

## Mean-field theory for critical phenomena in bilayer systems

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Magnetic systems consisting of a layer with finite thickness coupled to a semi-infinite bulk are studied with Ginzburg-Landau (GL) mean-field theory. The critical point of the system is determined in terms of the coefficients in the GL free-energy functional and the thickness of the thin layer. From the correlation functions within the semi-infinite bulk, an effective extrapolation length is able to be defined, although only the continuity of magnetization is assumed as the interfacial condition. This extrapolation length describes the coupling between the two subsystems and diverges with a new exponent  $\bar{\nu}=1$ . The critical behavior of spontaneous magnetization according to the variance of the layer thickness is derived as  $m_c \sim \sqrt{L/L_c - 1}$  with the critical thickness  $L_c$  and a new critical exponent  $\beta_L = \frac{1}{2}$ .

### I. INTRODUCTION

Multilayer systems have been attracting considerable attentions from the viewpoints of both theories and applications. Giant magnetoresistance was observed in ferromagnetic/nonmagnetic/ferromagnetic sandwiches.<sup>1-3</sup> Magnetic multilayer systems are found to be very useful for magneto-optical recording. Resorting to the differences among the Curie points and the magnitudes of magnetization, a magnetic trilayer disk of reading/switching/recording films, has been developed successfully in order to increase the recording density and to reduce the recording time for magneto-optical recording.<sup>4</sup> Coupling a magnetic thin film with in-plane anisotropy to the bulk medium for memory of information with vertical anisotropy reduces significantly the recording field.<sup>5</sup> Multilayer systems in other materials have also become important. A layer of He<sup>4</sup> deposited on a system of He<sup>3</sup> has been reported to change the properties of the He<sup>3</sup> system significantly.<sup>6,7</sup> A system of a layered structure of superconducting (and nonsuperconducting) materials is found to show interesting behaviors according to the electron-electron interactions, diffusion constants, and the layer thicknesses.<sup>8-11</sup>

As the present technologies are as sophisticated as to resort to the values of critical points and the detailed temperature dependences of relevant physical quantities, theoretical analyses up to now became insufficient. Even the most fundamental Ginzburg-Landau (GL) mean-field theory<sup>12</sup> has not been worked out in multilayer systems to provide the necessary information. In the bilayer system for magneto-optical recording, for example, the thickness of the layer with in-plane anisotropy is typically of the order of 100 Å. Therefore, this layer can neither be treated simply as the surface to the bulk medium, nor be approximated as a semi-infinite bulk, both of which have been studied extensively.<sup>13-20</sup> One has to treat a system consisting of a layer with finite thickness coupled to a semi-infinite bulk.<sup>21-24</sup> This structure can be a sufficient approximation for other systems, such as those mentioned above, and will be investigated in the present study. In the theoretical point of view, it is the simplest structure

which will show the most important aspects in multilayer structures. Since the inhomogeneity in general multilayer structures is taken into account in the bilayer system, the competition between the fluctuations in the neighboring subsystems via the interface can be discussed. On the other hand, the possible accumulated effects from other subsystems can be expressed formally by the extrapolation length defined at the top surface of the thin layer in the present bilayer system.

Since the semi-infinite bulk is one of the two subsystems of this structure, it is useful to review briefly the established results for the critical phenomena in semi-infinite systems. The theoretical approach in this field was begun by Mills two decades ago.<sup>13</sup> He set up first a microscopic molecular-field theory for magnetic systems with a surface and derived the relations for the magnitudes and derivatives of the order parameter at the surface. These relations are then clarified to be the boundary conditions for the continuum GL free-energy formalism in semi-infinite systems. Binder and Hohenberg studied the Ising model in a semi-infinite system by means of GL theory, high-temperature series expansion, and scaling theory for several thermodynamic quantities and correlation functions.<sup>14,15</sup> Critical exponents characteristic in semi-infinite systems are defined. They also introduced the concept of extrapolation length in magnetic systems. Lubensky and Rubin investigated the positive, negative, and infinite extrapolation length cases.<sup>16</sup> They presented analytic expressions for correlation functions and the susceptibilities. Bray and Moore studied in detail the special case of infinite extrapolation length.<sup>17</sup> Renormalization-group study was also performed by Lubensky and Rubin<sup>18</sup> for the case of positive extrapolation length. With a formalism in wave-number space,  $\epsilon$  expansions were carried out to the first order. Cordery and Griffin made  $\epsilon$  expansion study with another formalism where real-space notation is used for the direction vertical to the surface, while wave-number notations are adopted for the parallel directions.<sup>19</sup> They were successful in making the analyses very transparent.

Critical phenomenon in thin-layer systems have not been revealed as fully as those in semi-infinite systems. In

GL free-energy functional formalism, one introduces critical points *a priori* in the coefficients of quadratic terms of magnetization in the free-energy expression. However, the mean-field critical point is correct only for the corresponding bulk, namely for infinite thickness. For a layer of finite thickness and with arbitrary surface exchanges, phase transition occurs at a temperature deviating from the given critical point. This deviation of critical point was discussed by Kaganov and Omel'Yanchuk from the appearance of spontaneous magnetization.<sup>20</sup> Correlation functions, however, have not yet been studied.

The deviation of the critical point in GL mean-field theory is very important in magnetic multilayer systems where layers are of finite thicknesses. It is much more involved than that in a single thin layer, since the interfacial effects show complicated dependences on the exchange couplings within individual subsystems, the thicknesses of layers, and the exchange coupling on the open surfaces of the structures. To clarify these dependences and to determine the true mean-field critical point is the first purpose of the present study.

Generally, the critical point defined from the divergence of correlation length coincides with that defined from the symmetry breaking, namely the appearance of spontaneous magnetization. This property is taken into account *a priori* as an assumption in the usual GL formalism for bulk systems and semi-infinite bulk systems. However, this coincidence is not trivial in multilayer systems such as that studied in the present paper. To show this coincidence explicitly in GL mean-field theory for magnetic multilayer structures is the second purpose of the present study.

It has been revealed that phase transitions and critical phenomena in semi-infinite system can be investigated by

GL formalism analytically. Critical points, thermodynamic quantities, and correlation functions are expressed directly from the quantities given in the GL free-energy functional. It is of academic interest to see how far the multilayer systems can be approached analytically in the scheme of mean-field theory. This is the third purpose of the present paper.

The remaining part of the present paper is organized as follows: Formalism is given in Sec. II and the correlation functions are calculated. The critical point is determined and the mechanism for the critical-point shift is clarified. The critical behaviors of correlation functions are revealed. Susceptibilities are discussed briefly. Section III is devoted to discussions of spontaneous magnetization. The symmetry breaking is clarified as the thickness of the thin layer varies. Summary and discussions are given in Sec. IV.

## II. FORMULATION AND CORRELATION FUNCTIONS

The system studied in the present paper consists of a layer with finite thickness coupled to a semi-infinite bulk, as schematically shown in Fig. 1 where the  $z$  axis is taken to be vertical to the surface and the interface, and the origin is at the interface. The thickness  $L$  is finite but large on the scale of atomic lengths. However, the area of the surface and the interface are infinite so that true criticality occurs. The magnetic constants are taken to be uniform in the individual subsystems and change abruptly at the interface. The Ginzburg-Landau free-energy functional<sup>12</sup> for the present system under an external field can be given as

$$\begin{aligned} \frac{F}{T} = & \int d\mathbf{x}_{\parallel} \int_0^L \left[ \frac{1}{2} A_1 m^2 + \frac{1}{4} B_1 m^4 - H_{\text{ext}} m + \frac{1}{2} C_1 \left( \frac{\partial m}{\partial z} \right)^2 \right] dz + \int d\mathbf{x}_{\parallel} \frac{1}{2} C_1 \lambda^{-1} m^2(z=L) \\ & + \int d\mathbf{x}_{\parallel} \int_{-\infty}^0 \left[ \frac{1}{2} A_2 m^2 + \frac{1}{4} B_2 m^4 - H_{\text{ext}} m + \frac{1}{2} C_2 \left( \frac{\partial m}{\partial z} \right)^2 \right] dz, \end{aligned} \quad (1)$$

where  $\parallel$  is used to specify vectors parallel to the interface and the surface, with the following interface condition:

$$m(\mathbf{x}_{\parallel}, z=+0) = m(\mathbf{x}_{\parallel}, z=-0). \quad (2)$$

The first two integrals cover the free energy of the thin layer with a surface, and the last integral is for the semi-infinite bulk. The free-energy functional studied up to now can be obtained setting  $L=0$ .<sup>13-16</sup>

The coefficients of the quadratic terms are given by

$$A_1 = A'_1(T - T_{c1}) \quad \text{and} \quad A_2 = A'_2(T - T_{c2}), \quad (3)$$

where  $T_{c1}$  and  $T_{c2}$  are the mean-field critical points for the layer in the infinite-thickness limit and the semi-

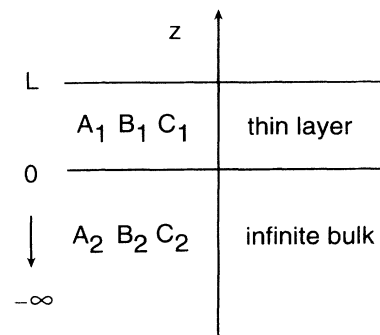


FIG. 1. Geometry of the present system: a thin layer coupled to a semi-infinite bulk.

infinite bulk, respectively, and  $A'_1$  and  $A'_2$  are positive constants. In the present paper we consider the case of  $T_{c1} > T_{c2}$ . The coefficients  $B_i$  and  $C_i$  ( $i=1,2$ ) are taken to be positive with weak temperature dependences. The extrapolation length  $\lambda$  is adopted to describe the surface condition of the thin layer.<sup>14,15</sup> The interface condition stands for systems with ferromagnetic interface coupling.

The differential equations for magnetization are derived from (1) and (2) by the variational method

$$\begin{cases} A_1 m + B_1 m^3 - H_{\text{ext}} = C_1 \frac{\partial^2 m}{\partial z^2}, & 0 < z < L, \\ A_2 m + B_2 m^3 - H_{\text{ext}} = C_2 \frac{\partial^2 m}{\partial z^2}, & z < 0 \end{cases} \quad (4)$$

with the following interface and boundary conditions:

$$\begin{cases} \left. \frac{\partial m(\mathbf{r})}{\partial z} \right|_{z=L} = -\lambda^{-1} m(\mathbf{x}_{\parallel}, z=L), \\ \left. \frac{\partial m(\mathbf{r})}{\partial z} \right|_{z=-\infty} = 0, \\ m(\mathbf{x}_{\parallel}, z=+0) = m(\mathbf{x}_{\parallel}, z=-0), \\ C_1 \left. \frac{\partial m(\mathbf{r})}{\partial z} \right|_{z=+0} = C_2 \left. \frac{\partial m(\mathbf{r})}{\partial z} \right|_{z=-0}. \end{cases} \quad (5)$$

Involving the relation  $S(\mathbf{r}, \mathbf{r}') = \delta m(\mathbf{r}) / \delta H(\mathbf{r}')$ , we obtain the following equations for the spin-spin correlation functions above the critical point and in the absence of external field:

$$\begin{cases} -\Delta S(\mathbf{r}, \mathbf{r}') + \frac{1}{\xi_1^2} S(\mathbf{r}, \mathbf{r}') = \frac{1}{C_1} \delta(\mathbf{r} - \mathbf{r}'), & 0 < z < L, \\ -\Delta S(\mathbf{r}, \mathbf{r}') + \frac{1}{\xi_2^2} S(\mathbf{r}, \mathbf{r}') = 0, & z < 0 \end{cases} \quad (6)$$

for  $z' > 0$  and

$$\begin{cases} -\Delta S(\mathbf{r}, \mathbf{r}') + \frac{1}{\xi_1^2} S(\mathbf{r}, \mathbf{r}') = 0, & 0 < z < L, \\ -\Delta S(\mathbf{r}, \mathbf{r}') + \frac{1}{\xi_2^2} S(\mathbf{r}, \mathbf{r}') = \frac{1}{C_2} \delta(\mathbf{r} - \mathbf{r}'), & z < 0 \end{cases} \quad (7)$$

for  $z' < 0$ , with the boundary and interface conditions

$$\begin{cases} \left. \frac{\partial S(\mathbf{r}, \mathbf{r}')}{\partial z} \right|_{z=L} = -\lambda^{-1} S(\mathbf{r}, \mathbf{r}') \Big|_{z=L} \quad \text{and} \quad \left. \frac{\partial S(\mathbf{r}, \mathbf{r}')}{\partial z} \right|_{z=-\infty} = 0, \\ S(\mathbf{r}, \mathbf{r}') \Big|_{z=+0} = S(\mathbf{r}, \mathbf{r}') \Big|_{z=-0} \quad \text{and} \quad \left. \frac{\partial S(\mathbf{r}, \mathbf{r}')}{\partial z} \right|_{z=+0} = \left. \frac{\partial S(\mathbf{r}, \mathbf{r}')}{\partial z} \right|_{z=-0}, \end{cases} \quad (8)$$

where

$$\xi_1^{-2} = \frac{A_1}{C_1} \quad \text{and} \quad \xi_2^{-2} = \frac{A_2}{C_2}. \quad (9)$$

Both  $\xi_1^{-2}$  and  $\xi_2^{-2}$  can be positive and negative as well, according to the temperature.

Since the system is uniform in the directions parallel to the surface and the interface, one can perform a Fourier transformation for the correlation function in these directions. In this way, one can solve the above equations and arrive at

$$\begin{aligned} S(\mathbf{Q}_{\parallel}; z, z') &= \frac{1}{2C_1\gamma_1} \frac{(C_1\gamma_1 - C_2\gamma_2)\exp(-\gamma_1 z) + (C_1\gamma_1 + C_2\gamma_2)\exp(\gamma_1 z)}{(C_1\gamma_1 + C_2\gamma_2)(\gamma_1 + \lambda^{-1})/(\gamma_1 - \lambda^{-1}) - (C_1\gamma_1 - C_2\gamma_2)\exp(-2\gamma_1 L)} \\ &\quad \times \left\{ \exp[-\gamma_1(2L - z')] + \frac{\gamma_1 + \lambda^{-1}}{\gamma_1 - \lambda^{-1}} \exp(-\gamma_1 z') \right\} \end{aligned} \quad (10)$$

for  $0 \leq z \leq z' \leq L$ ,

$$S(\mathbf{Q}_{\parallel}; z, z') = \frac{(\gamma_1 + \lambda^{-1})/(\gamma_1 - \lambda^{-1})\exp(-\gamma_1 z + \gamma_2 z') + \exp[-\gamma_1(2L - z) + \gamma_2 z']}{(C_1\gamma_1 + C_2\gamma_2)(\gamma_1 + \lambda^{-1})/(\gamma_1 - \lambda^{-1}) - (C_1\gamma_1 - C_2\gamma_2)\exp(-2\gamma_1 L)} \quad (11)$$

for  $z' \leq 0 \leq z \leq L$ , and

$$S(\mathbf{Q}_{\parallel};z,z') = \frac{1}{2C_2\gamma_2} \exp(-\gamma_2|z-z'|) + \frac{1}{2C_2\gamma_2} \times \frac{(C_1\gamma_1 + C_2\gamma_2)\exp[-2\gamma_1L + \gamma_2(z+z')] - (C_1\gamma_1 - C_2\gamma_2)(\gamma_1 + \lambda^{-1})/(\gamma_1 - \lambda^{-1})\exp[\gamma_2(z+z')]}{(C_1\gamma_1 + C_2\gamma_2)(\gamma_1 + \lambda^{-1})/(\gamma_1 - \lambda^{-1}) - (C_1\gamma_1 - C_2\gamma_2)\exp(-2\gamma_1L)} \quad (12)$$

for  $z', z \leq 0$ , where

$$S(\mathbf{r}, \mathbf{r}') = \int \frac{d^{d-1}Q_{\parallel}}{(2\pi)^{d-1}} S(\mathbf{Q}_{\parallel};z,z') \exp[i\mathbf{Q}_{\parallel} \cdot (\mathbf{x}_{\parallel} - \mathbf{x}'_{\parallel})], \quad (13)$$

and  $\gamma_1^2 = \xi_1^{-2} + Q_{\parallel}^2$  and  $\gamma_2^2 = \xi_2^{-2} + Q_{\parallel}^2$ .

In order to clarify the critical behaviors of the system, we transform the above correlation functions back into real space. To this end, we make the following transformation:

$$\frac{1}{2\gamma} \exp(-\gamma|z|) \times \frac{1}{f(\gamma)} = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{\exp(ik|z|)}{k^2 + \gamma^2} \times \frac{1}{f(-ik)} - \frac{i}{2} \sum_n \frac{\exp(ik_n|z|)}{k_n^2 + \gamma^2} \left[ \frac{df(-ik)}{dk} \Big|_{k=k_n} \right]^{-1} - i \sum_l \frac{\exp(ik_l|z|)}{k_l^2 + \gamma^2} \left[ \frac{df(-ik)}{dk} \Big|_{k=k_l} \right]^{-1}, \quad (14)$$

where the summation on  $n$  is for the zero point of function  $f(-ik)$  just on the real axis of  $k$  space, and the one on  $l$  is for those in the upper-half  $k$  space. The above transformation reveals all the poles of the correlation function, which contains the information about the critical point of the system. Since the distribution and the expressions of the poles show complicated dependences on the magnetic constants, the layer thickness, and the extrapolation length  $\lambda$ , we will concentrate on the positive  $\lambda$  case in the present paper.

**A. Correlation function in the thin layer**

We first analyze the denominator of  $S(\mathbf{Q}_{\parallel};z,z')$  in (10):

$$\begin{aligned} &(-ik + C'_2\gamma_2) \frac{-ik + \lambda^{-1}}{-ik - \lambda^{-1}} - (-ik - C'_2\gamma_2)\exp(2ikL) \\ &= \sqrt{k^2 + (C'_2\gamma_2)^2} \exp[i(kL - \theta_1)] \\ &\quad \times 2i \sin(kL + \theta_1 + \theta_2), \end{aligned} \quad (15)$$

where  $C'_2 = C_2/C_1$  and

$$\theta_1 = \tan^{-1}(k/C'_2\gamma_2) \quad \text{and} \quad \theta_2 = \tan^{-1}(k\lambda). \quad (16)$$

Thus, the poles for  $S(\mathbf{Q}_{\parallel};z,z')$  should satisfy

$$kL + \tan^{-1} \frac{k}{C'_2\gamma_2} + \tan^{-1}(k\lambda) = n\pi, \quad \text{for } n = \pm 1, \pm 2, \dots \quad (17)$$

The above equation is solved graphically in Fig. 2, where the negative poles have been omitted from the symmetry. The distribution of poles is schematically shown in Fig. 3. The interval between the nearest-neighboring poles approaches  $\pi/L$  for large  $n$  from (17). There is no pole in the upper half  $k$  space for positive  $\lambda$ .

Knowing the poles, we proceed to make a transformation on the correlation functions  $S(\mathbf{Q}_{\parallel};z,z')$  in (10) according to (14). For example, one has

$$\begin{aligned} &\frac{1}{2\gamma_1} \frac{(C_1\gamma_1 - C_2\gamma_2)\exp[-\gamma_1(2L + z - z')]}{(C_1\gamma_1 + C_2\gamma_2)(\gamma_1 + \lambda^{-1})/(\gamma_1 - \lambda^{-1}) - (C_1\gamma_1 - C_2\gamma_2)\exp(-2\gamma_1L)} \\ &= \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{-\exp[ik(L + z - z') + i\theta_1 + i\theta_2]}{(k^2 + \gamma_1^2)2i \sin(kL + \theta_1 + \theta_2)} + \sum_{n=1,2,\dots} \frac{\cos[k_n(z - z')]}{2(L + \theta'_1 + \theta'_2)} \frac{1}{k_n^2 + \gamma_1^2}, \end{aligned} \quad (18)$$

where  $\theta'_1$  and  $\theta'_2$  are derivatives of  $\theta_1$  and  $\theta_2$  with respect to  $k$ . Similarly, one can evaluate the remaining part of (10) and arrive finally at

$$\begin{aligned} S(\mathbf{Q}_{\parallel};z,z') &= \frac{1}{C_1} \sum_{n=1,2,\dots} \frac{\cos[k_n(z - z')] - \cos[k_n(z + z') + 2\theta_1]}{L + \theta'_1 + \theta'_2} \times \frac{1}{k_n^2 + \gamma_1^2} \\ &= \frac{1}{C_1} \sum_{n=1,2,\dots} \frac{\sin(k_n z + \theta_1)\sin(k_n z' + \theta_1)}{L + \theta'_1 + \theta'_2} \times \frac{1}{k_n^2 + \gamma_1^2}. \end{aligned} \quad (19)$$

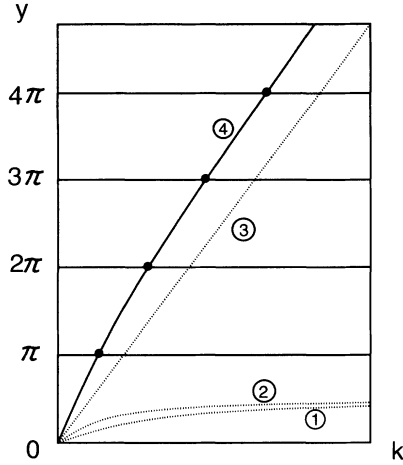


FIG. 2. Graphical solution for (17): curves 1 and 2:  $y = \tan^{-1}(k/C_2'\gamma_2)$  and  $y = \tan^{-1}(k\lambda)$ ; curve 3:  $y = kL$ , and curve 4 is the summation of curves 1, 2, and 3.

The integrals, such as that in (18), cancel with each other. As will be revealed more explicitly in the following, the complete cancellation among the integral terms is the responsible mechanism of the shift of critical point in the layer of finite thickness.

After the above reduction of the correlation function, we are ready to make an inverse Fourier transformation to obtain the following correlation function in real space:

$$S(\mathbf{r}, \mathbf{r}') = \frac{2}{C_1} \sum_{n=1,2,\dots} \frac{\sin(k_n z + \theta_1) \sin(k_n z' + \theta_1)}{L + \theta_1' + \theta_2'} \times G_{d-1}(\mathbf{x}_{\parallel} - \mathbf{x}'_{\parallel}, k_n^2 + \xi_1^{-2}), \quad (20)$$

where the function  $G_{d-1}(\mathbf{r}, t)$  is defined as<sup>16</sup>

$$G_d(\mathbf{r}, t) = \frac{\Gamma(d/2-1)}{4\pi^{d/2} |\mathbf{r}|^{d-2}} g_d(|\mathbf{r}| \sqrt{t}) \quad (21)$$

and

$$g_d(u) = \frac{1}{\Gamma(d/2-1)} \left[ \frac{u}{2} \right]^{d/2-1} K_{d/2-1}(u)$$

with the  $\Gamma$  function and the Bessel function  $K$ .

It is noted that the approximation  $\gamma_2 \simeq \xi_2^{-1}$  has been involved in the derivation of (20) from (19). This treatment is reasonable since we are concerned about the

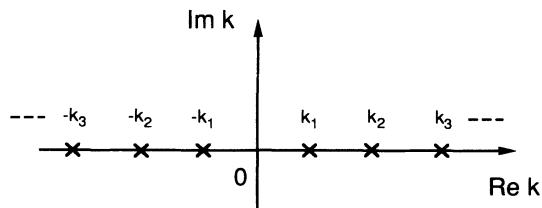


FIG. 3. Distribution of the poles for positive  $\lambda$ .

correlation function near the true mean-field critical point, where  $\xi_2^{-1}$  is generally finite and thus  $Q_{\parallel}^2$  in  $\gamma_2$  will contribute only as a higher-order infinitesimal in the asymptote of the correlation function.

Therefore, the critical point  $T_c$ , where the correlation length diverges, is determined from (20) by the relation  $k_1^2 + \xi_1^{-2} = 0$ , with  $k_1$  determined by (17) for  $n=1$  and  $\gamma_2 \simeq 1/\xi_2$ . As  $k_1^2 > 0$ , it is now transparent that  $T_c < T_{c1}$ . This inequality between the mean-field critical points is reasonable from the relation between the strengths of the exchange couplings on the top surface, in the layer and in the semi-infinite bulk.

Recalling the definitions of the correlation lengths  $\xi_1$  and  $\xi_2$  in (9), one arrives at the following equation for the critical point  $T_c$ :

$$L \sqrt{-A_1/C_1} = \cot^{-1} \sqrt{-A_1 C_1 / A_2 C_2} + \cot^{-1}(\lambda \sqrt{-A_1/C_1}). \quad (22)$$

This equation should be compared with (12) in Ref. 20 for a single layer with finite thickness. Then, one finds the explicit expression, such as the first term in the right-hand side (rhs) of (22), for the effect of interfacial coupling on the shift of the critical point.

The above analyses for the correlation function are established only in the temperature region where no magnetization exists in the total system. Thus, the critical point  $T_c$  derived as the solution of (22) is meaningful only if it is above the critical point  $T_{c2}$  of the semi-infinite bulk. From (22), the condition for  $T_c > T_{c2}$  is

$$L \sqrt{-A_1(T_{c2})/C_1(T_{c2})} > \cot^{-1}[\lambda \sqrt{-A_1(T_{c2})/C_1(T_{c2})}]. \quad (23)$$

This point will also be discussed from the correlation function within the semi-infinite bulk.

As will be revealed in what follows, the critical point  $T_c$  is also the temperature at which spontaneous magnetic ordering occurs in the thin layer and magnetization is induced in the semi-infinite bulk. We note that this coincidence is not trivial in the present bilayer system. Hereafter,  $T_c$  will be referred simply as the critical point of the system, although the bulk phase transition occurs at  $T_{c2}$ . The correlation function can only be evaluated analytically above  $T_c$  in the present inhomogeneous system.

There are two limits for the layer thickness, namely  $L \rightarrow 0$  and  $L \rightarrow \infty$ . They correspond to a semi-infinite system with a surface and a system consisting of two coupled semi-infinite bulks, respectively. The system in the limit of  $L \rightarrow 0$  is equivalent to the system obtained by putting the magnetic constants in the layer equal to those for the semi-infinite bulk. In this way, we obtain from (10)

$$S(Q_{\parallel}; z, z') = \frac{1}{2C_2\gamma_2} \exp[-\gamma_2(z' - z)] + \frac{1}{2C_2\gamma_2} \frac{\gamma_2 - \lambda^{-1}}{\gamma_2 + \lambda^{-1}} \exp[-\gamma_2(2L - z' - z)]. \quad (24)$$

This expression is nothing but (4.5) in Ref. 16, except for a trivial difference in the definition of the  $z$  axis.

In the limit  $L \rightarrow \infty$ , the interval between the nearest-neighbor poles in (17) approaches zero and one obtains the continuous integral path, the real axis of  $k$  space. Ex-

plicitly, we have from (17)

$$(L + \theta'_1 + \theta'_2)dk = \pi(n + 1 - n) = \pi, \quad \text{for } L \gg 1. \quad (25)$$

Then, the summation in (19) is replaced by the following integration:

$$\begin{aligned} S(\mathbf{Q}_{\parallel}; z, z') &= \frac{1}{C_1} \int_0^{\infty} \frac{dk}{\pi} \frac{\cos[k(z - z')] - \cos[k(z + z') + 2\theta_1]}{k^2 + \gamma_1^2} \\ &= \frac{1}{C_1} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{\exp[ik(z - z')]}{k^2 + \gamma_1^2} - \frac{1}{C_1} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{\exp[ik(z + z') + i2\theta_1]}{k^2 + \gamma_1^2}. \end{aligned} \quad (26)$$

This expression is equivalent to (4.5) in Ref. 16. From the correspondence between these two expressions, it is found that the effect on the top bulk from the bottom one can be described by an effective extrapolation length  $C_1 \xi_2 / C_2$ . The present effective extrapolation length is positively finite as far as the temperature is above  $T_{c2}$  and diverges with the critical exponent  $\nu = \frac{1}{2}$  for the correlation length. However, the effect of the finite layer on the semi-infinite bulk is significantly different, as will be revealed in what follows. As  $L \rightarrow \infty$ , the minimal pole approaches zero and the critical point is then restored to  $T_{c1}$ , as it should be by definition.

### B. Correlation function in the bulk

There are two terms in (12). The former describes the direct correlation and has been discussed in previous publications.<sup>16</sup> As for the second term, it is found that there are three typical temperatures:

$$\begin{cases} \lambda \sqrt{A'_1(T_{[1]} - T_{c1})} / C_1 = 1, \\ L \sqrt{A'_1(T_{c1} - T_{[2]})} / C_1 + \tan^{-1} \lambda \sqrt{A'_1(T_{c1} - T_{[2]})} / C_1 = \frac{\pi}{2}, \\ L \sqrt{A'_1(T_{c1} - T_{[3]})} / C_1 + \tan^{-1} \lambda \sqrt{A'_1(T_{c1} - T_{[3]})} / C_1 = \pi, \end{cases} \quad (27)$$

which satisfy  $T_{[3]} < T_{[2]} < T_{c1} < T_{[1]}$ . There is no pole in the upper-half  $k$  space in the temperature region  $T > T_{[2]}$ , while there is one for  $T < T_{[2]}$ .

With the approximation

$$\frac{1}{C_2} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{\exp[-ik(z + z')]}{k^2 + Q_{\parallel}^2 + \xi_2^{-2}} \frac{k - i\kappa}{k + i\kappa} \simeq \frac{-1}{C_2} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{\exp[-ik(z + z') + i2k\kappa^{-1}]}{k^2 + Q_{\parallel}^2 + \xi_2^{-2}}, \quad (28)$$

we have from (12)

$$S(\mathbf{r}, \mathbf{r}') \simeq \begin{cases} \frac{1}{C_2} G_d(\mathbf{r} - \mathbf{r}', \xi_2^{-2}) - \frac{1}{C_2} G_d[\mathbf{r} - \mathbf{vr}' - \hat{\mathbf{z}} 2C'_2 \xi_1 \coth(\beta + L \xi_1^{-1}), \xi_2^{-2}], & T \geq T_{[1]}, \\ \frac{1}{C_2} G_d(\mathbf{r} - \mathbf{r}', \xi_2^{-2}) - \frac{1}{C_2} G_d[\mathbf{r} - \mathbf{vr}' - \hat{\mathbf{z}} 2C'_2 \xi_1 \tanh(\tilde{\beta} + L \xi_1^{-1}), \xi_2^{-2}], & T_{[1]} \geq T \geq T_{c1}, \\ \frac{1}{C_2} G_d(\mathbf{r} - \mathbf{r}', \xi_2^{-2}) - \frac{1}{C_2} G_d[\mathbf{r} - \mathbf{vr}' - \hat{\mathbf{z}} 2C'_2 |\xi_1| \tan(\theta + L |\xi_1|^{-1}), \xi_2^{-2}], & T_{c1} \geq T > T_{[2]}, \end{cases} \quad (29)$$

where the notation of  $\mathbf{vr} = (\mathbf{x}_{\parallel}, -z)$  in Ref. 16 is used and

$$\beta = \coth^{-1} \frac{\lambda}{\xi_1}, \quad \tilde{\beta} = \tanh^{-1} \frac{\lambda}{\xi_1}, \quad \text{and } \theta = \tan^{-1} \frac{\lambda}{|\xi_1|}. \quad (30)$$

The above expressions of correlation function hold when  $|\mathbf{r} - \mathbf{vr}'|$  is much larger than the substractors in the arguments of the second terms. For  $T_{[2]} > T > T_{[3]}$ , we have

$$\begin{aligned}
S(\mathbf{r}, \mathbf{r}') \simeq & \frac{1}{C_2} G_d(\mathbf{r} - \mathbf{r}', \xi_2^{-2}) - \frac{1}{C_2} G_d[\mathbf{r} - \mathbf{r}' + \hat{\mathbf{z}} 2C_2' |\xi_1| \cot(\theta + L |\xi_1|^{-1} - \pi/2), \xi_2^{-2}] \\
& + \frac{2}{C_2} C_1' |\xi_1|^{-1} \tan(\theta + L |\xi_1|^{-1} - \pi/2) \exp[(z + z') C_1' |\xi_1|^{-1} \tan(\theta + L |\xi_1|^{-1} - \pi/2)] \\
& \times G_{d-1}\{\mathbf{x}_\parallel - \mathbf{x}'_\parallel, \xi_2^{-2} - [C_1' |\xi_1|^{-1} \tan(\theta + L |\xi_1|^{-1} - \pi/2)]^2\}, \quad (31)
\end{aligned}$$

with  $C_1' = C_1/C_2$ .

The critical point  $T_c$  is identified as the following. In the case  $T_{c2} > T_{[2]}$ , one has  $T_c = T_{c2}$  from (29) and the phase transition is the bulk one. This is the same conclusion as that given in Sec. II A. In the case  $T_{[2]} > T_{c2} > T_{[3]}$ ,  $\xi_2^{-2}$  drops from a positive value to zero as temperature decreases from  $T_{[2]}$  to  $T_{c2}$ . Meanwhile,  $\tan(\theta + L |\xi_1|^{-1} - \pi/2)$  increases from zero to positive. Thus, there is a temperature between  $T_{[2]}$  and  $T_{c2}$ , where

$$\xi_2^{-1} = \frac{C_1'}{|\xi_1|} \tan \left[ \theta + \frac{L}{|\xi_1|} - \frac{\pi}{2} \right] \quad (32)$$

and the correlation function in (31) changes its asymptote.

The above relation is equivalent to (22). Therefore, the temperature thus determined is the critical point  $T_c$  derived in the discussion for the correlation function within the thin layer. It is then concluded  $T_{[2]} > T_c > T_{c2}$ . The correlation length in the semi-infinite bulk parallel to the interface diverges as the one in the thin layer diverges. In the case  $T_{c2} < T_{[3]}$ , one finds that as the temperature decreases from  $T_{[2]}$  to  $T_{[3]}$ ,  $\tan(\theta + L |\xi_1|^{-1} - \pi/2)$  increases from zero to infinite, while  $\xi_2^{-2}$  remains finite in this region. Then we arrive at  $T_{[2]} > T_c > T_{[3]} > T_{c2}$ . Recall that the expressions of the correlation function for temperatures below the critical point  $T_c$  are meaningless.

### C. Correlation function between subsystems

The correlation function (11) is equivalent to

$$S(\mathbf{Q}_\parallel; z, z') = 2 \frac{\partial}{\partial z'} \left\{ \frac{(\gamma_1 + \lambda^{-1})/(\gamma_1 - \lambda^{-1}) \exp(-\gamma_1 z) + \exp[-\gamma_1(2L - z)]}{(C_1 \gamma_1 + C_2 \gamma_2)(\gamma_1 + \lambda^{-1})/(\gamma_1 - \lambda^{-1}) - (C_1 \gamma_1 - C_2 \gamma_2) \exp(-2\gamma_1 L)} \times \frac{\exp(\gamma_2 z')}{2\gamma_2} \right\}. \quad (33)$$

The expression of correlation function for  $T \geq T_{[1]}$  is given as

$$S(\mathbf{r}, \mathbf{r}') \simeq \frac{2\xi_1}{C_1} \frac{\cosh[\beta + (L - z)\xi_1^{-1}]}{\sinh(\beta + L\xi_1^{-1})} \frac{\partial}{\partial z'} G_d\{\mathbf{x}_\parallel - \mathbf{x}'_\parallel + \hat{\mathbf{z}}[z' - C_2' \xi_1 \coth(\beta + L\xi_1^{-1})], \xi_2^{-2}\}, \quad (34)$$

where the approximation  $1/(a + ik) \simeq \exp(-ik/a)/a + O(k^2)$  has been involved.

The subtractions in (29) are replaced by a derivative operation in the above correlation function. This results in the same critical exponent for the two correlation functions as will be discussed later.

Similarly, we have for  $T_{[1]} \geq T \geq T_{c1}$

$$S(\mathbf{r}, \mathbf{r}') \simeq \frac{2\xi_1}{C_1} \frac{\sinh[\tilde{\beta} + (L - z)\xi_1^{-1}]}{\cosh(\tilde{\beta} + L\xi_1^{-1})} \frac{\partial}{\partial z'} G_d\{\mathbf{x}_\parallel - \mathbf{x}'_\parallel + \hat{\mathbf{z}}[z' - C_2' \xi_1 \tanh(\tilde{\beta} + L\xi_1^{-1})], \xi_2^{-2}\}, \quad (35)$$

for  $T_{c1} \geq T > T_{[2]}$

$$S(\mathbf{r}, \mathbf{r}') \simeq \frac{2|\xi_1|}{C_1} \frac{\sin[\theta + (L - z)|\xi_1|^{-1}]}{\cos(\theta + L|\xi_1|^{-1})} \frac{\partial}{\partial z'} G_d\{\mathbf{x}_\parallel - \mathbf{x}'_\parallel + \hat{\mathbf{z}}[z' - C_2' |\xi_1| \tan(\theta + L|\xi_1|^{-1})], \xi_2^{-2}\}, \quad (36)$$

and finally for  $T_{[2]} > T \geq T_{[3]}$

$$\begin{aligned}
S(\mathbf{r}, \mathbf{r}') \simeq & \frac{2|\xi_1|}{C_1} \frac{\sin[\theta + (L - z)|\xi_1|^{-1}]}{\cos(\theta + L|\xi_1|^{-1})} \frac{\partial}{\partial z'} G_d\{\mathbf{x}_\parallel - \mathbf{x}'_\parallel + \hat{\mathbf{z}}[z' + C_2' |\xi_1| \cot(\theta + L|\xi_1|^{-1} - \pi/2)], \xi_2^{-2}\} \\
& + \frac{2}{C_2} \frac{\sin[\theta + (L - z)|\xi_1|^{-1}]}{\sin(\theta + L|\xi_1|^{-1})} G_{d-1}\{\mathbf{x}_\parallel - \mathbf{x}'_\parallel, \xi_2^{-2} - [C_1' |\xi_1|^{-1} \tan(\theta + L|\xi_1|^{-1} - \pi/2)]^2\} \\
& \times C_1' |\xi_1|^{-1} \tan(\theta + L|\xi_1|^{-1} - \pi/2) \exp[z' C_1' |\xi_1|^{-1} \tan(\theta + L|\xi_1|^{-1} - \pi/2)]. \quad (37)
\end{aligned}$$

As pointed out previously, the expressions of correlation functions are meaningless for temperatures below the critical point.

As revealed so far, the correlation functions change their expressions as temperature varies. This is a characteristic feature of the multilayer structures.

#### D. Critical behaviors of correlation functions

Knowing the expression of the correlation function and the critical point, we are now ready to investigate the critical phenomena in the system. Let us start to investigate the correlation function (20) for  $0 \leq z < z' \leq L$  and  $\lambda > 0$ . Near the critical point determined by (22), the first term in the summation in (20) becomes dominant. Its asymptotic behavior changes from an exponential decay to a power-law one at the critical point. The divergence of the correlation length parallel to the interface  $\xi_{\parallel,1}^{-1} = \sqrt{k_1^2 + \xi_1^{-2}}$  is ready to be evaluated from (21) as  $\xi_{\parallel,1}^{-1} \sim \sqrt{T/T_c - 1}$  for  $T \geq T_c$ . The critical exponent is  $\nu = \frac{1}{2}$ .

At the critical point, one has

$$S(\mathbf{r}, \mathbf{r}') \simeq \frac{2}{C_1} \frac{\sin(k_1 z + \theta_1) \sin(k_1 z' + \theta_1)}{L + \theta_1 + \theta_2} \times \frac{\Gamma[(d-3)/2]}{|\mathbf{x}_{\parallel} - \mathbf{x}'_{\parallel}|^{d-3}}, \quad (38)$$

as  $|\mathbf{x}_{\parallel} - \mathbf{x}'_{\parallel}| \rightarrow \infty$ .

Thus, one arrives at the critical exponent  $\eta_{\parallel,1} = -1$ , the same as that in (4.29) in Ref. 16.

Since  $0 < k_1 z + \theta_1 \leq k_1 L + \theta_1 = \pi - \theta_2$  at the critical point and  $\theta_2 > 0$ , we have  $\sin(k_1 z + \theta_1) \sin(k_1 z' + \theta_1) > 0$ . Therefore, the correlation function is positive, as it should be in the present ferromagnetic system. Two linear asymptotes are derived from (38):

$$\begin{cases} S((\mathbf{x}_{\parallel}, z), (\mathbf{x}_{\parallel}, L)) \sim \sin\theta_1 \sin\theta_2 \\ + k_1 \cos\theta_1 \sin\theta_2 \times z, \quad \text{for } z \geq 0, \\ S((\mathbf{x}_{\parallel}, 0), (\mathbf{x}_{\parallel}, z)) \sim \sin\theta_1 \sin\theta_2 \\ + k_1 \sin\theta_1 \cos\theta_2 \times (L - z), \quad \text{for } z \leq L \end{cases} \quad (39)$$

near the top surface and the interface.

As for the correlation function within the semi-infinite bulk, it is found from (29) that

$$\begin{cases} S(\mathbf{r}, \mathbf{0}) \sim \frac{1}{|\mathbf{r}|^{d-1}}, \quad \text{as } |\mathbf{r}| \rightarrow \infty, \\ S((\mathbf{x}_{\parallel}, 0), \mathbf{0}) \sim \frac{1}{|\mathbf{x}_{\parallel}|^d}, \quad \text{as } |\mathbf{x}_{\parallel}| \rightarrow \infty, \end{cases} \quad (40)$$

which state  $\eta_{1,2} = 1$  and  $\eta_{\parallel,2} = 2$ , respectively. They are the same as those in (4.21) of Ref. 16.

Comparing the expression of the correlation function in (4.19) in Ref. 16 and those in (29) for the present system, one finds that the effect from the thin layer to the semi-infinite bulk can be expressed effectively by an extrapolation length. Extrapolation length has been incor-

porated *a priori* in the models for semi-infinite systems up to now and the type of phase transition is classified by the value of this parameter.<sup>14-17</sup> However, in the present system, the effective extrapolation length at the interface shows a clear temperature dependence. This effect manifests itself most significantly in the correlation function (31). The third term in (31) behaves as  $\exp(-|z+z'|/\bar{\lambda})/\bar{\lambda}$  in the vertical direction, where  $\bar{\lambda}$  is the effective extrapolation length and is given explicitly by

$$\bar{\lambda} = |\xi_1| \cot \left[ \tan^{-1} \frac{\lambda}{|\xi_1|} + \frac{L}{|\xi_1|} - \frac{\pi}{2} \right], \quad (41)$$

comparing the third term of (31) with (4.28) of Ref. 16.

From (27), it is found that the effective extrapolation length  $\bar{\lambda}$  diverges as temperature approaches  $T_{[2]}$ . The divergence of this extrapolation length is characterized by a new exponent  $\tilde{\nu}$

$$\bar{\lambda} \sim (1 - T/T_{[2]})^{-\tilde{\nu}}, \quad T \nearrow T_{[2]}, \quad (42)$$

with  $\tilde{\nu} = 1$ . This exponent should be compared with the well-known critical exponent  $\nu = \frac{1}{2}$  for the correlation length.

Since  $|\tan[\tan^{-1}(\lambda/|\xi_1|) + L/|\xi_1|]| \rightarrow \infty$  as  $T \rightarrow T_{[2]}$ , the last asymptote in (29) and the second term in (31) become invalid. It is not difficult to obtain the following correlation function at  $T = T_{[2]}$ :

$$S(\mathbf{r}, \mathbf{r}') = \frac{1}{C_2} G_d(\mathbf{r} - \mathbf{r}', \xi_2^{-2}) + \frac{1}{C_2} G_d(\mathbf{r} - \nu \mathbf{r}', \xi_2^{-2}). \quad (43)$$

As the thickness  $L$  of the layer approaches infinity, the typical temperatures  $T_{[2]}$  and  $T_{[3]}$  coincide with  $T_{c1}$  as seen from (27). The correlation function in this limiting case can be seen in (26), where  $\theta_1 \simeq kC_1' \xi_2$  for small  $k$ . The length  $C_1' \xi_2$  diverges at  $T = T_{c2}$  with the critical exponent  $\nu = \frac{1}{2}$  for the correlation length. Therefore, the divergence of the interfacial extrapolation length with the exponent  $\tilde{\nu} = 1$  at a temperature above the critical point is a characteristic phenomenon in systems where a layer with finite thickness is present.

Finally, we discuss the correlation functions for  $z' \leq 0 \leq z \leq L$ . The asymptotic behaviors  $z \rightarrow 0$  and  $z \rightarrow L$  are similar to those given in (39). From (36), one has

$$\begin{aligned} S((\mathbf{x}_{\parallel}, z), (\mathbf{x}_{\parallel}, z')) & \sim \frac{\partial}{\partial z'} G_d \{ \hat{\mathbf{z}}[z' - C_2' |\xi_1| \tan(\theta + L |\xi_1|^{-1})], \xi_2^{-2} \} \\ & \sim \frac{1}{|z' - C_2' |\xi_1| \tan(\theta + L |\xi_1|^{-1})|^{d-1}}, \\ & \text{as } z' \rightarrow -\infty. \end{aligned} \quad (44)$$

Thus, one arrives at  $\eta_{1,2} = 1$ , same as that for the correlation function for  $z, z' \leq 0$ . The other critical exponents are also the same with those obtained previously.<sup>14-16</sup>

#### E. Susceptibilities

Susceptibilities can be evaluated from the corresponding correlation functions via the following relations:<sup>16</sup>



$$\begin{cases} \chi(z, z') = \int d^{d-1} \mathbf{x}' S(\mathbf{r}, \mathbf{r}') = S(\mathbf{0}; z, z'), \\ \chi(z) = \int d^d \mathbf{r}' S(\mathbf{r}, \mathbf{r}') = \int dz' S(\mathbf{0}; z, z'). \end{cases} \quad (45)$$

Therefore, we have from (19)

$$\begin{aligned} \chi(z, z') &= \frac{1}{C_1} \sum_{n=1,2,\dots} \frac{\sin(k_n z + \theta_1) \sin(k_n z' + \theta_1)}{L + \theta_1' + \theta_2'} \\ &\quad \times \frac{1}{k_n^2 + \xi_1^2} \\ &\sim \frac{1}{k_1^2 + \xi_1^2} \sim \frac{1}{T - T_c}, \quad \text{as } T \rightarrow T_c, \end{aligned} \quad (46)$$

for  $0 \leq z, z' \leq L$  where  $T_c$  is determined in (22). The critical exponent for the susceptibility is  $\gamma = 1$ . In the same way we can evaluate other susceptibilities. It is not difficult to see that the critical exponents are same with those in Refs. 14, 15, and 16. The evaluation for susceptibility  $\chi(z)$  is straightforward, although involved.

We notice that the susceptibility does not diverge at  $T_{[2]}$ , since there exists a prefactor  $\bar{\lambda}^{-1}$  in the third term of (31). Although no new critical exponent can be obtained for the susceptibility in the present bilayer system, experimental measurement for it is still very important which identifies the critical point of the multilayer system.

### III. SPONTANEOUS MAGNETIZATION

In order to integrate the simultaneous differential equations (4) for  $H_{\text{ext}} = 0$ , we use the magnitude of magnetization at the surface,  $m_L \equiv m(z=L)$ , as a parameter. One then obtains from the first equation (4)

$$\begin{aligned} \frac{1}{2} C_1 \left[ \frac{\partial m}{\partial z} \right]^2 &= \frac{1}{2} A_1 m^2 + \frac{1}{4} B_1 m^4 \\ &\quad - \left( \frac{1}{2} A_1 m_L^2 + \frac{1}{4} B_1 m_L^4 \right) + \frac{1}{2} C_1 \left[ \frac{\partial m}{\partial z} \right]^2 \Big|_{z=L} \\ &= \frac{1}{2} A_1 m^2 + \frac{1}{4} B_1 m^4 \\ &\quad - \left( \frac{1}{2} A_1 m_L^2 + \frac{1}{4} B_1 m_L^4 \right) + \frac{1}{2} C_1 \lambda^{-2} m_L^2, \end{aligned} \quad (47)$$

where the first condition in (5) has been involved in the last step.

From the discussion in the foregoing section, it is clear that the critical point  $T_c$  of the system is below  $T_{c1}$ . As far as the critical phenomena of the system are concerned, we can normalize the lengths, such as  $L$  and  $\lambda$ , and magnetizations by  $\sqrt{C_1 / -A_1}$  and  $\sqrt{-A_1 / B_1}$ , respectively. Hereafter, we denote these normalized quantities by a tilde.

There is a maximal magnitude of spontaneous magnetization in the thin layer for the positive  $\lambda$  case. This maximum  $\tilde{m}_*$  is given by

$$\tilde{m}_*^2 = 1 - \sqrt{1 - \tilde{a}^4}, \quad (48)$$

where  $\tilde{a}^4 = -\tilde{m}_L^4 + 2\tilde{m}_L^2 + 2\tilde{\lambda}^{-2}\tilde{m}_L^2$ .

The magnetization in the thin layer can be expressed using these quantities. Especially, one has

$$\frac{\tilde{L}}{\sqrt{2}} = \int_{\tilde{m}_0}^{\tilde{m}^*} \frac{d\tilde{m}}{\sqrt{\tilde{m}^4 - 2\tilde{m}^2 + \tilde{a}^4}} + \int_{\tilde{m}_L}^{\tilde{m}^*} \frac{d\tilde{m}}{\sqrt{\tilde{m}^4 - 2\tilde{m}^2 + \tilde{a}^4}}. \quad (49)$$

The above integral depends strongly on the value of  $\tilde{a}^4$ , which in turn is determined by the value of  $\tilde{m}_L$ . There are five regions for  $\tilde{m}_L$ , as shown in Fig. 4, which correspond to different values of  $\tilde{a}^4$  and thus one has different final expressions for (49).

In the region (a), where  $0 \leq \tilde{m}_L \leq (\sqrt{4 + 2/\tilde{\lambda}^2} - \sqrt{2/\tilde{\lambda}})/2$ , (49) is integrated and the expression for  $\tilde{m}_0$  as a function of  $\tilde{m}_L$  can be obtained in terms of the Jacobian elliptic functions as

$$\tilde{m}_0 = \mu \operatorname{sn}[x, k] \quad (50)$$

with

$$\begin{cases} \mu^2 = 1 - \sqrt{1 - \tilde{a}^4}, \quad \nu^2 = 1 + \sqrt{1 - \tilde{a}^4}, \quad k^2 = \frac{\mu^2}{\nu^2}, \\ x = 2F\left[\frac{\pi}{2}, k\right] - F\left[\arcsin\left[\frac{\tilde{m}_L}{\mu}\right], k_1\right] - \frac{1}{\sqrt{2}} \tilde{L} \nu. \end{cases} \quad (51)$$

The function  $F[x, k]$  is the first type of elliptic integral. It is ready to see that  $\mu$  is proportional to  $\tilde{m}_L$  near the critical point where  $\tilde{m}_L$  is small.

The first derivative is evaluated as

$$\begin{aligned} C_1 \frac{dm}{dz} \Big|_{z=+0} &= \sqrt{B_1 C_1 / 2} \frac{-A_1}{B_1} \sqrt{(\mu^2 - \tilde{m}_0^2)(\nu^2 - \tilde{m}_0^2)} \\ &= \sqrt{B_1 C_1 / 2} \frac{-A_1}{B_1} \frac{\mu}{k} \operatorname{cn}[x, k] \operatorname{dn}[x, k]. \end{aligned} \quad (52)$$

On the other hand, from (4) and (5) the first derivative at the interface from the semi-infinite bulk side is

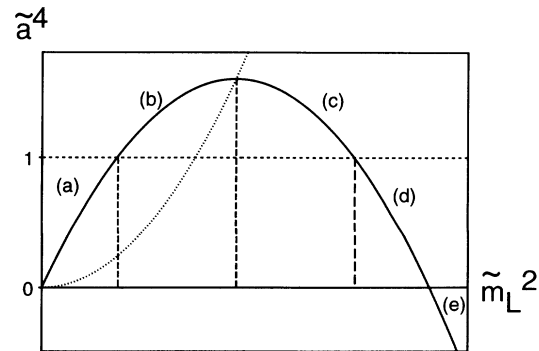


FIG. 4. Quantity  $\tilde{a}^4$  as the function of  $\tilde{m}_L$ . There are five regions for the value of  $\tilde{m}_L$ . The dot curve is for  $\tilde{a} = \tilde{m}_L$ .

$$C_2 \frac{dm}{dz} \Big|_{z=-0} = \sqrt{\frac{B_2 C_2}{2} \frac{-A_1}{B_1}} \times \bar{m}_0 \sqrt{\bar{m}_0^2 + 2(A_2/B_2)/(-A_1/B_1)}. \tag{53}$$

Finally, we obtain from the last condition in (5) the following equation for the magnetization at the surface  $\bar{m}_L$ :

$$\sqrt{B_2 C_2 / B_1 C_1} = \frac{\nu}{\sqrt{\mu^2 \text{sn}^2[x, k] + 2(A_2/B_2)/(-A_1/B_1)}} \times \frac{\text{cn}[x, k] \text{dn}[x, k]}{\text{sn}[x, k]}. \tag{54}$$

It is strongly nonlinear and involves the Jacobian elliptic functions. However, since the left hand side (lhs) of the above equation is independent from the thin-layer thickness, it is very convenient for discussions about the thickness dependence of the system. It can be reduced to a very simple equation for the critical points of the system.

Similarly, we can integrate (49) in the other regions of  $\bar{m}_L$ . Now we are ready to investigate the temperature and thin-layer thickness dependences of the spontaneous magnetization in the system. For this purpose, we define a function  $y(\bar{m}_L, \tilde{L})$  by the rhs of (54) and corresponding ones in other regions:

$$\sqrt{B_2 C_2 / B_1 C_1} = y(\bar{m}_L, \tilde{L}). \tag{55}$$

Two typical curves are shown in Fig. 5. We notice the following features for function  $y(\bar{m}_L, \tilde{L})$ : First, it is a monotonic decreasing function of  $\bar{m}_L$  and assume its maximum at  $\bar{m}_L=0$ . Second, the maximal value increases with  $L$ . Therefore, for a system of  $\sqrt{B_2 C_2 / B_1 C_1} = 0.6$ , for example, Eq. (55) has no solution at all for the system with the smaller thickness  $\tilde{L} = 1.0$ , since there is no crosspoint between  $y = 0.6$  and

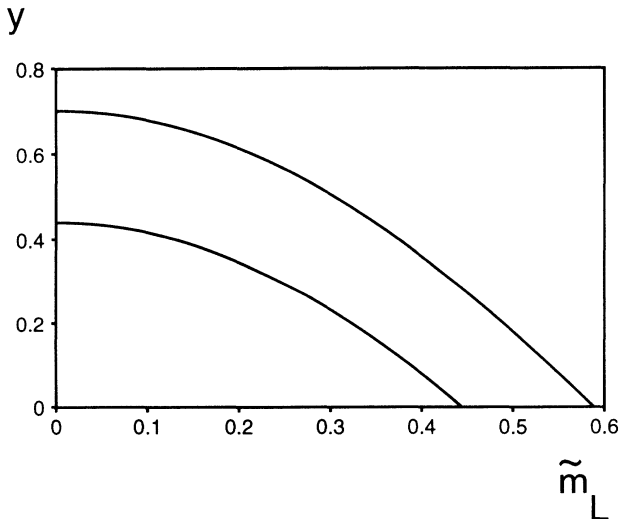


FIG. 5. Dependence of  $y(\bar{m}_L, \tilde{L})$  in (55) with  $L\sqrt{-A_1/C_1} = 1.2$  (upper curve) and 1.0 (lower curve),  $\lambda\sqrt{-A_1/C_1} = 1.5$  and  $(A_2/B_2)/(-A_1/B_1) = 1$ .

the lower curve in Fig. 5. This situation corresponds to  $m_L=0$ , and thus  $m(z)=0$  in the total system. On the other hand, there is a finite solution of  $\bar{m}_L$  for (55) for the system with the larger thickness  $\tilde{L} = 1.2$ . This indicates that there exists spontaneous magnetization in the system. Thus, there is a phase transition from a paramagnetic phase to a ferromagnetic phase of the system, as the thickness  $L$  increases. This phenomenon should be observed during an epitaxial growth process, where the temperature of the system is fixed at adequate value.

Now let us evaluate the critical thickness of the phase transition. As  $\bar{m}_L \rightarrow 0$ , one has  $\mu \rightarrow 0, \nu \rightarrow \sqrt{2}, k \rightarrow 0$  and

$$\frac{\bar{m}_L}{\mu_1} \rightarrow \frac{1}{\sqrt{1+1/\tilde{\lambda}^2}} \tag{56}$$

from (51). With the relations  $\text{sn}[x, k] \rightarrow \sin x, \text{cn}[x, k] \rightarrow \cos x, \text{dn}[x, k] \rightarrow 1$ , and  $F[x, k] \rightarrow x$  as  $k \rightarrow 0$ , we derive from (54) the following equation for the critical thickness  $L_c$ :

$$L_c \sqrt{-A_1/C_1} = \cot^{-1} \sqrt{-A_1 C_1 / A_2 C_2} + \cot^{-1}(\lambda \sqrt{-A_1/C_1}). \tag{57}$$

It is not difficult to see that transition from the paramagnetic phase to the ferromagnetic phase takes place also as the temperature is reduced, while the thickness  $L$  is fixed. Giving the temperature dependences of the magnetic parameters and fixing the thickness of the thin layer at a definite value, the above equation serves as the equation for the critical temperature for the system. Equation (57) coincides with (22) from the discussion on correlation functions. Therefore, we have shown explicitly that the temperatures where the correlation length diverges and where the spontaneous symmetry breaking occurs are the same. This coincidence is not trivial in multilayer structures, such as the bilayer system studied in the present paper. Knowing  $m_L$ , one can evaluate the magnetization profile  $m(z)$  easily. The magnetization profile is displayed in Fig. 6.

Let us investigate the critical behavior of spontaneous magnetization according to the thickness  $L$ . For  $L \geq L_c$  the magnetization  $\bar{m}_L$  should be very small. Paying attention to the fact that  $k \sim O(\bar{m}_L)$  as  $\bar{m}_L \rightarrow 0$  in (51), one can expand the rhs of (54) for  $L \geq L_c$  up to the second order of  $\bar{m}_L$  as

$$\bar{m}_L^2 \sim \sqrt{-A_1 B_2 / A_2 B_1} \cot(\arctan \tilde{\lambda} - \tilde{L}) - \sqrt{B_2 C_2 / B_1 C_1}, \tag{58}$$

where the relations

$$\begin{cases} \text{sn}[x, k] \approx \sin x - \frac{1}{4} k^2 (x - \frac{1}{2} \sin 2x) \cos x, \\ \text{cn}[x, k] \approx \cos x + \frac{1}{4} k^2 (x - \frac{1}{2} \sin 2x) \sin x, \\ \text{dn}[x, k] \approx 1 - \frac{1}{2} k^2 \sin^2 x, \end{cases} \tag{59}$$

for  $0 < k^2 \ll 1$  have been used. The expansion (58) is consistent with Fig. 5, where the curves show parabolic forms near  $\bar{m}_L=0$ . From (57) and (58), we have

$$\bar{m}_L^2 \sim \tilde{L} - \tilde{L}_c, \quad L \geq L_c. \tag{60}$$

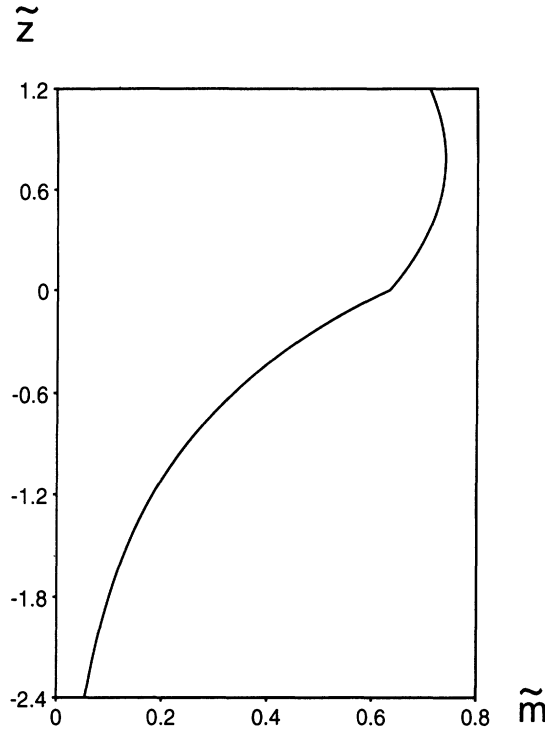


FIG. 6. Magnetization profile with  $L\sqrt{-A_1/C_1} = 1.2$ ,  $\lambda\sqrt{-A_1/C_1} = 5.0$ ,  $\sqrt{(B_2C_2)/(B_1C_1)} = 0.4$ , and  $(A_2/B_2)/(-A_1/B_1) = 1$ .

Thus, the critical behavior for  $m_L$  is

$$m_L \sim \sqrt{L/L_c - 1}, \quad L \geq L_c. \quad (61)$$

It is easy to see that the magnetization in the whole system shows the same critical behavior, with amplitude varying with the position. This critical exponent  $\beta_L = \frac{1}{2}$  for the spontaneous magnetization according to the variance of the layer thickness is derived for the first time. The critical behavior in (61) is schematically shown in Fig. 7.

Regarding (57) as the equation for the critical temperature, one can obtain the following critical behavior for the spontaneous magnetization:

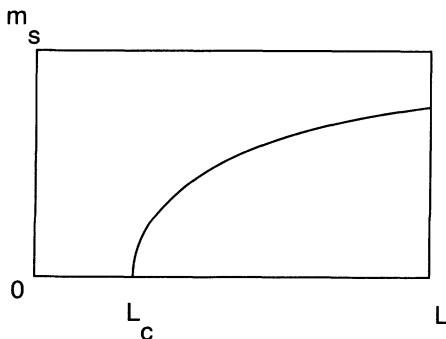


FIG. 7. Schematic critical behavior of magnetization with  $\beta_L = \frac{1}{2}$  as  $L$  increases and  $T$  fixed.

$$m_s(z) \sim \sqrt{T_c/T - 1}, \quad T \leq T_c. \quad (62)$$

The critical exponent is  $\beta_T = \frac{1}{2}$ . In the present mean-field theory, the above two critical exponents for the spontaneous magnetization coincide with each other.

#### IV. SUMMARY

We have studied the critical phenomena in the ferromagnetic system consisting of a layer with finite thickness coupled to a semi-infinite bulk in terms of Ginzburg-Landau free-energy functional formalism. Continuity of the magnitude of magnetization at the interface is assumed and the top surface of the thin layer is characterized by an extrapolation length. The exchange coupling within the layer is supposed to be stronger than that within the semi-infinite bulk. Therefore, as temperature drops, magnetic ordering occurs first in the layer and magnetization is induced in the semi-infinite bulk. Bulk phase transition will occur as the temperature is reduced further.

In the GL free-energy functional formalism, two critical points  $T_{c1}$  and  $T_{c2}$ ,  $T_{c1} > T_{c2}$ , are introduced for the materials in the layer and the semi-infinite bulk, respectively. Although  $T_{c2}$  describes the phase transition in the semi-infinite bulk appropriately,  $T_{c1}$  is correct only in the infinite layer-thickness limit. In order to eliminate this fictitious critical point for the present system with a finite layer and to find the true one, we have evaluated explicitly the correlation function within the layer. We have observed a complete cancellation among the integrals which show singularities at  $T_{c1}$ , as far as the thickness of the layer is finite. The true critical point is then determined up to the nonlinear equation, which is satisfied by the coefficients of the GL free-energy functional and the layer thickness. The shift of mean-field critical point for magnetic layer with finite thickness is studied from the correlation function.

From the correlation function within the semi-infinite bulk, we have found that an effective extrapolation length can be defined at the interface, which expresses the effect from the thin layer. This extrapolation length shows strong temperature dependence and diverges at a temperature above the critical point. The divergence is described by a new exponent  $\tilde{\nu} = 1$ , which should be compared with the well-known critical exponent  $\nu = \frac{1}{2}$  for the correlation length. This phenomenon is characteristic of systems in which a layer with finite thickness is coupled to a semi-infinite bulk, and is observed by the present authors. The susceptibility remains finite at this temperature.

The correlation functions in the present bilayer system can be expressed analytically. They change the detailed expression at several typical temperatures which, in turn, are solutions of nonlinear equations. Evaluation of correlation functions below the critical point is intractable analytically in the present inhomogeneous structure, since the magnitude of magnetization in the thin layer and that induced in the semi-infinite bulk show complicated dependences on the distance from the interface.

We have successfully reduced the simultaneous

differential equations derived from the GL free-energy functional by the variational method to a single nonlinear equation for the magnitude of magnetization at the top surface of the thin layer. The profile of magnetization in the whole system is then expressed in terms of the solution of this equation via the elliptic integrals and Jacobi-elliptic functions. The present formalism has been shown to be very useful for the clarification of the instability responsible for the para-ferro magnetic transition according to the variance of thin-layer thickness. The equation for the critical thickness is derived as the temperature is fixed. A new critical behavior of magnetization has been obtained and the critical exponent is  $\beta_L = \frac{1}{2}$ .

This equation also governs the critical temperature for fixed thickness. We have found that it coincides with that derived from the correlation functions. Therefore, it is concluded that the onset of the spontaneous magnetization and the divergence of the correlation length take place at the same temperature. This property is not trivial in the present inhomogeneous system, even in GL mean-field theory, and is verified in the present work.

The exchange coupling between subsystems plays an essential role in multilayer structures. In the present study, this coupling has been incorporated via the continuous condition for the magnitude of magnetization at the interface. It is not difficult to see that this condition can be relaxed to the following one, namely the magni-

tudes of magnetization in the two materials are proportional to each other at the interface, without significant modification for the present results. Antiferromagnetic interfacial coupling can also be treated similarly.

Although we have restricted ourselves in the case  $T_{c1} > T_{c2}$ , it is almost straightforward to extend the present formalism to the case  $T_{c2} > T_{c1}$ . The extrapolation length  $\lambda$  can take negative value, which implies that the exchange coupling at the top surface is stronger than that within the thin layer. The analysis is much more complex for the negative  $\lambda$  case, since a strong surface coupling is able to make the critical temperature above the bulk Curie point  $T_{c1}$ . It is found that the equation for the critical point changes its form according to the magnetic parameters and the thickness of the finite layer.<sup>25</sup> The critical behaviors, however, remain the same as those presented in this paper.

In spite of that the present study is for the system consisted from a thin layer and a semi-infinite bulk, it presents quite sufficient approximations for double layer systems used in magneto-optical recording, as briefly mentioned in the Introduction. The present formalism can also be applied to magnetic multilayer systems.

As a future work, Monte Carlo simulation will be carried out to test the predictions given in the present paper and to investigate numerically the critical exponents. Renormalization-group study based on the present mean-field results is also highly expected.

<sup>1</sup>M. N. Baibich, J. M. Broto, A. Fert, F. Nguyen van Dau, F. Petroff, P. Etienne, G. Creuzet, A. Friederich, and J. Chazelas, *Phys. Rev. Lett.* **61**, 2472 (1988).  
<sup>2</sup>G. Binash, P. Gruenberg, F. Saurenbach, and W. Zinn, *Phys. Rev. B* **39**, 4828 (1989).  
<sup>3</sup>S. S. P. Parkin, *Phys. Rev. Lett.* **71**, 1641 (1993).  
<sup>4</sup>M. Kaneko, K. Aratani, Y. Mutoh, A. Nakaoki, K. Watanabe, and H. Makino, *Jpn. J. Appl. Phys. Suppl.* **28-3**, 927 (1989).  
<sup>5</sup>S. Ohnuki, K. Shimazaki, N. Ohta, and H. Fujiwara, *J. Magn. Soc. Jpn. Suppl.* **15**, 399 (1991).  
<sup>6</sup>M. R. Freeman, R. S. Germain, E. V. Thuneberg, and R. C. Richardson, *Phys. Rev. Lett.* **60**, 596 (1988).  
<sup>7</sup>D. Kim, N. Nakagawa, O. Ishikawa, T. Hata, T. Kodama, and H. Kojima, *Phys. Rev. Lett.* **71**, 1581 (1993).  
<sup>8</sup>H. Meissner, *Phys. Rev.* **117**, 672 (1960).  
<sup>9</sup>P. G. de Gennes, *Rev. Mod. Phys.* **36**, 225 (1964).  
<sup>10</sup>S. Takahashi and M. Tachiki, *Phys. Rev. B* **33**, 4620 (1986); **34**, 3162 (1986).  
<sup>11</sup>K. Tanaka, *J. Magn. Mater.* **126**, 619 (1993).  
<sup>12</sup>V. L. Ginzburg and L. D. Landau, *Zh. Eksp. Teor. Fiz.* **20**, 1064 (1950).  
<sup>13</sup>D. L. Mills, *Phys. Rev. B* **3**, 3887 (1971).  
<sup>14</sup>K. Binder and P. C. Hohenberg, *Phys. Rev. B* **6**, 3461 (1972).

<sup>15</sup>K. Binder, in *Phase Transitions and Critical Phenomena*, edited by C. Bomb and J. L. Lebowitz (Academic, New York, 1983), Vol. 8, p. 1; references therein.  
<sup>16</sup>T. C. Lubensky and M. H. Rubin, *Phys. Rev. B* **12**, 3885 (1975).  
<sup>17</sup>A. Bray and M. A. Moore, *J. Phys. A* **10**, 1927 (1977).  
<sup>18</sup>T. C. Lubensky and M. H. Rubin, *Phys. Rev. B* **11**, 4533 (1975).  
<sup>19</sup>R. Cordery and A. Griffin, *Ann. Phys.* **134**, 411 (1981).  
<sup>20</sup>M. I. Kaganov and A. N. Omel'Yanchuk, *Sov. Phys. JETP* **34**, 895 (1972).  
<sup>21</sup>X. Hu and Y. Kawazoe, *Phys. Rev. B* **49**, 3294 (1994).  
<sup>22</sup>Y. Kawazoe, X. Hu, and S. Honma, in *Magnetic Ultrathin Films: Multilayers and Surfaces/Interfaces and Characterization*, edited by B. T. Jonker, S. A. Chambers, R. F. C. Farrow, C. Chappert, R. Clarke, W. J. M. de Jorge, T. Egami, P. Grünberg, K. M. Krishnan, E. E. Marinero, C. Rau, and T. Tsunashima, MRS Symposia Proceedings No. 313 (Materials Research Society, Pittsburgh, 1993), p. 513.  
<sup>23</sup>X. Hu, T. Yoroze, Y. Kawazoe, S. Ohnuki, and N. Ohta, *IEEE Trans. Magn.* **29**, 3790 (1993).  
<sup>24</sup>X. Hu and Y. Kawazoe, *J. Appl. Phys.* **75**, 6486 (1994).  
<sup>25</sup>X. Hu and Y. Kawazoe (unpublished).