Resistance fluctuations in diffusive transport at high magnetic fields in narrrow Si transistors

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(Received 22 February 1994; revised manuscript received 27 July 1994)

We have studied the aperiodic resistance fluctuations at millikelvin temperatures in 90-nm-wide silicon metal-oxide-semiconductor field-effect transistors at high magnetic fields. In the absence of the quantum Hall effect, the correlation length of the fluctuations in magnetic field shows dips commensurate with the Shubnikov–de Haas oscillations in the resistance. These can be explained by the formation of phase-coherent electron trajectories that contain both diffusive parts in the bulk and skipping parts along the boundaries.

In small, disordered metallic samples whose size is much larger than the elastic scattering length l but comparable with the phase-coherence length L_ϕ, one sees reproducible aperiodic fluctuations in the transport coefficients as a function of magnetic field B and Fermi energy E_F resulting from quantum interference.[1–5] At low B, such that $\omega_c\tau \lesssim 1$, where ω_c is the cyclotron frequency and τ is the bulk elastic-scattering time, the fluctuations in the conductance have an amplitude close to the universal value e^2/h. Their correlation length B_c is independent of B, being roughly equal to the flux quantum $\phi_0 = h/e$ divided by the typical area perpendicular to the field enclosed by a phase-coherent pair of electron trajectories. It is therefore a function of L_ϕ and the sample geometry.

At higher B ($\omega_c\tau \gtrsim 1$), in two-dimensional (2D) samples, Landau quantization produces Shubnikov–de Haas oscillations (SdHO's) which coexist with the aperiodic fluctuations in the resistance. Consequently, fluctuations in this regime have only been studied in transport-related properties which do not show large SdHO's, such as the microwave photo-emf,[6] the nonlocal resistance,[7] and the rectification and Hall voltages.[8] In all cases B_c was found to increase rapidly with B, in qualitative agreement with a diminishing diffusion coefficient.[9] At the same time, the amplitude of the fluctuations always remained surprisingly constant, defying the expected reduction due to self-averaging associated with a decreasing L_ϕ.

We have investigated the fluctuations of the resistance itself in narrow Si inversion layers, avoiding the SdHO's by determining the field-antisymmetric component of the four-terminal resistance. At low fields B_c is constant. At high B and low filling factor ν there are quantum Hall zeros in the longitudinal resistance, and the fluctuations take the form of resonancelike peaks adjacent to the zeros, similar to those observed in GaAs systems.[10–14] However, our focus here is on the regime at higher ν in which the SdHO's do not go to zero, where we find that B_c oscillates commensurately with the SdHO's. In line with recent theories, we explain this effect by the formation of electron trajectories which combine edge-state-like and bulk-diffusive components.[15,16]

The devices were Si metal-oxide-semiconductor field-effect transistors (MOSFET's) with polysilicon gates patterned by electron-beam lithography and reactive ion etching[17] into a narrow Hall bar geometry, as indicated in the inset to Fig. 1(a). The width of each segment was $W = 90$ nm, the probe separation was $L = 1.0$ μm, and the length of the narrow part of each probe was $L_p = 0.5$ μm. The gate oxide thickness was 210 Å, giving a carrier concentration of 1.0×10^{12} cm^{-2} per volt on the gate above the threshold voltage of about 1 V at 50 mK. The Hall mobility μ_H at $V_g = 4$ V was about 1.5 m^{2}V^{-1}s^{-1}, and the contact resistance was 1.5 kΩ. Standard ac techniques were used to measure the four-terminal resistances

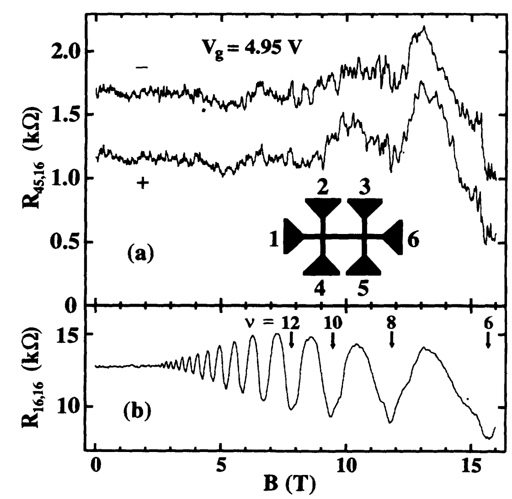

FIG. 1. (a) Variation of $R_{45,16}$ with magnetic field B for positive (+) and negative (−) field directions at $V_g = 4.95$ V. The upper trace is offset by 0.5 kΩ. Inset: gate geometry, showing labeling of probes. (b) Two-terminal resistance $R_{16,16}$ vs B, indicating positions of integer filling factor.

$R_{ij,kl}=V_{ij}/I_{kl}$, where V_{ij} is the voltage between probes i and j when current I_{kl} is passed between probes k and l. It was constantly checked that the results were independent of signal level, the voltage difference between probes always being less than 10 μV.

Figure 1(a) shows $R_{45,16}$ for one device at positive and negative B and a fixed gate voltage $V_g=4.95$ V. Reproducible fluctuations can be seen over the whole range of B. Figure 1(b) shows the two-terminal resistance of the device, $R_{16,16}$, which as expected is symmetric about $B=0$. The SdHO's here are associated with the long, wider 2D probe regions in series with the wire. They are visible for $B\gtrsim 2.4$ T, at which $\omega_c\tau\approx 3.6$ (the small-angle scattering time and momentum relaxation time are almost the same in Si inversion layers,[18] so that $\omega_c\tau=\mu_H B$). In $R_{45,16}$ they are suppressed until $B\approx 6$ T ($\omega_c\tau\approx 9$), where the cyclotron radius r_c becomes less than $W/2$. The minima are at even values of ν because of the valley degeneracy of 2 (the spin degeneracy is lifted at all fields of interest).

The symmetric and antisymmetric components of the four-terminal resistances are defined by $R^{s,a}_{ij,kl}=[R_{ij,kl}(B)\pm R_{ij,kl}(-B)]/2$. The SdHO's are almost absent in $R^a_{ij,kl}$. This is not surprising, because the antisymmetric component can be nonzero only if there is a lack of microscopic spatial symmetry, which is not the case for the SdHO's. Meanwhile, the fluctuations in $R^s_{ij,kl}$ and $R^a_{ij,kl}$ are of similar amplitude: about 0.1 kΩ. This is consistent with L_ϕ being roughly equal to the probe separation, so that in both cases the amplitude takes approximately the universal value of $R^2_\phi e^2/h$,[3,4] where R_ϕ is the resistance of a phase-coherent segment of the channel, which is about 1.5 kΩ for $L_\phi\approx 2$ μm.

Figure 2(a) shows traces of $R^a_{45,16}$ at a series of gate voltages spaced by 0.1 V. There is little similarity between adjacent traces, as the correlation scale in V_g is around 20 mV. The dotted lines join positions on the traces of constant integer filling factor, deduced from the minima of the SdHO's in the bulk resistance. Figure 2(b) shows the correlation field B_c as a function of B obtained from the data in Fig. 2(a) together with traces at four other nearby gate voltages. As usual, B_c was defined by[1] $F(B_c)=\frac{1}{2}F(0)$, where $F(\Delta B)=\langle R(B)R(B+\Delta B)\rangle_{\text{av}}-\langle R(B)\rangle^2_{\text{av}}$. B_c was calculated over a window of width 1 T centered on each value of B. The results for different V_g but equal filling factors were then scaled to match the B axis at $V_g=4.10$ V and averaged together. Similar results were obtained for $V_{23,16}$, and in other devices. For comparison, Fig. 2(c) shows the symmetric resistance, $R^s_{45,16}$, smoothed and averaged over the same set of gate voltages.

For $B\le 3$ T, B_c is constant at $B_c(0)=20$ mT. The devices are one dimensional with respect to phase coherence, so using $B_c=\beta\phi_0/L_\phi W$ with $\beta=1.2$ (Ref. 1) we obtain $L_\phi\approx 2$ μm. The bulk mean free path 1 deduced from μ_H at low B is 0.3 μm, so in the wire elastic scattering occurs largely at the boundaries, and indeed one-dimensional quantization has been reported in similar structures.[17] Flux cancellation under these conditions is known to increase B_c.[19] On the other hand, since L_ϕ is longer than L_p, the opening out of the leads into wide regions where the trajectories link more area can reduce B_c.[5] These corrections, together with other geometrical complications and the theoretical uncertainty about the appropriate value of β, may make the above value of L_ϕ inaccurate by a factor of 2 or 3. Unfortunately the weak-localization correction could not be used to obtain a corroborating value because it was obscured by the superconducting transition of the aluminum interconnects at low magnetic fields.

An increase in B_c implies a decreasing typical area enclosed by phase-coherent trajectories, which in the absence of edge effects should result from a reduction in $L_\phi=\sqrt{D\tau_\phi}$ via a changing diffusion coefficient D or phase-coherence time τ_ϕ.[9] In thin n^+-GaAs epilayers,[7,8] the behavior of B_c up to $\omega_c\tau\approx 3$ was found to be close to $B_c(B)=B_c(0)[1+(\omega_c\tau)^2]$. This is consistent with $B_c=\phi_0/L^2_\phi=\phi_0/D\tau^2_\phi$, assuming the semiclassical result $D(B)=D(0)[1+(\omega_c\tau)^2]^{-1}$ and constant τ_ϕ. By analogy, for $L_\phi>W$, so that $B_c\propto 1/L_\phi$, one might expect $B_c(B)=B_c(0)[1+(\omega_c\tau)^2]^{1/2}$, as shown by the dotted line in Fig. 2(b). However, the semiclassical result does not

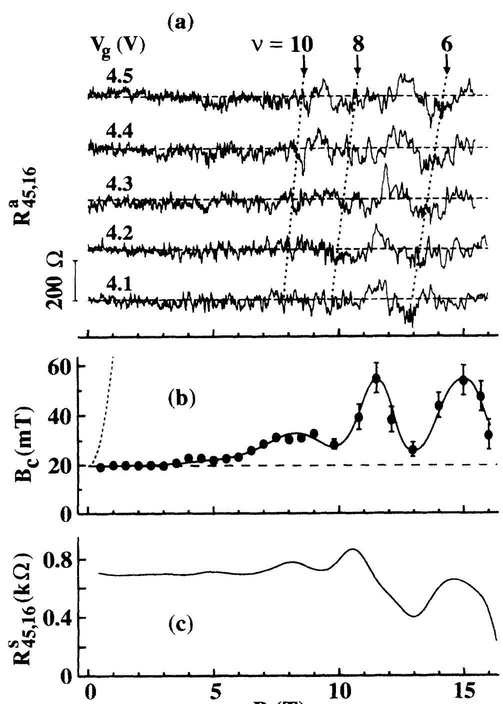

FIG. 2. (a) Antisymmetric component of $R_{45,16}$ vs B at the gate voltages labeled. The dashed horizontal line on each trace is zero. The dotted lines cross zero on each trace at the indicated filling factor ν. (b) Dependence of B_c on magnetic field, averaged over several gate voltages (see text). The error bars indicate the standard deviation where it is larger than the symbol size. The solid line is a guide to the eye, the dashed line denotes the low-field value $B_c(0)$, and the dotted line is a plot of $B_c(B)=B_c(0)[1+(\omega_c\tau)^2]^{1/2}$. (c) Four-terminal symmetric resistance $R^s_{45,16}$ averaged over the same gate voltages and smoothed on a scale of 0.4 T.

apply for $r_c \geq W/2$ ($B \leq 3$ T) when scattering is mainly at the boundaries. In this regime the typical phase-coherent area does not change and B_c is constant. For 4 $T < B < 8$ T, however, r_c becomes less than $W/2$, bulk scattering reduces the diffusivity, the phase-coherent area decreases, and B_c starts to increase.

For $B > 6$ T, we begin to see oscillations in B_c which for $B > 9$ T are clearly correlated with the SdHO's in $R^s_{45,16}$ and so with the movement of Landau levels past E_F. The transport mechanism in this regime must be intermediate between bulk diffusion, at low B, and the quantum Hall effect, at high B (and low υ), where nonscattering edge channels carry the current. At lower carrier densities in these devices there are indeed quantum Hall zeros in $R_{45,16}$ around $\nu=1$ and 2 (the valley degeneracy is lifted for the first Landau level), as can be seen in Fig. 3. Adjacent to the zeros, when the electrons are not completely confined to the edges, there is random resonancelike structure as a function of B. Similar resonances can be seen by sweeping V_g at constant B [inset to Fig. (3)]. Their existence implies that transmission of electrons between opposite edges is possible only at certain discrete energies (or filling factors). This can result from resonant tunneling via individual localized states in the bulk,[12] as sketched in Fig. 4(a). It leads to finite backscattering, and hence a finite four-terminal resistance, at these energies only.

Xiong and Stone[9] showed that the universality of the fluctuations is not affected by Landau quantization if localization and edge effects are neglected, but only very recently have the consequences of edges been investigated. Khmelnitskii and Yosefin[15] showed that, in a wire with $l \ll W \ll L$, the presence of boundaries enhances the diffusivity of the lowest mode of the diffusion equation. Using a simple Monte Carlo simulation of classical noninteracting electrons near a boundary, Brown *et al.*[8] found that the diffusion coefficient in the direction parallel to the boundary was almost independent of B, and was therefore greatly enhanced over the isotropic bulk coefficient at high B. Both these results suggest that in a diffusive system at high B phase coherence should be maintained for a longer distance along a boundary than in the bulk.

Reference 15 predicted that $B_c \propto N\rho_{xx}$, where N is the density of states and ρ_{xx} is the bulk diagonal resistivity. In Fig. 2 the peaks and dips in B_c line up approximately with those in $R^s_{45,16}$ and are proportionately of comparable amplitude, but it is not possible to obtain a fit to this equation, because N is unknown. Meanwhile, Maslov and Loss[16] presented us with a way of understanding the data qualitatively. They studied the diffusion equation for the Landau-level guiding centers with boundary conditions which allow the electrons to scatter in and out of "sliding" orbits at the edges. They predicted that in narrow channels, under conditions where the average sliding length along the edge is longer than $\sqrt{2}r_c$ but shorter than L, a kind of electron trajectory exists which circulates around a loop between the opposite edges, as sketched in Fig. 4(b). The loop trajectory comprises alternate sliding and diffusive parts. We note that in time τ_ϕ one of these chiral loop trajectories can link an area which is considerably larger than for a purely diffusive trajectory. This area may, when the sliding length is similar to L, approach the total area WL between the contact probes. One therefore expects such trajectories to produce oscillations in the conductance with a minimum period similar to $B_c(0)$.

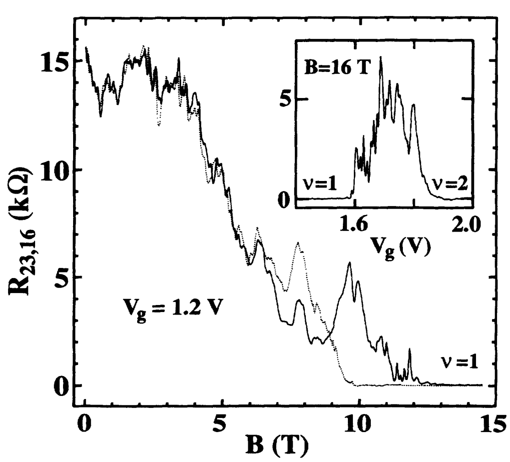

FIG. 3. Resonant structure adjacent to zero at low filling factor in the positive (solid line) and negative (dotted line) field directions. Inset: structure obtained by sweeping V_g in between adjacent quantum Hall zeros.

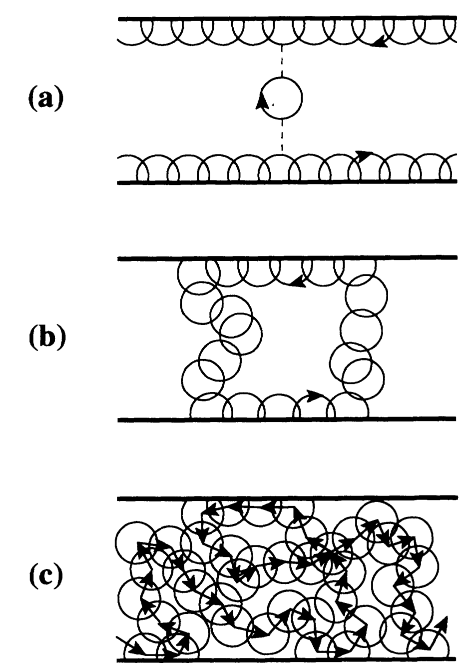

FIG. 4. Electron trajectories associated with fluctuations at high magnetic fields. (a) Resonant tunneling between well-formed edge states through a single localized bulk state whose energy level is close to E_F. (b) Loop trajectory when edge states are incipient. (c) Diffusive trajectories when edge states are absent.

For $\nu \gtrsim 4$ in these devices, transport is never in completely decoupled edge states because $R_{45,16}$ never goes to zero. Still, near the resistance minima (ν=even integer), where E_F in the channel center lies between Landau levels, the edge states are at least partially formed and loop trajectories are possible, giving values of B_c which dip close to $B_c(0)$. On the other hand, near the resistance maxima, where E_F lies near the center of a Landau level, the electrons never travel along an edge much further than $\sqrt{2}r_c$ and the trajectories are mainly diffusive, as indicated in Fig. 4(c). Their typical areas can then be much smaller, and B_c shows corresponding peaks. Moreover, because the scattering is not boundary limited as at it was at low B, and the bulk diffusivity is lower, the peak values can be much greater than $B_c(0)$.

In the absence of phase breaking, Ref. 16 predicts that the loop trajectories of Fig. 4(b) should cause modulations of the order of 10–20 % in the fluctuation amplitude as a function of B. Such modulations are too small and on too fine a scale in B to be resolved here. We see no dramatic variation in the fluctuation amplitude up to 16 T. To try to explain the constancy of the fluctuation amplitude while B_c increases in larger systems, it has been argued that the self-averaging effect is reduced because the phase-coherent regions near the edges do not become shorter at high B, as discussed above. In our narrow silicon devices, L_ϕ is never smaller than W and possibly does not become much smaller than L, but the fact that the amplitude does not vary while B_c does is still a puzzle.

In summary, we have measured the reproducible resistance fluctuations in a narrow disordered conductor in the regime intermediate between diffusive conduction and the quantum Hall effect. We observed an oscillatory behavior of the correlation field which constitutes strong evidence for the existence near integer filling factors of coherent electron trajectories which combine the properties of edge and bulk states.

We thank C. J. B. Ford, A. K. Geim, D. E. Khmelnitskii, D. L. Maslov, J. T. Nicholls, and M. Yosefin for help and discussions, and acknowledge the support of the UK Science and Engineering Research Council. The devices were fabricated under the Silicon Towards 2000 scheme of the DTI.

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