PHYSICAL REVIEW B VOLUME 50, NUMBER 18 1 NOVEMBER 1994-II

Effect of competition between point and columnar disorder on the behavior of flux lines in (1 + 1) dimensions

T. P. Devereaux, R. T. Scalettar, and G. T. Zimanyi
Department of Physics, University of California, Davis, California 95616
(Received 8 June 1994)

We investigate the behavior of flux lines in the presence of both columnar and point disorder in (1+1) dimensions using renormalization-group (RG) and world-line quantum Monte Carlo (QMC) techniques. In particular, we calculate the transverse wandering correlation function for a single boson and recover known results for point and columnar disorder separately. We then examine the existence of a localization transition of a flux line in the simultaneous presence of both types of disorder. We also performed RG calculations for interacting flux lines. The RG calculations indicate that the vortex glass is unstable with respect to an arbitrary small amount of columnar disorder. Using QMC techniques we find that the Bose glass transition temperature is reduced by the point disorder, in agreement with recent RG calculations. Further we find that the region posited to be an Anderson glass is completely destroyed.

I. INTRODUCTION

There has been a great deal of interest lately concerning the creation of a deliberately disordered environment as a method to pin flux lines and create a true superconductor with a vanishing linear resistivity. Two types of disorder have been employed: point disorder, which is uncorrelated in space, and columnar disorder (produced, e.g., by heavy-ion irradiation) which is correlated in a preferred direction.[1] While much theoretical attention has been lavished on the behavior of flux lines in the presence of either point or columnar disorder and their respective pinned glassy phases, the interplay of the two types of disorder is still an open topic. Both types of disorder lead to pinning of flux lines at low temperatures, but the nature of the phases is quite different. In the vortex glass[2] the flux line is pinned in a tortuous meandering path by the point disorder. In the Bose glass,[3,4] the flux line remains relatively straight, locked to the columnar pins. However, the interface of the Vortex and Bose glass phases, where point and columnar disorder compete, remains beyond the reach of present analytical approaches.[5] The only guidance comes from a recent study, where it has been argued that a single flux line will always be localized for arbitrary weak columnar disorder, irrespective of the strength of the point disorder.[6] Interactions between lines could then further limit wandering and enhance the localizing tendencies.

In this paper we analyze the localization of single and interacting flux lines in the presence of both point and columnar disorder. In particular, we employ a mapping of the flux line problem in $d+1$ dimensions onto the problem of disordered quantum bosons in d dimensions. We measure the transverse flux line wandering of the single flux line and discuss the possible existence of a critical ratio of the columnar to point disorder strength to localize the boson. Turning then to the interacting boson problem, we study the phase boundary of the Bose glass phase and its stability to point disorder. We combine these simulations with a renormalization-group (RG) treatment of a disorder potential, which interpolates between point and columnar disordered potentials, and we find evidence for trajectories scaling away from the vortex glass phase. We also find evidence that a localized phase at weak boson repulsion, the "Anderson-glass" region, is destroyed by the dephasing effect of the point disorder.

II. FORMALISM

A. Mapping

Our Monte Carlo analysis is based on the mapping of flux lines in $d+1$ dimensions onto the quantum problem of interacting bosons in d dimensions. This mapping can be understood qualitatively as arising from the similarity between the flux lines aligned parallel to the applied field and the boson world lines, which appear in a path integral representation. More specifically, for flux lines confined in a plane, the classical de Gennes – Matricon free energy in the absence of disorder is[7]

$$F_{\rm class} = \int_0^L dz \left\{ \sum_{j=1}^{N} \frac{1}{2}\tilde{\epsilon}_1 [dr_j(z)/dz]^2 + \frac{1}{2}\sum_{i\neq j} V[\| r_i(z) - r_j(z) \|] \right\}. \quad (1)$$

Here the displacement field $r_i(z)$ describes the position of flux line i in a direction perpendicular to that of the applied field ($\hat{z}$). The interaction potential V between the flux lines is often simplified to a δ function that prevents fluxes from touching. Then one can define a discrete field that takes on only integer values

$$A(x,z) = \sum_n \Theta(x - r_n(z)), \quad (2)$$

where Θ is the Heaviside step function, and rewrite the classical free energy as

$$F_{\rm class} = \int dx \int_0^L dz \left\{ \frac{1}{2}\tilde{\epsilon}_1 (dA/dz)^2 + \frac{V}{2}(dA/dx)^2 \right\}. \tag{3}$$

The partition function in the canonical ensemble is then

$$Z_{\rm class} = \int D[A] e^{-F_{\rm class}/T}. \tag{4}$$

Consider now the quantum-mechanical free energy for bosons in d spatial dimensions. In the continuum limit

$$F_{\rm quantum} = \int_0^{\hbar\beta} d\tau \int dx \left\{ -\frac{t}{2}(\Psi^\dagger d^2\Psi/dx^2 + {\rm c.c.}) + \frac{U}{2} \mid \Psi^\dagger \Psi \mid^2 \right\} \tag{5}$$

plus terms that can be absorbed into a chemical potential. Ψ is the second quantized field representing the bosons. Following Haldane,[8] we can write the Bose density operator

$$\hat{N}(x) = \Psi^\dagger(x)\Psi(x) = [\partial_x \hat{B}(x)] \sum_m e^{-im\hat{B}(x)}, \tag{6}$$

where $\partial_x \hat{B}(x) = N_0 + \hat{\Pi}(x)$ is chosen to ensure the discrete nature of the Bose field. Here N_0 is the average boson density and $\hat{\Pi}$ is a small fluctuation. Linearizing in $\hat{\Pi}$ we obtain

$$F_{\rm quantum} = \int_0^{\hbar\beta} d\tau \int dx \left\{ \frac{\hbar^2}{t}(d\hat{B}/d\tau)^2 + \frac{U}{2}(d\hat{B}/dx)^2 \right\}, \tag{7}$$

where we discard nonlinear terms representing commensuration effects. Thus we find that the partition function

$$Z_{\rm quantum} = \int D[Q] e^{-F_{\rm quantum}/\hbar}. \tag{8}$$

Comparison of the two partition functions leads to the analogy

Classical fluxes	Quantum bosons ($\hbar$)
T	$\hbar$
L	$\hbar/T$
$\tilde{\epsilon}_1/2$	$\hbar^2/t$
V	U

or, if one wants to set $\hbar = 1$, then the mapping is

Classical fluxes	Quantum bosons ($\hbar = 1$)
L	$1/T$
$\tilde{\epsilon}_1/2T$	$1/t$
V/T	U

and all classical energy scales get divided by T. The same holds true when one includes the chemical and disorder potentials.

B. Simulation algorithm

In this paper we consider a discrete lattice version of the interacting boson problem, the Bose-Hubbard Hamiltonian[4]

$$H = -t\sum_i (a_{i+1}^\dagger a_i + a_i^\dagger a_{i+1}) + V\sum_i n_i^2 + \sum_i n_i \epsilon_c(i). \tag{9}$$

As we shall see, ϵ_c represents columnar disorder. The inclusion of point disorder will be discussed below.

The partition function Z can be written by discretizing the imaginary time $\beta = L\Delta\tau$, separating the exponentials of the kinetic and potential energies, and inserting compete sets of occupation number states:[9]

$$\begin{aligned} Z &= {\rm Tr}\, e^{-\beta H} = {\rm Tr}\,[e^{-\Delta\tau K} e^{-\Delta\tau P}]^L \\ &= \sum_{n(i,\tau)} \langle n(i,1)|e^{-\Delta\tau K}e^{-\Delta\tau P}|n(i,2)\rangle \\ &\qquad \times \langle n(i,2)|e^{-\Delta\tau K}e^{-\Delta\tau P}|n(i,3)\rangle \\ &\qquad \times \cdots \langle n(i,L)|e^{-\Delta\tau K}e^{-\Delta\tau P}|n(i,1)\rangle. \end{aligned} \tag{10}$$

The sum is over a classical occupation number field whose first index runs over the spatial sites in the lattice and whose second index runs from 1 to L and labels the "imaginary time slice," that is, the position of the complete set in the expression for Z. Since H conserves particle number, and K moves bosons only locally, the allowed configurations of $n(i,\tau)$ form continuous world-line trajectories, which are the analogs of the flux lines. The sum over all configurations necessary to evaluate Z is performed stochastically.

Since P is diagonal in an occupation number representation,

$$\begin{aligned} Z = \sum_{n(i,\tau)} & e^{-S_b} \langle n(i,1)|e^{-\Delta\tau K}|n(i,2)\rangle \\ & \times \langle n(i,2)|e^{-\Delta\tau K}|n(i,3)\rangle \\ & \times \cdots \langle n(i,L)|e^{-\Delta\tau K}|n(i,1)\rangle, \end{aligned} \tag{11}$$

$$S_b = V\Delta\tau \sum_{i,\tau} n(i,\tau)^2 + \Delta\tau \sum_{i,\tau} \epsilon_c(i) n(i,\tau).$$

The matrix elements of the kinetic energy operator K can be evaluated using the checkerboard decomposition.[10] We are now in a position to see that "point disorder" can naturally be incorporated as an additional imaginary time-dependent random term in S_b, which takes the form

$$S_b^p = \Delta\tau \sum_{i,\tau} \epsilon_p(i,\tau) n(i,\tau). \tag{12}$$

All the standard methods of world line simulations can now be employed. In addition to Monte Carlo moves, which locally distort the world lines, we also include moves that shift the entire world-line position spatially. This significantly helps equilibration in the presence of columnar disorder. The point and columnar disorder are bounded and uniformly distributed, i.e., $-c, -p \leq \epsilon_{c,p} \leq c, p$, where c, p stands for columnar and point disorder strengths, respectively.

III. SINGLE FLUX LINE

In this section we consider a single boson in both a random point and columnar disorder potential and investigate the localization of the boson as a function of the strength of the respective disorders, c/p. In particular, we focus on the transverse wandering correlation function,

$$R(\tau) = \langle [x(\tau) - x(0)]^2 \rangle, \tag{13}$$

which measures the transverse extension of the boson world line from its initial position as it traverses the imaginary time axis. It is well known that in the absence of disorder the boson dynamics are diffusive, with the diffusion constant governed by t, or equivalently, the flux line tension[11] such that for large τ,

$$R(\tau) = t\ \tau, \quad \text{no disorder.} \tag{14}$$

Point disorder has been shown to promote flux line wandering due to the presence of favorable site energies which encourage excursions, and consequently,

$$R(\tau) \sim \tau^{2\zeta}, \quad \text{point disorder,} \tag{15}$$

with the wandering exponent $\zeta = 2/3$.[12] Lastly, in the case of columnar disorder simple quantum mechanics leads to the localization of the flux line for arbitrary pinning and thus the transverse extension of the flux line is bounded[3,4]

$$R(\tau) \sim l_c, \quad \text{columnar disorder.} \tag{16}$$

Since our boson must satisfy periodic boundary conditions, we study the wandering of the world line at imaginary time $\tau = \beta/2$ after positioning the boson at its lowest-energy site at time slice $\tau = 0$. Our runs use on average 100 realizations of the disorder. We first show our results, which recover the above limiting behaviors in Figs. 1 and 2. For the case of no disorder (Fig. 1) we recover both the exponent 1 and the value of the diffusion constant t for the case of 32 site chain, in agreement with Ref. 11. The finite size of the chain limits the correlation function for large values of hopping t and for large imaginary time separation β. For the case of point disorder (again for a 32-site chain, hopping t set equal to 1), we find that the wandering exponent ζ increases from 1/2 to 2/3 for arbitrary point disorder strength, which is shown in Fig. 2 for $p = 10$ and 25.

Turning on the point disorder p with a fixed value of columnar pinning strength c, we see in Fig. 3 that R lifts up at higher β reflecting the nature of the point disorder, which encourages flux line wandering. Our behavior is consistent with that of Kardar[13] who via another method found numerical evidence that for $p/c < 1.2$, the flux line is localized. However, it has been shown that a simple extrapolation of the localization length to infinity is inadequate and the presence of rare regions due to the point disorder leads to slow glassy dynamics, which keeps the localization length large but finite.[6] In Fig. 4 we show the value of the "localization length," i.e., the saturation of the wandering correlation function R, for runs at different values of p/c, which indeed seems to indicate a crossover to delocalized behavior at a value consistent with that of Kardar. However, our simulations are limited to β values, which cannot provide a closer analysis near $(p/c)_{\text{critical}}$ to reveal whether the localization length slowly turns over and remains finite at higher values of p/c, as argued in Ref. 6.

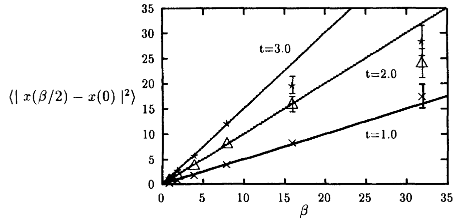

FIG. 1. The transverse wandering correlation function for different positions in imaginary time β along the boson world line for the case of no disorder, Eq. (14). A turnover from linear diffusive behavior occurs when the flux line begins to see the finite lattice size.

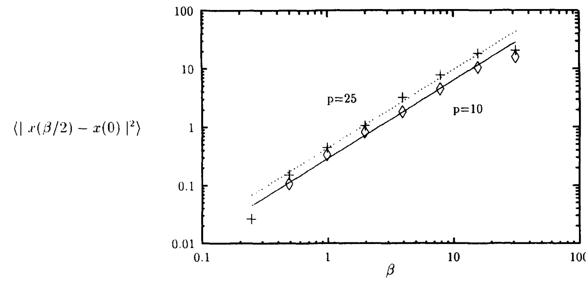

FIG. 2. The transverse wandering correlation function for different positions in imaginary time β along the boson world line for the case of two point disorder strengths. The slope of the line is 1.33 ± 0.03.

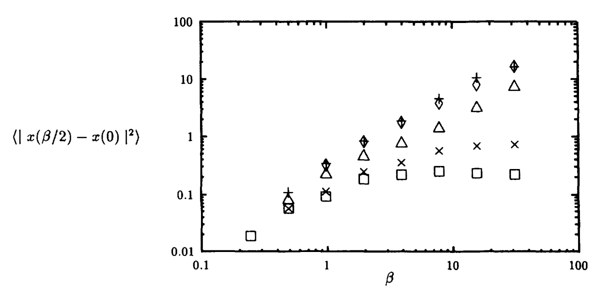

FIG. 3. Transverse wandering correlation function at imaginary time slice $\beta/2$ for $p = 10$, $c = 0$ (crosses); $p = c = 0$ (diamonds); $p = 10$, $c = 5$ (triangles); $p = c = 10$ ($\times$'s); $p = 0$, $c = 10$ (squares) for a 32-site chain.

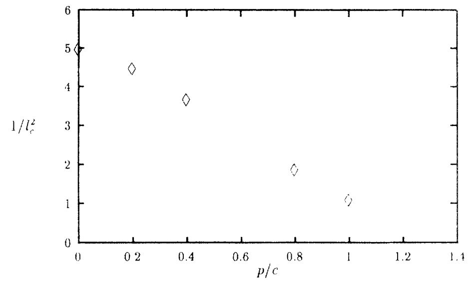

FIG. 4. Square of the inverse localization length as a function of p/c for a 32-site chain.

IV. INTERACTING FLUX LINES

We have seen in the preceding section how the interplay between point and columnar disorder affects the dynamics of flux lines in the absence of interactions. We now turn on the flux line repulsion V to study the various pinned phases of interacting flux lines. The advantage of using a boson world-line approach is that we can take the on-site interactions into account using the same numerical techniques as before. However, since the boson world lines are indistinguishable we found it more useful to measure the superfluid density ρ_s rather than the transverse correlation function as an indicator of the Bose glass ($\rho_s = 0$) and either the vortex glass or line liquid ($\rho_s \neq 0$) phases. In particular, we extend the results of our previous study of spatially disordered bosons by adding point disorder.[14] It was shown in Ref. 14 that there were two localized regions separated by a superfluid phase as a function of boson repulsion. It was posited that the physics of the region at small repulsion (Anderson glass) was qualitatively different than the phase at larger values of the repulsion (Bose glass). Therefore, the two phases may behave differently as point disorder is introduced. Localization of noninteracting bosons in one dimension is a consequence of the phase coherent (back)scattering of bosons and therefore the presence of a time-dependent potential will dephase the boson wave functions and thus allow the boson to propagate.[15] However, in the case of strongly interacting bosons, the boson repulsion governs localization, and it could be that the effects of point disorder are suppressed.

Recently, a renormalization-group (RG) approach has been applied[5] to the problem of flux lines with point and columnar disorder. This technique is able to capture the Bose and vortex glass phases as well as the line liquid phase which exists at high temperatures. It was shown that the location of the Bose glass transition is shifted towards higher repulsions (i.e., lower temperatures) by the point disorder, while the vortex glass phase is unaffected by the columnar disorder. Also, a phase diagram has been constructed based on RG calculations, suggesting a vortex glass–to–Bose glass transition at a critical value of c/p. However, since the RG treatment was confined to small disorder, this transition could not be accessed in Ref. 5.

We consider two approaches towards understanding the nature of the Bose and vortex glasses in the presence of both point and columnar disorder. First we will reconsider the RG approach from Ref. 5, where we introduce a disorder potential which interpolates between columnar and point disorder potentials. This introduces another small parameter which allows us to construct the scaling trajectories. Similar ideas were used by Weinrib and Halperin on a related problem.[16] Then we perform quantum Monte Carlo simulations to test the results of the RG analysis.

A. Renormalization-group approach

We reconsider the coarse-grained free energy in a continuum description,[5]

$$\mathcal{F} = \int dxdz \Bigg(\frac{U}{2}(\partial_x u)^2 + \frac{\tilde{\epsilon}_1}{2}(\partial_z u)^2 - V_0(x,z)\partial_x u - 2V_0(x,z)\rho\cos\{2\pi[x\rho + u(x,z)]\} \Bigg), \quad (17)$$

where u is the flux displacementlike field, ρ is the average line density, U is the on-site flux repulsion, and $\tilde{\epsilon}_1$ is the line tension. The disorder potential V_0 we take to represent both point and columnar potentials by writing

$$\langle V_0(x,z)V_0(x',z')\rangle \sim \Delta\delta(x-x')h(\mid z-z'\mid), \quad (18)$$

where we take

$$h(z-z') = \frac{z_0^a}{z_0^a + \mid z-z'\mid^a},$$

where z_0 is a short-distance cutoff. The Fourier transform of $h(z)$ is given by

$$h(k_z) \sim c_0 + c_1 k_z^{-(1-a)} \quad (19)$$

for small k_z, and $c_{0,1}$ are positive constants. We see that if $a > 1$ the potential is governed by short-range correlated disorder and thus describes point disorder, while for $a < 1$ the potential is governed by long-range correlations. Thus disorder with long-range correlations, which decay at larger distances faster than $1/z$ leads to the same critical behavior as that due to short-range correlated disorder.[16]

Replicating the above free energy and performing the disorder average leads to

$$\mathcal{F}_n/T = \int dxdz \sum_{\alpha,\beta,j} \Bigg(\frac{1}{2}K_j\ \partial_j u_\alpha \cdot \partial_j u_\beta - \int dz' h(\mid z-z'\mid)(\Delta_x \partial_x u_\alpha(x,z)\partial_x u_\beta(x,z+z') + g\cos\{2\pi[u_\alpha(x,z) - u_\beta(x,z+z')]\}) \Bigg), \quad (20)$$

where $\alpha, \beta \in \{1, \ldots, n\}$ are replica indices, and $j = \{x, z\}$. The bulk and tilt moduli are given by $K_x = U/T$ and $K_z = \tilde{\epsilon}_1/T$, respectively, and the bare coupling constant of the nonlinear cosine interactions is $g = \rho^2\Delta/T^2$. Lastly, $\Delta_x = \Delta/T^2$.

We have performed a renormalization-group analysis of Eq. (20) using $\delta = 1 - a$ and g as small parameters. We developed a technique which is an extension of the sine-Gordon scaling theory as discussed, e.g., by Kogut.[17] We performed our calculations up to next-to-leading order in the couplings. This involved considering terms with eight displacement fields. High scaling dimensions eliminate many of the newly generated terms. As a novelty, in this order the renormalization of the disorder correlation exponent a became necessary. Upon rescaling by a factor e^l, tedious but straightforward calculations lead to the recursion relations as

$$\frac{dK_x}{dl} = 0, \qquad \frac{dK_z}{dl} = C_0 g_1, \qquad \frac{dg_0}{dl} = g_0(2 - K^{-1}),$$
$$\frac{dg_1}{dl} = g_1\{3 - a - K^{-1}[1 + \Delta_x c_0/2K_x]\} - C_1 g_1^2, \quad \frac{da}{dl} = -C_2 g_1^2 \delta, \tag{21}$$

with $g_{0,1} = c_{0,1} g$, $C_{0,1,2}$ are positive constants, and $K^{-1} = 2\pi/\sqrt{K_x K_z} \sim T$. The recursion relations recover previous results on the Bose glass and vortex glass transitions, namely, for $a = 0$ we have the Bose glass transition temperature $K_{\mathrm{BG}}^{-1} = 3$ without point disorder.[18] This critical value is decreased by both point disorder as found in Ref. 5, and by the exponent a. The vortex glass transition $K_{\mathrm{VG}}^{-1} = 2$ is unmodified by columnar disorder or the exponent of the decay of the disorder correlations. Let us denote the dimensionless measure of the forward scattering induced by the point disorder by $D = \Delta_x c_0/2K_x$. It was argued in Ref. 5 that when the forward scattering is strong enough, $D > 1$, then the point disorder, measured by g_0, scales faster to strong coupling than the columnar disorder (g_1), indicating a transition to a vortex glass phase. However, two remarks are in order. First, strictly speaking when deriving the scaling equations the coefficients of *both* g_0 and g_1 (or equivalently the scaling dimensions) should be small. Second, in typical real systems and numerical simulations the bare values of the forward and backward scattering are expected to be comparable. While there is no restriction on D in the theory, g_0 has to be small, restricting the applicability of the results for $D \ll 1$ for typical systems or at least those which are not in too small magnetic fields, i.e., ρ on the order ~ 1. Therefore conclusions about a transition around $D \sim 1$ have to be viewed with some caution.

Both of these problems are remedied by the present method. Frist we see that for $a \approx 1$ the point at which the Bose glass becomes unstable occurs for small disorder, $D = (1 - a)/2 \ll 1$. Second, in this region both scaling dimensions are indeed small, justifying a proper expansion. This formulation thus constitutes a technically sound description of the transition to the vortex glass phase.

However, let us observe that the disorder exponent a scales in second order: for $a < 1$ ($\delta > 0$), a scales to zero, while for $a > 1$ ($\delta < 0$), a scales to large values. Thus the system, which started out on the vortex glass side of the transition $1 > D > (1 - a)/2 \ll 1$, will scale *into* the Bose glass phase for $(1 - a)/2$.

This flow into the Bose glass phase for *all* $a < 1$ indicates an instability of the vortex glass phase towards the addition of arbitrarily weak columnar pinning. While the "weakness" is characterized by the critical exponent a being arbitrary close to the point-disorder limit 1, we expect that this result also translates to a critical value of $c/p = 0$. This result is also plausible as the same instability has been argued for single flux lines,[6] and adding interactions will further localize the vortices. However, the Bose glass phase itself is unstable to point disorder, although admittedly only on astronomically long length scales.[5] The most likely picture emerging from the RG is therefore that the trajectories flow towards a strong-coupling fixed point(s), which is characterized by simultaneous finite values of c and p. Because of the presence of columnar disorder, this feature would most likely translate to a finite localization length.

B. Numerical approach

To test this scenario we have employed our Monte Carlo algorithm to investigate the phase boundary between the insulating and superconducting phases of the boson Hamiltonian, corresponding to the Bose (large V) or Anderson (small V) glass and either the line liquid or the vortex glass phase, respectively. No attempt was made to distinguish between the vortex glass and the line liquid phases, since it has been difficult to find the vortex glass phase from static properties.[19] Rather, we instead measure the superfluid density ρ_s as a function of both boson repulsion $V/t = 2(\pi K)^2$ and c/p to map out the Bose or Anderson glass phase boundary.

Results of our runs are shown in Fig. 5 for the case of a 64-site chain with 40 bosons at a temperature $\beta = 16$. Values of ρ_s are obtained by averaging over several realizations of the columnar disorder (fixed at $c = 2$) and point disorder, although each run showed only slight vari-

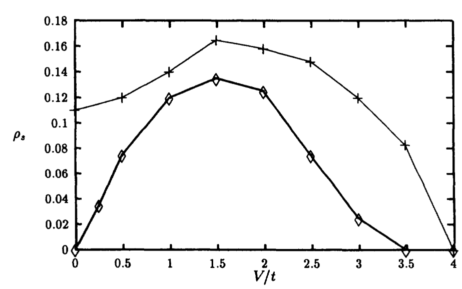

FIG. 5. Superfluid density ρ_s for various repulsion strengths V/t for the case $c = 2$, $p = 0$ (bottom line), and $c = p = 2$ (top line).

ations in the superfluid density and appears to be self averaging. Typical runs were over 50 000 warmup and 100 000 measurement sweeps. The bottom curve in Fig. 5 represents new measurements at lower temperatures of our previous results for ρ_s.[14] The transition from the superfluid phase to the localized one at large repulsions occurs at a slightly greater value than the Giamarchi-Shultz fixed point but is reduced compared to the runs that were performed at higher temperatures.[14] This may indicate that the localization length is nearing the chain size at this value of disorder. Further the localized phase at small repulsion (Anderson glass) becomes more pronounced at these lower temperatures.

Turning on the point disorder shows that the superfluid phase becomes enhanced, which is shown in the upper curve in Fig. 5 for $c/p = 1$. The Bose glass transition now occurs at a larger boson repulsion, which is in agreement with the RG results from Eq. (21) and Ref. 5. Further, the Anderson glass is even more adversely affected than the Bose glass phase as there is no evidence of a localized phase at small boson repulsion. It may be possible that the localization length is beyond the length of the chain at this temperature although runs performed at larger $\beta = 32$ do not support this. This would indicate that the Anderson glass is destroyed due to the dephasing effect of the presence of random site energies changing in time, and is in agreement with previous results concerning Mooij anomalies.[15] In addition, we have performed runs at $c/p = 0.1$ and have found no evidence for a localized phase for any value of boson repulsion.

V. DISCUSSION AND CONCLUSIONS

In this paper we have presented the analysis of independent and interacting flux lines in the presence of competing point and columnar disorder. Our quantum Monte Carlo simulations have verified that, for point disorder only, the exponent ζ characterizing line wandering changes from 1/2 to 2/3, in agreement with theoretical predictions. In the presence of columnar disorder a single flux line becomes localized. The localization length grows with the introduction of point disorder. It was argued in Ref. 6 that the localization length remains finite but very large for large values of c/p. Our results are not inconsistent with this picture. We found clear evidence for a finite localization length for $c/p < 0.8$, but for larger values of c/p the localization length crosses over to very large values, which are inaccesible with our method.

For interacting flux lines we studied the same issues with the help of renormalization-group and Monte Carlo techniques. We developed a variable exponent description of the disorder, which allows for a technically sound scaling treatment of the problem. The scaling trajectories indicate the instability of the vortex glass phase for arbitrarily small values of columnar disorder towards a Bose glass, which is presumably characterized by the localization of the vortices. Furthermore as the single flux line appears to be localized for arbitrarily small columnar disorder as well, it is possible that adding interactions between the lines could further enhance localization. Our Monte Carlo technique found a Bose glass phase at medium values of the columnar disorder. That this localized phase does not extend to very small values of c might be due to system size restrictions and the importance of rare regions. The localization length could thus be large but finite. It was also shown that the Bose glass phase is unstable towards the addition of point disorder, but only on astronomically long length scales. Thus, presumably the ultimate asymptotic behavior is governed by a fixed point with *both* c and p finite. As $c > 0$, the main physical characteristic of this phase is expected to be the localization of the vortex lines.

We continue by noting that previously it has been implicitly assumed that the relevant quantity, which characterizes the disordered environment of flux lines is the ratio of the columnar–to–point disorder strengths, c/p. However, the recursion relations of Eq. (21) seem to prevent a reduction in the scaling to a single dimensionless coupling due to the presence of the second-term in the recursion relation for g_1, which predicts the reduction of the Bose glass transition temperature. Further, since the RG of Eq. (21) is limited to small values of the coupling constants, the second-order terms could also lead to the conclusion that both c and p (g_1 and g_0) are relevant parameters, leading to a three-dimensional phase diagram in K^{-1}, g_0 and g_1. This can be tested numerically and is a subject for future consideration. Lastly, further numerical simulations are necessary to resolve the nature of the strong-coupling fixed point(s).

In conclusion, we have studied the behavior of free and interacting fluxlines in the simultaneous presence of point and columnar disorder. Renormalization-group analysis indicates in both cases that the addition of even weak columnar defects localizes the flux lines. However, the physics seems to be described by a strong-coupling fixed point, the nature of which has to be clarified by numerical methods. To this end, quantum Monte Carlo simulations were also performed, which show such a localization behavior for medium and strong columnar disorder only. The possible reason for that may be that the localization length exceeds the system size in the case of weak columnar pinning. It was also demonstrated that the transition temperature to the Bose glass is reduced by the presence of point disorder. Further work is necessary (for instance from dynamical quantities) to distinguish between the vortex and line liquid phases.

ACKNOWLEDGMENTS

We would like to thank T. Hwa for useful and enlightening discussions. This work was supported by N.S.F. Grant No. 92-06023.

[1]R. H. Koch, V. Foglietti, W. J. Gallaher, G. Koren, A. Gupta, and M. P. A. Fisher, Phys. Rev. Lett. **63**, 511 (1989); P. L. Gammel, L. F. Schneemeyer, and D. J. Bishop, *ibid.* **66**, 953 (1991); L. Civale, A. D. Marwick, T. K. Worthington, M. A. Kirk, J. R. Thompson, L. Krusin-Elbaum, Y. Sum, J. R. Clem, and F. Holtzberg, *ibid.* **67**, 648 (1991).

[2]M. P. A. Fisher, Phys. Rev. Lett. **62**, 1415 (1989); D. S. Fisher, M. P. A. Fisher, and D. Huse, Phys. Rev. B **43**, 130 (1991).

[3]D. R. Nelson and V. M. Vinokur, Phys. Rev. Lett. **68**, 2398 (1992); Phys. Rev. B **48**, 13 060 (1993); I. F. Lyuksyutov, Europhys. Lett. **20**, 273 (1992).

[4]M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, Phys. Rev. B **40**, 546 (1989).

[5]T. Hwa, D. R. Nelson, and V. M. Vinokur, Phys. Rev. B **48**, 1167 (1993).

[6]L.-H. Tang and I. F. Lyuksyutov, Phys. Rev. Lett. **71**, 2745 (1993); L. Balents and M. Kardar, Phys. Rev. B **49**, 13 030 (1994); T. Hwa and T. Natterman (unpublished).

[7]P. G. de Gennes and J. Matricon, Rev. Mod. Phys. **36**, 45 (1964).

[8]F. D. M. Haldane, Phys. Rev. Lett. **47**, 1840 (1981); M. P. A. Fisher and D. H. Lee, Phys. Rev. B **39**, 2756 (1989).

[9]J.E. Hirsch, R.L. Sugar, D.J. Scalapino, and R. Blankenbecler, Phys. Rev. B **26**, 5033 (1982).

[10]M. Barma and B.S. Shastry, Phys. Rev. B **18**, 3351 (1978).

[11]D. R. Nelson and H. S. Seung, Phys. Rev. B **39**, 9153 (1989).

[12]D. A. Huse and C. L. Henley, Phys. Rev. Lett. **54**, 2708 (1985); M. Kardar, *ibid.* **55**, 2923 (1985); D. A. Huse, C. L. Henley, and D. S. Fisher, *ibid.* **55**, 2924 (1985).

[13]M. Kardar, Phys. Rev. Lett. **55**, 2235 (1985); M. Zapotocky and T. Halpin-Healy, *ibid.* **67**, 3463 (1991).

[14]R. T. Scalettar, G. G. Batrouni, and G. T. Zimanyi, Phys. Rev. Lett. **66**, 3144 (1991).

[15]D. A. Evensky, R. T. Scalettar, and P. G. Wolynes, J. Phys. Chem. **94**, 1149 (1990).

[16]A. Weinrib and B. I. Halperin, Phys. Rev. B **27**, 413 (1983).

[17]J. Kogut, Rev. Mod. Phys. **51**, 659 (1979).

[18]T. Giamarchi and H. J. Shulz, Phys. Rev. B **37**, 325 (1988).

[19]G. G. Batrouni and T. Hwa, Phys. Rev. Lett. **72**, 4133 (1994); T. Hwa and D. S. Fisher, *ibid.* **72**, 2466 (1994).