Current scaling in the integer quantum Hall effect

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We report a current (I)-dependent transport measurement in between two adjacent integer quantum Hall plateaus at different bath temperatures (T_b) for 10 nA $< I <$ 10 μA. We find the maximum slope in ρ_{xy} scales with I when I is larger than a characteristic value. Combined with previous T scaling results, we find the current-dependent effective temperature of the two-dimensional electron gas in the integer quantum Hall effect regime, $T_e \sim I^{0.5}$, *independent of Landau levels and the spin degeneracy.* This result allows us to deduce that $p=2$ for the T exponent of the inelastic scattering length ($l_{\rm in} \sim T_e^{-p/2}$). Therefore, the values of the localization exponent ν and of the dynamical exponent z can be obtained directly and independently.

Scaling studies[1,2] in the transition regions between plateaus of the integer quantum Hall effect (IQHE) have drawn a great deal of interest in recent years both experimentally[3] and theoretically.[4,5] In a two-dimensional electron gas (2DEG) at low temperature (T) and high magnetic fields (B), the appearance of the quantum Hall plateaus in the Hall resistance $\rho_{xy}=h/ie^2$ (where h is the Planck constant, e the electron charge, and i an integer) is now understood in terms of a continuous metal-insulator transition with critical singularities in the resistances ρ_{xx} and ρ_{xy} as T approaches zero. More specifically, it has been demonstrated[1] that both the maximum in $d\rho_{xy}/dB$, $(d\rho_{xy}/dB)^{\rm max}$, and the inverse of the half width in ρ_{xx} $(\Delta B)^{-1}$, between two adjacent quantum Hall plateaus at low T follow the power law $T^{-\kappa}$ ($\kappa=0.42$), for spin-polarized Landau levels and independent of the Landau-level index. This critical behavior is a direct consequence of the existence of a diverging localization length (ξ) described by $\xi \sim (E_F-E_c)^{-\nu}$, where E_F is the Fermi energy, E_c the center of a Landau level, and ν a universal critical exponent.[2] Since the localization is due to quantum interference of the electron, the microscopic effective sample size is determined by a T-dependent inelastic-scattering length ($l_{\rm in}$).[6] One can conceptually cast the T into a length scale. If we assume that $l_{\rm in} \sim T^{-p/2}$ in high B, similar to that in weak localization,[7] the experimentally measured exponent is $\kappa=p/2\nu$. The power-law dependence is then a manifestation of scaling for the electronic transport of 2DEG in high B. For spin degenerate levels, κ was found to be 0.21,[3(d)] half that for spin-polarized levels. However, the T-dependent measurement alone cannot determine p and ν independently.

It is known that an electron heating experiment addresses the issue of $l_{\rm in}$ within a model proposed by Anderson, Abrahams, and Ramakrishnan.[8] In the absence of B, electric-field- (E-) dependent transport measurements in thin metal films[9(a)] and in Si-metal-oxide-semiconductor devices[9(b)] at low temperatures showed that the conductance $\sigma \sim \ln(E)$ when E was large enough, and $\sigma \sim \ln(T)$ when E was small. The latter T-dependent behavior was a characteristic of scaling in the weak-localization regime. Anderson, Abrahams, and Ramakrishnan[8] also discussed the effect of electric field using an electron heating model. When an electric field (E) is applied, energy is being put into the sample. The effective temperature (T_e) of the electron gas is then different from the bath temperature (T_b). The conductance measured at finite E hence reflects the conductance at T_e instead of T_b. Since the conductance scales with the length, or the temperature, it should also scale with E. Roukes *et al.*[10] obtained the T_e as a function of E using noise measurement in thin metal films and extracted the value of p following a more detailed calculation.[11] Recently, the effect of E, or equivalently the applied current I, was discussed in a model of scaling in the IQHE regime by Polyakov and Shklovskii.[5(a)] In particular, this model discussed the electron heating from the insulator side and predicted power laws for the E-dependent width of ρ_{xx} and T_e.

In this paper, we report current (I) scaling in the IQHE regime that addresses the issue of $l_{\rm in}$ at high B in the context of electron heating on the metal side. Most strikingly, we obtain $T_e \sim I^{0.5}$ *independent of Landau levels and the spin degeneracy.* From the electron heating model,[8,11] we deduce $p=2$ for the 2DEG in the IQHE regime. This value of p enables us to obtain, independently of any theoretical models, the localization exponent $\nu=2.4$ and 4.8 for spin-polarized and spin-degenerate Landau levels, respectively, and the dynamical exponent $z=1$.

The sample used in this work is an $In_{0.53}Ga_{0.47}As/InP$ heterostructure described in Ref. 1. The Hall bar pattern on the sample has a channel width of 600 μm. It has an electron density $n=3.3\times10^{11}$ cm^{-2} and a mobility $\mu=35\,000$ cm^2/V s at 4.2 K. In a recently built cryostat, the sample is glued onto a copper rod which is attached to a ^{3}He chamber in a cancellation coil, and the T_b is regulated via a heater. Transport coefficients are measured

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by standard ac lock-in technique. The circuit for the ac current source consists of the voltage source of the lock-in amplifier, an operational amplifier, and a decade resistor box. We have also checked that the result described in this paper does not change when a dc current source is used. The I range in our experiment is 10 nA $< I <$ 10 μA. At higher I ($>20\ \mu$A at $T_b = 0.3$ K), the bath starts to warm up, indicative of insufficient cooling power of the refrigerator. We also checked the relation

$$(d\rho_{xy}/dB)^{\max} \sim T_b^{-0.42} , \tag{1}$$

in the T range $0.3 < T_b < 4.2$ K. For T lower than 0.3 K, the sample is immersed in the ^{3}He and ^{4}He mixture in a dilution refrigerator. We use the voltage source of the lock-in amplifier connected in series with a 10 MΩ resistor as the current source. Since the results from the two different cryostats are consistent with each other, the thermal coupling of the sample to the bath does not play any role in our results.

The top panel of Fig. 1 shows ρ_{xy} and ρ_{xx} as a function of B at $I = 10$ nA for $T_b = 3.3$, 1.2, and 0.3 K. The bottom panel shows the I dependence of ρ_{xy} and ρ_{xx} at $T_b = 0.3$ K. We emphasize that the T and I dependent evolutions of ρ_{xx} and ρ_{xy} are qualitatively the same. In particular, identical ρ_{xx} oscillations in low B taken at $I = 10$ nA at any fixed bath temperature $T_b > 0.3$ K can be obtained at $T_b = 0.3$ K by increasing I (for example, the $T_b = 1.2$ K trace in the upper panel and the $I = 3.5$ μA trace in the lower panel). This fact, indicative that at sufficiently high I the electron system has an effective temperature above that of the crystal lattice, has been widely used to study the electron temperature T_e of the 2DEG.[12] The data indicates that, at $I = 13$ μA, T_e is less than 3.3 K, and thus, in our range of 10 nA $< I <$ 10 μA, the electron temperature does not exceed 4.2 K. This sample shows T scaling for $T < 4.2$ K when $I \leq 10$ nA. So, the I dependent $(d\rho_{xy}/dB)^{\max}$ is expected to show power-law behavior since T_e is determined by I.

In Fig. 2, we plot the I dependent $(d\rho_{xy}/dB)^{\max}$ measured at $T_b = 25$ mK (solid circles), 0.3 K (open circles), 0.85 K (squares), and 1.5 K (triangles) for Landau levels with indexes $N = 0\downarrow$ (upper part), $1\uparrow$ (middle part), and $1\downarrow$ (lower part). To avoid cluttering of data points, we have shifted downward all the data for the $N = 1\uparrow$ Landau level, and upward the solid circles for the $N = 0\downarrow$ Landau level by multiplying each data point by 0.5 and 1.6, respectively. For the $N = 0\downarrow$ Landau level, there is slight mismatch in the data from two different runs. But the linear dependence to be described below is the same in both runs. It is clear that there exists a characteristic current I_0 for each T_b, below which $(d\rho_{xy}/dB)^{\max}$ remains constant, and above which $(d\rho_{xy}/dB)^{\max}$ merges into one linear curve on our log-log plot. We obtain, from the data, for $I > I_0$,

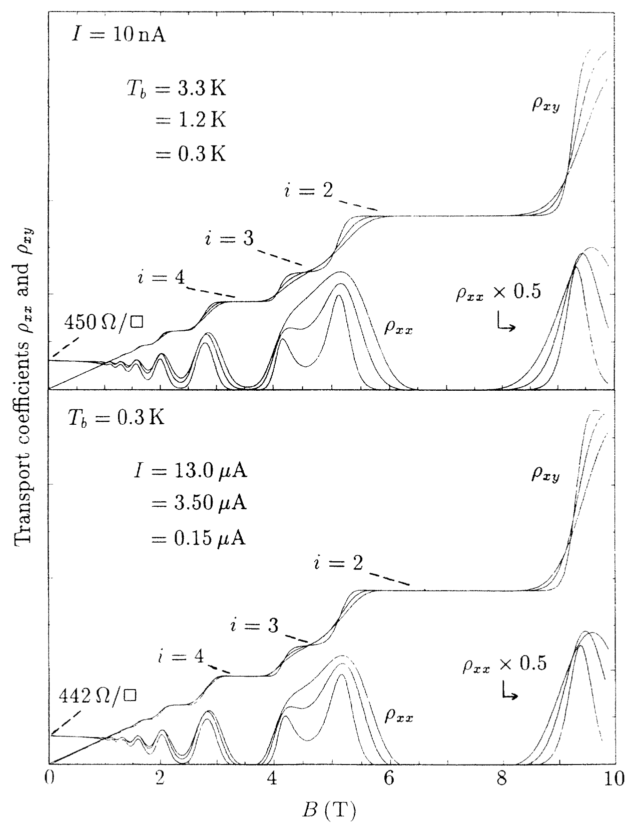

FIG. 1. Magnetotransport coefficients ρ_{xy} and ρ_{xx} versus B at $T_b = 3.3$, 1.2, and 0.3 K in a low mobility $In_xGa_{1-x}As/InP$ heterostructure for $I = 10$ nA (upper panel). The lower panel is with $I = 13$, 3.5, 0.15 μA at $T_b = 0.3$ K.

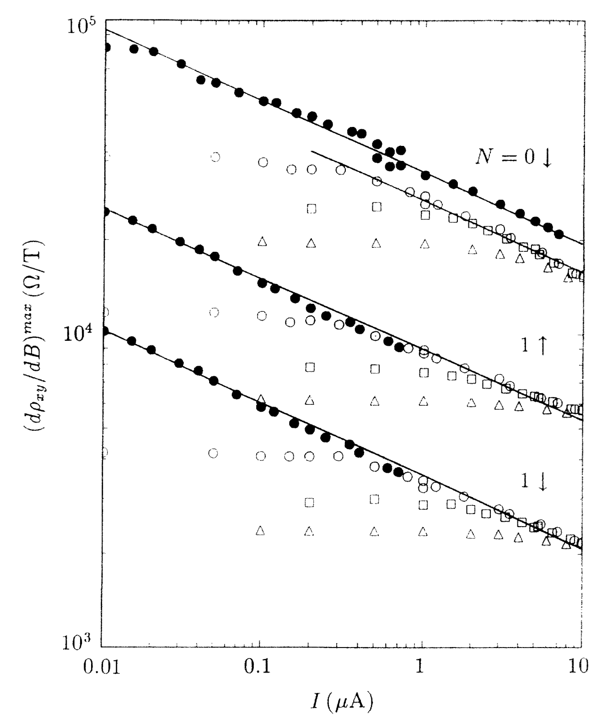

FIG. 2. $(d\rho_{xy}/dB)^{\max}$ versus I at $T_b = 25$ mK (solid circles), 0.3 K (open circles), 0.85 K (squares), and 1.5 K (triangles) for three Landau levels $N = 0\downarrow$, $1\uparrow$, and $1\downarrow$. The data for $N = 1\downarrow$ and the top solid circles of the $N = 0\downarrow$ are scaled by 0.5 and 1.6, respectively, for clarity. All the lines drawn through the data have a slope of 0.23.

$$(d\rho_{xy}/dB)^{\max} \sim I^{-b} \quad (2)$$

over three decades in I and $b=0.23\pm0.02$, independent of the Landau-level index. From our previous discussion, we interpret this single power-law dependence as a consequence of the T scaling in Eq. (1). Therefore, our data implies that

$$T_e \sim I^a \quad (3)$$

and $a=0.5$ within the experimental uncertainty of about 10%. This 10% uncertainty applies to all the exponents measured in this work.

In analogy to the electron heating model for thin metal films,[8,11] the smearing of the Fermi distribution function is characterized by T_e ($>T_b$) and is given by the energy acquired by the electron diffusing a distance l_{in} in an electric field E. Therefore, $k_B T_e \sim eEl_{\text{in}}$, and $T_e \sim I^{2/(2+p)}$. From our data, we obtain $p=2$, independent of Landau levels. If we identify this l_{in} as the relevant length scale for scaling, from $p=2$ and $\kappa=0.42=p/2\nu$, we obtain consequently $\nu=2.4$, which agrees with existing values.[3(b),4] This value of p has another consequence. If we combine the results from the T scaling experiment[1] and the more recent frequency scaling experiment,[3(c)] we find $pz=2$, where z is the dynamical exponent. Our result of $p=2$ gives $z=1$, independently of the value of ν.

The energy acquired by the electrons has to be dissipated to the bath through presumably the coupling between the 2DEG and the substrate lattice in our sample. One may therefore be tempted to identify the l_{in} described above as due to electron-phonon scattering. However, based on a semiclassical approximation for 2DEG in high B in a smooth random potential, Zhao and Feng[5(c)] obtained $p=3$ for the electron-phonon scattering rate. This is different from $p=2$ in our experiment, suggesting that the electron-phonon scattering is not relevant. In the weak-localization regime, Lin *et al.*[13] found that $p=1$, in agreement with the theory for electron-electron scattering in the dirty limit.[14] Similar theoretical studies in high B are currently lacking, and the understanding of our $p=2$ result awaits further investigations.

Another possible approach to explain the current scaling result in Eq. (2) is to consider a length scale caused by suppression of quantum interference as E is applied, which was first proposed by Fu.[15] Using the E dependent l_{in} deduced from his theory, we obtain $(d\rho_{xy}/dB)^{\max} \sim I^{-0.14}$ for spin-polarized Landau levels, which does not agree with our experiment. However, this theory is for bulk metals in the absence of B, and there is no extension to 2DEG in high B.

The experimental exponents in Eqs. (2) and (3) are apparently the same as those predicted in the recent model of scaling based on variable range hopping with a Coulomb gap.[5(a)] The approach for understanding the scaling in this model is fundamentally different from ours. In particular, l_{in} plays no role in explaining the scaling behavior within this model which considers the transport from the insulator side, i.e., in the plateau region. It assumes $k_B T_e \sim eE\xi$. The predicted exponents $b=\kappa/2$ and $a=\frac{1}{2}$ in this model are a result of equating $eE\xi$ to the Coulomb energy $\sim e^2/\epsilon\xi$. We would like to point out that our T_e is obtained from $(d\rho_{xy}/dB)^{\max}$ measured at the center of a Landau level, i.e., on the metal side. Therefore, it is legitimate to talk about the concept of l_{in}.[6]

The same measurement has also been done for spin-degenerate levels $N=2$ and 3 in our sample. The T scaling shows $(d\rho_{xy}/dB)^{\max} \sim T^{-0.21}$ [3(d)] The I dependence $(d\rho_{xy}/dB)^{\max}$ is plotted in Fig. 3 at $T_b=25$ mK (solid circles), 0.3 K (open circles), 0.85 K (squares), and 1.5 K (triangles) for the $N=2$ (upper part) and $N=3$ (lower part) Landau levels. The data for the $N=3$ level have been shifted downward by multiplying the data point by 0.5. When I is larger than a characteristic current, the $(d\rho_{xy}/dB)^{\max}$ data show the same linear dependence on this log-log plot over three decades in I. The straight line passing through the data points has a slope of 0.1 ± 0.01. Using the same heating model, we obtain $T_e \sim I^{0.5}$ and $p=2$ independent of the Landau-level index. Since $p=2$ and $\kappa=0.21$, thus $\nu=4.8$ for the spin-degenerate levels.

From our result, it is tempting to say that this spin-degenerate case belongs to a different universality class. However, it was found that the odd integer plateau would still appear in the limit of zero T in an $In_xGa_{1-x}As/InP$ sample with a small Zeeman energy $\Delta=0.025$ meV.[3(d)] It implies that at extremely low T there are two consecutive metal-insulator transitions each characterized by $\nu=2.4$, even though it appears to have only one transition at high T. This is confirmed in a recent numerical study,[16] in which Δ is assumed to be small compared with the width

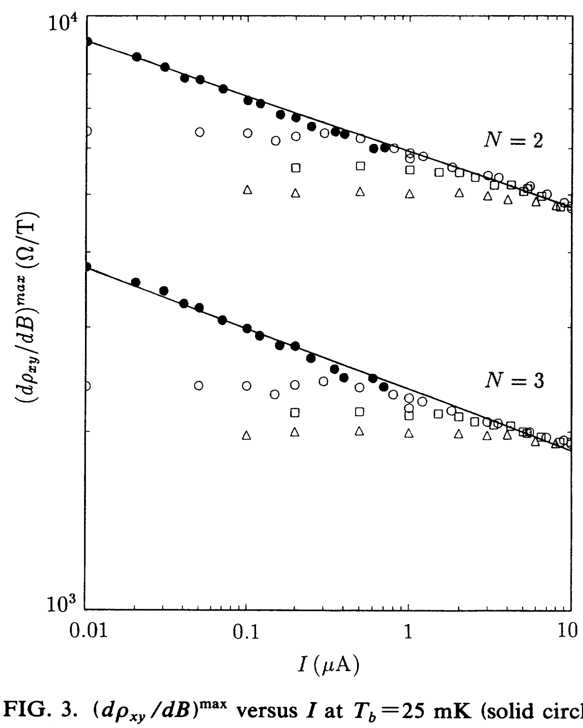

FIG. 3. $(d\rho_{xy}/dB)^{\max}$ versus I at $T_b=25$ mK (solid circles), 0.3 K (open circles), 0.85 K (squares), and 1.5 K (triangles) for two spin-degenerate Landau levels $N=2$ and 3. The data for $N=3$ are scaled by 0.5 for clarity. The lines passing through the data points have slopes of 0.10 ± 0.01.

of the disorder broadened Landau level. These observations are consistent with the three-dimensional conductance flow diagram proposed in Ref. 3(d). Namely, there is a T-dependent effective g factor that drives the system from the completely spin-degenerate case to the spin-polarized case. The apparent power laws $(d\rho_{xy}/dB)^{\max} \sim T^{-0.21}$ and $(d\rho_{xy}/dB)^{\max} \sim I^{-0.1}$ are crossover effects. In this case, it is surprising that the exponent does not depend on the Landau-level index and that the power law persists over three decades in I. Moreover, if T is high enough, the effective g factor is small. It is possible that the system is very close to the completely spin-degenerate case and we are in effect probing the critical behavior of a completely spin-degenerate level. In any case, it is not known theoretically at present if the spin degree of freedom in a completely spin-degenerate level can result in a different universality class. A more definitive experimental test should be done in a sample with zero Zeeman splitting.

Comments about applying this study to $GaAs/Al_xGa_{1-x}As$ heterostructures are also in order. It was found that there is no universal (i.e., independent of Landau levels) scaling behavior unless T is low enough.[17] In this case, the experimentally measured exponent κ at high T is not the critical exponent $p/2\nu$, even though there may be an apparent power law in $(d\rho_{xy}/dB)^{\max}$. However, one can still use the T dependence of $(d\rho_{xy}/dB)^{\max}$ as a thermometer to extract T_e from the I dependent $(d\rho_{xy}/dB)^{\max}$ and to study $l_{\rm in}$ in high B. Research along this line is currently in progress. We would also like to mention that our result $\nu=2.4$ for spin-polarized levels is the same as that obtained from sample-size scaling measurement by Koch *et al.* in $GaAs/Al_xGa_{1-x}As$ heterostructures.[3(b)] On the other hand, $\nu=4.8$ for spin-degenerate levels is different from their result. The reason for this disagreement is not clear at present.

In summary, in a study on I-dependent transport in the IQHE regime, we find a current scaling: $(d\rho_{xy}/dB)^{\max} \sim I^{-b}$, when I is larger than a characteristic current. The exponent $b=0.23\pm0.02$ and 0.1 ± 0.01 for spin-polarized and spin-degenerate Landau levels, respectively. We interpret this result as due to T scaling and obtain $T_e \sim I^{0.5}$. Assuming that $k_B T_e \sim eEl_{\rm in}$, we deduce $p=2$ independent of Landau levels and the spin degeneracy. This value of p yields $\nu=2.4$ and 4.8 for spin-polarized and spin-degenerate Landau levels, respectively. In addition, $z=1$ is a natural consequence.

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