Unconventional superconducting order parameters without nodes: The density of states and impurity scattering

Gianfranco Preosti, Heesang Kim, and Paul Muzikar
Department of Physics, Purdue University, West Lafayette, Indiana 47907
(Received 6 June 1994)

We compute the density of states $N_s(E)$ for a superconductor having an order parameter $\Delta(\hat{\mathbf{k}})$ with the following two properties: (i) $|\Delta(\hat{\mathbf{k}})|$ is a constant, independent of $\mathbf{k}$; (ii) the average of $\Delta(\hat{\mathbf{k}})$ over the Fermi surface vanishes: $\langle \Delta(\hat{\mathbf{k}}) \rangle = 0$. We particularly stress the effect of impurity scattering. We can do the calculation for an arbitrary Fermi surface, and any order parameter with the above two properties. Since our family of order parameters has no nodes, our results pinpoint the effects due to the vanishing Fermi surface average, $\langle \Delta(\hat{\mathbf{k}}) \rangle = 0$.

I. INTRODUCTION

The properties of superconductors with unconventional order parameters have received much theoretical attention recently,[1,2] in the context of heavy-fermion and high-T_c materials. An unconventional order parameter, $\Delta(\hat{\mathbf{k}})$, is one with less rotational symmetry than the normal-state lattice. Such an order parameter has a nontrivial $\hat{\mathbf{k}}$ dependence, which can lead to interesting properties. These novel effects are generally due to two related factors: (a) the average of $\Delta(\hat{\mathbf{k}})$ over the Fermi surface vanishes—$\langle \Delta(\hat{\mathbf{k}}) \rangle = 0$ and (b) in many cases $\Delta(\hat{\mathbf{k}})$ vanishes at points or lines on the Fermi surface.

Superconductors with such order parameters are sensitive in many ways to scattering by ordinary, nonmagnetic impurities.[1-5] However, it is difficult to disentangle what role is played by each of the two factors listed above. Thus, in this paper we consider a wide family of order parameters in which (a) is present but not (b). To be precise, we consider a class of order parameters satisfying the following two conditions.

(i) The magnitude of $\Delta(\hat{\mathbf{k}})$ is independent of $\hat{\mathbf{k}}$: $|\Delta(\hat{\mathbf{k}})| = \Delta$, for all $\mathbf{k}$. Thus there are no nodes.

(ii) The Fermi surface average vanishes: $\langle \Delta(\hat{\mathbf{k}}) \rangle = 0$.

Then, the effects that separate our class of order parameters from conventional ones are strictly due to factor (a); the order parameters we consider have no nodes.

We consider systems with an arbitrary Fermi surface, which may have several disconnected pieces. For this general case, we are able to compute[6] $N_s(E)$, for all order parameters satisfying conditions (i) and (ii) listed above. Some examples of $\Delta(\hat{\mathbf{k}})$'s within our family are: we can consider a Fermi surface made up of several disconnected pieces, such that $\Delta(\hat{\mathbf{k}}) = +\Delta$ on some pieces and $\Delta(\hat{\mathbf{k}}) = -\Delta$ on others, giving $\langle \Delta(\hat{\mathbf{k}}) \rangle = 0$, and we may consider a circular Fermi surface, parametrized by the polar angle $\phi(\hat{\mathbf{k}})[0 \rightarrow 2\pi]$ with $\Delta(\hat{\mathbf{k}}) = \Delta e^{in\phi(\hat{\mathbf{k}})}$.

It should be stressed that, in the absence of impurity scattering, the density of states for our family of unconventional order parameters is exactly the same as for a conventional order parameter with $\Delta(\hat{\mathbf{k}}) = \Delta$ for all $\hat{\mathbf{k}}$. Impurity scattering drastically changes the density of states for the unconventional case. The interesting structure we find at low energies has nothing to do with the gap nodes.

Conditions (i) and (ii) are appropriate for a spin-singlet order parameter of the form $\Delta_{\alpha\beta}(\hat{\mathbf{k}}) = i\sigma^y_{\alpha\beta}\Delta(\hat{\mathbf{k}})$. Our results also apply to triplet order parameters, $\Delta_{\alpha\beta}(\hat{\mathbf{k}})$, which satisfy the following conditions: $\Delta_{\alpha\tau}(\hat{\mathbf{k}})\Delta^*_{\beta\tau}(\hat{\mathbf{k}}) = \Delta^2\delta_{\alpha\beta}$, $\langle \Delta_{\alpha\beta}(\hat{\mathbf{k}}) \rangle = 0$, where Δ is independent of $\hat{\mathbf{k}}$.

Buchholtz and Zwicknagl,[7] in a pioneering paper, considered a particular triplet order parameter satisfying the above conditions. They discovered much interesting structure in the density of states resulting from impurity scattering. Hirschfeld *et al.*[8] treated the optical conductivity and density of states for our family of order parameters. Our work generalizes and extends these results.

It is also of interest to point out that the problem considered in our paper has an exact mathematical parallel to a quite different physical situation, namely, the problem of classical-spin impurities in a conventional *s*-wave superconductor. This problem has been treated by Shiba,[9] by Rusinov,[10] and by Chaba and Nagi[11]; the Born limit has been treated by Abrikosov and Gor'kov,[12] and by Skalski, Betbeder-Matibet, and Weiss.[13]

Our general problem can be mapped into Shiba's, for example, by identifying our parameters σ and τ with Shiba's parameters γ and τ_s in the following way:

$$\tau_s \Longleftrightarrow 2\tau \ ,$$

$$\gamma^2 \Longleftrightarrow 1-\sigma \ .$$

Then, Shiba's equation (3.9) is exactly the same as our (28). In the Born limit, our parameter τ, with $\sigma = 0$, is identified with the parameter Γ of Skalski *et al.* in the following way:

$$\frac{1}{2\tau} \Longleftrightarrow \Gamma \ .$$

Equation (2.13) of Skalski *et al.* is then equivalent to our (23). The interested reader can find points of contact between our work and the work presented in these five papers.

II. CALCULATION OF $N_s(E)$

In this section we explain how to calculate the density of states for a singlet gap $\Delta(\hat{\mathbf{k}})$ satisfying conditions (I) and (II) given in the previous section. $N_s(E)$ is given in terms of the $\hat{\tau}_3$ component of the quasiclassical propagator, $\hat{\mathbf{g}}(\varepsilon,\hat{\mathbf{k}})$:[14]

$$N_s(E)=\frac{N(0)}{\pi}\mathrm{Im}\int_{\mathrm{FS}} d^2\hat{\mathbf{k}}\, n(\hat{\mathbf{k}})g_3(i\varepsilon\rightarrow E-i\eta,\hat{\mathbf{k}}) \quad (1)$$

$$=\frac{N(0)}{\pi}\mathrm{Im}\langle g_3(i\varepsilon\rightarrow E-i\eta,\hat{\mathbf{k}})\rangle \ . \quad (2)$$

Here, $N(0)$ is the density of states at the Fermi surface in the normal state, ε is a Matsubara frequency, and $n(\hat{\mathbf{k}})$ is the density of states at $\hat{\mathbf{k}}$, normalized to one:

$$1=\int_{\mathrm{FS}} d^2\hat{\mathbf{k}}\, n(\hat{\mathbf{k}}) \ . \quad (3)$$

The Fermi surface integral of any quantity, such as $\Delta(\hat{\mathbf{k}})$, is denoted by $\langle\Delta(\hat{\mathbf{k}})\rangle$. Then, for our class of order parameters we have

$$\langle\Delta(\hat{\mathbf{k}})\rangle=\int_{\mathrm{FS}} d^2\hat{\mathbf{k}}\, n(\hat{\mathbf{k}})\Delta(\hat{\mathbf{k}})=0 \ . \quad (4)$$

For our class of order parameters, the propagator $g_3(\varepsilon,\hat{\mathbf{k}})$ is in fact independent of $\hat{\mathbf{k}}$; it is given by the following formula:

$$g_3(\varepsilon,\hat{\mathbf{k}})=g_3(\varepsilon)=\frac{-i\pi(\varepsilon+ia_3)}{[(\varepsilon+ia_3)^2+\Delta^2]^{1/2}} \ . \quad (5)$$

Thus,

$$N_s(E)=\frac{N(0)}{\pi}\mathrm{Im}\{g_3(i\varepsilon\rightarrow E-i\eta)\} \ . \quad (6)$$

The impurity self-energy $\hat{a}(\varepsilon)$ is given by $\hat{a}=c\hat{t}(\varepsilon)$, where c is the density of impurities and the $\hat{t}$ matrix is given by

$$\hat{t}(\varepsilon)=v+v\int d^2\hat{\mathbf{k}}\,\hat{g}(\varepsilon,\hat{\mathbf{k}})\hat{t}(\varepsilon)n(\hat{\mathbf{k}}) \ . \quad (7)$$

Here v is the strength of the impurity potential, assumed to be an s wave. Since $\langle\Delta(\hat{\mathbf{k}})\rangle=0$, t_1, and t_2 are zero; a_3 is determined from the following equation:

$$a_3=\frac{cN(0)v^2g_3}{1-[N(0)vg_3]^2} \ . \quad (8)$$

Since g_3 is given in terms of a_3 by Eq. (5), Eq. (8) constitutes a nonlinear, self-consistent equation, which should be solved for a_3.

We note that the fact that t_1 and t_2 (and so a_1 and a_2) are zero leads to the particularly simple solution for g_3 given by (5). We also note that if the impurity potential were not taken to be s wave, then t_1 and t_2 would not, in general, vanish.

So, we must solve (8) for a_3, and then compute $N_s(E)$ from (6). In general, this requires numerical work, but analytic progress is possible in several ways, as will be explained in future sections.

Instead of using c and v, it is sometimes useful to calibrate impurity effects in terms of two parameters,[7] given by

$$\frac{1}{2\tau}=\frac{N(0)\pi cv^2}{1+[N(0)\pi v]^2} \ , \quad (9)$$

$$\sigma=\frac{[N(0)\pi v]^2}{1+[N(0)\pi v]^2} \ . \quad (10)$$

Here, τ is the normal-state impurity scattering time, and σ is proportional to the cross section for an electron at the Fermi surface scattering off a single impurity.

We note here that the magnitude Δ will in general be a function of the impurity scattering. Thus the reader should bear in mind that whenever Δ appears in our equations, it should be interpreted as the actual value of the order parameter at the particular value of σ and τ in question.

In Fig. 1 we show a plot of $N_s(E)$, computed for a par-

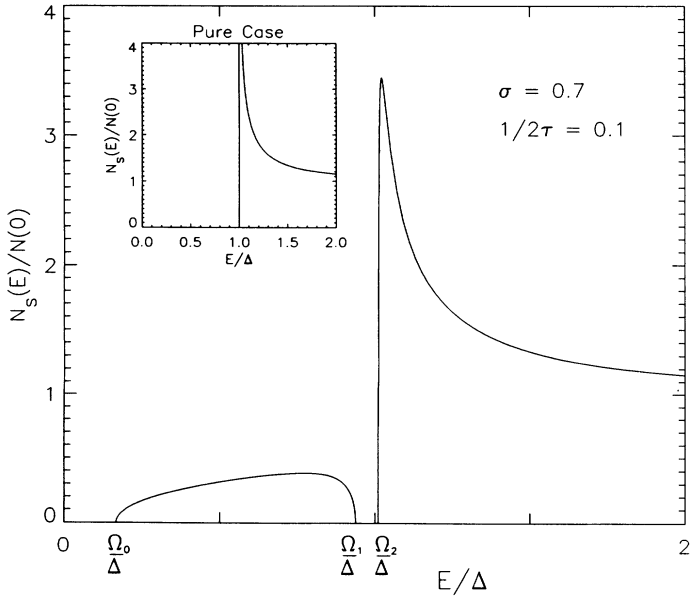

FIG. 1. Plot of the density of states as a function of energy for $\sigma=0.7$ and $1/2\tau\Delta=0.1$. We use this picture to define the three energies, Ω_0, Ω_1, and Ω_2, which are functions of the parameters σ and τ. For some parameter ranges, there are no Ω_1 and Ω_2 points, while for some parameter values there is no Ω_0 point. The inset shows the density of states with no impurities.

ticular choice of σ and τ. We use this picture to define the three energies Ω_0, Ω_1, and Ω_2, which separate regions of E in which $N_s(E)$ is zero from regions of E in which $N_s(E)$ is not zero. As we shall see, these energies are functions of σ and τ. Furthermore, for some choices of σ and τ, there are no Ω_1 and Ω_2 points, while for some choices there is no Ω_0 point.

III. UNITARITY LIMIT

This limit corresponds to $v\to\infty$, $\sigma\to 1$, with τ held fixed The equation for a_3 becomes

$$a_3=\frac{-c}{N(0)g_3}=\frac{[(\varepsilon+ia_3)^2+\Delta^2]^{1/2}}{i2\tau(\varepsilon+ia_3)} \tag{11}$$

By combining (11) with (6), we can see that $N_s(E)$ is given in terms of $a_3(\varepsilon)$ by

$$N_s(E)=-\frac{N(0)}{2\tau}\mathrm{Im}\frac{1}{a_3(i\varepsilon\to E-i\eta)} . \tag{12}$$

Note that (11) and (12) are exact in the unitarity limit.

We first study $N_s(E=0)$. Equation (11) can be solved for $a_3(0)$, yielding

$$ia_3(0)=\left\{\frac{1}{2}\left[\frac{1}{2\tau}\right]^2+\frac{1}{2}\left[\frac{\Delta^2}{\tau^2}+\left[\frac{1}{2\tau}\right]^4\right]^{1/2}\right\}^{1/2} . \tag{13}$$

This gives

$$\frac{N_s(E=0)}{N(0)}=\frac{1}{\left[\frac{1}{2}+\frac{1}{2}(1+16\tau^2\Delta^2)^{1/2}\right]^{1/2}} . \tag{14}$$

So, in the unitarity limit there is always a nonzero density of states at zero energy, when $c>0$. If the concentration c is small, so that $1/2\tau\Delta\ll 1$, we may approximate (14) as

$$\frac{N_s(E=0)}{N(0)}=\sqrt{1/2\tau\Delta} . \tag{15}$$

At small values of c, $N_s(E=0)$ is proportional to $c^{1/2}$. Note that no energy plays the role of Ω_0 in the unitarity limit.

We now turn to the problem of $N_s(E)$ at general values of E. Equation (11) can be turned into the following polynomial equation for $a_3(E)$, where $E=-i\varepsilon$,

$$a_3^4-2Ea_3^3+\left[E^2+\frac{1}{(2\tau)^2}\right]a_3^2-\frac{2E}{(2\tau)^2}a_3+\frac{(E^2-\Delta^2)}{(2\tau)^2}=0 . \tag{16}$$

At values of E for which this equation has four real roots, the density of states is zero.

For small values of $1/2\tau\Delta$, the density of states vanishes for energies between $\Omega_1(\tau\Delta)$ and $\Omega_2(\tau\Delta)$. As $1/2\tau\Delta$ increases, Ω_1 and Ω_2 come together at a critical value of $1/2\tau\Delta$, given by

$$\frac{1}{2\tau\Delta}=\frac{2\sqrt{3}}{9} , \tag{17}$$

at which the values of Ω_1 and Ω_2 are

$$\frac{\Omega_1}{\Delta}=\frac{\Omega_2}{\Delta}=\tfrac{4}{3}\sqrt{\tfrac{2}{3}} . \tag{18}$$

For values of $1/2\tau\Delta$ greater than the critical one given by (17), $N_s(E)$ does not vanish for any value of E. This behavior is illustrated by the numerical results shown in Fig. 2. Analytic formulas, in terms of a parametric equation, can be derived for $\Omega_1(\tau\Delta)$ and $\Omega_2(\tau\Delta)$. We write

$$\begin{aligned}&\frac{1}{2\tau\Delta}=y^2\sqrt{1-y^2},\quad 0\le y\le 1 ,\\ &\frac{\Omega_1}{\Delta}=2y-y^3\quad 0\le y\le\sqrt{\tfrac{2}{3}}\\ &\frac{\Omega_2}{\Delta}=2y-y^3,\quad \sqrt{\tfrac{2}{3}}\le y\le 1 .\end{aligned} \tag{19}$$

As the parameter y varies between 0 and 1, these formulas give the exact value of Ω_1 and Ω_2 in terms of $\tau\Delta$. Note that Ω_2/Δ is always greater than one.

Figure 3 shows the behavior of Ω_1 and Ω_2 as a function of τ.[9] By using (19), we can derive asymptotic expressions for Ω_1 and Ω_2 in terms of $\tau\Delta$, which is quite accurate when $1/\tau\Delta$ is small. These are

$$\begin{aligned}&\frac{\Omega_1}{\Delta}\approx 2\left[\frac{1}{2\tau\Delta}\right]^{1/2}-\frac{1}{2}\left[\frac{1}{2\tau\Delta}\right]^{3/2},\\ &\frac{\Omega_2}{\Delta}\approx 1+\frac{1}{2}\left[\frac{1}{2\tau\Delta}\right]^2+\frac{3}{8}\left[\frac{1}{2\tau\Delta}\right]^4 .\end{aligned} \tag{20}$$

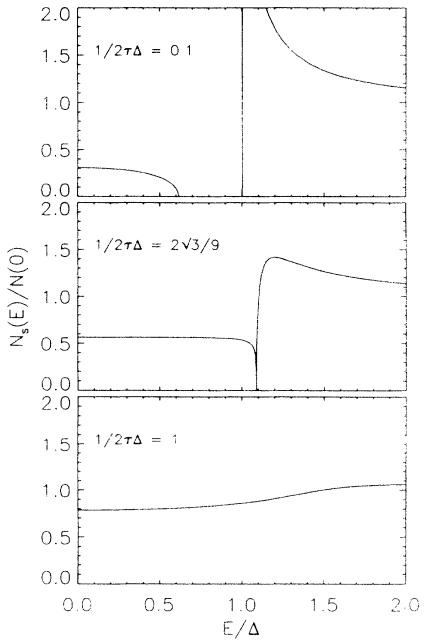

FIG. 2. Density of states in the unitarity limit ($\sigma=1$), for three different choices of $1/2\tau\Delta$. At the critical value of $1/2\tau\Delta=2\sqrt{3}/9$, the region of E over which the density of states is zero disappears.

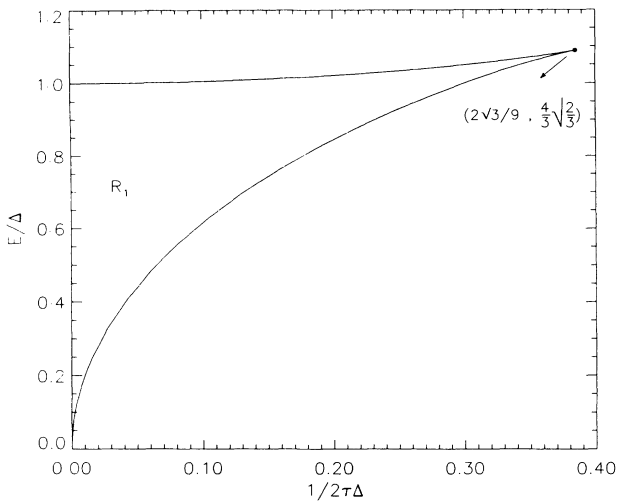

FIG. 3. Phase diagram in the E-τ plane, for the unitarity limit ($\sigma=1$). The region labeled R_1 has $N_s(E)=0$. The upper line shows Ω_2/Δ, while the lower line shows Ω_1/Δ.

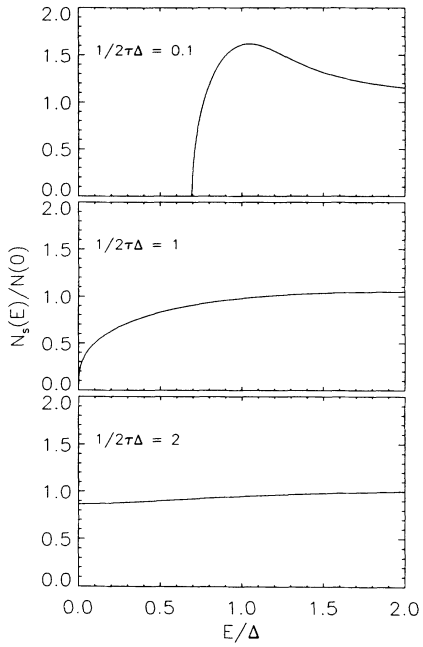

FIG. 4. Density of states in the Born limit ($\sigma=0$), for three different choices of $1/2\tau\Delta$. Note that $N_s(E=0)$ becomes nonzero when $1/2\tau\Delta>1$.

For small values of $1/2\tau\Delta$, we can derive the following approximation for $a_3(\varepsilon)$, which is valid at low energies:

$$a_3(E)=\frac{E}{2}-i\left[\frac{\Delta}{2\tau}-\frac{E^2}{4}\right]^{1/2} . \tag{21}$$

Using Eq. (12), we can then calculate the following approximate formula for the density of states at low energy:

$$\frac{N_s(E)}{N(0)}\approx\begin{cases}\frac{1}{2}\left[\frac{2}{\Delta\tau}-\frac{E^2}{\Delta^2}\right]^{1/2}, & E\le\sqrt{2\Delta/\tau}\\ 0, & E\ge\sqrt{2\Delta/\tau}\end{cases} . \tag{22}$$

This formula is an accurate description of the branch of $N_s(E)$ below Ω_1, for small values of $1/\tau\Delta$. Then, at small values of c, the low-energy piece of $N_s(E)$ has a height proportional to $\sqrt{c}$ and width proportional to $\sqrt{c}$. The total area is then proportional to c, as we expect; this point is discussed further in Sec. VII.

IV. BORN LIMIT

This limit corresponds to $v\to\infty$, $\sigma\to 0$, with τ held fixed. The equation for $a_3(\varepsilon)$ becomes

$$a_3=cN(0)v^2g_3=-\frac{i}{2\tau}\frac{(\varepsilon+ia_3)}{[(\varepsilon+ia_3)^2+\Delta^2]^{1/2}} . \tag{23}$$

The density of states is then given by

$$\frac{N_s(E)}{N(0)}=2\tau\,\mathrm{Im}[a_3(i\varepsilon\to E-i\eta)] . \tag{24}$$

Figure 4 illustrates the Born limit behavior of $N_s(E)$ as $1/2\tau$ is increased. The structure is quite different from the unitarity limit. As $1/2\tau$ is increased, the value of E/Δ at which the density of states is nonzero decreases; when $1/2\tau\Delta$ reaches the value of unity, $N_s(E=0)$ becomes nonzero. Thus, in the Born limit,[12,13] the superconductor is not gapless until $1/2\tau\Delta=1$. In addition, there are no energies corresponding to Ω_1 and Ω_2.

We can understand this result analytically in the following way. We can rewrite Eq. (23) for a_3 as follows, with $i\varepsilon\to E$:

$$a_3^4-2Ea_3^3+\left[E^2-\Delta^2+\left[\frac{1}{2\tau}\right]^2\right]a_3^2-\frac{2E}{(2\tau)^2}a_3+\frac{E^2}{(2\tau)^2}=0 . \tag{25}$$

When this equation has four real roots (at a particular value of E), then the density of states at that energy is zero. When two of the real roots become a complex conjugate pair, then the density of states is nonzero.

Let us call the lowest energy for which the density of states is nonzero Ω_0. Then it is straightforward to deduce that Ω_0 is given in terms of $1/2\tau$ by the following formula:[12,13]

$$\frac{\Omega_0}{\Delta}=\left[1-\left[\frac{1}{2\tau\Delta}\right]^{2/3}\right]^{3/2} . \tag{26}$$

As this equation indicates, in the Born limit, the superconductor becomes gapless when $1/2\tau\Delta>1$; the density of states at zero energy is then given by[12]

$$\frac{N_s(E=0)}{N(0)}=(1-4\tau^2\Delta^2)^{1/2} . \tag{27}$$

V. GENERAL CASE

For general values of c and v, or σ and τ, the equation for $a_3(E=i\varepsilon)$ becomes

$$a_3(E)=-\frac{1}{2\tau}\frac{[1/(1-\sigma)]\{(E-a_3)/[\Delta^2-(E-a_3)^2]^{1/2}\}}{1-[\sigma/(1-\sigma)]\{(E-a_3)^2/[\Delta^2-(E-a_3)^2]\}} \tag{28}$$

which can be turned into a sixth-order polynomial equation for a_3. The density of states in terms of a_3 is given by

$$N_s(E)=\frac{N(0)}{2\pi\sigma}\mathrm{Im}\left[\frac{1}{a_3}\left\{-\frac{1}{2\tau}+\left[\left[\frac{1}{2\tau}\right]^2+4a_3^2\sigma(1-\sigma)\right]^{1/2}\right\}\right]. \tag{29}$$

The above equation shows that, as in the Born and unitarity limits, if a_3 is real at a particular value of E, then $N_s(E)$ is zero.

It is straightforward to examine the special case $E=0$, and to see when the polynomial for a_3 has complex roots. We find that $N_s(E=0)$ is nonzero when the following condition is satisfied:[9]

$$N(0)\pi cv^2>\Delta \tag{30}$$

or, in terms of σ and τ,

$$\left[\frac{1}{2\tau}\right]\left[\frac{1}{1-\sigma}\right]>\Delta\ . \tag{31}$$

Equation (31) reproduces our previous results for the Born and unitarity limits: when $\sigma=1$ (unitarity) the superconductor is always gapless, and when $\sigma=0$ (Born) it is gapless when $1/2\tau>\Delta$.

We now consider general values of E. As σ and τ vary there is an evolving pattern such that certain regions of E have $N_s(E)$ vanishing, and other regions have it nonzero; see Fig. 5. To understand the trend, we present the plots of Figs. 6 and 7. These show, for particular values of σ, regions in the E-τ plane in which $N_s(E)$ vanishes, and regions in which it is nonzero. The boundary lines separating these areas can be written in parametric form as follows:

$$\frac{E}{\Delta}=\frac{y^3(1+\sigma-y^2)}{1-y^2-\sigma(1-2y^2)}\ ,\qquad \frac{1}{2\tau\Delta}=\frac{(1-\sigma-y^2)^2\sqrt{1-y^2}}{1-y^2-\sigma(1-2y^2)}\ , \tag{32}$$

where y satisfies

$$0\le y\le 1\ . \tag{33}$$

Note that a given value of $1/2\tau\Delta$ may be produced by several values of y, each value of y then having a separate value of E.

The general structure in the E-τ plane is as follows. For $0<\sigma<1$ (i.e., except for the Born and the unitarity limit) there are two separate regions in which $N_s(E)$ is zero; these are labeled R_1 and R_2. In the Born limit ($\sigma=0$), region R_1 vanishes, and in the unitarity limit ($\sigma=1$) R_2 vanishes.

Figure 5 shows a generic E-τ plane diagram, with various special points labeled. Analytic formulae can be derived, showing how these special points move about as σ and τ change.[9,10] Point A has coordinates

$$\left[\frac{1}{2\tau\Delta}\right]_A=0\ ,\qquad E_A=\Delta\sqrt{1-\sigma}\ . \tag{34}$$

The physical explanation for this value of E_A will be given in the final section. Point B has the following coordinates:

$$\left[\frac{1}{2\tau\Delta}\right]_B=1-\sigma\ ,\qquad E_B=0\ . \tag{35}$$

When $1/2\tau\Delta>(1/2\tau\Delta)_B$, we have $N_s(E=0)>0$. Point C is given by

$$\left[\frac{1}{2\tau\Delta}\right]_C=0\ ,\qquad E_C=\Delta\ . \tag{36}$$

Point D has a rather complicated analytic formula:

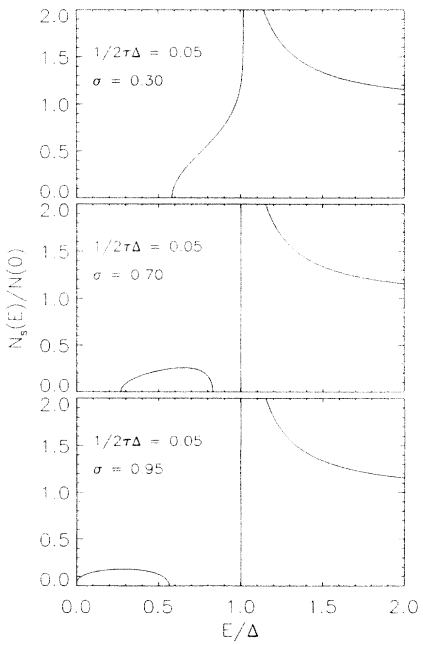

FIG. 5. The density of states as a function of E, for $1/2\tau\Delta=0.05$, and three different values of σ. Note that when $\sigma=0.3$, no energies correspond to Ω_1 and Ω_2 as defined in Fig. 1. When $\sigma=0.95$, no energy corresponds to Ω_0.

$$\left[\frac{1}{2\tau\Delta}\right]_D=\frac{\sigma\sqrt{\sigma(3-\sigma-\sqrt{12-12\sigma+\sigma^2})}(-6+7\sigma+\sqrt{12-12\sigma+\sigma^2})^2}{3^{3/2}(-1+2\sigma)^{5/2}(\sigma+\sqrt{12-12\sigma+\sigma^2})},$$
$$E_D=\frac{(-5\sigma+\sqrt{12-12\sigma+\sigma^2})(3-3\sigma-\sigma^2-\sigma\sqrt{12-12\sigma+\sigma^2})^{3/2}}{3^{3/2}(1-2\sigma)^{5/2}(\sigma+\sqrt{12-12\sigma+\sigma^2})}. \quad (37)$$

We note that point D moves to the right of point B when $\sigma\geq 0.776\,631\ldots$. When $1/2\tau\Delta>(1/2\tau\Delta)_D$ the stateless region between Ω_1 and Ω_2 disappears; $N_s(E)$ curves then have no Ω_1 and Ω_2 points.

VI. GAP EQUATION

In this section we briefly discuss the weak-coupling gap equation for our family of order parameters. For gaps satisfying conditions (i) and (ii) (given in Sec. I) we find that the equations simplify remarkably. We start with the basic self-consistent equation:

$$\Delta(\hat{\mathbf{k}})=T\sum_{\varepsilon}{}'\int d\hat{\mathbf{k}}'\frac{n(\hat{\mathbf{k}}')V(\hat{\mathbf{k}},\hat{\mathbf{k}}')\Delta(\hat{\mathbf{k}}')}{[(\varepsilon+ia_3)^2+\Delta^2]^{1/2}}. \quad (38)$$

The prime indicates that the sum on ε needs a cutoff.

Within weak-coupling theory, we take

$$V(\hat{\mathbf{k}},\hat{\mathbf{k}}')=\lambda\sum_j \mathcal{F}_j(\hat{\mathbf{k}})\mathcal{F}_j^*(\hat{\mathbf{k}}'). \quad (39)$$

Here, the $\{\mathcal{F}_j\}$ are the basis functions from a particular irreducible representation of the crystalline point group. We assume they are normalized, so that

$$\langle\mathcal{F}_j(\hat{\mathbf{k}})\mathcal{F}_k^*(\hat{\mathbf{k}})\rangle=\delta_{jk}. \quad (40)$$

Then the order parameter is a linear combination of these functions:

$$\Delta(\hat{\mathbf{k}})=\sum_j \alpha_j\mathcal{F}_j(\hat{\mathbf{k}}), \quad (41)$$

satisfying, according to our basic assumption, $\langle\Delta(\hat{\mathbf{k}})\rangle=0$ and $|\Delta(\hat{\mathbf{k}})|=\Delta$. The self-consistent equation then reduces to the following:

$$1=\lambda T\sum_{\varepsilon}\frac{1}{[(\varepsilon+ia_3)^2+\Delta^2]^{1/2}}. \quad (42)$$

Thus, Δ will be a universal function of σ, τ, and the temperature T:

$$\Delta=\Delta(\sigma,\tau,T). \quad (43)$$

Any details about the Fermi surface have dropped out.

VII. DISCUSSION

We have seen that our basic equations for a_3 and g_3, (5) and (8), lead to a very rich spectrum of behavior as σ and τ are varied. To understand these results a little better, we bring into the discussion one key fact. An isolated s-wave impurity, in a host superconductor whose order parameter satisfies (i) and (ii), has a bound state at the following energy:[7,9,10]

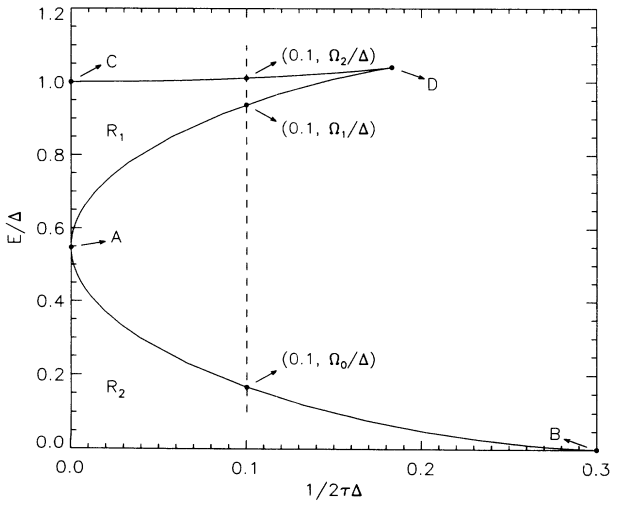

FIG. 6. Phase diagram in the E-τ plane for $\sigma=0.7$. In the regions labeled R_1 and R_2, $N_s(E)=0$. The vertical dashed line represents a particular choice of τ, namely, $1/2\tau\Delta=0.1$. The energies Ω_0, Ω_1, and Ω_2 correspond to those in Fig. 1. The special points A, B, C, and D are discussed in the text.

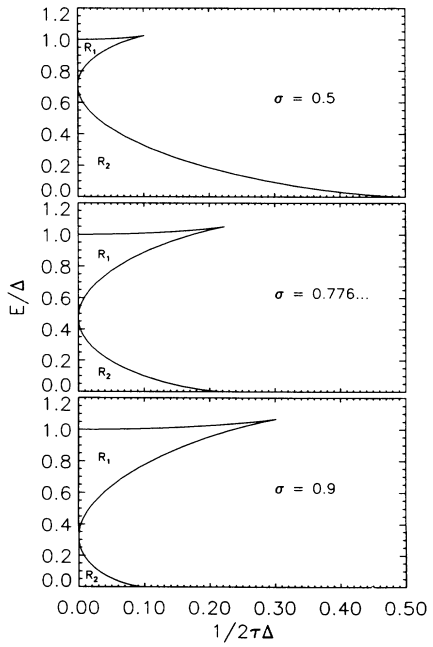

FIG. 7. Phase diagram in the E-τ plane, for three different values of σ. In the regions labeled R_1 and R_2, we have $N_s(E)=0$.

$$E_0 = \frac{\Delta}{\sqrt{1+N^2(0)\pi^2 v^2}} = \Delta\sqrt{1-\sigma} \ . \tag{44}$$

So, if σ is close to one (i.e., near the unitarity limit) this bound state is well below Δ. Thus, at small concentrations we have[9] an impurity band well separated from the continuum starting near $E=\Delta$. This band at low energy is not due to gap nodes, since our family of order parameters has no nodes.

Near the Born limit (σ close to zero) this bound state is close to Δ, and so an impurity band separated from the upper continuum is harder to achieve. Finally we note that the single impurity bound state at $E_0=\Delta\sqrt{1-\sigma}$ precisely accounts for the position of the point A in the E-τ plane diagram.

ACKNOWLEDGMENT

This work was supported by the DOE, Grant No. DE-FG02-90ER45427, through the Midwest Superconductivity Consortium.

[1]For a recent review, with many references, see P. Muzikar, D. Rainer, and J. A. Sauls, in *Vortices in Superfluids,* NATO Advanced Study Institute (Norwell, MA, in press).

[2]L. Gorkov, Sov. Sci. Rev. A **9**, 1 (1987).

[3]H. Kim, G. Preosti, and P. Muzikar, Phys. Rev. B **49**, 3544 (1994).

[4]P. J. Hirschfeld and N. Goldenfeld, Phys. Rev. B **48**, 4219 (1993).

[5]P. Arberg, M. Maasor, and J. P. Carbotte, Solid State Commun. **86**, 671 (1993).

[6]Recent references, which discuss the density of states in unconventional superconductors, include P. Hirschfeld, P. Wolfe, and D. Einzel, Phys. Rev. B **37**, 83 (1988); L. S. Borkowski and P. J. Hirschfeld, *ibid.* **49**, 15 404 (1994); G. Preosti, H. Kim, and P. Muzikar, Phys. Rev. B **50**, 1259 (1994); and R. Fehrenbacher and M. R. Norman (unpublished).

[7]L. J. Buchholtz and G. Zwicknagl, Phys. Rev. B **23**, 5788 (1981).

[8]P. J. Hirschfeld, P. Wolfe, J. A. Sauls, D. Einzel, and W. O. Putikka, Phys. Rev. B **40**, 6695 (1989).

[9]H. Shiba, Prog. Theor. Phys. **40**, 435 (1968).

[10]A. I. Rusinov, Sov. Phys. JETP **29**, 1101 (1969).

[11]A. N. Chaba, and A. D. Singh Nagi, Can. J. Phys. **50**, 1736 (1972).

[12]A. A. Abrikosov and L. P. Gor'kov, Sov. Phys. JETP **12**, 1243 (1961).

[13]S. Skalski, O. Betbeder-Matibet, and P. R. Weiss, Phys. Rev. **136**, A1500 (1964).

[14]We follow the formalism and notation of C. H. Choi and P. Muzikar, Phys. Rev. B **39**, 11 296 (1989).