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Quantum magnetotransport in a nondegenerate two-dimensional electron gas under extremely strong magnetic fields

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The quantum magnetotransport properties of a nondegenerate two-dimensional gas of electrons interacting with helium-vapor atoms above a liquid-helium surface is studied in magnetic fields up to 20 T by the ac capacitive coupling technique. The data and the theoretical analysis performed show that in the ultraquantum limit the generalized or effective collision frequency ν_{eff} of the electrons increases faster with magnetic field than the cyclotron frequency ω_c. Under extremely strong magnetic fields, where the Landau level width becomes comparable to or larger than the thermal energy, the high-cyclotron-frequency approximation $\omega_c \gg \nu_{\mathrm{eff}}$, usually assumed in quantum transport theories, is no longer valid. The self-consistent Born-approximation theory is extended to be valid for any ratio of ω_c to ν_{eff}. Then it describes the data perfectly without any adjustable parameter. The results reported here also give strong support to the universality of the linear Hall resistivity.

I. INTRODUCTION

In the presence of a strong quantizing magnetic field B the surface state electrons (SSE's) on liquid helium represent a two-dimensional (2D) system of mobile particles, which has a completely discrete energy spectrum if interactions are neglected. The unique quality of the helium surface and the simplicity of the interaction potential of an electron with available scatterers make the SSE's very attractive for testing different approaches to the description of quantum magnetotransport phenomena in two dimensions. At high enough temperatures ($T > 1$ K) the main scatterers are helium-vapor atoms, which can be considered as short range and elastic. The experimental densities of SSE's $n < 2\times 10^{13}$ m^{-2}, are low compared to other 2D electron systems and the Landau level filling factor is much smaller than one. Therefore, in the ultraquantum limit $\hbar\omega_c \gg k_B T$ ($\hbar$ is Planck's constant, ω_c is the cyclotron frequency, T is the temperature, and k_B is the Boltzmann constant), easily achieved experimentally, electrons can be scattered only within the lowest Landau level. Under strong magnetic fields, the Landau level width Γ, determined by the interaction with the vapor atoms, is much smaller than the Landau level separation $\hbar\omega_c$. Therefore the usual ultraquantum limit might be subdivided in two possible cases: (i) at $k_B T \gg \Gamma$ the electrons are smoothly distributed within the ground level; (ii) at $k_B T \ll \Gamma$ the electrons populate the lower tail states of the density of states function.

The first experimental studies of quantum magnetotransport properties of the SSE's interacting with helium-vapor atoms or ripplons were performed both on a solid-hydrogen[1] and on liquid-helium[2–4] surfaces in magnetic fields up to 5 T. They showed that the electrical transport of SSE's is strongly influenced by quantum effects. In each study, however, an adjustable parameter had been necessary to fit the data to the most established theory,[5] based on the self-consistent Born approximation (SCBA). Data of the magnetoconductivity measurements under extremely strong magnetic fields up to 20 T reported in Ref. 6 also were fitted to the SCBA theory by introducing an adjustable parameter in the definition of the Landau level width.

An additional interest for the investigation of SSE transport in the ultraquantum limit is to have another test case of the universality of the linear Hall resistivity observed in semiconductor 2D electron systems.[7,8] The original SCBA theory[5] applied to nondegenerate systems[9] appeared to predict a Hall resistivity ρ_{xy}, which differs from the classical value $\rho_{xy}^{(\mathrm{cl})} = B/ne$ (e is the elementary charge) at extremely strong magnetic fields, where the Landau level width $\Gamma \propto B^{1/2}$ becomes comparable to or larger than $k_B T$. This condition can be reached experimentally in the gas-atom scattering regime in fields above 10 T.

The purpose of the present work is the detailed study of the quantum magnetotransport properties of SSE's at extremely strong magnetic fields. In this extreme limit a new qualitative feature of the field dependence of the conductivity tensor becomes pronounced. Theories of quantum galvanomagnetic effects, including the famous center migration theory,[10] usually assume that at strong and extremely strong magnetic fields B the longitudinal conductivity component σ_{xx} is much smaller than the Hall conductivity σ_{yx}. This so-called case of strong magnetic fields in a quasiclassical treatment corresponds to the high-cyclotron-frequency limit $\omega_c \gg \nu$, where ν is a collision frequency. The theoretical assumption implies that for a real system the effective collision frequency (to be exact, the total momentum loss per second) in the quantum limit would not increase with B more rapidly than ω_c. In the present paper, we would like to emphasize that at least for SSE's this is not true. The analysis presented and the data obtained give a strong evidence that for SSE's at $T > 1$ K the high-cyclotron-frequency approximation ($\omega_c \gg \nu$) is inconsistent with the real strong-field limit.

Because it is impossible to attach electrical leads to the SSE system, measurements of its transport properties are usually done with ac capacitive coupling techniques.[11] These work very well at zero magnetic field, but, with the exception of ideally circular symmetric geometries used for σ_{xx} measurements, are complicated at high magnetic fields due to the effects of edge magnetoplasmons.[12,13] Actually, at first we aimed at measuring the Hall resistivity ρ_{xy} by studying these propagating modes in the high-field region. However, the behavior of the effective collision frequency in extremely strong magnetic fields mentioned above makes the data dependent on both ρ_{xx} and ρ_{xy} and a detailed numerical calculation of the response of the 2D transmission line formed by the electrons and the electrodes is required to compare the data with theory. The extended SCBA theory established here for any relation between longitudinal and Hall conductivities without any adjustable parameter perfectly describes the data.

A preliminary presentation of the data and a qualitative analysis has been presented elsewhere.[14] After a more rigorous and quantitative analysis, the present interpretation of the data partly differs from that given in Ref. 14.

II. EXPERIMENTAL METHOD

For the experiments an electrode array (14 mm diam), shown in the inset of Fig. 1, was placed at a distance $d = 0.5$ mm below the helium surface. A circular guard ring at a negative potential surrounds the electrode array. The electron sheet is charged to saturation and hence the density is given by the top plate potential. The distance between the electrode array and the upper plate is 3.0 mm. The electron pool diameter (13 mm) was calculated for the confining potentials on the electrodes.[15] Electrode 1 was excited with an ac voltage V_{ac}. The currents in and out of phase with respect to the driving voltage, induced on one of the outer electrodes (usually diametrically opposite to the driving one, e.g., 5), were measured by a

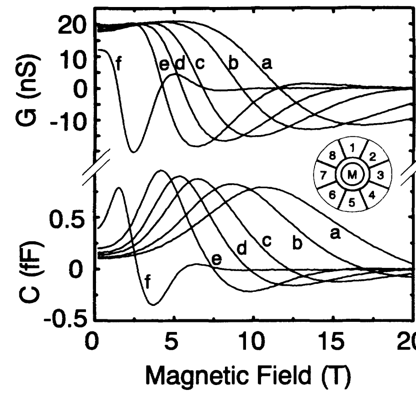

FIG. 1. The components C and G of the complex admittance $Y = G + j\omega C$ measured between electrodes 1 and 5 versus magnetic field. Data are shown for electron densities of 10.5 (a), 8.2 (b), 5.8 (c), 4.7 (d), 3.5 (e), and 1.2 (f) (in units of 10^{11} m^{-2}). The temperature is 2.0 K and the measuring frequency $\omega/2\pi$ is 5 kHz. The electrode geometry (14 mm diam) is shown in the inset.

current amplifier in conjunction with a dual-phase lock-in amplifier. For each driving frequency, the currents measured with the uncharged surface were subtracted from the data to account for stray resistances and capacitances. The phase shifts are corrected for phase shifts of the current amplifier. At low frequencies the experimental system behaves like an RC circuit, so that the convention is to write the measured complex current as $I = YV_{ac}$, where the complex admittance Y is defined as $Y = G + j\omega C$. The phase shift φ is defined as $\varphi = \arctan(G/\omega C)$. Zero phase shift therefore corresponds to a pure capacitive coupling. The homogeneity of the resistive magnet was sufficient to ignore helium level variations with the magnetic field.

To compare the experimentally measured phase shifts with theory, a detailed calculation of the phase shift was made by numerically solving the basic equations that describe the ac transport in the sheet.[12] As in Ref. 12, the transmission line model was used, which means that the local ac density in the electron pool is related to the ac potential in the pool and to the ac potential on the lower electrodes according to the local capacitance approximation. The full inhomogeneous density and capacitance profile at the edge are taken into account in the calculations.[16] To obtain convergence of the iterative procedure for solving the equations at high fields, a proper approximation of differentials should be used.[17] The earlier quantitative discrepancy between calculations and data[16] can now be attributed to convergence problems. The ratio σ_{yx}/σ_{xx} (equal to ρ_{xy}/ρ_{xx}) and the conductivity σ_{xx} (or equivalently any one of the other conductivity or resistivity parameters) appear as parameters in the transmission line model. These calculations are crucial for the interpretation of the present experiments.

To enhance the investigated effects and to avoid possible broadening of the Landau levels due to Coulomb interactions, the experiments were performed at relatively high temperatures (~ 2 K). Higher temperatures imply an increased vapor-atom density and therefore an increased scattering rate. This leads to broad levels with negligible contribution from electron-electron interactions. The largest ratio of $\Gamma/k_B T$ that has been obtained experimentally is about 2.

III. THEORETICAL CONCEPT

To describe the quantum magnetotransport properties of SSE's on the liquid-helium surface, we will follow the approach presented briefly in Ref. 18, which is analogous to the method of the quantum-mechanical momentum balance equation.[19] This approach allows a direct comparison with the center migration theory and reveals the physical cause of the peculiar behavior of the conductivity tensor of the SSE's in the extremely strong magnetic-field limit. First we use a very simple phenomenological analysis, which then will help us to find the solution at an arbitrary relation between σ_{xx} and σ_{yx} for strongly correlated electrons. Let us consider an infinitely large isotropic 2D electron system moving along the surface in crossed magnetic $\mathbf{B}$ and electric $\mathbf{E}$ fields. The total kinetic friction $\mathbf{F}_{\rm fr}$ acting on the electrons by the helium-vapor atoms (or the total momentum loss per second) is an unknown function of the current density $\mathbf{j}$ and possibly of $\mathbf{E}$. In the low driving field limit the expansion of this function starts with linear terms: $\mathbf{F}_{\rm fr}=-a\mathbf{j}-b\mathbf{E}$. This is the most general expression of a linear expansion of $\mathbf{F}_{\rm fr}$. The term proportional to $\mathbf{E}$ leads to the effect of screening of the external electric field and can be omitted. The other one can be rewritten in a more physical way: $\mathbf{F}_{\rm fr}=-N_e m\nu_{\rm eff}\mathbf{u}$, where N_e is the total number of electrons, m is the electron mass, $\mathbf{u}$ is the mean electron velocity, and $\nu_{\rm eff}$ is an effective or generalized collision frequency, which in general is a function of B and electron concentration n.

The general structure of the conductivity tensor can be easily found by the use of the balance of forces equation $\mathbf{F}_{\rm fr}=-\langle\mathbf{F}_{\rm ext}\rangle$, where

$$\mathbf{F}_{\rm ext}=-N_e e\mathbf{E}+\sum_e m[\mathbf{v}_e\times\boldsymbol{\omega}_c] \tag{1}$$

($\mathbf{v}_e$ is an electron velocity; $\langle\ \rangle$ means total, including quantum-mechanical, averaging; $\langle\mathbf{v}_e\rangle=\mathbf{u}$, and vector $\boldsymbol{\omega}_c$ is directed along the magnetic field). Here we assume that there are no localized states of the SSE's, typical for the integer quantum Hall effect. This equation together with the linear expansion of $\mathbf{F}_{\rm fr}$ gives elementary expressions for the conductivity and resistivity tensor components:

$$\begin{aligned}\sigma_{xx}&=(e^2n/m)\{\nu_{\rm eff}(B,n)/[\omega_c^2+\nu_{\rm eff}^2(B,n)]\}\ ,\\ \sigma_{xy}&=-[\omega_c/\nu_{\rm eff}(B,n)]\sigma_{xx}\ ,\\ \rho_{xx}&=m\nu_{\rm eff}(B,n)/ne^2,\quad \rho_{xy}=B/ne\ .\end{aligned} \tag{2}$$

It should be emphasized that here $\nu_{\rm eff}$ describes a property of the whole 2D electron system and may have no relation to a single-electron collision frequency.

This general structure of the conductivity tensor based on the simple and direct argument leads to two very important conclusions. First, the Hall resistivity ρ_{xy}, which follows from Eq. (2), is independent of the detailed dependence of $\nu_{\rm eff}$ on B and n and is exactly the linear Hall resistivity $\rho_{xy}^{(\rm cl)}$ even in extremely strong magnetic fields. This result is in agreement with recent evidence of the universality of the Hall resistivity in degenerate 2D electron systems at semiconductor heterostructures,[7,8,20] where, apart from the quantum Hall plateau regions, it retains its classical value even in the localized Wigner lattice regime. In the inset of Fig. 2, a comparison with the ρ_{xy} from the existing SCBA theory[9] is given.

The second important conclusion can be seen by comparison of Eq. (2) with the previously used SCBA expressions[2,5] derived for $\sigma_{xx}\ll\sigma_{xy}$. Let us suppose $\omega_c\gg\nu_{\rm eff}$, which means $\sigma_{xx}\propto\nu_{\rm eff}/\omega_c^2$. Then the field dependence of $\sigma_{xx}\sim B^{-1/2}$, which follows from the result of Ref. 2 at $\Gamma\ll k_B T$, gives us the field dependence of $\nu_{\rm eff}\sim B^{3/2}$, which is faster than the field dependence of $\omega_c\sim B$. It means that the high-cyclotron-frequency approximation ($\omega_c\gg\nu_{\rm eff}$ or $\sigma_{xx}\ll\sigma_{yx}$) might fail in the case of extremely strong magnetic fields.

Qualitatively, an expression for $\nu_{\rm eff}(B,n)$ can be simply found by comparing Eq. (2) with the result of the SCBA theory in the high-cyclotron-frequency limit. But there is one limiting case for which we can calculate $\mathbf{F}_{\rm fr}$ or $\nu_{\rm eff}$ directly.

Since the properties of the system of helium-vapor atoms are well known (compared with the properties of the 2D electron liquid), it is easier to calculate $\mathbf{F}_{\rm fr}$ as a momentum acquired by the vapor gas per second. In the most general case the interaction Hamiltonian of the SSE's with helium-vapor atoms can be written as

$$H_{\rm int}=\sum_{\mathbf{K},\mathbf{K}'}V_{-\mathbf{K}}\langle 1|e^{ikz}|1\rangle\rho_{-\mathbf{q}}a^\dagger_{\mathbf{K}'-\mathbf{K}}a_{\mathbf{K}'}\ . \tag{3}$$

Here $V_{\mathbf{K}}$ is a 3D Fourier transform of the electron–helium-atom interaction potential $V(\mathbf{R})$; $\mathbf{R}=\{\mathbf{r},z\}$; $\mathbf{K}=\{\mathbf{q},k\}$; $\rho_{\mathbf{q}}=\sum_e\exp(i\mathbf{q}\cdot\mathbf{r}_e)$ is a 2D Fourier transform of the electron density; $a^\dagger_{\mathbf{K}}$ is a creation operator of the ^{4}He atom; $\langle 1|\ |1\rangle$ means the average over the ground surface state level. The same Hamiltonian can be used also for scattering on static impurities, and in this case we should take the mass of the atom $M=\infty$ in the final formulas.

By the use of Eq. (3) the momentum acquired by the vapor atoms per second and the kinetic friction $\mathbf{F}_{\rm fr}$ can be found in the Born approximation as a function of the dynamic form factor $S(\mathbf{q},\omega)$ of the 2D electron liquid:

$$\mathbf{F}_{\rm fr}=\frac{N_e}{\hbar}\sum_{\mathbf{K},\mathbf{K}'}|\langle 1|e^{ikz}|1\rangle|^2|V_{-\mathbf{K}}|^2 n^{(a)}_{\mathbf{K}'}\mathbf{q}S(\mathbf{q},\Delta\omega)\ , \tag{4}$$

where

$$S(\mathbf{q},\omega)=\frac{2\pi\hbar}{N_e}\left\langle\sum_{j'}\left|\langle j'|\rho_{-\mathbf{q}}|j\rangle\right|^2\delta(E_{j'}-E_j-\hbar\omega)\right\rangle \equiv\frac{1}{N_e}\int_{-\infty}^{\infty}dt\,\exp(-i\omega t)\langle\rho_{\mathbf{q}}\rho_{-\mathbf{q}}(t)\rangle\ .$$

E_j is the electron energy spectrum unknown in the general case; $\hbar\Delta\omega=\varepsilon^{(a)}_{\mathbf{K}'}-\varepsilon^{(a)}_{\mathbf{K}'-\mathbf{K}}$ is the energy exchanged at a collision; $\varepsilon_{\mathbf{K}}^{(a)}=\hbar^2K^2/2M$ is the gas atom spectrum; $n_{\mathbf{K}}^{(a)}$ is the vapor atom distribution function. In the general case we should keep $\hbar\Delta\omega\neq 0$, but at temperatures $T>1.3$ K, the broadening $\Gamma \gg \hbar\Delta\omega$, and we can consider an electron-atom scattering as completely elastic ($\hbar\Delta\omega=0$).

The main problem here is to calculate the dynamic form factor $S(\mathbf{q},\omega)$, since the density operator which should be used for averaging is not an equilibrium one in the presence of a driving field. We could of course use the linear-response theory, which would be the approach of the center migration theory. But there is another more simple approach applicable to strongly correlated systems. There is an analogy for such approach in usual quasiclassical kinetic theory. For strongly correlated electrons we can consider the real distribution function to be a shifted Fermi function $f(E_{\mathbf{k}}-\hbar\mathbf{k}\cdot\mathbf{u})$ instead of calculating a correction to the equilibrium Fermi distribution function caused by the driving field. Then the current or the drift velocity $\mathbf{u}$ is to be found from the momentum conservation law equation. For electron–helium-vapor atom scattering both these approaches give the same result [21], which hereby is independent on correlation.

According to the analogy mentioned above, let us consider that due to the mutual electron-electron interaction the electron liquid is in equilibrium in the center-of-mass frame moving along the surface with drift velocity $\mathbf{u}$. Mathematically this means that the dynamic form factor of the electron liquid in the laboratory frame $S(\mathbf{q},\omega)=S_0(q,\omega-\mathbf{q}\cdot\mathbf{u})$,[18,21] where $S_0(q,\omega)$ is the equilibrium dynamic form factor. If we take into account basic properties of the equilibrium form factor such as $S_0(q,-\omega)=\exp(-\hbar\omega/k_BT)S_0(q,\omega)$, we can transform Eq. (4) in order to obtain a more convenient form for the low-velocity expansion:

$$\mathbf{F}_{\mathrm{fr}}=\frac{N_e}{2\hbar}\sum_{\mathbf{q},\mathbf{q}'}\sum_{k,k'}|\langle 1|e^{ikz}|1\rangle|^2|V_{-\mathbf{q},-k}|^2 n_{\mathbf{q}',k'}^{(a)} \times S_0(q,\Delta\omega-\mathbf{q}\cdot\mathbf{u})\mathbf{q} \times\{1-\exp[\hbar\mathbf{q}\cdot\mathbf{u}/k_BT]\} \;. \tag{5}$$

This expression of $\mathbf{F}_{\mathrm{fr}}$ as a function of the dynamic form factor is a basic one for studying quantum transport phenomena for strongly correlated 2D electron systems, including the 2D Wigner solid. At low driving fields $\hbar\mathbf{q}\cdot\mathbf{u}\ll k_BT$ we can expand $\mathbf{F}_{\mathrm{fr}}$ and find the effective collision frequency as it was defined just above Eq. (1). In this paper we intend to test the approach and will therefore consider the more simple case, when electron-electron collisions affect only the distribution of the SSE's and do not substantially change their energy spectrum. It means that we will neglect the level broadening caused by the Coulomb interaction (as shown in Ref. 22, this effect will be most significant below 1.5 K and 5 T). At such conditions we can represent $S_0(q,\omega)$ as a trace in the one-electron space and insert it into the expression of ν_{eff}, which follows from Eq. (5). For a short-range interaction potential $[V(R)=V_0\delta(\mathbf{R})]$ and for elastic scattering ($\Delta\omega=0$) one can find

$$\nu_{\mathrm{eff}}=\frac{3\pi\hbar V_0^2 n^{(\mathrm{gas})}\gamma}{8N_e m k_B T A}\sum_{\mathbf{q}} q_y^2\int dE\, f(E)[1-f(E)]\langle \mathrm{Tr}[\delta(E-H_e)\exp(i\mathbf{q}\cdot\mathbf{r})\delta(E-H_e)\exp(-i\mathbf{q}\cdot\mathbf{r})]\rangle_s \;, \tag{6}$$

where $n^{(\mathrm{gas})}$ is the helium-vapor density; A is the surface area; H_e is the single-electron Hamiltonian, which includes the interaction with helium-vapor atoms; $f(E)$ is the equilibrium electron distribution function; γ is the parameter of the electron wave function $\langle 1|z\rangle \sim z\exp(-\gamma z)$; and $\langle\;\rangle_s$ means the average with respect to the scatterers' variables. If we would use a two-dimensional plane-wave electron state $|\mathbf{k}\rangle$ for calculating the trace in (6), we would obtain the zero-magnetic-field result for the collision frequency,[21] which together with Eq. (1) would give a semiclassical Drude formula. In the quantum limit, the Landau quantum states $|n,X\rangle$ should be used.

If the high-cyclotron-frequency approximation is valid ($\omega_c\gg\nu_{\mathrm{eff}}$), Eq. (6) together with the expression for the conductivity tensor component $\sigma_{xx}\simeq(e^2n/m)\nu_{\mathrm{eff}}/\omega_c^2$ gives the same result as the center migration theory,[10] which is the starting point for the SCBA theory.[5] Therefore all details of the calculations of ν_{eff} are the same as those of σ_{xx} in Ref. 5. The final expression for ν_{eff} can be written as

$$\nu_{\mathrm{eff}}=(2\omega_c/\pi)\coth(\hbar\omega_c/2k_BT) \times[\cosh(\Gamma/k_BT) -(k_BT/\Gamma)\sinh(\Gamma/k_BT)]/I_1(\Gamma/k_BT)\;, \tag{7a}$$

where $I_1(x)$ is the modified Bessel function of order 1, $\Gamma=\hbar\{(2/\pi)\omega_c\nu_0\}^{1/2}$, and $\nu_0=(3V_0^2n^{(\mathrm{gas})}\gamma m)/8\hbar^3$. Here a semielliptic shape for the density of states function of a Landau level is used. For the Gaussian shape the collision frequency will have a slightly different form:

$$\nu_{\mathrm{eff}}=\omega_c\sqrt{\pi}(\Gamma/4k_BT) \times\exp[-(\Gamma/4k_BT)^2]\coth(\hbar\omega_c/2k_BT)\;. \tag{7b}$$

As expected, in the high-cyclotron-frequency limit these expressions for ν_{eff} and Eq. (2) give the same result as in the approaches in Refs. 1, 2, and 6. It means that in the quantum transport theory of electrons scattering on vapor atoms, as in the case of the quasiclassical kinetic theory, there is no dependence on electron correlation as long as the Coulomb broadening is neglected.

The SCBA for the evaluation of Eq. (6) is valid if the parameter $\Gamma/\hbar\omega_c$ is small,[5] while the high-cyclotron-frequency approximation requires another parameter Γ/k_BT to be small, as follows from Eq. (7). Since the Landau level broadening caused by the interaction with vapor atoms increases with B as $\Gamma\sim B^{1/2}$, the first parameter $\Gamma/\hbar\omega_c$ decreases with B, but the second Γ/k_BT increases with B so that the high-cyclotron-frequency approximation will eventually fail. As can be seen from Eq.

(7), $\nu_{\rm eff}(B)$ increases with B faster than ω_c up to $\Gamma \simeq k_B T$. At higher Γ, according to Eq. (7a), the ratio $\nu_{\rm eff}/\omega_c$ still increases, while according to Eq. (7b) it attains a maximum. In Fig. 2 the ratio $\nu_{\rm eff}/\omega_c = \rho_{xx}/\rho_{xy}$ is plotted for different temperatures as a function of B. It should be emphasized that for $T \leq 1.6$ K the difference between Eqs. (7a) and (7b) is numerically negligible up to 20 T.

It should also be mentioned that the level broadening Γ [see below Eq. (7a)] is expressed in terms of well-established basic parameters of the electron–helium-atom interaction (including V_0 and γ), which cannot be changed without strong arguments. The broadening usually is expressed in terms of the theoretical collision frequency ν_0 or corresponding mobility calculated in Ref. 23, which, however, should not be used as an adjustable parameter.

IV. RESULTS AND DISCUSSION

A representative set of data for the components C and G of the complex admittance Y as a function of the magnetic field B is shown in Fig. 1 at a frequency $\omega/2\pi$ of 5 kHz for several electron densities. Both components oscillate as a function of the magnetic field with a gradually decaying amplitude. Resonances are inhibited because of the strong damping. Roughly, these data are similar to the previously published data taken in the field range below 4 T.[13] Somewhat larger ratios n/ω were used for the present data, in order to obtain sufficient signal strength at higher fields. Consequently, the oscillation period is larger too. The oscillations are a result of the Hall effect in strong magnetic fields which leads to a localized edge mode ("edge magnetoplasmon"[24]), even for very low frequencies.[13] The propagation constant, which is proportional to the Hall resistivity, was previously directly obtained from the phase shift. Here this technique was planned to probe the Hall resistivity, but the analysis in the presence of very strong magnetic fields turns out be much more complicated and also to be dependent on ρ_{xx}.

For a further analysis, the phase shifts, obtained from the data of Fig. 1 for different electron concentrations, are plotted in Fig. 3(a) as a function of the magnetic field. In Fig. 3(b) the measured phase shifts for several frequencies are shown at a fixed electron density of 5.8×10^{11} m^{-2}. The curves d in Fig. 3(a) and h in Fig. 3(b) are taken for such a low n/ω ratio, that the amplitude is unmeasurably small in fields higher than 10 T. These curves are representative for the low-field regime. In this regime, the $\varphi(B)$ curves are linear, except for the region close to the origin. The maximum phase shift before the signal has decayed is then about 10 rad. An extensive

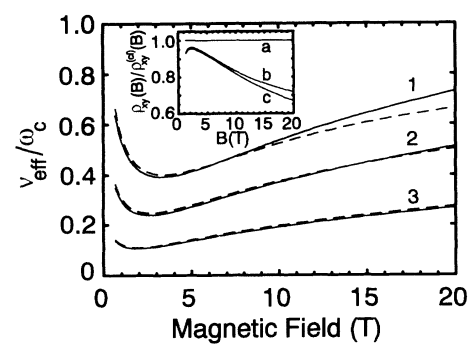

FIG. 2. The ratio $\nu_{\rm eff}/\omega_c$ ($=\rho_{xx}/\rho_{xy}$) is shown as a function of the magnetic field for the three temperatures: 2 K [mobility 1.7 m^2/V s (Ref. 23), curve 1], 1.6 K [mobility 5.7 m^2/V s (Ref. 23), curve 2], and 1.2 K [mobility 39 m^2/V s (Ref. 23), curve 3]. Full curves correspond to Eq. (7a) and dashed lines to Eq. (7b). The inset shows the calculated ratio of Hall resistivity to its classical value $\rho_{xy}^{(\rm cl)} = B/ne$ versus magnetic field for the different theories for the temperature of Fig. 1 [2 K, mobility 1.7 m^2/V s (Ref. 23)]. Line a is ρ_{xy} from Eq. (2). Lines b and c are the Hall resistivities calculated from the original SCBA theory, where line b is with a Gaussian density of states and line c with a semielliptical density of states.

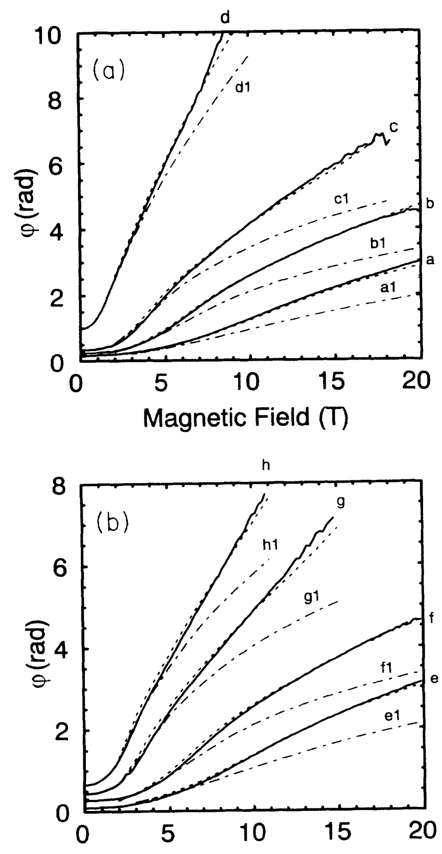

FIG. 3. (a) The full lines are the measured phase shifts $\varphi = \arctan(G/\omega C)$ for some of the electron densities in Fig. 1. 10.5 (a), 5.8 (b), 3.5 (c), and 1.2 (d) (in units of 10^{11} m^{-2}). The dashed lines are the calculated phase shifts which use the ρ_{xx} and ρ_{xy} from the new theory for the same parameters as the experimental data. The dash-dotted lines, numbered ($a1$)–($d1$), are the calculated phase shifts with resistivities from the original SCBA theory, for the same parameters as the experimental data. (b) Same as (a), but now for a fixed electron density of 5.8×10^{11} m^{-2} for frequencies of 3.0 kHz (e), 5.0 kHz (f), 10.0 kHz (g), and 15.0 kHz (h). The curves calculated according to the original SCBA theory and corresponding to the data (e)–(h) are designated by ($e1$)–($h1$).

series of measurements in the field range up to 10 T has verified this behavior. It has been shown[13] that in the linear $\varphi(B)$ regime the phase shift φ is proportional to the edge mode propagation constant q_{EMP} ($\varphi=q_{\mathrm{EMP}}l$, where l is the distance along the perimeter of the electron sheet from the driving electrode to the detecting electrode). The experimentally determined propagation constant, which is in agreement with the theory of Volkov and Mikhailov,[24] is given by

$$q_{\mathrm{EMP}}=\frac{\varepsilon\varepsilon_0\omega}{\alpha\sigma_{yx}}\simeq\frac{\varepsilon\varepsilon_0\omega\rho_{xy}}{\alpha}\ , \tag{8}$$

where the dimensionless constant α describes the inhomogeneous edge of the electron layer and $\varepsilon\varepsilon_0$ is the dielectric permittivity of liquid helium. The second equality is valid if $\sigma_{xx}\ll\sigma_{yx}$. The experimentally determined propagation constant in fields up to approximately 10 T is in good agreement with Eq. (8). With $\rho_{xy}=B/ne$, the constant $\alpha=0.27\pm0.05$ in agreement with Ref. 13.

A linear $\varphi(B)$ relation is no longer observed for data in the field range beyond 10 T [curves a–c in Fig. 3(a) and e–g in Fig. 3(b)]. The maximum phase shift that can be obtained for nonzero signal strengths at fields near 20 T is limited to about 5 rad. However, the simple interpretation of the data according to Eq. (8) fails when σ_{xx} becomes comparable to σ_{yx} (or equivalently $\rho_{xx}\sim\rho_{xy}$), which is the case here (see Fig. 2). Under such conditions, the decay length, wavelength, and width of the edge mode all become comparable and the idea of an edge mode loses its meaning. The data can then only be analyzed by a rigorous numerical calculation of the phase shifts for the given experimental electrode geometry and applied dc potentials.

The numerically calculated phase shifts, which use the ratio $\beta\equiv\rho_{xx}/\rho_{xy}$ and ρ_{xx} as parameters, are plotted also in Fig. 3, along with the experimental curves for both the original SCBA theory[2,5] and the new theory [Eqs. (2) and (7)]. The data are very well described by the new theory without any adjustable parameters and significantly deviate from the original SCBA theory. For both theories, the calculated phase shifts increase nonlinearly with field as do the experimental curves. The phase shifts from Eqs. (2) and (7) are larger than from the original SCBA. The nonlinear behavior observed here, as opposed to the linear behavior observed at lower fields, is characteristic for the value of β approaching unity and is not due to a nonlinear Hall resistivity as suggested earlier.[14] It should be noted from Eq. (2) that the ratio β is the same in both the original SCBA theory and in the theory presented here. The difference between the theories corresponds to neglecting ν_{eff}^2 in the denominator of the expression for σ_{xx} in Eq. (2) in the original SCBA theory. This affects the individual expressions for the conductivities, but not their ratio.

With β being the same in the two theories, the difference in the calculated curves can be attributed to the difference in the individual values of the absolute conductivities (or resistivities) in both theories. As suggested by the simple limiting case corresponding to Eq. (8), the most natural other transport parameter for the present experiments is ρ_{xy}. This is illustrated in the inset of Fig. 2, where the nonlinear $\rho_{xy}(B)$ curves from the original SCBA theory (b and c) are compared to the linear $\rho_{xy}(B)$ behavior which follows from Eq. (2). The curves b and c are for a Gaussian and elliptical density of states (DOS), respectively. Note that in the present field range with the theoretical level broadening, the results barely depend on the shape of the DOS. This is also true for the calculated curves in Fig. 3, which are therefore only drawn for an elliptical DOS. As shown before,[14] the shape of the DOS becomes important in the limit $\Gamma\gg k_BT$.

Since the longitudinal conductivity can be written as $\sigma_{xx}=\beta[\rho_{xy}(1+\beta^2)]^{-1}$ with β being the same in both theories, the relative differences between the σ_{xx} in both theories are the same as the relative differences in the ρ_{xy}. The present experiments therefore are equally sensitive to test the differences between the two theories as would be a σ_{xx} experiment (for instance, Ref. 6). The two types of experiments are independent and complementary.

V. CONCLUSION

We have investigated the quantum magnetotransport phenomena of the SSE's interacting with helium-vapor atoms under extremely strong magnetic fields. It was found that in the quantum limit the effective collision frequency ν_{eff}, which was defined to express the total momentum loss of the 2D electron system, has a very strong field dependence. The ratio $\nu_{\mathrm{eff}}/\omega_c=\rho_{xx}/\rho_{xy}$, which decreases with B in usual systems, is shown to attain a minimum for SSE's in the strong quantizing field regime and then increases to a value of the order of one in high fields. Therefore the high-cyclotron-frequency approximation ($\omega_c\gg\nu_{\mathrm{eff}}$ or $\sigma_{xx}\ll\sigma_{yx}$) eventually fails in the limit of extremely strong magnetic fields and the more general expression for the conductivity tensor valid beyond the approximation must be used. This conclusion and the theoretical concept presented here can be applied also to other 2D electron systems interacting with impurities or with static distortions of interfaces.

The data and the theoretical analysis performed show that quantum magnetotransport phenomena in two dimensions can be perfectly (without any adjustable parameter) described by the use of the generalized SCBA theory in which the specific field dependence of the ratio $\nu_{\mathrm{eff}}(B)/\omega_c$ is taken into account. The investigation of the Hall effect of the SSE's has given an additional proof of the universality of the linear behavior of the Hall resistivity, observed in degenerate 2D electron systems.

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