Transverse ac susceptibility of strips and disks with complex linear resistivity

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The linear ac susceptibility in a transverse magnetic ac field is calculated for thin strips and disks with arbitrary complex resistivity $\rho_{\rm ac}(\omega)$. The exact result is approximated with high precision by a finite sum. This analytic expression allows one to extract $\rho_{\rm ac}(\omega)$ from contact-free magnetic measurements on high-T_c superconductors in search for scaling laws and phase transitions. As an example the susceptibility of a superconducting disk with pinning and thermally assisted flux flow is given.

The linear ac resistivity $\rho_{\rm ac}$ of high-T_c superconductors (HTSC's) observed in a certain range of applied fields H_a and temperatures T may be used to investigate pinning and viscous motion of Abrikosov vortices[1–5] and to search for predicted phase transitions and scaling properties.[6–11] Preferably, $\rho_{\rm ac}(\omega)$ should be measured by a contact-free method via the magnetic response of a HTSC platelet or film to a small perpendicular ac field $\mathbf{H}_0 e^{i\omega t}$. This transverse geometry yields much larger signals than the demagnetization-free longitudinal geometry. However, as yet no theory has been given which would allow to extract $\rho_{\rm ac}(\omega)$ from the transverse ac susceptibility $\chi(\omega) = \mu(\omega) - 1$ of realistic specimens with finite size and constant thickness,[12,13] which in general differs from the ac susceptibility of ellipsoids considered recent.[14,15] The present paper fills this gap, presenting the transverse susceptibility of long strips and circular disks as a function of the complex ac resistivity $\rho_{\rm ac}(\omega)$. With this result, extraction of $\rho_{\rm ac}(\omega)$ from the measured complex $\mu(\omega)$ is readily performed by inverting this relationship.

In longitudinal geometry, the linear response is known. For slabs of thickness $2a$ and cylinders of radius a in a parallel ac field one has the permeabilities[3–5]

$$\mu_{\rm slab}(\omega) = \tanh(u)/u\,, \tag{1}$$

$$\mu_{\rm cyl}(\omega) = 2I_1(u)/uI_0(u)\,, \tag{2}$$

where $I_0(u)$ and $I_1(u) = I_0'(u)$ are modified Bessel functions and

$$u = a/\lambda_{\rm ac} = [i\omega a^2\mu_0/\rho_{\rm ac}(\omega)]^{1/2}\,. \tag{3}$$

The complex ac penetration depth $\lambda_{\rm ac} = (\rho_{\rm ac}/i\omega\mu_0)^{1/2}$ is related to the skin depth $\delta = (2\rho/\mu_0\omega)^{1/2}$, which is real for real (Ohmic) ρ, by $\lambda_{\rm ac} = (1-i)\delta/2$ or $\lambda_{\rm ac}^{-1} = (1+i)\delta^{-1}$. In terms of the relaxation time $\tau_0 = 4a^2\mu_0/\pi^2\rho_{\rm ac}(\omega)$ of the slab one has $u^2 = (\pi^2/4)i\omega\tau_0$. $\mu_{\rm slab}(\omega)$ [Eq. (1)] is a special case of the longitudinal permeability of a bar with rectangular cross section $b \times d$,[13]

$$\mu_{\rm rect}(\omega) = \frac{64}{\pi^4}\sum_\mu\sum_\nu \frac{b^2\nu^{-2} + d^2\mu^{-2}}{b^2\mu^2 + d^2\nu^2 + i\omega\pi^2\mu_0/\rho_{\rm ac}} \tag{4}$$

(μ, $\nu = 1, 3, 5, \ldots$), yielding (1) in the limit $b \gg d = 2a$.

Formulas (1)–(4) are obtained from the diffusion equation $\partial\mathbf{j}/\partial t = (\rho/\mu_0)\nabla^2\mathbf{j}$ for the current density $\mathbf{j}(\mathbf{r},t)$ using appropriate boundary conditions and the definition

$$\mu(\omega) = 1 + \chi(\omega) = 1 - M(\omega)/M(\omega\to\infty)\,, \tag{5}$$

where $\mathbf{m}(\omega) = -\frac{1}{2}\int \mathbf{j}(\mathbf{r},\omega)\times\mathbf{r}\,d^3r = V\mathbf{H}_0 M(\omega)$ is the magnetic moment of the specimen of volume V.

To determine the magnetic moment of thin specimens of thickness d in a perpendicular ac field $\mathbf{H}_0 e^{i\omega t}$ we have to solve an integral equation which describes nonlocal diffusion of the sheet current $\mathbf{J}(y,z) = \mathbf{j}(y,z)d$; here complete penetration $|\delta| > d$ will be assumed, equivalent to $\omega < 2|\rho_{\rm ac}|/\mu_0 d^2$. This integral equation follows by inserting the nonlocal relation between $\mathbf{J}(y,z)$ and the perpendicular field $H_x(y,z)$ it generates (Ampère's law) into the induction law $\nabla\times\mathbf{E} = -\dot{\mathbf{B}}$ using the material laws $\mathbf{E} = \rho_{\rm ac}\mathbf{j}$ and $\mathbf{B} = \mu_0\mathbf{H}$ (for nonmagnetic materials). For disks of radius a ($r \le a$) and strips of half width a ($|y| \le a$) this integral equation is *one-dimensional*,[12,13]

$$J(y,\omega) = w\Big[2\pi y + \int_0^1 K(y,u)\,J(u,\omega)\,du\Big]\,, \tag{6}$$

$$J(r,\omega) = w\Big[\pi r + \int_0^1 Q(r,u)\,J(u,\omega)\,du\Big]\,, \tag{7}$$

$$w = \frac{ad}{2\pi\lambda_{\rm ac}^2} = \frac{i\omega ad\mu_0}{2\pi\rho_{\rm ac}(\omega)} = i\omega\tau(\omega)\,. \tag{8}$$

Here the length unit a and field unit H_0 are chosen to obtain a dimensionless prefactor w [Eq. (8)], which is the only parameter of our theory. In the Ohmic case, $\rho_{\rm ac} = \rho$ is real; thus w is purely imaginary even as the longitudinal parameter u^2 [Eq. (3)], and $\tau(\omega) = ad\mu_0/2\pi\rho$ is a real relaxation time. In the Meissner state or for rigid pinning, $\lambda_{\rm ac} = \lambda$ is real (magnetic penetration depth), and thus w and u^2 are real and frequency independent. The integral kernels in (6) and (7) are $K(y,u) = \ln|(y-u)/(y+u)|$ for the strip and $Q(r,u) = -q(r/u)$ for the disk, with

$$q(x) = \int_0^\pi \frac{\cos\phi}{(1-2x\cos\phi+x^2)^{1/2}}\,d\phi\,. \tag{9}$$

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The integral (9) may be computed and tabulated, or expressed by complete elliptic integrals.[13]

Expanding $J(y,\omega)$ in terms of the eigenfunctions $f_n(y)$ of the eigenvalue problem for the strip,

$$f_n(y) = -\Lambda_n \int_0^1 K(y,u)\, f_n(u)\, du\,, \tag{10}$$

one gets $J(y,\omega) = \sum_n a_n(\omega) f_n(y)$ with

$$a_n(\omega) = \frac{2\pi\, w\, b_n}{1 + w/\Lambda_n}, \quad b_n = \int_0^1 f_n(y) y\, dy\,, \tag{11}$$

$$M(\omega) = 2\int_0^1 J(y,\omega) y\, dy = 2\sum_n a_n(\omega) b_n\,. \tag{12}$$

Similarly, for the disk $J(r,\omega) = \sum_n a_n(\omega) f_n(r)$ with

$$f_n(r) = -\Lambda_n \int_0^1 Q(r,u)\, f_n(u)\, du\,, \tag{13}$$

$$a_n(\omega) = \frac{\pi\, w\, b_n}{1 + w/\Lambda_n}, \quad b_n = \int_0^1 f_n(r) r^2\, dr\,, \tag{14}$$

$$M(\omega) = \pi \int_0^1 J(r,\omega) r^2\, dr = \pi \sum_n a_n(\omega) b_n\,. \tag{15}$$

Different orthonormalities hold for the strip and disk, $\int_0^1 f_n(y) f_m(y)\, dy = \int_0^1 f_n(r) f_m(r) r\, dr = \delta_{mn}$. Finally, one obtains μ and $\chi = \mu - 1$ in the form (Fig. 1)

$$\mu(\omega) = \sum_n \frac{c_n}{\Lambda_n + w}, \quad \chi(\omega) = -w \sum_n \frac{c_n/\Lambda_n}{\Lambda_n + w}\,, \tag{16}$$

with $c_n = 4 b_n^2 \Lambda_n^2$ for the strip and $c_n = (3\pi^2/8) b_n^2 \Lambda_n^2$ for the disk. From $\mu(0) = 1$ and $\mu(\infty) = 0$ follow sum rules for the strip (disk), $\sum b_n^2 = 1/3$ $(1/4)$ and $\sum b_n^2 \Lambda_n = 1/4$ $(8/3\pi^2)$. From the asymptotics below we further find $\sum b_n^2/\Lambda_n = 1/2$ $(4/15)$. For $n \gg 1$ one has $b_n^2 \approx 1/2\pi^2 n^3$ $(8/\pi^4 n^3)$ and $c_n \approx 2/\pi^2 n$ $(3/\pi^2 n)$ since for all $n = 1, 2, \ldots$ the eigenvalues are $\Lambda_n \approx \Lambda_1 + n - 1$. Here $\Lambda_1 = 0.638\,57$ $(0.876\,87)$ is the lowest eigenvalue, which also determines the transverse relaxation time of an Ohmic strip or disk, $\tau_0 = a d \mu_0/(2\pi\rho\Lambda_1)$.[12,13]

Using physical arguments related to the dissipation at low and high frequencies,[13] one obtains the asymptotic behavior of the complex $\mu(\omega)$ at $|w| \ll 1$ and $|w| \gg 1$:

$$\begin{aligned} &\mu \approx 1 - (4/3)w + 2w^2\,, \\ &\mu \approx (2/\pi^2 w)\, \ln(16.2w) \quad \text{(strip)}, \end{aligned} \tag{17}$$

$$\begin{aligned} &\mu \approx 1 - (3\pi^2/32)w + (\pi^2/10)w^2\,, \\ &\mu \approx (3/\pi^2 w)\, \ln(11.3w) \quad \text{(disk)}. \end{aligned} \tag{18}$$

In the Ohmic case $w = ix$ $(x = \omega\tau = \omega a d\mu_0/2\pi\rho)$ this means $\mu(x \ll 1) = 1 - 2x^2 - \frac{4}{3}ix$, $\mu(x \gg 1) = [1 - \frac{2}{\pi} i \ln(16.2x)]/\pi x$ (strip) and $\mu(x \ll 1) = 1 - \frac{\pi^2}{10}x^2 - \frac{3\pi^2}{32} ix$, $\mu(x \gg 1) = [1 - \frac{2}{\pi} i \ln(11.3x)] 3/2\pi x$ (disk). Using this asymptotic behavior one may construct an excellent approximation for all x, with an error less than 1.6% (Fig. 2),

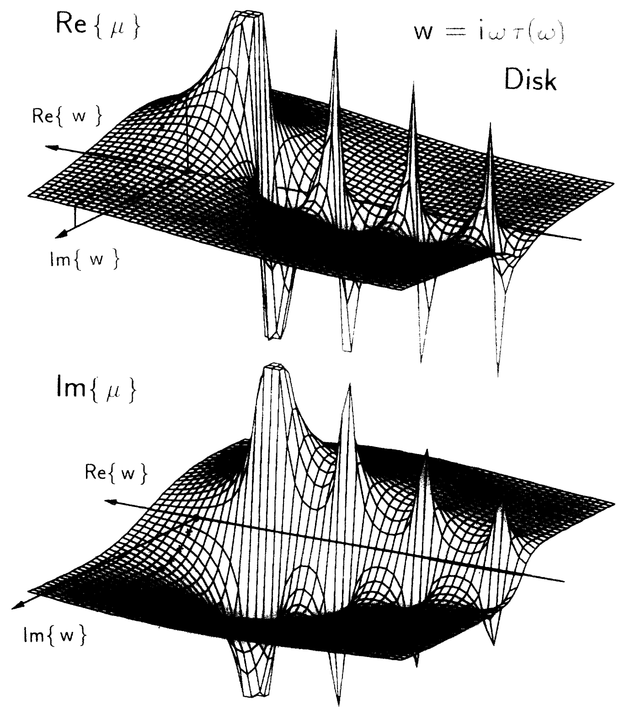

FIG. 1. Real (top) and imaginary (bottom) parts of the permeability $\mu = \mu' - i\mu''$ [Eq. (16)] of a circular disk in transverse magnetic field as functions of the complex variable $w = i\omega\tau(\omega)$ [Eq. (8)] from $w = -4.6 - 2.1i$ (right corner) to $w = 0.6 + 2.7i$ (left corner). The line spacing is 0.1. The scale may also be seen from the positions of the poles in Eq. (16). The peaks are cut off at heights ± 2. The cross sections yielding the Ohmic case $w = i\omega$ are marked by crosses.

$$\mu' \approx [1 - c_1 + (c_1^2 + c_2 \pi^2 x^2)^{1/2}]^{-1}\,, \tag{19}$$

$$\mu'' \approx \left[\frac{c_3}{x} + \frac{c_4 \pi^2 x}{\ln(1 + x^2) + c_5}\right]^{-1}, \tag{20}$$

where for the strip (disk) $c_1 = \pi^2/4$ $(20/9)$, $c_2 = 1$ $(4/9)$, $c_3 = 3/4$ $(32/3\pi^2)$, $c_4 = 1$ $(2/3)$, and $c_5 = 5.57 = 2\ln 16.2$ $(4.85 = 2\ln 11.3)$. Notice that these approximations contain *no fit parameter* except for c_5, which was fitted at $\omega \to \infty$. The Ohmic dissipation has a peak $\mu'' = 0.4488$ at $x = 1.108$ (strip) and $\mu'' = 0.4411$ at $x = 1.169$ (disk).

The general result (16) expresses $\mu(\omega)$ as an infinite sum $(n = 1, \ldots, \infty)$ of terms which have first order poles at real positions Λ_n in the complex w plane, or at complex positions in the ω plane since $w = i\omega\tau(\omega)$ [Eq. (8)], (Fig. 1). Similarly, the above longitudinal permeabilities may be expressed as sums of the form (16), namely, μ_{slab} [Eq. (1)] by putting $c_n = 2$ and $\Lambda_n = \pi^2(n - \frac{1}{2})^2$, and μ_{cyl} [Eq. (2)] by putting $c_n = 4$ and $\Lambda_n = x_n^2$, where $x_n \approx \pi(n - \frac{1}{4}) + [8\pi(n - \frac{1}{4})]^{-1}$ are the zeros of the Bessel function $J_0(x)$; in both cases $w = u^2$ [Eq. (3)]. This means that, in contrast to the transverse permeabilities of the strip and disk [Eq. (16)], which in the $\omega\tau$ plane have nearly equidistant poles at $\omega\tau = i\Lambda_n \approx ni$, the longitudinal permeabilities of the slab [Eq. (1)] and cylinder [Eq. (2)] have poles at nonequidistant points

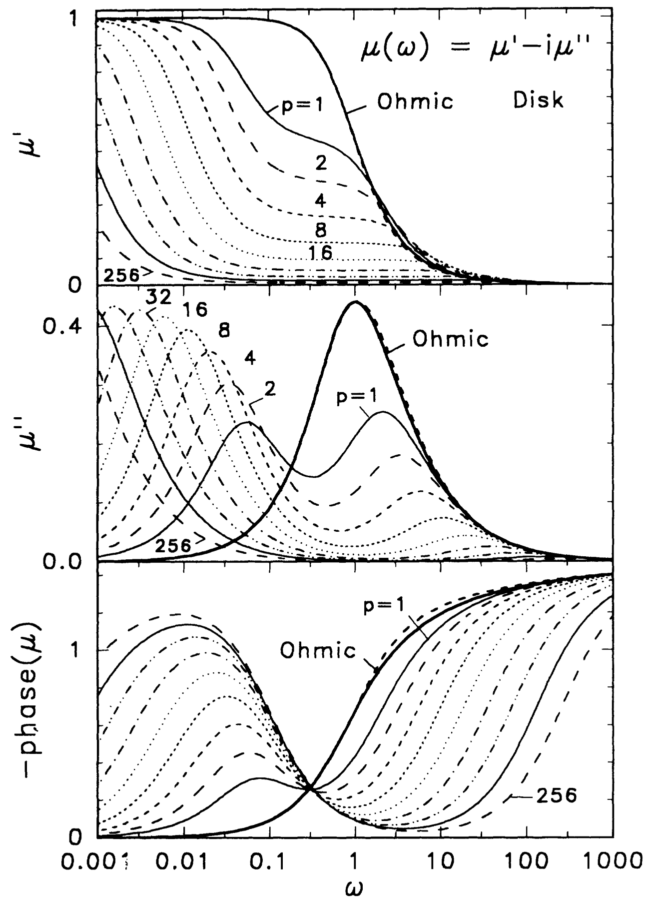

FIG. 2. The transverse permeability $\mu(\omega) = \mu' - i\mu''$ of a superconducting disk with complex resistivity [Eq. (23)] which models thermally assisted flux flow, pinning, and free flux flow, yielding $w = i\omega(p+i\omega)/(q+i\omega)$ [Eq. (24)]. Parameters of this example are $p = 1, 2, 4, 8, 16, 32, 64, 128, 256$ and $q = 0.1$. Also shown is $\mu(\omega)$ for an Ohmic disk with $w = i\omega$ (bold solid line) and its approximation [Eqs. (19), (20)] (bold dashed line). Time unit is $\tau_\perp = ad\mu_0/2\pi\rho_{\rm FF}$.

$\omega\tau = i\Lambda_n \approx n^2 i$.

I show now how the amplitudes c_n and eigenvalues Λ_n may be calculated and how the infinite sum (16) can be approximated with high precision by a finite sum of N terms ($n = 1, \ldots, N$). After discretizing the continuous variables y, r, and u as described in Ref. 13, the eigenvalue problems (10) and (13) are equivalent to the diagonalization of an $N \times N$ matrix K_{ij} or Q_{ij} defined by

$$K_{i\neq j} = w_j \ln\left|\frac{u_i - u_j}{u_i + u_j}\right|, \quad K_{jj} = w_j \ln\frac{w_j}{4\pi u_j}, \tag{21}$$

$$Q_{i\neq j} = -w_j\, q\left(\frac{u_i}{u_j}\right), \qquad Q_{jj} = w_j \ln\frac{0.923\,63 w_j}{2\pi u_j}. \tag{22}$$

Here the $u_i = u(x_i)$ span a nonequidistant grid obtained by inserting equidistant $x_i = (i - \frac{1}{2})/N$ ($i = 1, \ldots, N$) into an appropriate substitution function, e.g., $u(x) = (35x - 35x^3 + 21x^5 - 5x^7)/16$, which yields a weight function $w(x) = u'(x) = (35/16)(1 - x^2)^3$ and weights $w_i = w(x_i)/N$ vanishing at the edge $x = u = 1$. This grid and the diagonal terms K_{jj} [Eq. (21)] and Q_{jj} [Eq. (22)], are chosen to maximize the accuracy of the sums which approximate integrals like $\int_0^1 f(y)\,dy \approx \sum f_i w_i$ or $\int_0^1 K(y_i, u) f(u)\,du \approx \sum K_{ij} f_j$ with an error $\sim N^{-3}$ even when $f'(y \approx 1) \sim \ln(1 - y)$ diverges at the edge.

The matrices K_{ij} and Q_{ij} have exactly N eigenvalues Λ_n and N eigenfunctions f_n. Inserting these into (11) and (14) one gets the coefficients b_n and the N amplitudes c_n entering the sums (16) for μ and χ; see Table I. The finite sums obtained in this way present the exact solution of the discretized problem and are very good approximations to the original problem, the magnetic response of strips and disks. For example, in the Ohmic case $\rho_{\rm ac}(\omega) = \rho$ the choice $N = 4$ (6, 10, 16, 20) yields $\mu = \mu' - i\mu''$ with relative deviation of less than 2% up to high frequencies $x = |w| = |\omega|\tau = 30$ (60, 100, 300, 1000), except near the poles $w = \Lambda_n$. This may be checked from the asymptotic expressions (17) and (18). Notice that the condition for full transverse penetration ($|\delta| > d$) means $x = |\omega|\tau < a/d\pi$; thus, large x values apply only to thin films.

To illustrate how the permeability of a strip or disk may look like when a frequency-dependent resistivity is inserted, I consider the model resistivity

$$\rho_{\rm ac}(\omega) = \rho_{\rm FF}\frac{\tau_{\rm TAFF}^{-1} + i\omega}{\tau_{\rm FF}^{-1} + i\omega}, \tag{23}$$

which has been derived for HTSC's in three different

TABLE I. Positions of the poles Λ_n and amplitudes c_n entering the transverse susceptibility [Eq. (16)] of strips and circular disks for $N = 20$. With these numbers inserted, the finite sum (16) approximates the complex functions μ and χ of the complex argument $w = i\omega\tau$ [Eq. (8)] with high precision up to $|w| \approx 1000$. Notice the close similarity of the numbers for strips and disks.

	Strip		Disk	
n	Λ_n	c_n	Λ_n	c_n
1	0.638523	0.509196	0.876827	0.635477
2	1.629850	0.158763	1.874281	0.232998
3	2.618692	0.090685	2.866420	0.137091
4	3.589607	0.061974	3.841950	0.094590
5	4.523368	0.045581	4.783496	0.070120
6	5.391483	0.034153	5.664069	0.052806
7	6.142144	0.023513	6.446688	0.040108
8	6.623936	0.012743	7.207646	0.041054
9	7.199416	0.024916	8.099109	0.000342
10	8.238317	0.033578	8.251990	0.052417
11	9.799197	0.041631	9.809897	0.065132
12	12.14073	0.050459	12.15068	0.078494
13	15.76635	0.060820	15.77640	0.094003
14	21.68446	0.073688	21.69528	0.113146
15	32.10710	0.090618	32.11942	0.138270
16	52.56298	0.114443	52.57783	0.173639
17	99.72189	0.151083	99.74098	0.228158
18	240.8422	0.215510	240.8689	0.324328
19	937.0992	0.360493	937.1396	0.541420
20	16278.65	1.024858	16278.61	1.539020

ways[2,3,5] and which means that the flux-line lattice in a periodic[2] or random[3,5] pinning potential behaves viscoelastically: At high frequencies $\omega \gg \tau_{\rm FF}^{-1} = \alpha_L/\eta$ (α_L = Labusch parameter[16] and η are the pinning-caused elastic and the viscous restoring forces per unit volume of the flux-line lattice, respectively), and $\rho_{\rm ac}$ equals the Ohmic resistivity $\rho_{\rm FF} = B^2/\eta$ of free flux flow.[17] At low $\omega \ll \tau_{\rm TAFF}^{-1} = \tau_{\rm FF}^{-1}\exp(-U/kT) \ll \tau_{\rm FF}^{-1}$, thermally assisted flux flow (TAFF) Ref. (4) again leads to Ohmic resistivity $\rho_{\rm ac}(\omega) = \rho_{\rm TAFF} = \rho_{\rm FF}\exp(-U/kT) \ll \rho_{\rm FF}$. At intermediate frequencies, the imaginary part of $\rho_{\rm ac}$ dominates due to elastic pinning, and $\lambda_{\rm ac}(\omega)$ becomes real and frequency independent, equal to Campbell's penetration depth $\lambda_C = (B^2/\alpha_L\mu_0)^{1/2}$.[16] A different model $\rho_{\rm ac}(\omega)$ follows from vortex-glass scaling.[6–11] With (23) inserted, the complex variable w (8) becomes

$$w = i\omega\tau_\perp \frac{\tau_{\rm FF}^{-1} + i\omega}{\tau_{\rm TAFF}^{-1} + i\omega} \quad \text{or} \quad w = i\omega\,\frac{p + i\omega}{q + i\omega}\,, \qquad (24)$$

where in the second version $\tau_\perp = ad\mu_0/(2\pi\rho_{\rm FF})$ is the time unit, yielding two dimensionless parameters $p = \tau_\perp/\tau_{\rm FF} = ad/(2\pi\lambda_C^2)$ and $q = \tau_\perp/\tau_{\rm TAFF} = p\exp(-U/kT) \ll p$. Figure 2 shows the resulting permeabilities $\mu(\omega) = \mu' - i\mu''$ of a disk for the examples $q = 0.1$ and $p = 1, 2, 4, 8, \ldots, 256$. Note the two peaks in the dissipative part $\mu''(\omega)$, which are caused by thermally activated and normal flux flow.

In conclusion, the linear susceptibilities $\chi(\omega)$ or permeabilities $\mu(\omega) = 1 + \chi$ were calculated for strips and disks in perpendicular ac magnetic field as functions of the complex resistivity $\rho_{\rm ac}(\omega)$. The asymptotic behavior at high frequencies is $\mu(\omega) \sim (\rho_{\rm ac}/i\omega)\ln(i\omega/\rho_{\rm ac})$ [(16)–(18)] while longitudinal permeabilities exhibit $\mu(\omega) \sim (\rho_{\rm ac}/i\omega)^{1/2}$ [(1)–(4)]. The exact infinite sums (16) are approximated with high precision by finite sums. An example demonstrates the nontrivial frequency dependence of $\mu(\omega)$ in HTSC's with pinning and thermally assisted flux flow. The coefficients [Eqs. (11) and (14)] (which follow from Table I) and the general $\mu(\omega)$ [Eq. (16)] allow one to calculate the current distribution and the linear response for arbitrary time-dependent applied field $H_a(t)$ by Fourier transformation. This *linear* response should be compared with the *nonlinear* magnetic response of superconducting disks[18] and strips[19] with strong pinning in a perpendicular field,[20] for which analytical solutions were obtained recently.

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