

High-frequency random telegraph voltage noise in high- T_c thin films

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The vortex mechanism, leading to very high-frequency random telegraph voltage noise signals switching with MHz frequencies, is discussed. The mechanism assumes that vortices do not flow freely across dc-current-biased superconducting films but undergo subsequent processes of trapping and releasing from pinning centers. Random transitions of vortices between pinned and flow states result in the appearance of a Lorentzian component in a fluctuating-voltage power spectrum. It is shown that fluctuations of randomly distributed Abrikosov vortex density, rigidly moving across the strip, contribute an oscillating component to the noise spectrum. The distance between oscillating peaks corresponds to the time of flight of vortices across the strip.

I. INTRODUCTION

One of the most characteristic features of high-temperature superconductors (HTSC's) is the strong manifestation of various flux processes. In fact, the existence of many flux phases and the possibilities of transitions between them are the major features distinguishing HTSC materials from "classical" superconductors. Although the physics of vortex states and processes in oxide superconductors attracts the attention of many research groups, we are still far from having definite answers to many fundamental questions concerning the dynamics of vortex movements and, in particular, we are still unable to identify proper physical mechanisms leading to pronounced noise voltages showing in current-biased HTSC thin films.¹

In general, movement of flux vortices leads to voltages across current-biased superconducting samples, voltages possessing a dc voltage and a fluctuating component. Fluctuating voltages in HTSC's typically manifest themselves as current-, temperature-, and magnetic-field-dependent strong $1/f$ -like noise and/or pronounced random telegraph noise signals (RTS's). RTS noise and $1/f$ noise fluctuations are closely related. According to the widely accepted Dutta-Horn-Dimon model,² $1/f$ noise in solid-state systems originates from actions of many elementary two-level fluctuators (ETLF's) undergoing thermally activated or tunnel transitions between their energy wells, thus generating elementary RTS contributions to $1/f$ noise. The time-domain wave form of a random telegraph signal demonstrates random transitions

between two different values of the fluctuating physical quantity, usually referred to as *up* and *down* levels, whereas RTS power spectra in the frequency domain have a Lorentzian shape. Incoherent superposition of many RTS's possessing a proper distribution of cutoff frequencies of their Lorentzian spectra leads to $1/f$ noise.

There are several reports on experimental observations of low-frequency RTS noise in HTSC materials.^{1,3} Telegraph signals were detected as a straightforward RTS flux noise in free-standing HTSC samples at temperatures close to the superconducting transition,³ as well as RTS voltage wave forms superimposed on a constant voltage developing across dc current-biased HTSC macrobridges,⁴ superconducting quantum interference devices (SQUID's),⁵ and thin film strips.⁶⁻¹¹ In general, high levels of $1/f$ and low-frequency RTS noise that are observed in HTSC's can be ascribed to pronounced random movements of flux lines and high concentration of defects, frequently acting as intrinsic Josephson junctions. High temperature of operation, strong anisotropy, and relatively low flux-pinning energies of HTSC's result in easy random movements of flux vortices. Spontaneous flux noise converts into observable voltages by means of an intrinsic flux-to-voltage conversion mechanism. The generation of low-frequency voltage noise in HTSC's seems therefore to be a complex and indirect process most likely involving two separate mechanisms, the *fluctuator* and the *detector* mechanisms. The fluctuator action is responsible for the kinetics of random movements of flux lines between TLF wells, while the detector action couples these fluctuations to observable quantities.⁸⁻¹¹

On the basis of our experimental observations we have tentatively associated origins of macroscopic low-frequency RTS's to mechanisms depending on the sample crystalline microstructure. In oriented and epitaxial HTSC films TLF fluctuators cause random changes of a number of vortices participating in flux-flow or flux-creep dissipation processes, thus causing fluctuations in sample resistance. Under a constant current flow resistance fluctuations convert into fluctuating voltages.¹¹ Generation of telegraph voltages in granular films possibly involves another fluctuator and detector mechanism. The fluctuator mechanism relays activated jumps of flux lines between distinct pinning sites, while the detector action consists in direct detection of flux changes by intrinsic Josephson quantum interferometers.^{8,9}

Apart from the low-frequency telegraph signals, switching with frequencies from mHz up to the kHz range, one of us has reported on exotic manifestations of very high-frequency (VHF) RTS's, switching with rates well in the MHz range.¹² In a marked difference to the usual *low-frequency* RTS's, the Lorentzian shape of the VHF telegraph noise power spectrum is modified by an additional oscillating component.¹³ This is a feature that is not observed in the low-frequency RTS spectra.¹⁰ Very high switching frequencies, to say nothing about the excess components in the voltage spectra, cannot be explained in the framework of any mechanism previously invoked to explain the nature of low-frequency RTS voltages.

In the present paper we propose an intrinsic mechanism which accounts for the generation of VHF telegraph signals in superconducting films, and simultaneously predicts the appearance of additional periodic components in the frequency spectra. The mechanism relays on an assumption that vortices do not flow freely across dc current-biased superconducting films but undergo subsequent acts of trapping and releasing from pinning centers. Random transitions of vortices between pinned and flow states lead directly to the appearance of Lorentzian components in the voltage power spectrum.

II. BASIC EQUATIONS FOR FLUX MOTION

The total voltage due to vortex motion can be represented as a superposition of contributions from individual vortices. The relation between voltage V_i produced by the i th vortex and its velocity is determined by a resolution function $g(\rho_i)$,

$$V_i = g(\rho_i)v_i, \quad (1a)$$

where ρ_i stands for the i th vortex radius vector in the laboratory reference frame.¹⁴ The detailed form of $g(\rho_i)$ depends on the sample geometry and on the layout of the voltage measuring circuit,¹⁴

$$g(\rho) = \frac{\phi_0}{4\pi I_m} [b_{mt}(\rho) - b_{mb}(\rho)]. \quad (1b)$$

Here v_i is the vortex velocity, and b_{mt} and b_{mb} are the values of magnetic induction due to the current flow I_m in the measuring circuit on the top and on the bottom of

the superconducting slab, respectively. Let us define the time-dependent vortex-flow density $J(\rho, t)$ that contains all the information about the vortex dynamics,¹⁵

$$J(\rho, t) = \sum_i v_i(t) \delta_2[\rho - \rho_i(t)], \quad (2)$$

and express the total voltage due to the vortex motion using this quantity,

$$V(t) = \int d^2\rho g(\rho) J(\rho, t). \quad (3)$$

Obviously, voltage (3) possesses also a fluctuating component,

$$\delta V(t) = V(t) - \langle V(T) \rangle_t = \int d^2\rho g(\rho) \delta J(\rho, t), \quad (4)$$

where

$$\delta J(\rho, t) = J(\rho, t) - \langle J(\rho, t) \rangle_t$$

and $\langle \dots \rangle_t$ stand for time averages. We can now write the voltage autocorrelation function as

$$\begin{aligned} \Psi_V(\tau) &= \langle \delta V(t) \delta V(t + \tau) \rangle_t \\ &= \int d^2\rho \int d^2\rho' \sum_{\alpha\beta} g_\alpha(\rho) g_\beta(\rho') K_{\alpha\beta}(\rho, \rho', \tau), \end{aligned} \quad (5)$$

where the vortex-flux correlation function

$$K_{\alpha\beta}(\rho, \rho', \tau) = \langle \delta J_\alpha(\rho, t) \delta J_\beta(\rho', t + \tau) \rangle_t \quad (6)$$

is expressed in Cartesian coordinates α and β .

III. VORTEX STATES IN CURRENT-BIASED FILM

In the following we shall consider a superconducting slab in a current-induced dissipative state. In real samples possessing a certain distribution of pinning energies and for typical current flow levels used in experiments, i.e., for currents not exceeding the sample critical current by more than 2–3 times, it is physically reasonable to assume that we deal with two subsystems of vortices in the sample. Part of the vortices, with an average concentration n_p , will be pinned, while another part, with an average concentration n_f , will be in the flow state due to the Lorentz force of the bias current. Let us assume that on average vortices are uniformly distributed in the sample and that the probabilities of transitions of vortices between the two subsystems obey the Poisson law. This means that the probabilities of finding a vortex in a trap, P_{pin} , and the probability of vortex depinning, P_{dep} , during a time interval t are given by

$$\begin{aligned} P_{\text{pin}}(t) &= 1 - \exp(-t/\tau_f), \\ P_{\text{dep}}(t) &= 1 - \exp(-t/\tau_p), \end{aligned} \quad (7)$$

where τ_f and τ_p are the average lifetimes in the flow and pinned state, respectively. The average concentrations of vortices in both states, n_p and n_f , are related to the relevant average lifetimes, τ_f and τ_p ,

$$n_p = \frac{N_p}{S} = \frac{B}{\phi_0} \frac{\tau_p}{\tau_p + \tau_f}, \quad n_f = \frac{N_f}{S} = \frac{B}{\phi_0} \frac{\tau_f}{\tau_p + \tau_f}, \quad (8)$$

where N_p and N_f are the numbers of pinned and flowing

vortices, W and L stand for film width and length, respectively, and B is the value of the magnetic induction vector applied perpendicular to the film surface $S = WL$. According to Eq. (5) voltage noise can be expressed in terms of a vortex-flux correlation function. If the dimensions of the measuring circuit are large with respect to the inter-vortex spacing, we can treat both vortex subsystems as a continuum. In the following we assume that bias-current fluctuations are negligible; therefore, consequently, we neglect velocity fluctuations of the flowing vortices. Under these assumptions we obtain, within the first-order approximation, the change in the vortex-flow density $\delta J_f(\rho, t)$:

$$\delta J_f(\rho, t) = v_0 \delta n_f(\rho, t), \quad (9)$$

where v_0 is the average flow velocity of the vortices.

Let us define the geometry of the problem as illustrated in Fig. 1. The assumed geometry is close to the typical experimental configuration.¹² The thin-film superconducting strip possesses length L in the x direction, width W in the y direction, and thickness D in the z direction, such that $L \gg W \gg D$. Measuring leads are attached at points $[0, 0]$ and $[L, 0]$, and kept far away from the surface elsewhere. Current flow is directed along the strip length; consequently vortices flow perpendicularly to the strip, in the y direction. Neglecting the effects of the edges of a strip, we assume for the geometrical factor $g(\rho)$ the form appropriate for a thin-film sample:^{15,16}

$$g(\rho) = \frac{\phi_0}{cW} y, \quad (10)$$

where y is the unit vector in the y direction.

Fluctuations of vortex concentration around the mean value $n_f(\rho, t)$ at a given point of the sample may result from the randomness in the distribution of flowing vortices in space and from fluctuations in the ratio of the number of pinned to the number of flowing vortices, as determined by Eq. (8). Taking this into account we write

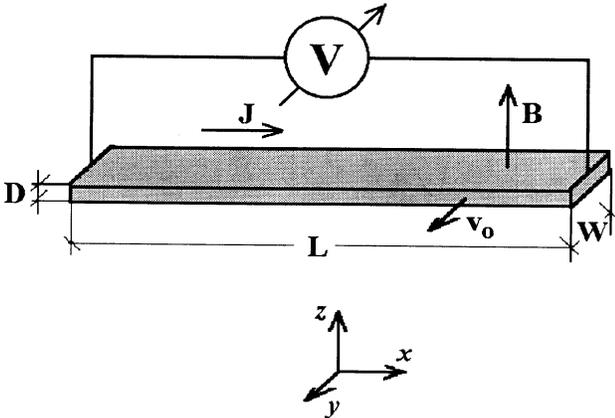


FIG. 1. Geometry of the problem.

for the vortex-flow correlation function

$$K_{\alpha\beta}(\rho, \rho', t) = v_0^2 \delta_{\alpha y} \delta_{\beta y} \langle \delta n_f(\rho, t_0) \delta n_f(\rho', t_0 + t) \rangle, \quad (11)$$

where $\delta_{\alpha y}$ and $\delta_{\beta y}$ are Kronecker symbols.

Calculations of the density correlator $\langle \delta n_f(\rho, t_0) \delta n_f(\rho', t_0 + t) \rangle$ for an arbitrary vortex concentration and for an arbitrary relation between the lifetimes τ_f and τ_p and the time of flight of vortices across the sample, $\tau = W/v_0$, is a complicated and difficult mathematical task. However, the problem may be significantly simplified by neglecting interactions between the vortices. We adopt the approximation of noninteraction throughout the paper. Representing the average concentration of flowing vortices as

$$n_f(\rho, t) = \sum_i \delta(\rho - \rho_i(t)), \quad (12)$$

where index i counts all flowing vortices, we obtain for the density correlator

$$\begin{aligned} & \langle \delta n_f(\rho, t_0) \delta n_f(\rho', t_0 + t) \rangle \\ &= \langle n_f(\rho, t_0) n_f(\rho', t_0 + t) \rangle - \langle n_f(\rho, t) \rangle^2 \\ &= N_f \langle [\delta(\rho - \rho_i(t_0)) \delta(\rho' - \rho_i(t_0 + t)) \\ & \quad - \langle \delta(\rho - \rho_i(t_0)) \rangle^2] \rangle. \end{aligned} \quad (13)$$

Neglecting the nonsignificant constant term we write for the density correlator

$$\langle \delta(\rho - \rho_i(t_0)) \delta(\rho' - \rho_i(t_0 + t)) \rangle = p_f(\rho, t_0, \rho', t_0 + t). \quad (14)$$

$p_f(\rho, t_0, \rho', t_0 + t)$ is a two-coordinate distribution function for a flowing vortex. Since we deal with a spatially homogeneous one-dimensional problem, where vortices move only in the y direction, the function $p_f(\rho, t_0, \rho', t_0 + t)$ can be represented as follows:

$$\begin{aligned} p_f(\rho, t_0, \rho', t_0 + t) &= p_f(\rho - \rho', t) \\ &= \frac{1}{S} \delta(x - x') p_1(y - y', t), \end{aligned} \quad (15)$$

where $p_1(Y, t)$ is a one-coordinate probability distribution function for a flowing vortex, located at the moment $t=0$ at the coordinate $Y=0$. In order to find this function one has to derive and solve a set of differential equations for probability distribution functions for flowing, $p_1(Y, t)$, and pinned, $p_2(Y, t)$, vortices. We approach this problem by using the well-known method of a two-state random walk model^{17,18} and obtain the following set of differential equations for p_1 and p_2 :

$$\begin{aligned} \frac{\partial p_1}{\partial t} &= -v_0 \frac{\partial p_1}{\partial Y} - \frac{p_1}{\tau_f} + \frac{p_2}{\tau_p}, \\ \frac{\partial p_2}{\partial t} &= \frac{p_1}{\tau_f} - \frac{p_2}{\tau_p}. \end{aligned} \quad (16)$$

The initial conditions are set by the definitions of both functions, namely,

$$p_1(Y, 0) = \delta(Y) \quad \text{and} \quad p_2(Y, 0) = 0. \quad (17)$$

At this point, to simplify the calculations, we assume that the vortex system behaves in a symmetric way. That is, average lifetimes in the pinned and in the flowing states are equal, $\tau_f = \tau_p$. In this approximation we may obtain an analytical solution for $p_1(Y, t)$ from Eqs. (16):

$$p_1(Y, t) = \exp(-t/\tau_f) \left[\frac{Y}{(v_0\tau_f)^2} \frac{I_1(\alpha)}{\alpha} + \delta(Y - v_0t) \right] \times H(Y)H(v_0t - Y), \quad (18)$$

$$\alpha = \left[\left(\frac{2Y - v_0t}{v_0\tau_f} \right)^2 - \left(\frac{t}{\tau_f} \right)^2 \right]^{1/2},$$

where $I_1(\alpha)$ is the Bessel function of the first order of an imaginary argument and $H(Y)$ is the Heaviside step function.

Let us evaluate the form of the spectral density of voltage fluctuations due to the vortices that during their lifetime in the film undergo several subsequent acts of trapping and releasing from pinning centers. In order to obtain the equation for the spectral density in these conditions we proceed with the calculation of the voltage autocorrelation function (5) at the limit $v_0\tau_f \ll W$. It can be

easily shown that in this limit the function $p_1(Y, t)$ has a sharp peak in the vicinity of the point $Y = v_0t/2$. The characteristic width of this peak is much smaller than the sample width W . Observe that the geometrical factor $g(\rho) \equiv g(y)$ varies slowly, with respect to the variations of $p_1(Y, t)$, at distances of the order of W . Therefore the spatial dependence of the distribution function $p_1(Y, t)$ can be approximated by a δ function $\delta(Y - v_0t/2)$ while integrating Eq. (5) over y . Within this approximation one obtains

$$\Psi_V(t) = n_f L v_0^2 \int p_1(Y, |t|) dY \times \int_{-W/2}^{W/2} g_y(y) g_y(y + v_0|t|/2) dy, \quad (19)$$

where the quantity $\int p_1(Y, |t|) dY$ denotes the total probability of finding a vortex in the state of flow. This probability can be directly calculated by integrating Eq. (18) over Y ,

$$\int p_1(Y, |t|) dY = \frac{1 + \exp(-2t/\tau_f)}{2}. \quad (20)$$

Putting Eq. (20) into Eq. (19) we have

$$\Psi_V(t) = \frac{1}{2} n_f L v_0^2 \left[\int_{-W/2}^{W/2} g_y(y) g_y(y + v_0|t|/2) dy + \exp(-2|t|/\tau_f) \int_{-W/2}^{W/2} g_y(y) g_y(y + v_0|t|/2) dy \right]. \quad (21)$$

Since the exponential prefactor of the second integral term in (21) decays at times of the order of τ_f in the limit of $\tau_f \ll \tau$ we are entitled to substitute $g_y(y + v_0|t|/2)$ by $g_y(y)$. Putting in the geometrical factor for a thin-film narrow strip configuration from Eq. (10) we arrive at the final equation for the voltage autocorrelation function:

$$\Psi_V(t) = \frac{B\phi_0 v_0^2 L}{4Wc^2} [(1 - |t|/2\tau) + \exp(-2|t|/\tau_f)]. \quad (22)$$

Using the Wiener-Khinchine theorem we find the spectral density of voltage fluctuations due to the discussed vortex processes:

$$S_V(\omega) = \frac{B\phi_0 v_0 L}{c^2} \left[\frac{\sin^2(\omega\tau)}{(\omega\tau)^2} + \frac{\tau_f}{4\tau} \frac{1}{[(\omega\tau_f/2)^2 + 1]} \right]. \quad (23)$$

The shape of the power spectrum is determined by two contributions, oscillating and Lorentzian. The oscillating term is due to the motion of randomly distributed vortices across the strip width. The characteristic frequency of oscillations is determined by the inverse of the average time of flight of vortices τ . The Lorentzian term reflects processes of random transitions of vortices between pinned and flow states. The relative contribution of both terms to the total power spectrum depends on the actual frequency range. At low frequencies, $\omega \sim 1/\tau \ll 1/\tau_f$, spectrum (23) is dominated by the oscillating term, while

at high frequencies, $\omega \sim 1/\tau_f \gg 1/\tau$, the Lorentzian term prevails. The crossover frequency between these two regimes is of the order of $\omega \sim (\tau\tau_f)^{-1/2}$. Figure 2 demonstrates the evolution of the shape of the power spectrum described by Eq. (23). Parameter α , $\alpha = \tau_f/\tau = l/W$, where $l = v_0\tau_f$ is the mean distance between pinning centers and W is the strip width, describes the intensity of processes of random transition between different vortex states.

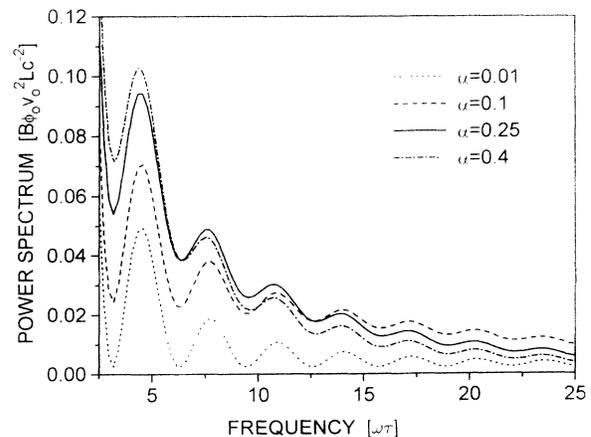


FIG. 2. Shape of the power spectrum of voltage fluctuations across dc current-biased superconducting thin-film strips as predicted by the model for various intensities of processes of random interruptions of vortex flow; see text.

IV. CONCLUSIONS

We have demonstrated that processes of randomly interrupted flow of vortices across dc current-biased thin-film superconducting narrow strips lead to the appearance of fluctuating voltages across the strip characterized by a power spectrum contributed by oscillating and Lorentzian components. As discussed by van Ooijen and van Garp and recently by us, flow of randomly distributed vortices,¹⁶ or flow of a vortex lattice with frozen density fluctuations,¹⁹ results in oscillatory character of the power spectra. The Lorentzian term may reflect a random telegraph shape of the noise wave form in the time domain. Indeed, random telegraph noise possesses a Lorentzian spectrum in the frequency domain. However, since the random telegraph noise is a strongly non-Gaussian process, the presence of the Lorentzian term in the power spectra does not allow one to unequivocally claim the existence of RTS's in the time domain. However, if one assumes that due to the collective pinning effects vortices move as bundles, coherently undergoing random transitions between pinned and flow states, then time-domain wave forms will take the form of the random telegraph voltages seen in the experiments. Formally, this means that in Eq. (12) index i should count now vortex bundles while the factor $g(\rho)$ should be multiplied by n_b , where n_b is the number of vortices in the bundle.

Let us estimate the characteristic switching frequency of the telegraph signal corresponding to the Lorentzian component in the power spectrum. Let us consider a typ-

ical geometry of a strip of width $W \sim 0.1$ mm and thickness D of the order of $1 \mu\text{m}$, biased with a current flow $I \sim 1$ mA. Assuming that the average distance between effective pinning sites is of the order of $l \sim 0.1 \mu\text{m}$ and taking for the viscosity $\eta \sim 10^{-6}$ g/cm s, we estimate $\tau_f = l/v_0 = l\eta W D c / \phi_0 I \sim 0.5 \mu\text{s}$. The cutoff frequency of the considered Lorentzian spectrum $\omega_c = 2/t_f$ falls thus into the VHF range as was observed in the experiments.^{12,13}

In conclusion, we have proposed a mechanism of vortex fluctuations that may be responsible for the generation of very high-frequency random telegraph voltages across dc current-biased superconducting strips. The mechanism consists in random transition of collectively pinned vortex bundles between pinned and flow states during their time of flight across the strip. Lorentzian power spectra for such processes are accompanied by an oscillating component, particularly dominating the low-frequency part of the spectra. An estimation of the telegraph signal switching frequency gives frequencies extending far into the MHz range. Therefore this mechanism may lay behind the exotic VHF's that show in HTSC thin-film strips.

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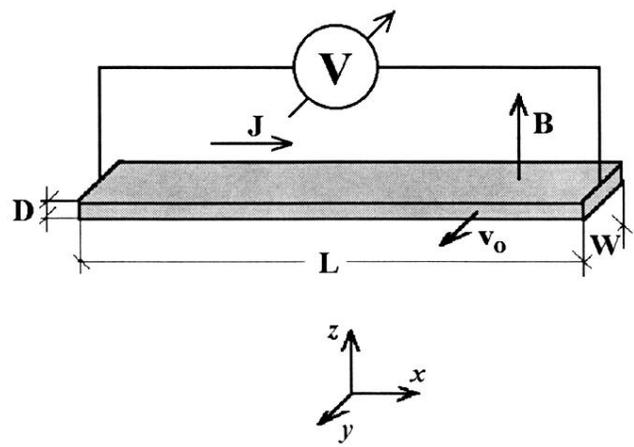


FIG. 1. Geometry of the problem.