of the curved 2DEG will vary with the thickness modulation. The feasibility of this method is, however, limited by the technological difficulties of the molecular-beam epitaxy regrowth techniques required. So far, the most simple technique, originally proposed by Rammer and Shelankov[4] for studying weak localization effects in inhomogeneous magnetic fields, employs a type-II superconducting gate on top of the heterostructure containing the 2DEG. For a type-II superconducting gate an applied magnetic field will create the so-called mixed state in the superconductor above the lower critical field B_{c1}. In this state the magnetic field penetrates the film as flux tubes. Each flux tube will contain an integral number of (superconductivity) flux quanta $\Phi_0=h/2e$. For a perfect type-II superconductor the mixed state is accomplished by the formation of a two-dimensional hexagonal lattice of vortices. In a real superconductor inhomogeneities will tend to pin the vortices, so a random distribution of flux tubes is more likely to occur rather than the regular lattice. The magnetoresistance of the type-II superconductor gated samples have been investigated experimentally in various limits of 2DEG properties. Bending *et al.*[5] and Geim[6] have studied the weak localization effects predicted by Rammer and Shelankov[4] for a low mobility $GaAs/Al_xGa_{1-x}As$ heterostructure with a Pb gate and a thin Bi film evaporated on a Nb/Mo substrate, respectively. Kruithof *et al.*[7] have studied the mechanisms of voltage induction in the 2DEG of a Si metal-oxide-semiconductor field-effect transistor caused by flux flow in a Nb/Mo superconducting gate. The above experiments have probed the diffusive properties of the 2DEG's. In the ballistic regime, where the electronic mean free path is much longer than the vortex diameter, a series of experiments were performed by Geim *et al.*,[8–10] and the results interpreted by treating the vortices as scatterers. The effect of single vortices has also been studied both experimentally and theoretically.[11–13]

In this paper we demonstrate a very simple technique for creating a strong magnetic field modulation. We also propose a semiclassical model based on the Boltzmann equation to explain the measured magnetoresistance caused by the inhomogeneous magnetic field. Our model gives excellent agreement with the experiment, and in addition is applicable to the results of Geim *et al.*[10]

The inhomogeneous magnetic field was achieved by means of small lead grains randomly distributed on the surface of a high mobility $GaAs/Al_xGa_{1-x}As$ heterostructure. The lead grains (approximated as spheres) used had a size (average diameter) distribution as shown in Fig. 1. For these grain sizes Pb is a type-I superconductor. Below the critical field given by

$$B_c(T)=B_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right], \qquad (1)$$

there will be (partial) flux expulsion from the grains, creating an inhomogeneous magnetic field in the 2DEG. Below $\frac{2}{3}B_c(T)$ the grains will no longer be in the intermediate state, and they will exhibit full Meissner effect. For lead $B_c(0)=80.3$ mT. The T dependence in (1) holds to a good approximation for our purpose with $T_c=7.2$ K. The mobility of the investigated samples was 93.5 $m^2/V\,s$ at the lowest temperature of 0.3 K in the experiment. At 7.2 K the mobil-

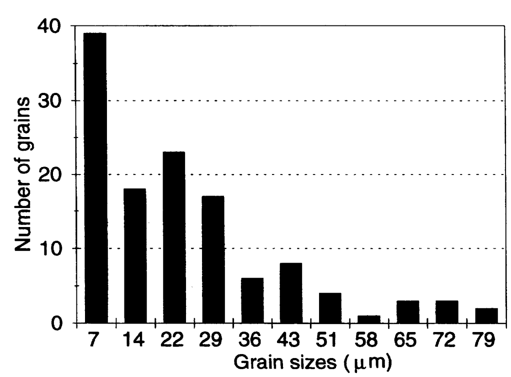

FIG. 1. Distribution of lead grain sizes on the sample for which data are presented in this paper. This distribution was simply obtained by measuring the grain sizes on the sample on a photograph taken through an optical microscope.

ity had degraded to 82.6 m^2/Vs. In the whole temperature range the carrier density of the 2DEG was 4.0×10^{15} m^{-2}. This corresponded to a mean free path of $l\approx9$ μm. The experiment was thus performed in a regime where the mean free path was comparable to the typical grain size l_{Pb} as well as the average distance between grains $n_{Pb}^{-1/2}$, i.e., $l\leq l_{Pb},n_{Pb}^{-1/2}$, with n_{Pb} being the density of lead grains. The sample geometry consisted of a standard 400 μm wide Hall bar with three pairs of voltage probes, each pair placed on opposite sides of the Hall bar. The voltage probe pairs were displaced a distance of 1600 μm (four squares) from each other. In addition the Hall bar also contained two current probes displaced another four squares from the voltage probes. The resistance was measured by conventional small signal lock-in techniques in a current controlled four-probe configuration. To check the homogeneity of the lead grain distribution we measured the longitudinal magnetoresistance between all three combinations of voltage probes along each side of the Hall bar. Such measurements always gave the same result within 5% when normalized with respect to the number of squares between the voltage probes. We also made control measurements on samples cut from the same heterostructure, but without lead grains. Such samples showed no magnetoresistance in fields below 0.1 T. The measurements were performed in the following way. First, we cooled down the sample in zero magnetic field to the relevant temperature. Then we swept the magnetic field to above the critical field while measuring the magnetoresistance. This was followed by consecutive down and up sweeps as exemplified in Fig. 3 below. Here we should emphasize that the first sweep after each cooldown procedure is fundamentally different from the following sweeps. This difference is caused by the trapping of flux in the superconductor. In fact, we believe that the magnetoresistance in the consecutive sweeps is dominated by the random magnetic field caused by the frozen flux. We exploit both experimental situations to test our theoretical model for two different realizations of a random magnetic field.

In Fig. 2 we show a set of "sweep-up" traces at different temperatures. The pronounced magnetoresistance peak is seen to vary in amplitude and width with temperature. Moreover, the peaks are asymmetric in the field. However, as seen in Fig. 3 the corresponding "sweep-down" traces are asymmetric as well but with the maximum resistance at negative magnetic fields. As indicated by the vertical dashed lines in Fig. 2 the magnetoresistance defined as $\Delta\rho_{xx}(B) = \rho_{xx}(B)-\rho_0$ goes to zero at $B=B_c(T)$. Here ρ_0 is the resistance at zero magnetic field prior to the first sweep after the cooldown procedure. It is also seen from Fig. 2 that the magnetoresistance peak vanishes for temperatures above the critical temperature of lead ($T_c=7.2$ K). The observed magnetoresistance shown in Figs. 2 and 3 cannot originate from any weak localization contribution to the magnetoresistance. The weak localization magnetoresistance is extremely weak for high mobility $GaAs/Al_xGa_{1-x}As$ samples and is practically extinguished for magnetic fields $B\geq4B_\phi\simeq10^{-3}$ T, where B_ϕ is the characteristic magnetic field corresponding to the phase breaking scattering time.[14] Weak localization effects will in addition not show the hysteresis effect displayed in Fig. 3. Moreover the observed magnetoresistance is fundamentally different from the curves reported in Ref. 10 with a continuous lead gate. This difference is most easily seen on the "first sweep" curve in Fig. 3, which for small magnetic fields has a $\Delta\rho_{xx}\propto B^2$ dependence, while the magnetoresistance observed in Ref. 10 exhibits a $\Delta\rho_{xx}\propto B$ dependence in the same regime of fields.

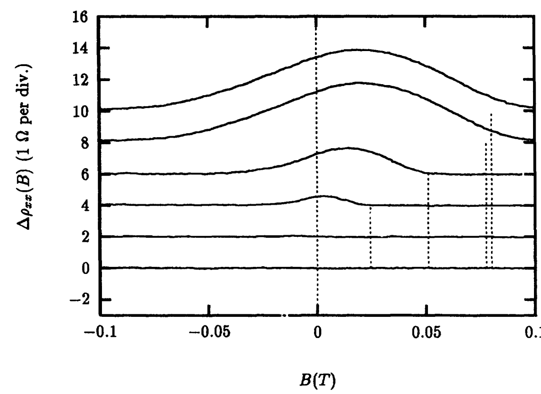

FIG. 2. A series of sweep-up curves at different temperatures, displaced for clarity. The temperatures were 0.3 K, 1.3 K, 4.4 K, 6.0 K, 7.2 K, and 8.5 K, where the upper curves correspond to the lowest temperatures. The magnetoresistance anomaly disappears when the sample is heated to above the critical temperature for lead ($T_c=7.2$ K). The vertical dashed lines indicate the critical magnetic field calculated with Eq. (1).

We now turn to describe the theory. Disregarding interference effects and wave-vector quantization imposed by sample boundaries, one can in general expect a classical approach to conduction in a magnetic field to be valid if $k_Fl\gg1$, where k_F is the Fermi wave vector and l is the electronic mean free path. A randomly modulated magnetic field

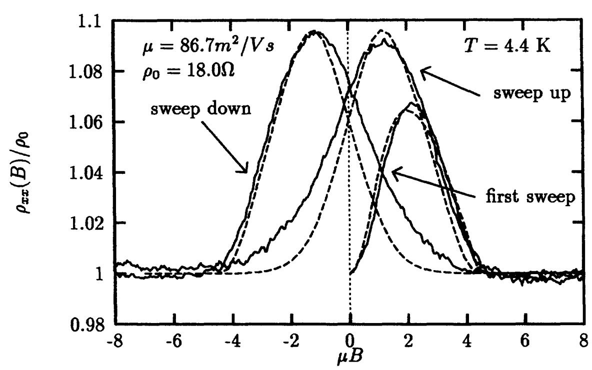

FIG. 3. A set of magnetoresistance traces taken at 4.4 K (solid lines). The "first sweep below T_c" curve is taken just after the sample was cooled from above the critical temperature in zero magnetic field. The sweep-down and sweep-up curves are the subsequent sweeps, after the first sweep where the magnetic field was taken to above the critical field. The dashed curves are calculated with Eqs. (10–14) with $l_{Pb}=11.5$ μm and $n_{Pb}=7.75\times10^8$ m^{-2}. The parameters are taken from the distribution of lead grain sizes shown in Fig. 1, and the position of the maximum (used as a fitting parameter) is consistent with the trapped flux interpretation. The vertical dashed line indicates the critical magnetic field at the relevant temperature.

can be included in the Boltzmann equation either in the driving force term or as an effective impurity cross section, depending on the correlation length a of the modulation (in our case given by the size of the lead grains). If $a \gg 1/k_F$, as definitely is the case in our experiment, a modulation δB of the magnetic field can be treated as an ordinary external field in the driving force term of the Boltzmann equation. However, if $a \simeq 1/k_F$, δB should be incorporated as an impurity cross section. (When $a \gg 1/k_F$ we could of course also treat δB as a scatterer, i.e., put it on the right-hand side of the Boltzmann equation. By contrast it would be inconsistent to put δB on the left-hand side when $a \sim 1/k_F$.)

Our starting point is thus the usual semiclassical Boltzmann equation in the relaxation time approximation. We introduce polar coordinates v,ϕ for the velocity and confine ourselves to $T=0$. Then v only enters through a δ function and can be put equal to v_F. Furthermore we will make the usual assumption of a constant, external driving field $\boldsymbol{E}$ and calculate to linear order in it. The resulting equation is [with $g(\boldsymbol{r},\phi,v)=\tilde{g}(\boldsymbol{r},\phi)\delta(v-v_F)$]

$$\left\{ v_F \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \frac{\partial}{\partial \boldsymbol{r}} + \omega_c(\boldsymbol{r}) \frac{\partial}{\partial \phi} + \frac{1}{\tau} \right\} \tilde{g}(\boldsymbol{r},\phi) = -\frac{e}{m} \boldsymbol{E} \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}. \tag{2}$$

Here $\omega_c(\boldsymbol{r})=eB(\boldsymbol{r})/m$ is a function of position. Writing $\omega_c(\boldsymbol{r})=\omega_0+\delta\omega(\boldsymbol{r})$, defined such that $\langle\delta\omega\rangle=0$ ($\langle\,\rangle$ denotes an average over random magnetic field configurations) we can write the Boltzmann equation as an operator equation

$$D\tilde{g} \equiv (D_0+W)\tilde{g}=\chi, \tag{3}$$

with $W=i\delta\omega(\boldsymbol{r})\partial/\partial\phi$ (we have multiplied the equation with i for convenience). The eigenfunctions $\langle \boldsymbol{r}\phi|\boldsymbol{k}n\rangle$ and the Green's function for D_0 are readily found. The current in terms of the full Green's function $G(\boldsymbol{r},\phi;\boldsymbol{r}',\phi') = \langle \boldsymbol{r}\phi|D^{-1}|\boldsymbol{r}'\phi'\rangle$ is

$$\boldsymbol{j}=\frac{2em^2}{(2\pi\hbar)^2}\int v\,dv\,d\phi\,v\begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}\tilde{g}(\boldsymbol{r},\phi)\,\delta(v-v_F), \tag{4}$$

$$\tilde{g}(\boldsymbol{r},\phi)=\int d\boldsymbol{r}'\,d\phi'\,G(\boldsymbol{r},\phi;\boldsymbol{r}',\phi')\chi(\phi'). \tag{5}$$

Calculating to second order in W using the Dyson equation

$$D^{-1} \simeq D_0^{-1}+D_0^{-1}\langle W D_0^{-1} W\rangle D^{-1}, \tag{6}$$

and performing the ensemble average $\langle\delta\omega(\boldsymbol{r})\delta\omega(\boldsymbol{r}')\rangle \to f(|\boldsymbol{r}-\boldsymbol{r}'|)$, D^{-1} becomes diagonal in $\boldsymbol{k}$, i.e., G is only a function of $\boldsymbol{r}-\boldsymbol{r}'$. From Eq. (5) we see that only $G_{\boldsymbol{k}=0}$ is needed for the conductivity. When $\boldsymbol{k}=\boldsymbol{0}$, D^{-1} is also diagonal in n and Eq. (6) is trivial to invert.

We find[15] the resistivity tensor

$$\boldsymbol{\rho}=\frac{m}{ne^2\tilde{\tau}}\begin{pmatrix} 1 & \tilde{\omega}\tilde{\tau} \\ -\tilde{\omega}\tilde{\tau} & 1 \end{pmatrix}, \tag{7}$$

where we have defined the renormalized quantities

$$\tilde{\omega}=\omega_0-\mathrm{Re}\,\Sigma_1, \tag{8}$$

$$\frac{1}{\tilde{\tau}}=\frac{1}{\tau}-\mathrm{Im}\,\Sigma_1, \tag{9}$$

with the "self-energy" Σ_1 given by

$$\Sigma_1=-\frac{i}{\omega_0\pi}\frac{1}{e^{2\pi/\omega_0\tau}-1}\int_0^{2\pi} f\left(2r_c\sin\frac{\theta}{2}\right)e^{(i+1/\omega_0\tau)\theta}d\theta. \tag{10}$$

Here $r_c=v_F/\omega_0$ is the average cyclotron radius and $f(r)=\langle\delta\omega(\boldsymbol{r})\delta\omega(0)\rangle$ is the correlation function, depending on the nature of the random magnetic field modulation. We see that the change in $\boldsymbol{\rho}$ is directly related to Σ_1:

$$\frac{\Delta\rho_{xx}}{\rho_{xx0}}=-\tau\,\mathrm{Im}\,\Sigma_1, \tag{11}$$

$$\frac{\Delta\rho_{xy}}{\rho_{xy0}}=-\omega_0^{-1}\mathrm{Re}\,\Sigma_1. \tag{12}$$

To proceed we now introduce a model for the modulated magnetic field. We start by treating the situation with perfect flux expulsion due to the Meissner effect applicable to the first sweep curves. The lead grains had a distribution of sizes as seen in Fig. 1. We represent this distribution by an average size l_{Pb}. We model the magnetic field modulation from a single lead grain by[16] $\delta b(r)=B[(r/l_{\mathrm{Pb}})^2-1]e^{-r^2/l_{\mathrm{Pb}}^2}$. This expression fulfills the necessary flux conservation condition $\int dr\, r\, \delta b(r)=0$. The magnetic field modulation can now be expressed by $\delta B(\boldsymbol{r})=\Sigma_i \delta b(\boldsymbol{r}-\boldsymbol{R}_i)$, where $\boldsymbol{R}_i$ is the position of the ith grain. $\boldsymbol{R}_i$ is randomly distributed, and the magnetic field modulation should be averaged over different distributions of lead grains on the surface of the semiconductor. This gives rise to the correlation function

$$f(r)=B^2\frac{n_{\mathrm{Pb}}l_{\mathrm{Pb}}\pi}{32}\left\{8-8\left(\frac{r}{l_{\mathrm{Pb}}}\right)^2+\left(\frac{r}{l_{\mathrm{Pb}}}\right)^4\right\}e^{-r^2/l_{\mathrm{Pb}}^2}. \tag{13}$$

At low magnetic fields we get $\Delta\rho_{xx} \propto B^2$ in accordance with the experiment. At higher fields ($>\frac{2}{3}B_c$ for spheres) the grains will be in the intermediate state, reducing the amplitude of f until it vanishes at $B=B_c$. This we model by multiplying the correlation function in (13) by a factor going to zero as $1-\sqrt{B/B_c(T)}$ when $B\to B_c$. The temperature dependence of $B_c(T)$ accounts for the observed temperature dependence of the magnetoresistance as shown in Fig. 2.

In the case when the magnetic modulation is caused by frozen flux, the correlation function (13) should be replaced by

$$f(r)=C(B)e^{-r^2/l_{\mathrm{Pb}}^2}, \tag{14}$$

where $C(B)$ is an asymmetric function of B going to zero for $|B|>B_c$, giving a phenomenological measure of the amount of flux trapped in the lead grains. The position of the maximum of C is used as a fitting parameter. In Fig. 3 we have shown a fit of the model to the experimental traces. The correspondence is seen to be quite satisfactory.

Finally we demonstrate that our theoretical model is applicable to the experiments in Ref. 8, where the magnetic modulation was produced by filaments of magnetic field emerging from a type-II superconducting gate. Each flux tube may be modeled by the following expression[17] $b(r) = (\Phi_0/2\pi\lambda_L^2)K_0(r/\lambda_L)$, where λ_L is the effective London length in the plane of the 2DEG and K_0 a modified Bessel function. In this case the correlation function turns out to be

$$f(r)=n_\nu \frac{\Phi_0^2}{4\pi\lambda_L^2}\frac{r}{\lambda_L}K_{-1}\left(\frac{r}{\lambda_L}\right), \tag{15}$$

where n_ν is the density of vortices. Since $n_\nu \propto B$ it is immediately seen that the longitudinal magnetoresistance will be proportional to B for small fields, as is also found in Ref. 8. We can also find the density dependence of $\Delta\rho_{xx}$ within this framework, and find that $\Delta\rho_{xx} \propto n^{-3/2}$. However, when Eq. (15) is used to fit the experimental traces with λ_L as the fitting parameter, the obtained values of λ_L are approximately a factor of 5–10 larger than estimated in Ref. 8. This may be a result of flux bundles containing several flux tubes, as also reported by Stoddart *et al.*[12] We would like to emphasize both that our calculation is purely semiclassical, and that we do not treat the vortices as scatterers to be included in a scattering cross section, but rather as a perturbation to the driving force term in the Boltzmann equation.

In conclusion we have measured the magnetoresistance of a 2DEG subject to a random shielding of the externally applied magnetic field. We have modeled our results by solving the semiclassical Boltzmann equation with an appropriately chosen random magnetic field in the sample. We have demonstrated that our model can be applied in electrical transport problems with other types of magnetic field modulation.

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