Cancellation of quasiparticle mass enhancement in the conductance of point contacts

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The high fraction of Andreev reflections that has been observed in noble-metal point contacts with the high-T_c oxides $YBa_2Cu_3O_7$ and $La_{2-x}Sr_xCuO_4$ as well as with the heavy-fermion superconductors UPt_3 and URu_2Si_2 suggests that the boundary condition at the interface involves Fermi velocities without mass-enhancement factors. We show theoretically that this is indeed the case for a self-energy $\Sigma(\omega,k)$ only weakly dependent on k. A comparison between the velocities derived from point-contact experiments with known values of the fully dressed quasiparticle velocities allows a determination of the mass-enhancement factor.

INTRODUCTION

The theory of point contacts where one electrode is a superconductor S and the other one a normal metal N has been developed by Blonder, Tinkham, and Klapwijk (BTK).[1] They have calculated the effect of different interface conditions on the shape of the $V(I)$ characteristics, going from the pure Andreev case of a perfect metallic contact to the Giaever tunneling case for which the conductance is limited by a dielectric barrier. The BTK theory successfully explains the variety of characteristics that can be obtained in a superconducting-normal metal point contact in terms of an effective barrier parameter Z that goes from zero in the pure Andreev case to infinity in the Giaever case:

$$Z^2=[Z_b^2+(1-r)^2/4r]\ , \tag{1}$$

where the first term in the right-hand side represents the contribution of a dielectric barrier and the second term expresses the effect of the mismatch of the Fermi velocities between the S and the N sides in terms of their ratio r:

$$r=(v_{FS}/v_{FN})\ . \tag{2}$$

This second term has its origin in a boundary condition for the electronic wave functions at the S/N interface, involving the dispersion relations on both sides. A detailed discussion of this boundary condition constitutes the main part of this paper, as will be seen later. The normal-state resistance of the contact is given by

$$R_n=R_0(1+Z^2)\ , \tag{3}$$

where R_0 is the Sharvin resistance of the point contact $(4/3\pi)(\rho l/a^2)$, ρ and l being, respectively, the normal-state resistivity and the mean free path, and a the radius of the contact. The Sharvin expression for R_0 is valid when $l \gg a$. R_0 is within numerical factors the resistance of $(k_F a)^2$ conducting channels in parallel each of them having a resistance $(\hbar/e^2)$.

We show in this paper that the Fermi velocities entering in the ratio r are not the quasiparticle velocities but rather the velocities without the mass enhancement factor. This explains the high fraction of Andreev reflection observed in contacts where in one of the electrodes the mass-enhancement factor is very large (a heavy fermion). This remark offers a method to determine the mass-enhancement factor by comparing velocities obtained from point-contact characteristics with quasiparticle velocities. We find for instance that it is at least equal to 5 in $La_{2-x}Sr_xCuO_4$ (LSCO) and $YBa_2Cu_3O_7$ (YBCO), and briefly comment on the significance of this result.

EXPERIMENTAL BACKGROUND

When one of the electrodes of a pure Sharvin contact ($Z=0$) is superconducting with a pair potential Δ, its conductance is enhanced at voltages $|eV|<\Delta$ by a factor of 2 compared to its high voltage value ($|eV|\gg\Delta$). This is due to the well-known Andreev reflections: at $|eV|<\Delta$ incoming electrons from the N side are reflected as holes, while the current in the S side is carried by Cooper pairs. At $|eV|>\Delta$ the normal single-particle current is progressively restored. For a superconducting state having the s symmetry and at $T \ll T_c$, the dynamical conductance remains constant up to $|eV|=\Delta$ and then drops sharply, returning to the normal-state conductance over an energy scale Δ.

BTK have shown that if $0<Z\leq 0.5$, the zero-bias conductance is reduced but remains larger than the normal-state conductance. Simultaneously, the conductance starts to develop peaks at $|eV|=\Delta$. For $Z\geq 1$, the characteristics are Giaever-like but with a finite zero-bias conductance.

For non-s symmetries and a small Z, Hasslebach, Kirtley, and Lejay have shown[2,3] that the conductance of the point contact has a broad maximum of width $\simeq 2\Delta$ centered at zero bias, instead of the sharply defined plateau characteristic of the s symmetry. There are no peaks at $|eV|=\Delta$. This result reflects the existence of nodes in

$\Delta(k)$. It is obtained from a summation of the conductance over all channels involving with equal weight all k vectors on the Fermi surface, in other words assuming that Z has no angular dependence. This is a reasonable approximation in the absence of a significant tunneling barrier ($Z_b \ll 1$).

Recently reported point-contact measurements on YBCO (Ref. 4) and LSCO (Ref. 5) using a gold tip oriented along the CuO_2 planes display an enhanced conductance at zero bias and peaks at $|eV|=\Delta$: they are characteristic of an s symmetry and $Z \leq 0.5$. A typical result is shown in Fig. 1. The heavy fermion URu_2Si_2 also shows an enhanced zero-bias conductance indicating ≤ 0.5, probably with a s symmetry.[2,3] For the heavy fermion UPt_3, the conductance shows a broad maximum centered at zero bias, that can be fitted to a non-s symmetry superconducting state.[6,7] We are pointing out here that these observations of an enhanced conductance at small bias imply that the Fermi velocities entering in the ratio r [Eq. (2)] cannot be the quasiparticle velocities.

In heavy fermions, the reduction of the quasiparticle velocity due to mass enhancement is known to be of the order of 100. For $r \simeq 100$, and assuming the absence of any dielectric barrier ($Z_b=0$), we obtain $Z \simeq 5$ from Eq. (1). According to BTK theory, only Giaever-like characteristics should be observed: the zero-bias conductance should be depressed, not enhanced, compared to the normal-state conductance. A similar conclusion can be reached concerning high-T_c oxides. As done in Refs. 4 and 5, the quasiparticle velocities can be calculated from the BCS expression $\xi=(\hbar v_F/\pi\Delta)$ using experimental in-plane values for ξ and the measured value for Δ. This procedure is justified below. For YBCO and LSCO, one then obtains[4,5] $v_F \simeq 1.5\times 10^7$ cm/s, about 10 times smaller than the Fermi velocity in the Gold point. Even in the absence of any dielectric barrier, Eq. (1) gives $Z \simeq 2$. Again, no enhancement of the zero-bias conductance should be observed, contrary to the experimental results.

Assuming that the finite value for Z that best fits their experiments ($Z \simeq 0.3$) is entirely due to a mismatch of the Fermi velocities at the interface, Hass and co-workers have calculated a lower bound for the Fermi velocity in YBCO (Ref. 4) and LSCO (Ref. 5) obtaining $v_F \geq 6\times 10^7$ cm/s. Eliashberg has already pointed out[8] that velocities measured when a particle is injected or extracted from a sample is not in general the quasiparticle velocity. We now ask what is the exact meaning of the velocity measured in a point-contact experiment, and what information can be extracted from its ratio to the quasiparticle velocity. These questions are answered in the next section.

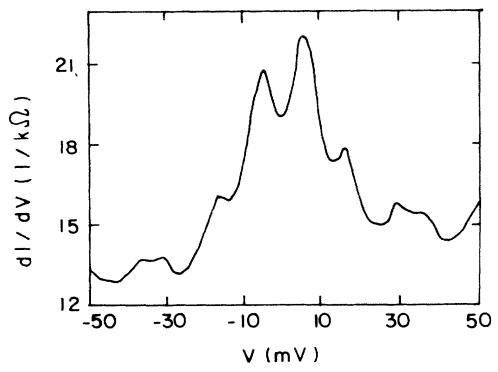

FIG. 1. Typical conductance of an Andreev point contact Au-LSCO. (After N. Hass, Ph.D thesis, Tel Aviv, 1993.)

THEORY

Consider an interacting Fermi liquid, possibly coupled to other degrees of freedom. The Hamiltonian is

$$H=\sum_i \frac{p_i^2}{2m}+V(x)+H_{\text{int}} , \tag{4}$$

where $V(x)$ is the one-body lattice periodic potential. In the normal state the single-particle Green's function is

$$G(k,\omega)=\frac{1}{(\hbar^2k^2/2m)-\omega-\Sigma_{\text{tot}}(k,\omega)} . \tag{5}$$

$\Sigma_{\text{tot}}(k,\omega)$ is the total self-energy, *which includes a frequency-independent part* $\Sigma_V(k)$ due to the lattice potential V, and a retarded part $\Sigma(k,\omega)$ due to interactions. In the absence of the latter the Bloch state bare energy is just

$$\epsilon_k=\frac{\hbar^2k^2}{2m}-\Sigma_V(k) . \tag{6}$$

One may equally well write G in the form

$$G(k,\omega)=\frac{1}{\epsilon_k-\omega-\Sigma(k,\omega)} \tag{7}$$

(we thus use Bloch states as a basis instead of plane waves). In a first stage we rely on (7), but we can clearly return to (5) if we include an *instantaneous* (ω-independent) Σ_v into Σ.

The interaction self-energy Σ is retarded: the normal quasiparticle energy ξ_k is a solution of

$$\xi_k=\epsilon_k-\Sigma(k,\xi_k) . \tag{8}$$

The Fermi level corresponds to

$$k=k_F,\quad \xi_k=\mu=\epsilon_F-\Sigma(k_F,\mu) .$$

Near the Fermi level we expand all quantities

$$k=k_F+\bar{k},\quad \omega=\mu+\bar{\omega},\quad \epsilon_k=\epsilon_F+\bar{k}v_{F_0} , \tag{9}$$

$$\xi_k=\mu+\bar{k}v_F .$$

It is easily shown that

$$v_F=\left[v_{F_0}-\frac{\partial\Sigma}{\partial k}\right]z ,\quad G(k,\omega)=\frac{z}{\xi_k-\mu-\bar{\omega}} , \tag{10}$$

where z is the wave-function renormalization

$$z=\frac{1}{1+(\partial\Sigma/\partial\omega)} . \tag{11}$$

In the heavy fermion regime z is very small, while $\partial\Sigma/\partial k$ is moderate, and eventually negligible.

Assume now that the Fermi liquid becomes superconducting, with an anomalous self-energy Δ_0 which we take independent of k and ω. The inverse Green's function matrix

$$G^{-1}=\begin{bmatrix} G^{-1}(\omega) & \Delta_0 \\ \Delta_0 & G^{-1}(-\omega) \end{bmatrix} \tag{12}$$

yields quasiparticle energies

$$E_k=\sqrt{(\xi_k-\mu)^2+\Delta^2}\ , \tag{13}$$

in which $\Delta=z\Delta_0$ is the real gap. The coherence length ξ corresponds to values of k for which E_k departs from ξ_k

$$\frac{\hbar}{\xi}=\frac{\Delta}{v_F}=\frac{\Delta_0}{v_{F_0}-(\partial\Sigma/\partial k)}\ . \tag{14}$$

Note that z disappears in (14): *only the nonlocality of* Σ *is relevant in* ξ— a hint of what we are looking for.

At this stage it is useful to stop and to survey all the velocities we have considered (they will play different roles). $\hbar k_F/m$ is the velocity of a *free particle;* $v_{F_0}=\hbar k_F/m-\partial\Sigma_v/\partial k$ is the velocity of an unrenormalized Bloch state (without interactions); $\bar{v}_F=v_{F_0}-\partial\Sigma/\partial k=\hbar k_F/m-\partial\Sigma_{\rm tot}/\partial k$ is an "effective velocity" that does not account for retardation (it will actually control reflection at an interface); $v_F=z\bar{v}_F$ is the actual *quasiparticle* velocity, as seen in low-temperature specific heat or in the coherence length. In that hierarchy it is a matter of taste whether one starts from free particles or from bare Bloch states.

We now consider the planar interface between our heavily renormalized superconductor (labeled "2") and a normal material "1" which, for simplicity, we take as nonrenormalized ($\Sigma_1=0$). A plane wave $\exp(i\mathbf{k}\cdot\mathbf{r})$ comes from the 1 side: it is partly reflected and partly transmitted. Since k_x and k_y are conserved the problem is one dimensional: we only consider the coordinate z normal to the interface.

For energies close to the Fermi level, the two materials are characterized by the Fermi wave vector k_{F_1} and the velocity $v_{F_{01}}$ on side 1, and the same quantities $k_{F_2}, v_{F_{02}}$ as well as z_2, Δ_2 on side 2.

The corresponding excitation spectra for real quasiparticles are depicted in Fig. 2. Consider an incident particle on side 1, with energy E, depicted as I in the figure. Superconductive pairing allows transformation of the particle into a hole, but two conditions must be fulfilled:

(i) Energy must be conserved;

(ii) Outgoing particles must depart from the interface, hence their group velocity must be positive for transmission, negative for reflection. As a result there may exist two transmitted states T_1 and T_2 (which of course disappear if $E<\Delta$) and two reflected states R_1 and R_2. The state R_1 is the standard specular reflection, while R_2 is Andreev's reflection. In order to obtain these four amplitudes we need matching conditions at the interface.

In their fundamental paper, Blonder, Tinkham, and Klapwijk[1] consider first two identical materials, unrenormalized, differing only by their gap $\Delta(\Delta_1=0)$. The interface is a sharp discontinuity. Within an error of order $\hbar/\xi$ all states with energy E have a wave vector $\approx+k_F$ on the right-hand side of Fig. 1, $\approx-k_F$ on the left-hand side. Such an error is negligible with an accuracy Δ/E_F. The only relevant quantity is then the *break of slope* of the wave function at the interface, here due to a possible potential barrier. If the slope is continuous only T_1 and R_2 are present, leading to the canonical Andreev conductance behavior. A break of slope, characterized by their factor Z, admixes modes T_2 and R_1: it changes the nature of the conductance characteristics.

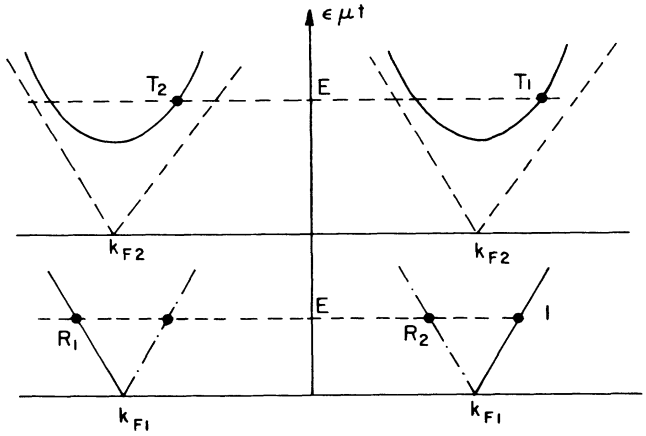

FIG. 2. The excitation spectrum. "1" is the normal electrode (full line is an electron excitation, dot-dash is a hole excitation). "2" is the superconducting electrode. The incident wave is denoted by I. R_1, R_2, T_1, T_2 are the various reflected and transmitted waves.

If the two materials are different, a change in Fermi velocities (v_1-v_2) also contributes a break of slope. What matters is the total break, characterized by the single parameter Z, which contains two terms, one due to the potential barrier, the other to (v_1-v_2). The important fact is that Z is controlled by *normal-state* properties only. Superconductivity changes the excitation spectrum, shifting wave vectors a little bit near the Fermi level. But these shifts are irrelevant anyway. The only important thing is the break of slope, i.e., what is conserved at the interface: momentum, bare velocity, dressed velocity, and in that case, which one?

A. Noninteracting particles

The Hamiltonian is a one-body operator. The most general Schrödinger equation can be written as

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+\int dx'\Sigma_V(x,x')\psi(x')\ . \tag{15}$$

In such an integral form, (15) is unambiguous. Complications in the barrier are hidden in the *nonlocal energy* $\Sigma_V(x,x')$, which is instantaneous (ω independent). The matching conditions follow from solving (15) through the transition region.

In the bulk electrodes, $\Sigma_V(x,x')$ is a function of $(x-x')$ modulated by the lattice. Its Fourier transform generates the bare particle energy ϵ_k. It k is small, one may try a Taylor expansion in powers of k, thereby introducing an effective mass m^* different in the two electrodes. It is tempting to generalize such a concept into a *mass profile* $m(x)$ that varies inside the barrier. The Schrödinger equation should then be written approximately as

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m(x)}\frac{\partial^2\psi}{\partial x^2}\ . \tag{16}$$

As it stands, (16) is definitely unacceptable, as the Hamiltonian is not Hermitian (the norm of ψ is not conserved). But, even if we cure that, (16) is ambiguous, since we are free to change the way in which $\partial/\partial x$ acts on $m(x)$. While the *integral* form (15) is neat (it embodies *both* inertial and potential changes in the barrier), a *differential* equation such as (16) is undefined. The issue was addressed before by Morrow and Brownstein.[9] We comment briefly on that point, as we feel that the problem is largely fictitious, depending on the way $\Sigma_V(x,x')$ is shuffled between potential and kinetic energy.

Reference 9 proposes a general form for the kinetic energy

$$T=-\frac{\hbar^2}{4}\int dx\left[m^{\alpha}\frac{\partial}{\partial x}m^{\beta}\frac{\partial}{\partial x}m^{\gamma}+\alpha\leftrightarrow\gamma\right] \tag{17}$$

with $\alpha+\beta+\gamma=-1$ in such a way as to recover the homogeneous case. The symmetrization $\alpha\leftrightarrow\gamma$ ensures Hermiticity. The corresponding contribution to the Schrödinger equation is easily found to be

$$\begin{aligned}T_{\psi}&=-\frac{\hbar^2}{2}\frac{\psi''}{m}+\frac{\hbar^2\psi'm'}{2m^2}+A\psi\ ,\\ A&=-\frac{\hbar^2m''}{m^2}\left[\frac{\alpha+\gamma}{4}\right]+\frac{\hbar^2m'^2}{m^3}\left[\frac{\alpha+\gamma+\alpha\gamma}{2}\right]\ .\end{aligned} \tag{18}$$

We see that the arbitrary choice of α and γ only produces a *localized extra potential* in the barrier region. Changing α and γ is equivalent to transferring terms between the barrier potential energy and the kinetic energy. Since the former is not fixed *a priori*, this is a matter of choice, irrelevant as long as the choice is *consistent* (i.e., a different barrier potential is defined for different α,γ). The ambiguity does not affect the current density, which is always defined as

$$J(x)=-\frac{i\hbar}{2m(x)}\left[\psi^*\frac{\partial\psi}{\partial x}-\psi\frac{\partial\psi^*}{\partial x}\right]\ .$$

Together with (18) such a choice guarantees the conservation law

$$\frac{\partial J}{\partial x}=-\dot{\rho}\quad\text{with }\rho=\psi^*\psi\ .$$

The simplest choice is to put *all* potential terms into an explicit potential energy, i.e., $A=0$. This is achieved when $\alpha=\gamma=0$, corresponding to a kinetic energy

$$T=-\frac{\hbar^2}{2}\frac{\partial}{\partial x}\left[\frac{1}{m(x)}\right]\frac{\partial}{\partial x}\ , \tag{19}$$

(19) is obviously Hermitian.

Assume that we make the choice (19) for a sharp interface [$m(x)$ is a step function]. If there is *no* potential barrier at interface, then the Schrödinger equation should not develop its own δ functions: hence $(1/m(x))(\partial\psi/\partial x)$ should be continuous. What is conserved is the *velocity* $\hbar k/m(x)$, not the momentum—a fairly obvious result. The corresponding reflection amplitude coefficient is

$$R=\frac{v_1-v_2}{v_1+v_2} \tag{20}$$

as used by BTK. Of course, (20) is modified if there is a potential barrier, whether genuine or due to a different choice of α and γ: what matters is the *net* reflection coefficient.

For a given $\Sigma_V(x,x')$, there should be no ambiguity, but for the fact that the concept of a "sharp" interface makes little sense for a nonlocal self-energy. The results (19) and (20) correspond to a specific limit, in which $\psi(x)$ is a slowly varying function of x on the range of Σ_V. Put another way, the incident wavelength k^{-1}, the scale of variation b of Σ_V and the range $(x-x')\sim a$ are such that

$$k^{-1}\gg b\gg a\ . \tag{21}$$

Then we can expand $\psi(x')$ in a Taylor series in the integral of (15) which becomes

$$\Sigma_0\psi(x)+\Sigma_1\psi'(x)+\Sigma_2\frac{\psi''(x)}{2}\ , \tag{22}$$

where Σ_0, Σ_1, Σ_2 are the successive moments of $\Sigma_V(x-x')$.

$$\begin{aligned}&\Sigma_0=\int dx'\Sigma_V(x,x')\ ,\quad \Sigma_1=\int dx'\Sigma_V(x,x')(x'-x)\ ,\\ &\Sigma_2=\int dx'\Sigma_V(x,x')(x'-x)^2\ .\end{aligned} \tag{23}$$

Σ_0 adds to the potential, which is slowly varying and hence does not produce reflection. If the following is met,

$$\Sigma_1=\frac{\Sigma_2}{2}\ , \tag{24}$$

the next two terms have the form (19) with a "mass correction:"

$$\delta\left[\frac{1}{m}\right]=-\Sigma_2\ . \tag{25}$$

(24) is indeed a consequence of *Hermiticity* as shown by the following algebra:

$$\Sigma_1(x)=\int d\xi\,\xi\Sigma_{\rm tot}(x,x+\xi)$$
$$=\int d\xi\,\xi\Sigma_{\rm tot}(x+\xi,x)$$
$$=\int d\xi\,\xi\left[\Sigma_{\rm tot}(x,x-\xi)+\xi\frac{\partial}{\partial x}\Sigma_{\rm tot}(x,x-\xi)+\cdots\right]. \quad (26)$$

In lowest order (24) follows immediately.

Our conclusion is that the transmission coefficient of noninteracting particles is controlled by the *spatial structure* of $\Sigma_V(x,x')$, i.e., by its *momentum k dispersion*. Precise results depend on details of the barrier, which are usually of atomic scale. (20) provides only an order of magnitude. The above detailed discussion is mostly formal since the barrier is not really controlled: we only give it in order to make contact with past work.[9]

B. Interacting particles

$\Sigma_V(x,x')$ is now complemented by an interaction self-energy $\Sigma(x,x',\omega)$ (we no longer assume translation invariance). The Green's function $G(x,x',\omega)$ describes the amplitude radiated at x at frequency ω by a source located at x'. Away from the source, that amplitude obeys the following "effective" Schrödinger equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}-\int dx'\Sigma_{\rm tot}(x,x',\omega)\psi(x')+V(x)\psi(x)=\omega\psi(x) \quad (27)$$

(m is the bare mass: effective masses are hidden in $\Sigma_{\rm tot}$). $\Sigma_{\rm tot}$ is supposed to depend very strongly on $\omega(z\ll 1)$, but moderately on k [range $(x-x')$ of atomic size]. We must solve (27) through the interface region for a *specific* value of ω. Away from the interface, $\Sigma_{\rm tot}$ depends only on $(x-x')$, and the solution is a plane-wave vector $k(\omega)$ such that

$$\xi_k=w \implies k=\pm\left[k_F+\frac{\omega-\mu}{v_F}\right].$$

v_F is the *quasiparticle* velocity, which involves z, but near the Fermi level that term is negligible and $|k|\approx k_F$ for all practical purposes (k_{F_1} and k_{F_2} on either side). What remains is a *matching problem*.

The spatial structure of $\Sigma_{\rm tot}$ is the same as that of Σ_V. Within a moment expansion, Σ_0 is an effective potential profile, Σ_1 and Σ_2 contribute to an "effective mass profile." The effect on the reflection coefficient is the same. For a *given* ω, and within the choice (19), what is conserved in the absence of a barrier is the effective velocity defined earlier, which involves $\partial\Sigma/\partial k$.

We conclude that the reflection coefficient of a renormalized quasiparticle at an interface depends on the nonlocality of Σ (its k dependence) and *not* on its retardation (its ω dependence). The conclusion holds for normal materials as well as for Andreev reflection on superconductors. It is especially relevant for heavy fermion materials in which a slow degree of freedom produces a very small z.

Such a result is not surprising: it holds whenever one considers the response to a *localized* probe. Consider for instance tunneling through an insulating barrier, described with a Bardeen Hamiltonian

$$ta^*b+\text{c.c.} \quad (28)$$

t is the hopping amplitude through the barrier, a and b refer to edge sites on either side. Second-order perturbation theory shows that the conductance involves *local* spectral densities on sites a, b:

$$\rho_a(\omega)=\frac{1}{\pi}\sum_k \mathrm{Im}G_a(k,\omega) \quad (29)$$

and similarly for ρ_b. In terms of the (total) self-energy $\Sigma(k,\omega)$ these spectral densities are

$$\rho_a(\omega)=\frac{1}{\pi}\sum_k \mathrm{Im}\frac{1}{\epsilon_{ka}-\omega-\Sigma_a(k,\omega)}. \quad (30)$$

Assume that Σ does not depend on k. We then carry the summation over k first. If the bare density of states

$$\rho_{ao}(\omega)=\sum_k \delta(\epsilon_{ka}-\omega) \quad (31)$$

does not vary too much, ImG behaves as a slightly broadened δ function of ϵ_k: hence

$$\rho_a\approx\rho_{a0} \quad (32)$$

The tunnel current is unaffected by retardation, i.e., by z. In contrast a k dependence of Σ affects the ϵ_k integral and it changes the conductance. In the end this result is nothing but the well-known Migdal theorem formulated forty years ago in the context of electron-phonon interactions.

CONCLUSIONS

What emerges from the above discussion is that the study of metallic contacts ($Z_b\ll 1$) gives access to the effective velocity $\bar{v}_F$, while the measurement of the coherence length gives access to the quasiparticle velocity $v_F=\bar{v}_F\cdot z$.

In principle, the measurement of the normal-state resistance of a pure metallic contact ($Z_b=0$) is sufficient to obtain the ratio between the effective Fermi velocities of the two electrodes: Z is determined from Eq. (3) and r from Eq. (1). But in practice the value of the Sharvin resistance R_0 is generally not known accurately (for instance because the size of the contact is not known), hence the value of Z cannot be deduced from the measured R_n.

The Andreev reflections that occur when one of the electrodes is superconducting provide us with an easier method to determine $\bar{v}_F$, because the shape of the $V(I)$ characteristics is then directly sensitive to the value of Z. One difficulty remains, which is that we do not have an independent determination of Z_b. Assuming $Z_b=0$ gives us only a bound on $\bar{v}_F$ (a lower bound in the case of heavy

fermions).

We note again that $\bar{v}_F$ is, in general, not equal to the bare (band) velocity, as shown in the previous section. Indeed, it has been found that for YBCO (Ref. 4) and LSCO (Ref. 5) the lower bound for $\bar{v}_F$ obtained with the above procedure is in fact somewhat larger than calculated band velocities,[10] suggesting that in these materials $\partial\Sigma/\partial k$ is not negligible.[8,11]

A comparison between $\bar{v}_F$ and the quasiparticle velocity v_F gives directly the mass renormalization factor z. v_F can, in principle, be obtained from a number of normal-state low-temperature measurements such as that of the electronic heat capacity. But in the case of high-T_c oxides the upper critical field H_{c2} is so high at low temperatures that these normal-state measurements are quite difficult. Also, low-temperature measurements are very sensitive to the presence of a small concentration of magnetic secondary phases. An easier method to obtain v_F is to determine the value of the coherence length. This can be done through a measurement of $H_{c2}(T) = [\phi_0/2\pi\xi^2(T)]$, and extrapolating to low temperatures using the theoretical temperature dependence of H_{c2}. $\xi(t)$ can also be obtained from proximity effect experiments.

Thus combining the study of Andreev reflections with a determination of the coherence length gives us a general method to obtain z, or rather a bound for z. It is precisely the discrepancy, by about a factor of 5, between the velocity obtained from point-contact measurements and that obtained from ξ for YBCO (Ref. 5) that raised originally the question of the exact meaning of the former. Our interpretation of this result is that for this oxide $z^{-1} > 5$. What part of the mass renormalization is due to the electron-phonon interaction, and what part is due to the electron-electron interaction is another question.

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