Fulde-Ferrell state in quasi-two-dimensional superconductors

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Inhomogeneous superconductivity proposed by Fulde and Ferrell (FF state) is studied in a strongly Pauli-limited type-II superconductor with a cylindrical Fermi surface. The phase diagram of the FF state, BCS state, and normal state is obtained. It is found that the FF state is remarkably enhanced owing to the two-dimensional Fermi-surface structure in comparison to the spherical symmetric case. We discuss that the Fermi-surface nesting is also advantageous to the FF state as well as nesting instabilities. Our result obtained in the cylindrical symmetric case is applicable in general to quasi-low-dimensional superconductors in which the nesting is sufficiently incomplete so that the nesting instabilities are suppressed. Relation to the exotic superconductors discovered recently is briefly discussed.

I. INTRODUCTION

In strongly Pauli-limited type-II superconductors under strong magnetic field, the possibility of inhomogeneous superconductivity with spatial oscillation of the gap function was pointed out by Fulde and Ferrell[1] and Larkin and Ovchinnikov.[2] The origin of the strong magnetic field could be assigned also to a molecular field due to magnetic impurities with ferromagnetic spin configuration, as well as an applied external field. Normal-state electrons under strong magnetic field have Fermi surfaces of up-spin electrons and down-spin electrons which are displaced because of the Zeeman energy. Therefore, attractive interactions of electrons near the Fermi surfaces with opposite spins may lead to formations of the pairs with nonzero total momentum. Then, the phase of the gap function varies spatially with the wave vector of this total momentum $\mathbf{q}$. We call this kind of inhomogeneous superconductivity Fulde-Ferrell (FF) superconductivity or the FF state.

In spite of many theoretical studies,[1-7] there have been few experimental observations which indicate the possibility of the existence of the FF state. This is considered to be because the FF state is easily destroyed by the normal impurity.[4] An FF superconductor has to be a clean limit of type-II superconductor, which seems to be impossible in ordinary metals. Recently, Gloos *et al.* have found the first-order phase transition below the upper critical field curve in heavy fermion UPd_2Al_3, and insisted that this indicates the existence of the FF state.[8] They argued that their discovery was possible because this heavy fermion superconductor is strongly Pauli-limited and represents the extremely clean limit of a type-II superconductor. For the same reason, sufficiently clean samples of exotic superconductors discovered recently, heavy fermion, organic and oxide superconductors, could be good FF superconductors under a strong magnetic field.

For FF superconductivity, the band structure of electrons is important in contrast to the ordinary BCS superconductivity, although FF superconductivity has been examined mainly in spherical symmetric systems so far.[1-4] When an electron pair with $\mathbf{k}$ and $-\mathbf{k}+\mathbf{q}$ is formed in an FF superconductor, this pair of electrons is scattered to that with $\mathbf{k}'$ and $-\mathbf{k}'+\mathbf{q}$ by pairing interactions, conserving total momentum $\mathbf{q}$, where $\mathbf{k}'$ is arbitrary in the range of the interaction in momentum space. Therefore, any pair of electrons on Fermi surfaces of up and down spins is necessarily mixed with the other pairs which are not on the Fermi surface in general. This is disadvantageous for FF superconductivity. However, in one-dimensional (1D) cases, the electron $(-\mathbf{k}+\mathbf{q},\downarrow)$ is always on the down-spin Fermi surface for any $(\mathbf{k},\uparrow)$ on the up-spin Fermi surface for an appropriate choice of $\mathbf{q}$. In fact, Machida and Nakanishi[6] and Suzumura and Ishino[7] found a large FF phase on their phase diagrams in a 1D case. If there is a flat portion of the Fermi surface even in the three-dimensional (3D) systems, it is expected that the FF phase is enhanced in the same way as in the 1D cases. Takada and Izuyama[3] argued that in this case the FF state is easily formed and the vector $\mathbf{q}$ becomes perpendicular to this part of the Fermi surface. Further, in more general cases, we could see by brief consideration that if the nesting condition of the Fermi surface is good, even if there is not a flat portion, the FF state is enhanced. Figure 1 describes the portions of the up-spin and down-spin Fermi surfaces with a good nesting condition. In this figure, $\mathbf{Q}$ denotes the nesting vector between the up-spin Fermi surfaces (a) and (d), and $\mathbf{q}$ denotes the difference of the up-spin and down-spin Fermi surfaces due to Zeeman energy. If the momentum dependence of the Fermi velocity is small on this Fermi-surface portion, the difference $\mathbf{q}$ does not depend on the momentum strongly. Then, a part of the up-spin Fermi surface (d) almost coincides with that of the down-spin Fermi surface (c) by the translation with a vector $\mathbf{q}$. Thus, suppose that $(\mathbf{k},\uparrow)$ is on a portion of the Fermi surface with a good nesting condition, $(-\mathbf{k}+\mathbf{q},\downarrow)$ is also near the Fermi surface, since $(-\mathbf{k},\uparrow)$ is always on the Fermi surface due to the symmetry of the crystal. Therefore, if the nesting condition of the Fermi surface is good, it is advantageous also to the FF superconductivity, as well as to the spin-density-wave (SDW) and charge-density-wave (CDW) states. In such cases, however, the FF state would com-

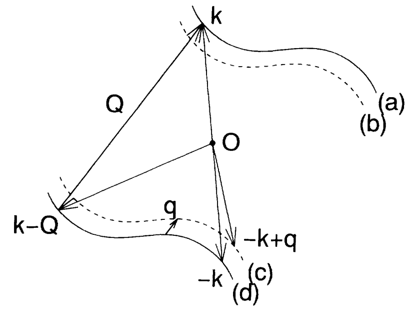

FIG. 1. Schematic diagram of the Fermi-surface portion of the good nesting condition under magnetic field. Solid lines (a) and (d), and broken lines (b) and (c) show the up-spin and down-spin Fermi surfaces. The point O represents $\mathbf{k}=\mathbf{0}$. $\mathbf{Q}$ is the nesting vector between the up-spin Fermi surfaces (a) and (d). $\mathbf{q}$ is the difference of the up-spin and down-spin Fermi surfaces due to the magnetic field.

pete with SDW and CDW states. Which state is most favorable depends on properties and strength of interactions. However, when the nesting becomes incomplete, the SDW and CDW instabilities are suppressed even at $T=0$ for sufficiently weak coupling, while the FF superconductivity survives. This is because the FF superconductivity could appear for arbitrary weak attractive interactions, even if there is no Fermi-surface nesting at all. Therefore, quasi-low-dimensional superconductors whose Fermi-surface nesting condition is good enough to enhance the FF state and bad enough to suppress the SDW and CDW instabilities could be good FF superconductors, if they are clean limits of strongly Pauli-limited type-II superconductors.

In this context, we consider superconductors with a cylindrical symmetric Fermi surface in this paper, as a model of quasi-two-dimensional (Q2D) superconductors. In this system, any finite surface portion of the Fermi surface could not touch any translated Fermi surface, while only a line on the Fermi surface could do. Thus, SDW and CDW instabilities do not occur for sufficiently weak coupling. On the other hand, the FF state is expected to be enhanced in the present system in comparison to the system with the spherical symmetric Fermi surface, owing to the nesting on a line. In this sense, the nesting condition of this system is moderately good. Thus the present system is interesting rather than the system with a spherical symmetric Fermi surface and the systems with complete Fermi-surface nesting. In application to real materials, we expect that our model describes Q2D organic superconductors, for example. In this paper, we implicitly assume the existence of three-dimensionality which is strong enough to justify mean-field treatment for sufficiently low temperatures and is weak enough to be neglected in the self-consistent equation. This is the reason why we call our system a quasi-two-dimensional system, not a two-dimensional system.

In Sec. II we derive the gap equation of the FF superconductivity from the condition of the free energy minimum. In Sec. III, we obtain the second-order transition curve and make a phase diagram on the temperature and magnetic-field plane in the present Q2D case. Section IV is devoted to the summary and discussion.

II. FREE ENERGY AND GAP EQUATION

We start with the model Hamiltonian defined by

$$H=\sum_{\mathbf{k}\sigma}\xi_{\mathbf{k}\sigma}a^{\dagger}_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma} -\frac{V}{N}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4}\delta_{\mathbf{k}_1-\mathbf{k}_2,\mathbf{k}_4-\mathbf{k}_3}a^{\dagger}_{\mathbf{k}_1+\mathbf{q}/2\uparrow}a^{\dagger}_{-\mathbf{k}_2+\mathbf{q}/2\downarrow} \times a_{-\mathbf{k}_3+\mathbf{q}/2\downarrow}a_{\mathbf{k}_4+\mathbf{q}/2\uparrow} \tag{2.1}$$

with $\xi_{\mathbf{k}\sigma}=\xi_{\mathbf{k}}-\sigma h$ and $\xi_{\mathbf{k}}=\epsilon_{\mathbf{k}}-\mu$, according to many authors.[1,3,4] Here, $a_{\mathbf{k}\sigma}$, $\epsilon_{\mathbf{k}}$, and μ are electron operators, one-particle energies, and the chemical potential, respectively. h is the Zeeman energy $\mu_0|\mathbf{H}|$, where $\mathbf{H}$ is the magnetic field and μ_0 is the magnetic moment of an electron. The interactions are assumed to exist only in a region $\pm\omega_D$ around the Fermi surface. We define the approximate Hamiltonian H_0 as

$$H_0=\sum_{\mathbf{k}\sigma}E_{\mathbf{k}\sigma}\alpha^{\dagger}_{\mathbf{k}\sigma}\alpha_{\mathbf{k}\sigma} \tag{2.2}$$

with

$$\begin{aligned} \alpha_{\mathbf{k}\uparrow}&=u_{\mathbf{k}}a_{\mathbf{k}+\mathbf{q}/2\uparrow}+v_{\mathbf{k}}a^{\dagger}_{-\mathbf{k}+\mathbf{q}/2\downarrow}\ ,\\ \alpha^{\dagger}_{-\mathbf{k}\downarrow}&=u_{\mathbf{k}}a_{\mathbf{k}+\mathbf{q}/2\uparrow}+u_{\mathbf{k}}a^{\dagger}_{-\mathbf{k}+\mathbf{q}/2\downarrow}\ ,\\ u_{\mathbf{k}}&=\cos\theta_{\mathbf{k}}\ ,\\ v_{\mathbf{k}}&=\sin\theta_{\mathbf{k}}\ , \end{aligned} \tag{2.3}$$

where $E_{\mathbf{k}\sigma}$, $\theta_{\mathbf{k}}$, and $\mathbf{q}$ are variational parameters. Here, we have ignored the orbital magnetism for simplicity. This is justified either for specimens smaller than the penetration depth or near the critical field in the strongly Pauli-limited superconductors, as discussed by Takada and Izuyama.[3] It was shown by Gruenberg and Gunther[5] that coexistence of the vortex state and FF state is possible. For the two-dimensional nature of the present system, the direction of the magnetic field is also important when we consider orbital magnetism. We leave this problem for future study.

We variationally minimize an approximate free energy $\hat{F}$ defined by

$$\hat{F}\equiv\langle H-H_0\rangle_0+F_0\ , \tag{2.4}$$

where $F_0\equiv -T\ln[\mathrm{Tr}(e^{-\beta H_0})]$ and $\langle\cdots\rangle_0 \equiv\mathrm{Tr}(e^{-\beta H_0}\cdots)/\mathrm{Tr}(e^{-\beta H_0})$. This is equivalent to the mean-field approximation of the Hamiltonian Eq. (2.1):

$$H_{\mathrm{MF}}=\sum_{\mathbf{k}\sigma}\xi_{\mathbf{k}\sigma}a^{\dagger}_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma}+\sum_{\mathbf{k}}\Delta_{\mathbf{q}}a^{\dagger}_{\mathbf{k}\uparrow}a^{\dagger}_{-\mathbf{k}-\mathbf{q}\downarrow} +\sum_{\mathbf{k}}\Delta^{*}_{\mathbf{q}}a_{-\mathbf{k}-\mathbf{q}\downarrow}a_{\mathbf{k}\uparrow} \tag{2.5}$$

with

$$\Delta_{\mathbf{q}}=-VN^{-1}\sum_{\mathbf{k}'}\langle a_{-\mathbf{k}'-\mathbf{q}'\downarrow}a_{\mathbf{k}'\uparrow}\rangle\ . \tag{2.6}$$

If we fix $\mathbf{q}=0$, our approximation reduces to the ordinary BCS approximation.

Taking the average, we obtain

$$\hat{F}=T\sum_{\mathbf{k}\sigma}\ln[1-f(E_{\mathbf{k}\sigma})]-\sum_{\mathbf{k}\sigma}E_{\mathbf{k}\sigma}f(E_{\mathbf{k}\sigma}) + \sum_{\mathbf{k}}(u_{\mathbf{k}}^2-v_{\mathbf{k}}^2)\xi_{\mathbf{k}}^{(s)}[f(E_{\mathbf{k}\uparrow})+f(E_{\mathbf{k}\downarrow})-1] + \sum_{\mathbf{k}}\xi_{\mathbf{k}}^{(a)}[f(E_{\mathbf{k}\uparrow}-f(E_{\mathbf{k}\downarrow})+1]-NVG_n^2-NVG_s^2 \tag{2.7}$$

with

$$G_n=\frac{1}{N}\sum_{\mathbf{k}}\{u_{\mathbf{k}}^2f(E_{\mathbf{k}\uparrow})+v_{\mathbf{k}}^2[f(E_{\mathbf{k}\downarrow})-1]\}$$
$$G_s=\frac{1}{N}\sum_{\mathbf{k}}u_{\mathbf{k}}v_{\mathbf{k}}[f(E_{\mathbf{k}\uparrow})+f(E_{\mathbf{k}\downarrow})-1] , \tag{2.8}$$

where $f(x)=1/(e^{\beta x}+1)$, and

$$\xi_{\mathbf{k}}^{(s)}=\tfrac{1}{2}(\xi_{\mathbf{k}+\mathbf{q}/2\uparrow}+\xi_{-\mathbf{k}+\mathbf{q}/2\downarrow}) ,$$
$$\xi_{\mathbf{k}}^{(a)}=\tfrac{1}{2}(\xi_{\mathbf{k}+\mathbf{q}/2\uparrow}-\xi_{-\mathbf{k}+\mathbf{q}/2\downarrow}) . \tag{2.9}$$

The term including G_n comes from normal decoupling of the interaction term: $V\langle a^\dagger a\rangle\langle a^\dagger a\rangle$, and contributes to the internal magnetic-field shift as well as the unimportant chemical potential shift. We ignore this term for a while and will discuss the effect of the shift of the internal field due to interactions later. We assume that $\epsilon_{\mathbf{k}}=k^2/2m$ in this paper. Thus, we have $\xi_{\mathbf{k}}^{(s)}=\xi_{\mathbf{k}}+q^2/8m$ and $\xi_{\mathbf{k}}^{(a)}=h(\bar{q}x-1)$, where $\bar{q}=v_Fq/2h$, $x=\cos\theta$, and θ is the angle between $\mathbf{k}$ and $\mathbf{q}$. $k=|\mathbf{k}|$ is equal to $\sqrt{k_x^2+k_y^2}$ in two dimensions and $\sqrt{k_x^2+k_y^2+k_z^2}$ in three dimensions. The q^2 term of $\xi_{\mathbf{k}}^{(s)}$ is negligible if $\Delta_{\mathbf{q}}/\omega_D\ll 1$, since $v_Fq\sim h\sim\Delta_{\mathbf{q}}$.

Variational conditions with respect to $E_{\mathbf{k}\sigma}$ and $\theta_{\mathbf{k}}$:

$$\frac{\partial\hat{F}}{\partial E_{\mathbf{k}\sigma}}=0 \quad\text{and}\quad \frac{\partial\hat{F}}{\partial\theta_{\mathbf{k}}}=0$$

lead to the gap equation

$$\Delta_{\mathbf{q}}=\frac{V}{N}\sum_{\mathbf{k}}\frac{1-f(E_{\mathbf{k}\uparrow})-f(E_{\mathbf{k}\downarrow})}{2E_{\mathbf{k}}}\Delta_{\mathbf{q}} \tag{2.10}$$

with

$$E_{\mathbf{k}\sigma}=\sigma\xi_{\mathbf{k}}^{(a)}+E_{\mathbf{k}} ,$$
$$\Delta_{\mathbf{q}}=-G_sV=-\frac{V}{N}\sum_{\mathbf{k}}u_{\mathbf{k}}v_{\mathbf{k}}[f(E_{\mathbf{k}\uparrow})+f(E_{\mathbf{k}\downarrow})-1] ,$$
$$u_{\mathbf{k}}=\left[\frac{1}{2}\left[1+\frac{\xi_{\mathbf{k}}^{(s)}}{E_{\mathbf{k}}}\right]\right]^{1/2} , \tag{2.11}$$
$$v_{\mathbf{k}}=\left[\frac{1}{2}\left[1-\frac{\xi_{\mathbf{k}}^{(s)}}{E_{\mathbf{k}}}\right]\right]^{1/2} ,$$
$$E_{\mathbf{k}}=\sqrt{\xi_{\mathbf{k}}^{(s)2}+\Delta_{\mathbf{q}}^2} .$$

We have to solve the gap equation for each $\mathbf{q}$ and evaluate $\hat{F}$ for this solution and then minimize $\hat{F}$ with respect to $\mathbf{q}$. The final solution in our approximation is that for a particular $\mathbf{q}$ which gives the minimum of $\hat{F}$.

Using the gap equation, we rewrite the difference $\Delta F\equiv\hat{F}-F_n$ as

$$\Delta F=T\sum_{\mathbf{k}\sigma}\ln\frac{1-f(E_{\mathbf{k}\sigma})}{1-f(\sigma\xi_{\mathbf{k}}^{(a)}+|\xi_{\mathbf{k}}^{(s)}|)} + \sum_{\mathbf{k}}\{|\xi_{\mathbf{k}}^{(s)}|-E_{\mathbf{k}}\}+\frac{N}{V}\Delta_{\mathbf{q}}^2 , \tag{2.12}$$

and further

$$\Delta F=T\sum_{\mathbf{k}\sigma}\ln\frac{1+e^{-\beta(\sigma\xi_{\mathbf{k}}^{(a)}+|\xi_{\mathbf{k}}^{(s)}|)}}{1+e^{-\beta E_{\mathbf{k}\sigma}}} -\frac{1}{2}NN(0)[\Delta_{\mathbf{q}}(T)]^2\left[1+2\ln\left|\frac{\Delta_0(0)}{\Delta_{\mathbf{q}}(T)}\right|\right] \tag{2.13}$$

in the weak-coupling limit.

In the FF state, there are regions of $\mathbf{k}$ where the quasiparticle excitation energy $E_{\mathbf{k}\sigma}$ is negative, which is denoted by R_{II}^{σ} here and called blocking region. The region where both excitation energies $E_{\mathbf{k}\uparrow}$ and $E_{\mathbf{k}\downarrow}$ are positive is denoted by R_{I}. Then, the ground state $|\text{g.s.}\rangle$ is expressed as

$$|\text{g.s.}\rangle=\prod_{\mathbf{k}\in R_{\text{I}}}(u_{\mathbf{k}}-v_{\mathbf{k}}a_{\mathbf{k}+\mathbf{q}/2\uparrow}^\dagger a_{-\mathbf{k}+\mathbf{q}/2\downarrow}^\dagger) \times\prod_{\sigma}\prod_{\mathbf{K}\in R_{\text{II}}^{\sigma}}a_{\sigma\mathbf{k}+\mathbf{q}/2,\sigma}^\dagger|0\rangle , \tag{2.14}$$

where $|0\rangle$ is the electron vacuum state. The blocking region has been examined in detail in Refs. 1 and 3. For $\bar{q}\geq 1$,

$$E_{\mathbf{k}\uparrow}\leq 0 \quad\text{for}\quad -1\leq x\leq\phi^-(\xi_{\mathbf{k}}) \quad\text{and}\quad |\xi_{\mathbf{k}}|\leq\xi_1$$
$$E_{\mathbf{k}\downarrow}\leq 0 \quad\text{for}\quad 1\geq x\geq\phi^+(\xi_{\mathbf{k}}) \quad\text{and}\quad |\xi_{\mathbf{k}}|\leq\xi_2 , \tag{2.15}$$

while for $\bar{q}<1$,

$$E_{\mathbf{k}\uparrow}\leq 0 \quad\text{for}\quad \begin{cases}-1\leq x\leq\phi^-(\xi_{\mathbf{k}}) \quad\text{and}\quad \xi_1\geq|\xi_{\mathbf{k}}|\geq\xi_2\\ -1\leq x\leq 1 \quad\text{and}\quad \xi_2\geq|\xi_{\mathbf{k}}|\geq 0 ,\end{cases} \tag{2.16}$$

where

$$\phi^{\pm}(\xi)=\frac{h\pm\sqrt{\xi^2+\Delta_{\mathbf{q}}^2}}{\bar{q}h} ,$$
$$\xi_1\equiv h(\bar{q}+1)x_1 \quad\text{with}\quad x_1\equiv\text{Re}\sqrt{1-\Delta_{\mathbf{q}}^2/\{h(\bar{q}+1)\}^2} , \tag{2.17}$$
$$\xi_2\equiv h|\bar{q}-1|x_2 \quad\text{with}\quad x_2\equiv\text{Re}\sqrt{1-\Delta_{\mathbf{q}}^2/\{h(\bar{q}-1)\}^2} .$$

At $T=0$, the gap equation in two dimensions is calculated as

$$\ln\frac{\Delta_0}{\Delta_{\mathbf{q}}}=\int_0^{\xi_1}d\xi\frac{1}{\pi}\arccos[-\phi^-(\xi)]\frac{1}{E_{\mathbf{k}}} +\int_0^{\xi_2}d\xi\frac{1}{\pi}\arccos[\phi^+(\xi)]\frac{1}{E_{\mathbf{k}}} \tag{2.18}$$

for $\bar{q} \geq 1$, and

$$\ln\frac{\Delta_0}{\Delta_{\mathbf{q}}} = \int_{\xi_2}^{\xi_1} d\xi \frac{1}{\pi}\arccos[-\phi^-(\xi)]\frac{1}{E_{\mathbf{k}}} + \frac{1}{2}\ln\left|\frac{1+x_2}{1-x_2}\right| \tag{2.19}$$

for $\bar{q} \leq 1$. In three dimensions,[3] we obtain

$$\ln\frac{\Delta_0}{\Delta_{\mathbf{q}}} = -\frac{\bar{q}+1}{4\bar{q}}\left[\ln\frac{1-x_1}{1+x_1}+2x_1\right] - \frac{\bar{q}-1}{4\bar{q}}\left[\ln\frac{1-x_2}{1+x_2}+2x_2\right] . \tag{2.20}$$

III. CRITICAL MAGNETIC FIELD OF FULDE-FERRELL SUPERCONDUCTIVITY

In this section, we calculate the critical magnetic field of Fulde-Ferrell superconductivity in strongly Pauli-limited superconductors with a cylindrical Fermi surface. The Fulde-Ferrell phase appears at low temperatures and high fields, between the BCS phase and the normal phase. It is known that the transition between the FF state and spin-polarized normal state is second order in the cases studied so far. We have confirmed it in the present case at $T=0$ by numerical calculation. From now on in this paper, we study only the second-order phase transition with respect to the FF state. We take the $\Delta_{\mathbf{q}} \to 0$ limit in the gap equation (2.10) in two dimensions:

$$1=\lambda\int_0^{\omega_D} d\xi\int_0^{\pi}\frac{d\theta}{\pi}\frac{\sinh(\beta\xi)}{\xi[\cosh(\beta\xi)+\cosh(\beta\zeta)]} , \tag{3.1}$$

with $\zeta = h(\bar{q}x-1)$, $x \equiv \cos\theta$, and $\lambda = VN(0)$. In the weak-coupling limit, this equation is written as

$$\ln\frac{T}{T_c^{(0)}} = -\int_0^{\pi}\frac{d\theta}{\pi}\sinh^2\frac{\beta\zeta}{2}\int_0^{\infty} dy \ln y\left[\frac{2\sinh^2 y}{\cosh^2 y+\sinh^2(\beta\zeta/2)} - \frac{1}{\cosh^2 y\,[\cosh^2 y+\sinh^2(\beta\zeta/2)]}\right] , \tag{3.2}$$

where $T_c^{(0)}$ is the superconducting transition temperature under zero magnetic field. At $T=0$, Eq. (3.1) is rewritten as

$$\ln\frac{2\hat{h}(\bar{q})}{\Delta_0} = \int_0^{\pi}\frac{d\theta}{\pi}\ln\left|\frac{1}{\bar{q}\cos\theta-1}\right| = \begin{cases} -\ln\dfrac{1+\sqrt{1-\bar{q}^2}}{2} & \text{for } \bar{q}\leq 1 \\ -\ln\dfrac{\bar{q}}{2} & \text{for } \bar{q}>1 , \end{cases} \tag{3.3}$$

where $\hat{h}(\bar{q})$ is the magnetic field at which a second-order transition to the FF state with given $\bar{q}$ occurs. Critical magnetic field $H_c = h_c/\mu_0$ is determined by

$$\ln\frac{2h_c}{\Delta_0} = \max_{\bar{q}}\left\{\ln\frac{2\hat{h}(\bar{q})}{\Delta_0}\right\} = \ln 2 . \tag{3.4}$$

Thus, we find that $h_c = \Delta_0$, i.e., $H_c = \Delta_0/\mu_0$ and it is given by $\bar{q}=1$, i.e., $|\mathbf{q}| = 2\Delta_0/v_F$. In the three-dimensional FF superconductor with a spherical symmetric Fermi surface,[1,3] $\hat{h}(\mathbf{q})$ is calculated as

$$\frac{\hat{h}(\bar{q})}{\Delta_0} = \frac{e}{2(\bar{q}+1)}\left[\frac{\bar{q}+1}{|\bar{q}-1|}\right]^{(\bar{q}-1)/2\bar{q}} . \tag{3.5}$$

Thus, $h_c = \max_{\bar{q}}\{\hat{h}(\bar{q})\}$ is

$$\frac{h_c}{\Delta_0} = \frac{e}{2(\bar{q}+1)}\left[\frac{\bar{q}_c+1}{|\bar{q}_c-1|}\right]^{(\bar{q}_c-1)/2\bar{q}_c} , \tag{3.6}$$

where $\bar{q}_c$ is given by

$$2\bar{q}_c = \ln\left|\frac{1+\bar{q}_c}{1-\bar{q}_c}\right| . \tag{3.7}$$

By numerical calculation, we obtain $h_c \approx 0.75\Delta_0$, i.e., $H_c \approx 0.75\Delta_0/\mu_0$. There, $\bar{q} \approx 1.2$, i.e., $|\mathbf{q}| \approx 1.8\Delta_0/v_F$. Comparing the above results at $T=0$, we find that the FF state is remarkably enhanced due to the two-dimensional structure of the Fermi surface.

For finite temperatures, we have to calculate numerically. Figure 2 shows the phase diagram of the FF state,

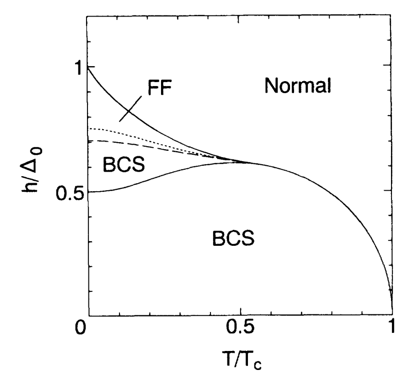

FIG. 2. Phase diagram of FF, BCS, and normal states in the Q2D system with a cylindrical symmetric Fermi surface. The solid lines are the second-order transition curve between the normal and the BCS phases for $T > T^*$, and that between the normal state and the FF phases for $T < T^*$. The broken line is the fictitious first-order transition curve between the normal and the BCS phases and could be practically regarded as the first-order transition curve between the FF and the BCS phases. The dotted line shows the second-order transition curve between the normal and the FF phases in the 3D system with a spherical symmetric Fermi surface.

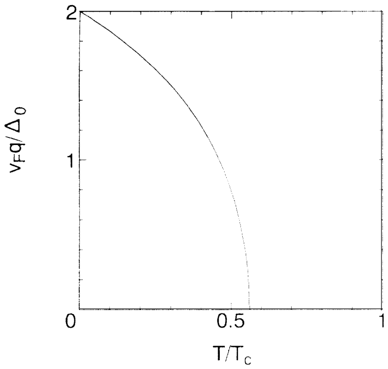

FIG. 3. Temperature dependence of the total pair momentum along the second-order transition curve.

BCS state, and normal state. The broken line is the fictitious first-order transition curve between the normal state and the BCS state assuming the absence of the FF state. The first-order transition curve between the FF state and the BCS state lies slightly below this line, because the free energy of the FF state is slightly smaller than that of the normal state. As in the three-dimensional case,[3] this difference is negligibly small also in the present case. Thus, the broken line could be practically regarded as the first-order transition curve between the FF state and the BCS state. The solid line shows the second-order transition curve between the BCS state and the normal state for $T > T^*$, and between the FF state and normal state for $T < T^*$, where T^* is the tricritical temperature equal to about $0.56T_c$. The solid line below the broken line for $T < T^*$ could be regarded as a supercooling critical field when the first-order transition to the BCS state does not occur.[3,9] In Fig. 2, we find that the FF phase is remarkably enhanced in the two-dimensional case, also for finite temperatures as well as in the ground state examined above. It is also found, however, that the maximum value of the transition temperature of the FF state, i.e., the tricritical temperature, is not affected by the band structure. This is plausible because $|\mathbf{q}|$ decreases to 0 continuously in the FF phase as one approaches the tricritical temperature. The magnitude of the wave vector $\mathbf{q}$ just below the critical magnetic field of the FF state depends on the temperature as shown in Fig. 3.

IV. SUMMARY AND DISCUSSION

We have studied a strongly Pauli-limited type-II superconductor with a cylindrical symmetric Fermi surface. In this system, the Fermi surface touches the appropriately translated Fermi surface on a line. Such a system is interesting from both an experimental viewpoint and a theoretical viewpoint, because the nesting condition is moderately good so that the critical magnetic field of the FF state is expected to be enhanced while simultaneously, SDW and CDW instabilities are suppressed for sufficiently weak coupling. We have calculated the critical magnetic field H_c for FF superconductivity, and obtained the phase diagram of the normal, BCS, and FF states on the H-T plane. The magnitude of the wave vector $\mathbf{q}$ at the critical field increases with decreasing temperature. At $T=0$, it is obtained that $H_c=\Delta_0/\mu_0$ and $|\mathbf{q}|=2h_c/v_F=2\Delta_0/v_F$. This value of $|\mathbf{q}|$ is just the same as the magnitude of the minimum momentum difference of the up-spin and down-spin Fermi surfaces and twice the inverse of the coherence length.

It is found from the phase diagram that the FF phase is much larger in the cylindrical symmetric case than in the spherical symmetric case. This is because the up-spin and down-spin Fermi surfaces could touch each other on a line by the inversion $\mathbf{k}\to-\mathbf{k}$ and translation $-\mathbf{k}\to-\mathbf{k}+\mathbf{q}$ by an appropriate wave vector $\mathbf{q}$ in the former case, while they could do so only on a point in the latter case. In Q2D superconductors, the situation is essentially the same as the present case, in the sense that the nesting holds only on a line of the Fermi surface. Hence, we could expect a large FF phase in such systems as we obtained in the cylindrical symmetric case. Therefore, the FF state would be likely to be observed experimentally in two-dimensional cases rather than in three-dimensional cases.

We have also found that the tricritical point is the same as in the three-dimensional case. This is plausible because the tricritical point is obtained in the limit of $\bar{q}\to 0$, and as $\bar{q}$ becomes smaller, the band structure becomes less important. Hence, we conjecture that the tricritical point does not depend on the band structure in general in a sufficiently weak-coupling limit. In the recent observation in the heavy-fermion UPd_2Al_3 system by Gloos *et al.*,[8] the tricritical point is much larger than the theoretical prediction,[10] if the observed phase is an FF phase as they insisted. We have shown that mixing of the singlet and the triplet order parameters, inherent in the FF state, enhances the tricritical temperature, and discussed the relation to the heavy-fermion compound.[11]

Now, we briefly discuss the effect of the internal magnetic-field shift due to the interactions. Attractive interactions between electrons near the Fermi surface reduce the internal field as is seen by a random phase approximation.[3] In real materials, there are Coulomb repulsive interactions, which enhance the internal field. Anyway, in the presence of the interactions, the internal field $\tilde{H}$ which electrons feel, is not the same as the external field H. Then, the magnetization M is written as $M=\chi\tilde{H}=\tilde{\chi}H$, where χ is the bare susceptibility and $\tilde{\chi}$ is renormalized susceptibility. For the second-order phase transition curve, this effect is taken into account only by replacing H with $\tilde{H}$. This means that the critical field of the FF superconductivity H_c becomes $\chi/\tilde{\chi}$ times the value in the absence of the internal field shift. On the other hand, for the first-order phase transition, it is misleading, if we take it into account in the same way. The critical field of the BCS state due to the first-order transition, H_P, is roughly determined by $N(0)\Delta_0^2/2=\tilde{\chi}H_P^2/2$ at $T=0$. Therefore, the actual Pauli limit H_P is $\sqrt{\chi/\tilde{\chi}}$ times the value in the absence of the internal field shift.[3] Therefore, if the interactions enhance the internal field, the FF phase becomes smaller and might vanish in three dimensions, because the FF phase is very narrow in three dimensions even in the absence of the internal field shift. On the other hand, it is hopeful that the FF phase survives in two dimensions, because it is much wider than three dimensions. From the estimations in the previous section, the FF phase survives in the phase diagram in the two dimensions, unless the external field $\tilde{H}$ exceeds approximately 2.04 times the internal field H, while it disappears at $\tilde{H} \approx 1.15H$ for the three dimensions.

In conclusion, we believe from our results that the FF state is likely to be observed in Q2D systems, such as organic superconductors. They could be clean type-II superconductors, since their coherence lengths of superconductivity are short in general because of small electron hopping energies. Copper oxide superconductors could also be good FF superconductors owing to the short coherence length and the two-dimensionality, if the sample is in a sufficiently clean limit.

Competition between the SDW or CDW states and the FF state, when the Fermi-surface nesting becomes much better, is an interesting problem to be examined. The impurity effect in the two-dimensional case is also to be examined.

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[1] P. Fulde and A. Ferrell, Phys. Rev. **135**, A550 (1964).
[2] A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **47**, 1136 (1964) [Sov. Phys. JETP **20**, 762 (1965)].
[3] S. Takada and T. Izuyana, Prog. Theor. Phys. **41**, 635 (1969).
[4] S. Takada, Prog. Theor. Phys. **43**, 27 (1970).
[5] L. W. Gruenberg and L. Gunther, Phys. Rev. Lett. **16**, 996 (1966).
[6] K. Machida and H. Nakanishi, Phys. Rev. B **30**, 122 (1984).
[7] Y. Suzumura and K. Ishino, Prog. Theor. Phys. **70**, 654 (1983).
[8] K. Gloos, R. Modler, H. Shimanski, C. D. Bredl, C. Geibel, F. Steglich, A. I. Buzdin, N. Sato, and T. Komatsubara, Phys. Rev. Lett. **70**, 501 (1993).
[9] K. Maki and T. Tsuneto, Prog. Theor. Phys. **31**, 68 (1964).
[10] G. Yin and K. Maki, Phys. Rev. B **48**, 650 (1993).
[11] S. Matsuo, H. Shimahara, and K. Nagai, J. Phys. Soc. Jpn. **63**, 2499 (1994).