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Anderson localization in strongly anisotropic metals

A. A. Abrikosov
Materials Science Division, Argonne National Laboratory, 9700 South Cass Avenue, Argonne, Illinois 60439
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Conditions for Anderson localization are derived for three cases: (1) anisotropic three-dimensional metal, (2) quasi-two-dimensional metal, and (3) quasi-one-dimensional metal. For all these cases the conductivity at $T=0$ as well as the interference correction are calculated. The simplest models are used. From the estimate $\Delta\sigma/\sigma\sim 1$, localization conditions are obtained. It is shown that localization takes place in all three cases but in cases (2) and (3) the critical value of the random potential is essentially reduced if the overlap integrals are small. In a two-dimensional metal this refers to the conductivity along the planes whereas for the conductivity perpendicular to the planes the three-dimensional condition applies, i.e., contrary to common wisdom localization in this direction is more difficult to reach than along the planes.

I. INTRODUCTION

As is well known, Anderson localization is a result of quantum interference of de Broglie waves representing the wave function of an electron scattered from lattice defects. The conductivity of an infinite sample with impurities vanishes at $T=0$ (see, e.g., in Ref. 1). The localization condition depends, however, on the dimensionality of the model. Whereas in a three-dimensional (3D) isotropic metal the localization condition is $l<\lambda$, l being the mean free path and λ the Fermi wavelength, in 1D and 2D metals the localization takes place at any magnitude of the random potential, and only the range of the localized state depends on the mean free path. For a 1D state this is proven exactly (see, e.g., Ref. 2), and for a 2D state there exist convincing arguments.[3]

At the same time with the exception of metal-oxide-semiconductor field-effect transistors (MOSFET's) real systems are quasi-1D or quasi-2D substances, i.e., they have a 3D but strongly anisotropic conductivity. A very popular example is the copper oxide high-T_c superconductors, and this makes the question about localization conditions in these more realistic models of considerable interest. We consider here three such cases: (1) a 3D metal with a highly anisotropic energy spectrum, (2) a quasi-2D metal with the Fermi surface being a slightly corrugated cylinder, and (3) a quasi-1D metal with a Fermi surface in the form of slightly corrugated planes.

We will obtain the localization condition in the following way. It is well known that in various cases the so-called interference correction to the conductivity can be easily calculated (see Ref. 1). If the calculation is done for an object of finite size at $T=0$, the condition that the correction becomes of the order of the main contribution permits one to obtain the "localization length" or the width of the wave function of the localized state. The validity of such an estimate is confirmed by exact calculations in cases where they are possible: 1D metal (see Ref. 2) and a long wire of finite thickness.[5] It is natural to assume that this general consideration applies also for an infinite sample with various relations between the mean free path and the parameters of the energy spectrum. This idea will be the basis of our consideration. Since our task is only the evaluation of the localization conditions, we will use the simplest models, permitting us to avoid unnecessary complications.

II. ANISOTROPIC 3D METAL

The condition $l<\lambda$ was found for an isotropic case. In order to trace the influence of anisotropy we consider the simplest axially symmetric quadratic energy spectrum

$$\varepsilon=p_l^2/2m_l+p_t^2/2m_t\ , \tag{1}$$

where $\mathbf{p}_t=(p_x,p_y)$. According to Ref. 6 the scattering time can be obtained from the self-energy due to impurity scattering (Fig. 1),

$$-\frac{i}{2\tau}\operatorname{sgn}\omega=\frac{|U|^2n_i}{(2\pi)^3}\int\frac{p_t dp_t dp_l d\varphi}{\omega-\xi+i\delta\operatorname{sgn}\omega}\ , \tag{2}$$

where $\xi=\varepsilon-\mu$, φ is the polar angle in the (p_x,p_y) plane, n_i is the impurity concentration, and U is the Born scattering amplitude, which we assume to be isotropic. A simple integration gives us

$$\tau^{-1}=\pi^{-1}n_i|U|^2m_tP\ , \tag{3}$$

where $P=(2m_l\mu)^{1/2}$ is the Fermi momentum along the symmetry axis.

Now we calculate the static conductivity in the zeroth approximation. According to Ref. 6 we have

$$\operatorname{Im}Q_{ij}=\frac{i\omega_0}{c}\sigma_{ij}(\omega_0)\ , \tag{4}$$

where Q_{ik} is the retarded function corresponding to the diagram in Fig. 2. Hence,

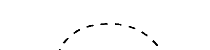

FIG. 1. Self-energy due to impurity scattering.

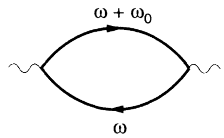

FIG. 2. Diagram for conductivity in the zeroth approximation.

$$\sigma_{ij}(0)=2e^2\lim_{\omega_0\to+0}\frac{\partial}{\partial\omega_0}\int\frac{d\omega\,d^3p}{(2\pi)^4}v_iv_k \times\left[\omega+\omega_0-\xi+\frac{i}{2\tau}\right]^{-1} \times\left[\omega-\xi-\frac{i}{2\tau}\right]^{-1}, \quad (5)$$

where $v_i=\partial\varepsilon/\partial p_i$.

Due to the symmetry of the problem it is evident that the tensor σ_{ik} has the principal values $\sigma_{zz}\equiv\sigma_l$ and $\sigma_{xx}=\sigma_{yy}\equiv\sigma_t$. Taking into account that $d^3p\to\frac{1}{2}d\varphi\,d(p_t^2)dp_l$ and integrating first by $d\omega\,d(p_t^2)$ and then by other variables we obtain

$$\sigma_0=\begin{Bmatrix}\sigma_l\\ \sigma_t\end{Bmatrix}=\frac{e^2P^3\tau}{3\pi^2}\times\begin{Bmatrix}m_t/m_l^2\\ 1/m_l\end{Bmatrix}. \quad (6)$$

The quantum interference correction can be calculated as the sum of diagrams presented in Fig. 3 (see Ref. 7). We will start with the "ladder" which is sometimes called "the cooperon." Since the impurity scattering is elastic, the electron lines in every link contain the same frequencies, $\omega+\omega_0$ in one line, and ω in the other. Let the momentum transferred along the ladder be $\mathbf{q}$. In this case a single link is equal to

$$Z=n_i|U|^2\int\frac{d^3p}{(2\pi)^3}\left[\omega+\omega_0-\xi(\mathbf{p})+\frac{i}{2\tau}\right]^{-1} \times\left[\omega-\xi(\mathbf{q}-\mathbf{p})-\frac{i}{2\tau}\right]^{-1}. \quad (7)$$

Assuming $\mathbf{q}$ to be small, substituting $\xi(\mathbf{q}-\mathbf{p})\approx\xi(\mathbf{p})-\mathbf{v}\cdot\mathbf{q}$ and expanding with respect to $\mathbf{v}\cdot\mathbf{q}$, we perform the integration over the momenta and find

$$Z=\frac{1}{\tau}\left[\left[-i\omega_0+\frac{1}{\tau}\right]^{-1}-\frac{\tau^3}{3}\frac{P^2}{m_l}\left[\frac{q_l^2}{m_l}+\frac{q_t^2}{m_t}\right]\right]. \quad (8)$$

This expression is correct only for sufficiently small q, such that the second term in Eq. (8) is smaller than the first term:[8]

$$q_l\ll m_l/P\tau,\quad q_t\ll(m_lm_t)^{1/2}/P\tau\,. \quad (9)$$

The whole ladder is a geometric progression. The sum with the "locking" impurity line is equal to

$$\frac{n_i|U|^2}{1-Z}=\frac{n_i|U|^2}{\tau}\left[-i\omega_0+\frac{\tau}{3}\frac{P^2}{m_l}\left[\frac{q_l^2}{m_l}+\frac{q_t^2}{m_t}\right]\right]^{-1} \quad (10)$$

(we assumed $\omega_0\ll1/\tau$). The expression obtained here is the well-known diffusion pole for the case under consideration (see Ref. 7).

Now we will find the remaining part of the diagram in Fig. 3. Similarly to the diagram in Fig. 2, we obtain

$$\Delta\sigma=-\frac{2e^2\tau^3m_t}{3\pi^2}n_i|U|^2\int(1-Z)^{-1}\frac{d^3q}{(2\pi)^3}P^3\times\begin{Bmatrix}m_l^{-2}\\ (m_lm_t)^{-1}\end{Bmatrix}$$
$$=-\frac{2e^2P^2\tau}{3\pi}\int\frac{d^3q}{(2\pi)^3}\left[-i\omega_0+\frac{\tau}{3}\frac{P^2}{m_l}\left[\frac{q_l^2}{m_l}+\frac{q_t^2}{m_t}\right]\right]^{-1}\times\begin{Bmatrix}m_l^{-2}\\ (m_lm_t)^{-1}\end{Bmatrix}. \quad (11)$$

Comparing with σ_0 and putting $\omega_0=0$ we get

$$\frac{\Delta\sigma}{\sigma_0}=\frac{3\pi}{P^3\tau}\frac{m_l}{m_t}\int\frac{d^3q}{(2\pi)^3}\left[\frac{q_l^2}{m_l}+\frac{q_t^2}{m_t}\right]^{-1}. \quad (12)$$

In the integral the q_l and q_t close to the upper limit are important, and they are defined by the conditions (9). In this case

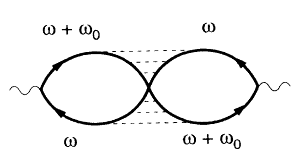

FIG. 3. Diagram with the "cooperon" representing the quantum interference correction.

$$\frac{q_l^2}{m_l}\sim\frac{q_t^2}{m_t}\sim\frac{m_l}{P^2\tau^2}\sim\frac{1}{\mu\tau^2},$$

and hence

$$\frac{\Delta\sigma}{\sigma_0}\sim\frac{m_l^2}{\tau^2P^4}\sim\frac{1}{(\mu\tau)^2}. \quad (13)$$

It follows that the localization condition is

$$\frac{1}{\tau}\gtrsim\mu\,. \quad (14)$$

For the principal directions this condition is equivalent to $l\lesssim\lambda$.

III. QUASI-2D METAL

The energy spectrum of a quasi-2D metal can be taken in the form

$$\varepsilon=\frac{p_t^2}{2m_t}+\alpha\cos(p_l d)\ , \tag{15}$$

with $\alpha \ll \mu$ (d is the period of the structure). According to formula (2) with the integration limits $-\pi/d \equiv -K/2 \le p_l \le K/2$ ($K=2\pi/d$ is the reciprocal lattice period along p_l), we obtain

$$\frac{1}{\tau}=\frac{m_t n_i}{2\pi}|U|^2K\ . \tag{16}$$

The conductivity to the first approximation is defined by formula (5). This time the velocity along l is

$$v_l=\partial\varepsilon/\partial p_l=-\alpha d\sin(p_l d)\ .$$

The velocity in the plane is $v_t=p_t/m_t$. Substituting these values and performing the integrations we get

$$\sigma_0=\begin{Bmatrix}\sigma_{t0}\\ \sigma_{l0}\end{Bmatrix}=\frac{e^2m_tK\tau}{4\pi^2}\times\begin{Bmatrix}\alpha^2d^2\\ v_t^2\end{Bmatrix}\ , \tag{17}$$

where v_t has the value at the Fermi boundary, i.e., $v_t=(2\mu/m_t)^{1/2}$.

Using formula (7) we obtain for the link of the cooperon

$$Z=\frac{m_tn_i|U|^2K}{2\pi}\left[\frac{1}{-i\omega_0+1/\tau}-\frac{v_t^2q_t^2}{2(1/\tau)^3}-a\frac{2\alpha^2\sin^2(q_ld/2)}{(1/\tau)^3}\right]\ . \tag{18}$$

Here we assumed that the second and third terms are small compared to the first. This means

$$q_t \ll 1/(v_t\tau),\quad \alpha\sin(q_ld)\ll 1/\tau\ . \tag{19}$$

The second limitation can be achieved at large values of q_ld, if $\alpha\tau\ll 1$. The analog of Eq. (10) for the cooperon will be in this case

$$\frac{n_i|U|^2}{1-Z}=\frac{n_i|U|^2}{\tau}\left[-i\omega_0+\tau\left[\frac{v_t^2q_t^2}{2}+2\alpha^2\sin^2(q_ld/2)\right]\right]^{-1}\ . \tag{20}$$

Calculating $\Delta\sigma$ according to the diagram in Fig. 3, we obtain

$$\Delta\sigma=-\frac{e^2m_tK\tau^3}{2\pi^2}\int\frac{d^3q}{(2\pi)^3}\frac{n_i|U|^2}{1-Z}\times\begin{Bmatrix}\alpha^2d^2\cos(q_ld)\\ v_t^2\end{Bmatrix}\ . \tag{21}$$

Using Eq. (18), putting $\omega_0=0$, and comparing with σ_0 [Eq. (17)], we get

$$\frac{\Delta\sigma}{\sigma_0}=\frac{4\pi}{m_tK\tau}\int\frac{d^3q}{(2\pi)^3}\left[\frac{v_t^2q_t^2}{2}+2\alpha^2\sin^2(q_ld/2)\right]^{-1}\times\begin{Bmatrix}\cos(q_ld)\\ 1\end{Bmatrix}\ . \tag{22}$$

Consider first the conductivity along l. In case $\alpha\tau\ll 1$ the integration over q_l is within the limits $-K/2\equiv-\pi/d\le q_l\le K/2$. Hence we obtain [see Eq. (20)]

$$\begin{aligned}\frac{\Delta\sigma_l}{\sigma_{0l}}&=\frac{2}{\pi m_tK\tau v_t^2}\int_0^{\pi/d}dq_l\cos(q_ld)\ln\left[\frac{1}{2\alpha^2\tau^2\sin^2(q_ld/2)}\right]\\ &=-\frac{4}{\pi^2m_tv_t^2\tau}\int_0^{\pi/2}dy(2\cos^2y-1)\ln(\sin y)=\frac{1}{\pi m_tv_t^2\tau}=\frac{1}{2\pi\mu\tau}\ .\end{aligned} \tag{23}$$

In the opposite case $\alpha\tau\gg 1$ only small values of q_ld are permitted [see Eq. (20)], and we get from Eqs. (22) and (19)

$$\frac{\Delta\sigma_l}{\sigma_{0l}}=\frac{1}{\pi m_tK\tau}\int\frac{d(q_t^2)dq_l}{v_t^2q_t^2+\alpha^2q_l^2d^2}=\frac{1}{2\pi\mu\alpha\tau^2}\ . \tag{24}$$

The localization condition is $\Delta\sigma_l/\sigma_{0l}\sim 1$. Using Eq. (24) we obtain $\alpha\sim(2\pi\mu\tau^2)^{-1}$. This contradicts the assumption $\alpha\tau\gg 1$. On the other hand, assuming $\alpha\tau\ll 1$ we get from Eq. (23) the condition $\mu\tau\sim 1$, fitting our assumption, since $\alpha\ll\mu$.[9]

For the conductivity in the plane σ_t we obtain, assuming $\alpha\tau\ll 1$,

$$\frac{\Delta\sigma_t}{\sigma_{0t}}=\frac{1}{2\pi m_tK\tau}\int_0^{2/(v_t^2\tau^2)}d(q_t^2)\int_{-K/2}^{K/2}dp_l\left[\frac{v_t^2q_t^2}{2}+2\alpha^2\sin^2(q_ld/2)\right]^{-1}=\frac{1}{\pi\mu\tau}\ln\frac{1}{\alpha\tau}\ . \tag{25}$$

The condition $\Delta\sigma_t/\sigma_{0t}\sim 1$ gives

$$\alpha\sim\tau^{-1}\exp(-\pi\mu\tau)\ll\tau^{-1},$$

or

$$\tau^{-1}\sim\pi\mu/\ln(\mu/\alpha)\ll\pi\mu . \tag{26}$$

Hence our assumption $\alpha\tau\ll 1$ is justified.

A strange situation arises. Localization is easier to reach for the conductivity along the planes than perpendicular to them. This can, of course, be explained by the consideration that the conductivity across the planes is a 3D phenomenon, and therefore the localization criterion has to be the same as for 3D systems. Nonetheless, this result leaves some uneasy feeling. In this connection it should be remembered that averaging over positions of impurities can in some cases be misleading. I have in mind, for example, the paradox with the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction in the presence of impurities (see Ref. 11). A simple averaging contributes a factor $\exp(-r/l)$, where r is the distance between spins and l the mean free path. If, however, the square of the RKKY interaction is averaged, then, according to Ref. 11, terms appear without the exponentially decreasing factor. In order to be on the safe side we consider the interference correction to σ_{0l}^2. It is represented by the diagram in Fig. 4. A calculation of this type was done by Al'tshuler[12] and Al'tshuler and Khmel'nitskii.[13] The result for our case will be

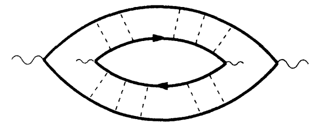

FIG. 4. Diagram for the main mesoscopic correction to the square of conductivity.

$$\left[\left[\frac{\Delta\sigma_l}{\sigma_{0l}}\right]^2\right]_{\text{av}}=\frac{2}{V^2}\frac{1}{(\pi\nu)^2}\sum_{\mathbf{q}}\frac{\cos^2(q_l d)}{\tau^2[(v_t q_t)^2/2+2\alpha^2\sin^2(q_l d/2)+1/\tau_\phi\tau]^2}$$

$$=\frac{1}{V}\frac{2}{(m_t v_t K\tau)^2}\int_0^{\pi/d}\frac{\cos^2(q_l d)dq_l}{2\alpha^2\sin^2(q_l d/2)+1/(\tau\tau_\phi)}, \tag{27}$$

where V is the normalization volume, $\nu=m_t K/(2\pi^2)$ is the density of states, and we have added to the cooperon pole the inelastic phase relaxation time as, e.g., in Ref. 13. The purpose of this is to make the integral convergent at small q_l.

Here the sequence of taking limits is important. In Refs. 12 and 13, mesoscopic effects in small samples were considered, and therefore the large size of the sample was assumed to be much less than the phase relaxation length [in the l direction $L_{\phi l}=\alpha d(\tau\tau_\phi)^{1/2}$]. Here, however, we consider an infinite sample at very low but nevertheless finite temperature. Therefore τ_ϕ is finite, the integral in Eq. (27) is convergent, and with $V\to\infty$ the correction vanishes, as it should, since at sizes larger than the phase relaxation length self-averaging has to be restored.[14]

IV. QUASI-1D METAL

We will suppose the energy spectrum in the form

$$\varepsilon=\frac{p_z^2}{2m_l}+\alpha_x\cos(p_x d_x)+\alpha_y\cos(p_y d_y), \tag{28}$$

with $\alpha_x,\alpha_y\ll\mu$. For the scattering time we get

$$-\frac{i}{2\tau}\operatorname{sgn}\omega=\frac{n_i|U|^2}{(2\pi)^3}\int\frac{d^3p}{\omega-\xi+i\delta\operatorname{sgn}\omega},$$

and hence

$$\frac{1}{\tau}=\frac{n_i|U|^2K_xK_y}{(2\pi)^2v_l}, \tag{29}$$

where $v_l=(2\mu/m_l)^{1/2}$, and K_x,K_y are the reciprocal lattice vectors. For the conductivity, as before, we get to the first approximation

$$\sigma_0=\frac{e^2K_xK_y\tau}{4\pi^3 v_l}\times\begin{Bmatrix} v_l^2\\ (\alpha_x d_x)^2/2\\ (\alpha_y d_y)^2/2\end{Bmatrix}. \tag{30}$$

The link of the cooperon is

$$Z=\frac{n_i|U|^2 2\pi i}{(2\pi)^3 v_l}\int dp_x dp_y(\omega_0+i/\tau-v_l q_l+\alpha_x\{\cos[(q_x-p_x)d_x]-\cos(p_x d_x)\}+\alpha_y\{\cos[(q_y-p_y)d_y]-\cos(p_y d_y)\})^{-1}.$$

Expanding in q_l, α_x, and α_y, putting $\omega_0=0$, and performing the integrations, we obtain

$$Z=\frac{n_i|U|^2K_xK_y\tau}{(2\pi)^2v_l}\{1-(v_l q_l\tau)^2-2[\alpha_x\tau\sin(q_x d_x/2)]^2-2[\alpha_y\tau\sin(q_y d_y/2)]^2\}, \tag{31}$$

with the conditions

$$v_l q_l \ll \tau^{-1}, \quad \alpha_x \sin(q_x d_x/2) \ll \tau^{-1}, \quad \alpha_y \sin(q_y d_y/2) \ll \tau^{-1} \; ; \tag{32}$$

therefore

$$\frac{n_i |U|^2}{1-Z} = \frac{n_i |U|^2}{\tau^2\{(v_l q_l)^2 + 2[\alpha_x \sin(q_x d_x/2)]^2 + 2[\alpha_y \sin(q_y d_y/2)]^2\}} \; . \tag{33}$$

As before, we calculate $\Delta\sigma$ and get

$$\frac{\Delta\sigma}{\sigma_0} = -\frac{v_l}{\pi K_x K_y \tau} \int d^3q \{(v_l q_l)^2 + 2[\alpha_x \sin(q_x d_x/2)]^2 + 2[\alpha_y \sin(q_y d_y/2)]^2\}^{-1} \times \begin{Bmatrix} 1 \\ \cos(q_x d_x) \\ \cos(q_y d_y) \end{Bmatrix} . \tag{34}$$

In the case $\alpha_x, \alpha_y \ll 1/\tau$ we can integrate over q_l within the limits $-\infty < q_l < \infty$, and over q_x, q_y over the whole Brillouin zone; after this, changing the variables, we obtain

$$\frac{\Delta\sigma}{\sigma_0} = 0 \frac{1}{\sqrt{2}\pi^2 \tau} \int_0^{\pi/2} d\phi \int_0^{\pi/2} d\psi (\alpha_x^2 \sin^2\phi + \alpha_y^2 \sin^2\psi)^{-1/2} \times \begin{Bmatrix} 1 \\ \cos 2\phi \\ \cos 2\psi \end{Bmatrix} \sim -\frac{1}{\alpha\tau} \tag{35}$$

(the precise coefficients are different for the longitudinal and the transverse components but our goal is only the estimate).

If we assume $\alpha\tau \gg 1$, we have to use the limits (32). Small values of q_x, q_y are then important, and after a simple integration we obtain

$$\frac{\Delta\sigma}{\sigma_0} \sim -\frac{1}{(\alpha\tau)^2} \; . \tag{36}$$

The criterion $\Delta\sigma/\sigma_0 \sim 1$ gives us $\alpha\tau \sim 1$, and it does not matter from what side we approach this limit. The condition is the same for all components of the conductivity, the same as in the 3D anisotropic metal.

V. CONCLUSIONS

The results obtained are not surprising with the exception of the 2D case, where the results are in some sense opposite to what could be expected. This is confirmed indirectly by the infrared properties of a layered quasicrystal having a true periodicity of the layer sequence and quasiperiodicity within a layer,[15] where the frequency dependence of the c-axis conductivity shows the usual 3D metallic behavior, contrary to the other components.

ACKNOWLEDGMENTS

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[8]This is actually the condition of choosing the cooperon diagrams as the main correction to the usual conductivity.

[9]A correction of the same order as Eq. (24) was obtained by Dorin (Ref. 10), but his result for other components differs from Eq. (25).

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[14]In this connection it is worthwhile to mention the difference from the case of averaging the square of the RKKY interaction. In Ref. 11 the same diagram was considered, but no excess V^{-1} factor appeared. The same would happen if we calculated the $\mathbf{q}=\mathbf{0}$ Fourier component of $[\sigma^2(\mathbf{r})]_{\rm av}$ (which is equal to $[\sum_{\mathbf{q}} \sigma(\mathbf{q})\sigma(-\mathbf{q})]_{\rm av}$), rather than $[\sigma(\mathbf{q}=\mathbf{0})^2]_{\rm av}$.

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