Statistical properties and statistical interaction for particles with spin: The Hubbard model in one dimension and a statistical spin liquid

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We derive the statistical distribution functions for the Hubbard chain with infinite Coulomb repulsion among particles and for the statistical spin liquid with an arbitrary magnitude of the local interaction in momentum space. Haldane's statistical interaction is derived from an exact solution for each of the two models. In the case of the Hubbard chain, the charge (holon) and the spin (spinon) excitations decouple completely and are shown to behave statistically as fermions and bosons, respectively. In both cases the statistical interaction must contain several components; a rule for the particles with the internal symmetry.

I. INTRODUCTION

It is well known that in the space of dimension higher than two the many-particle wave function is either symmetric or antisymmetric under a permutation group operation; this property leads to the division into the systems of bosons and fermions, respectively. As a consequence, the distribution function for the ideal gas is given either by the Bose-Einstein or by the Fermi-Dirac functions.[1] In low-dimensional systems ($d = 1$ and 2) the situation changes drastically because, e.g., a proper symmetry group in two dimensions for the hard-core particles is the braid group, the characters of which are complex numbers.[2] In such instances the distribution function has not been determined as yet. On the other hand, the distribution function can be changed by the interaction among particles. Such a situation arises, for instance, at the critical point when the system undergoes a phase transition. Below the critical temperature (e.g., in the superconducting phase) the distribution function changes its form from that in the normal state. So, the statistical properties of the particles are influenced by both system dimensionality and by the character of dynamical interaction between particles.

In his paper,[3] Haldane noted that the distribution function can also differ from the Bose-Einstein or the Fermi-Dirac form in the normal state. He generalized the Pauli exclusion principle by introducing the concept of statistical interaction which determines how the number of accessible orbitals changes when particles are added to the system. The paper dealt with the many-particle Hilbert space of finite dimension. The limitation turned out to be irrelevant. Namely, Murthy and Shankar showed[4] that the statistical interaction, when extended to the Hilbert space of infinite dimension, is proportional to the second virial coefficient.

Very recently, Wu[5] solved the problem of the distribution function for Haldane's fractional statistics. He found a general form of the equations for the distribution function for an arbitrary statistical interaction and discussed the thermodynamics of such a gas. Furthermore, Bernard and Wu[6] found the explicit form of the statistical interaction in the case of interacting scalar particles in one dimension. In the particular case of bosons they showed that as the amplitude of a local delta-function interaction changes from zero to infinity, the distribution function evolves from the Bose-Einstein to the Fermi-Dirac form.

In this paper we introduce the spin degrees of freedom into the problem and determine the statistical properties as well as the statistical interaction for particles in two situations. We consider first the Hubbard model in the space of one dimension and with an infinite on-site Coulomb repulsion. In this limit, we show rigorously that the charge excitations (holons) obey the Fermi-Dirac distribution, whereas the spin excitations (spinons) obey the Bose-Einstein distribution. The boson part leads to the correct entropy ($k_B \ln 2$ per carrier) in the Mott insulating limit. As a second example, we express the statistical spin liquid partition function[7] with the help of the statistical interaction concept. These two examples represent nontrivial generalizations of Haldane's fractional statistics to particles with internal symmetry such as spin. In both cases an explicit form of the multicomponent statistical interaction is required. We show that the statistical distributions are changed when the interaction between the particles diverges. For the spin liquid case the form of the distribution functions are also presented for intermediate values of the dynamical interaction. In both cases, the nonstandard statistics is due to the interaction between the particles.

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II. STATISTICAL INTERACTION FOR THE HUBBARD MODEL IN ONE DIMENSION

A. Thermodynamic limit for the Bethe-ansatz equations ($U \to \infty$)

We consider first the one-dimensional system of particles with a contact interaction. One of the simplest models of interacting spin one-half particles was introduced by Hubbard[8]. The Hamiltonian in this case is

$$H = -t \sum_{\langle i,j\rangle \sigma} a^{+}_{i\sigma} a_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}, \qquad (1)$$

where t is the hopping integral between the nearest neighboring pairs $\langle i, j\rangle$ of lattice sites, and U is the on-site Coulomb repulsion when the two particles with spin up and down meet on the same lattice site. We set $t = 1$. This model was solved in one dimension by Lieb and Wu.[9] The solution is given by the set of the Bethe-ansatz equations determining the rapidities $\{k_i\}$, $\{\Lambda_\alpha\}$; i.e.,

$$\frac{2\pi}{L} I_j = k_j - \frac{1}{L} \sum_{\beta=1}^{M} \Theta(2 \sin k_j - 2\Lambda_\beta),$$

$$\frac{2\pi}{L} J_\alpha = \Lambda_\alpha - \frac{1}{L} \sum_{j=1}^{N} \Theta(2\Lambda_\alpha - 2\sin k_j) + \frac{1}{L} \sum_{\beta=1}^{M} \Theta(\Lambda_\alpha - \Lambda_\beta) - \sum_{\beta=1}^{M} \Lambda_\beta \delta_{\Lambda_\alpha, \Lambda_\beta}, \qquad (2)$$

where N is the total number of particles in the system, M is the number of particles with spin down, L is the length of the chain, $j = 1, ..., N$, and $\alpha = 1, ..., M$. I_j is an integer (half-odd integer) for M even (odd), and J_α is an integer (half-odd integer) for $N - M$ odd (even). The phase shift function $\Theta(p)$ is defined by

$$\Theta(p) = -2 \tan^{-1} \left(\frac{2p}{U} \right). \qquad (3)$$

The second set of Eqs. (2) was written in the form better suited to our purposes. The basis in the Hilbert space which diagonalizes the Hamiltonian (1) is called the holon-spinon representation.

The Bethe-ansatz equations can be rewritten in such a way that all dynamical interactions are transmuted into the statistical interaction.[6] We determine explicitly the statistical interaction in the case of the Hubbard model. Our method is a straightforward generalization of the Bernard and Wu result[6] and is valid in the case of infinite interaction only. In this limit, the charge and the spin excitations decouple and there are no bound states in the system.[10] In other words, all bound states in the upper Hubbard subband are pushed out from the physical many-particle Hilbert space.

In the large U limit the Bethe-ansatz equations read[11]

$$\frac{2\pi}{L} I_j = k_j + \frac{1}{L} \sum_{\beta=1}^{M} \Theta(2\Lambda_\beta),$$

$$\frac{2\pi}{L} J_\alpha = \Lambda_\alpha - \frac{N}{L} \Theta(2\Lambda_\alpha) + \frac{1}{L} \sum_{\beta=1}^{M} \Theta(\Lambda_\alpha - \Lambda_\beta) - \sum_{\beta=1}^{M} \Lambda_\beta \delta_{\Lambda_\alpha, \Lambda_\beta}. \qquad (4)$$

We rewrite these equations in the thermodynamic limit, i.e., for $N \to \infty$, $L \to \infty$, and $N/L = \text{const}$. We divide the range of the momentum k and Λ into the intervals with an equal size Δk and $\Delta\Lambda$, as well as label each interval by its midpoints k_i and Λ_α, respectively. We treat the particles with the momenta in the ith or the αth interval as belonging to the ith or the αth group. As usual, the number of available bare single-particle states are $G^0_i = L\Delta k/2\pi$ and $G^0_\alpha = L\Delta\Lambda/2\pi$. These numbers follow from the decomposition of the Bethe-ansatz wave function in the $U \to \infty$ limit.[10] Next, we introduce the distribution functions (the densities of states) for the roots k_j and Λ_α of the Bethe-ansatz equations (4). Namely, we define $L\rho(k_i)\Delta k \equiv N^c_i$ as the number of k values in the interval $[k_i - \Delta k/2, k_i + \Delta k/2]$, and $L\sigma(\Lambda_\beta)\Delta\Lambda \equiv N^s_\beta$ as the number of Λ values in the interval $[\Lambda_\beta - \Delta\Lambda/2, \Lambda_\beta + \Delta\Lambda/2]$. Hence, the two quantities $2\pi\rho(k_i) = N^c_i/G^0_i \equiv n^c_i$ and $2\pi\sigma(\Lambda_\beta) = N^s_\beta/G^0_\beta \equiv n^s_\beta$, are, respectively, the occupation-number distributions for the holon and the spinon excitations in the Hubbard chain. In effect, the Bethe-ansatz equations in the intervals Δk and $\Delta\Lambda$ take the form

$$\frac{2\pi}{L} I(k_j) = k_j + \sum_{\beta} \Theta(2\Lambda_\beta)\sigma(\Lambda_\beta)\Delta\Lambda,$$

$$\frac{2\pi}{L} J(\Lambda_\alpha) = \Lambda_\alpha - \frac{N}{L}\Theta(2\Lambda_\alpha) + \sum_{\beta} \Theta(\Lambda_\alpha - \Lambda_\beta)\sigma(\Lambda_\beta)\Delta\Lambda - L \sum_{\beta} \Lambda_\beta \delta_{\Lambda_\alpha, \Lambda_\beta} \sigma(\Lambda_\beta)\Delta\Lambda. \qquad (5)$$

The function $\rho(k)$ does not appear explicitly in the large U limit.

The numbers of accessible states in each of the ith and the αth groups are

$$\tilde{D}^c_i(\{N^c_i\}, \{N^s_\beta\}) = I(k_i + \Delta k/2) - I(k_i - \Delta k/2), \qquad (6)$$

$$\tilde{D}^s_\alpha(\{N^c_i\}, \{N^s_\beta\}) = J(\Lambda_\alpha + \Delta\Lambda/2) - J(\Lambda_\alpha - \Delta\Lambda/2). \qquad (7)$$

Using the continuous form (5) of the Bethe-ansatz equations we find that $\tilde{D}^c_i = L\rho^c_t(k_i)\Delta k$ and $\tilde{D}^s_\alpha = L\rho^s_t(\Lambda_\alpha)\Delta\Lambda$, where in the thermodynamic limit ($\Delta k \to 0$, $\Delta\Lambda \to 0$) we have, respectively, the total densities of states for charge and spin excitations

$$\rho^c_t(k) = \frac{1}{2\pi}, \qquad (8)$$

and[12]

$$\rho_t^s(\Lambda) = \frac{1}{2\pi} - \frac{N}{2\pi L}\frac{\partial\Theta(2\Lambda)}{\partial\Lambda} + \frac{1}{2\pi}\int d\Lambda'\sigma(\Lambda')\frac{\partial\Theta(\Lambda-\Lambda')}{\partial\Lambda} - \int d\Lambda'\sigma(\Lambda')\Lambda'\frac{\partial\delta(\Lambda-\Lambda')}{\partial\Lambda}. \quad (9)$$

Substituting the form (3) for $\Theta(p)$ to the derivative $\partial\Theta/\partial\Lambda$ one can easily find that in the $U\to\infty$ limit

$$\rho_t^s(\Lambda) = \frac{1}{2\pi} + \sigma(\Lambda). \quad (10)$$

To derive (10) we utilized the fact that $\sigma(\Lambda)$ is a flat function of Λ in the large U limit.[13] We see that the numbers of accessible states for the holons and the spinons in the $U\to\infty$ limit are independent of each other. This result, as we show in the following, leads to the decomposition of the partition function into the holon and the spinon parts.

B. Statistical interaction for the Hubbard chain

We define the statistical interaction and the total number of states for spin one-half particles. For that purpose we work in the basis in which the Hamiltonian is diagonal, i.e., we choose the holon-spinon representation and observe that the dimensions D_i^c and D_α^s of the one-particle Hilbert spaces for the particle in the ith or the αth groups are functionals of both $\{N_i^c\}$ and $\{N_\alpha^s\}$, i.e., $D_i^c = D_i^c(\{N_i^c\},\{N_\beta^s\})$, and $D_\alpha^s = D_\alpha^s(\{N_i^c\},\{N_\beta^s\})$. Namely, starting from the Haldane definition[3] of the change of the number of the accessible states and adopting it to the present situation we obtain

$$\Delta D_i^c = -\sum_j g_{ij}^{cc}\Delta N_j^c - \sum_\alpha g_{i\alpha}^{cs}\Delta N_\alpha^s, \quad (11)$$

$$\Delta D_\alpha^s = -\sum_j g_{\alpha j}^{sc}\Delta N_j^c - \sum_\beta g_{\alpha\beta}^{ss}\Delta N_\beta^s, \quad (12)$$

where the four g parameters are called the statistical interactions. These difference equations can be transformed to the following differential forms:

$$(-g_{ij}^{cc})^{-1}\frac{\partial D_i^c}{\partial N_j^c} + (-g_{i\alpha}^{cs})^{-1}\frac{\partial D_i^c}{\partial N_\alpha^s} = 2, \quad (13)$$

$$(-g_{\alpha i}^{sc})^{-1}\frac{\partial D_\alpha^s}{\partial N_i^c} + (-g_{\alpha\beta}^{ss})^{-1}\frac{\partial D_\alpha^s}{\partial N_\beta^s} = 2. \quad (14)$$

This set of equations establishes the generalization of Haldane's equations for the number of accessible orbitals of the species α in the case of particles without internal symmetry. As before,[3] statistical interactions $\{g\}$ do not depend on the occupations N_α^s and N_i^c, since otherwise the thermodynamic limit would not be well defined. The factors 2 in the right-hand side of (13) and (14) are irrelevant because they can be incorporated into the g parameters. Then the solutions of Eqs. (13) and (14) are

$$D_i^c(\{N_i^c\},\{N_\beta^s\}) = G_i^0 - \sum_j g_{ij}^{cc}N_j^c - \sum_\alpha g_{i\alpha}^{cs}N_\alpha^s, \quad (15)$$

$$D_\alpha^s(\{N_i^c\},\{N_\beta^s\}) = G_\alpha^0 - \sum_j g_{\alpha j}^{sc}N_j^c - \sum_\beta g_{\alpha\beta}^{ss}N_\beta^s. \quad (16)$$

One should note that these solutions are well defined also in the boson limit, since then the corresponding g parameter(s) vanish. The relations $D_i^c(\{0\},\{0\}) = G_i^0$ and $D_\alpha^s(\{0\},\{0\}) = G_\alpha^0$ express the boundary conditions for this problem; the values G_α^s and G_i^c represent the maximal values of available one-particle states in the situation when the holon and the spinon bands are empty.

Additionally, the total number of microscopic configurations with the numbers $\{N_j^c\}$ and $\{N_\beta^s\}$ of holon and spinon excitations is given by

$$\Omega = \prod_{i=1}^{N}\frac{(D_i^c+N_i^c-1)!}{(N_i^c)!(D_i^c-1)!}\prod_{\alpha=1}^{M}\frac{(D_\alpha^s+N_\alpha^s-1)!}{(N_\alpha^s)!(D_\alpha^s-1)!}. \quad (17)$$

In this expression, the two products are in general interconnected via the relations (15) and (16). Each of the factors is defined in the same manner as in Ref. 3. In the fermionic bookkeeping for I_j and J_α the same Ω is obtained with the number of accessible states in the ith and αth groups taken to be[6]

$$\tilde{D}_i^c(\{N_i^c\},\{N_\beta^s\}) = D_i^c(\{N_i^c\},\{N_\beta^s\}) + N_i^c - 1 = G_i^0 + N_i^c - 1 - \sum_j g_{ij}^{cc}N_j^c - \sum_\alpha g_{i\alpha}^{cs}N_\alpha^s, \quad (18)$$

$$\tilde{D}_\alpha^s(\{N_i^c\},\{N_\beta^s\}) = D_\alpha^s(\{N_i^c\},\{N_\beta^s\}) + N_\alpha^s - 1 = G_\alpha^0 + N_\alpha^s - 1 - \sum_j g_{\alpha j}^{sc}N_j^c - \sum_\beta g_{\alpha\beta}^{ss}N_\beta^s. \quad (19)$$

Rewriting these equations for each of the intervals Δk and $\Delta\Lambda$ we easily find that in the $\Delta k\to 0$ and $\Delta\Lambda\to 0$ limits these four types of statistical interactions reduce to the following form:

$$g^{cc}(k,k') = \delta(k-k'), \quad (20)$$

$$g^{cs}(k,\Lambda) = g^{sc}(\Lambda,k) = g^{ss}(\Lambda,\Lambda') = 0. \quad (21)$$

Thus, the vanishing g functions in (21) simplify the expression (17) for the total number of available configurations, which is then

$$\Omega = \prod_{i=1}^{N} \frac{(G_i^0)!}{(N_i^c)!(G_i^0 - N_i^c)!} \prod_{\alpha=1}^{M} \frac{(G_\alpha^0 + N_\alpha^s - 1)!}{(N_\alpha^s)!(G_\alpha^0 - 1)!}. \quad (22)$$

The statistical weight Ω factorizes into the holon (Ω^c) and the spinon (Ω^s) parts. This, once again, expresses the fact that the spin and the charge degrees of freedom decouple in the $U \to \infty$ limit.[10] As a consequence, the entropy of the system is a sum of the two parts $S = S^c + S^s = k_B \ln \Omega^c + k_B \ln \Omega^s$, where the corresponding expressions calculated per particle are

$$S^c = -k_B \frac{1}{N_a} \sum_{i=1}^{N} [n_i^c \ln n_i^c + (1 - n_i^c) \ln(1 - n_i^c)], \quad (23)$$

and

$$S^s = -k_B \frac{1}{N_a} \sum_{\alpha=1}^{M} [n_i^s \ln n_i^s - (1 + n_i^s) \ln(1 + n_i^s)], \quad (24)$$

where N_a is the number of atomic sites.

We recognize immediately that the holon contribution to the system entropy coincides with that for spinless fermions, whereas the spinon contribution reduces to localized-spin moments ($k_B \ln 2$) in the Mott-insulator limit and in the spin disordered phase, i.e., when $n_\alpha^s = 1$ and $M = N_a/2$. In general, one may say that S^c provides the entropy of charge excitations (and vanishes in the Mott insulating limit $n_i^c = 1$), whereas S^s represent the spin part of the excitation spectrum. This demonstrates again that the holon (charge) excitations are fermions and the spinon (spin) excitations are bosons. In the $U \to \infty$ limit considered here the Heisenberg coupling constant ($J = 4t^2/U$) vanishes and the spin wave excitations do not interact with each other.[14] In other words, they are dispersionless bosons. Also, the charge excitations are spinless fermions. The total entropy of the system reduces in the Mott insulating spin-disordered limit to $S = k_B \ln 2$. This value is different from that for the Fermi liquid in the high-temperature limit, which is $2k_B \ln 2$. This difference confirms on statistical grounds the inapplicability of the Fermi liquid concept to the Hubbard model in one dimension.

III. STATISTICAL INTERACTION FOR THE SPIN LIQUID

In this section, we derive the statistical interaction for the so-called statistical spin liquid. This concept was introduced in Ref. 7 to describe the thermodynamic properties of strongly interacting electrons. The basic assumption in this approach is to exclude the doubly occupied configurations of electrons with spin up and down not only in real space but also in reciprocal space (with given $\mathbf{k}$). This assumption leads to a different class of universality for electron liquids. Its thermodynamics in the normal, magnetic, and superconducting states were examined in the series of papers.[7,15] A justification of this approach has as its origin in the concept of the singularity in the forward scattering amplitude due to interparticle interactions. Namely, it was noted by Anderson[16] and by Kveshchenko[17] that in two spatial dimensions this amplitude may diverge either due to the Hubbard on-site repulsion, or due to the long-distance current-current interaction mediated by the transfer gauge fields. With the assumption that in those situations the wave vector is still a good quantum number, one can write down the phenomenological Hamiltonian describing such liquid in the form

$$H = \sum_{\mathbf{k}\sigma} (\epsilon_{\mathbf{k}} - \sigma h) N_{\mathbf{k}\sigma} + U_s \sum_{\mathbf{k}} N_{\mathbf{k}\uparrow} N_{\mathbf{k}\downarrow}. \quad (25)$$

In this model, $\epsilon_{\mathbf{k}}$ is the dispersion relation for the particles with the wave vector $\mathbf{k}$ moving in the applied external magnetic field h, $N_{\mathbf{k}\sigma}$ is the number of electrons in the state $|\mathbf{k}\sigma\rangle$, and $\sigma = \pm 1$ is the projected spin direction. The number of double occupancies in given $\mathbf{k}$ state is $N_{\mathbf{k}d} = N_{\mathbf{k}\uparrow} N_{\mathbf{k}\downarrow}$. The nonvanishing $N_{\mathbf{k}d}$ causes an increase of the system energy by $U_s > 0$ for each doubly occupied $\mathbf{k}$ state. Finally, we will put $U_s \to \infty$ because this model is to represent the situation with the singular forward scattering amplitude. It turns out that this exactly solvable model[18] belongs to the class of models with Haldane's fractional statistics, as shown below. In this case, the statistical interaction is proportional to $\delta_{\mathbf{k}\mathbf{k}'}$ in $\mathbf{k}$ space, but is a nondiagonal matrix in the extended spin space. The results are valid for an arbitrary dimension of space.

To prove this we define the total size of the Hilbert space of the many-particle states determined by the number of physically inequivalent configurations

$$\Omega = \prod_{\mathbf{k}} \frac{(D_{\mathbf{k}\uparrow} + N_{\mathbf{k}\uparrow} - N_{\mathbf{k}d} - 1)!}{(N_{\mathbf{k}\uparrow} - N_{\mathbf{k}d})!(D_{\mathbf{k}\uparrow} - 1)!} \frac{(D_{\mathbf{k}\downarrow} + N_{\mathbf{k}\downarrow} - N_{\mathbf{k}d} - 1)!}{(N_{\mathbf{k}\downarrow} - N_{\mathbf{k}d})!(D_{\mathbf{k}\downarrow} - 1)!} \frac{(D_{\mathbf{k}d} + N_{\mathbf{k}d} - 1)!}{(N_{\mathbf{k}d})!(D_{\mathbf{k}d} - 1)!}. \quad (26)$$

Due to the local nature of the interaction in $\mathbf{k}$ space we must treat separately the singly occupied states as distinct from those with double occupancy in reciprocal space. Then, the statistical weight Ω expresses the possible ways of distributing $N_{\mathbf{k}\sigma} - N_{\mathbf{k}d}$ quasiparticles over $D_{\mathbf{k}\sigma}$ states and $N_{\mathbf{k}d}$ quasiparticles over the $D_{\mathbf{k}d}$ states. In general, the dimension of the one-particle Hilbert space for the singly ($D_{\mathbf{k}\sigma}$) and the doubly occupied ($D_{\mathbf{k}d}$) states is the function of the number of other quasiparticles $\{N_{\mathbf{k}\sigma}\}$ and $\{N_{\mathbf{k}d}\}$ (Refs. 7 and 18), i.e.,

$$D_{\mathbf{k}\alpha}(\{N_{\mathbf{k}\beta}\}) = G_{\mathbf{k}}^0 - \sum_{\beta} g_{\alpha,\beta}(\mathbf{k}, \mathbf{k}')(N_{\mathbf{k}'\beta} - \delta_{\alpha\beta}\delta_{\mathbf{k}\mathbf{k}'}), \quad (27)$$

where α and β label the configurations $\uparrow, \downarrow, d$; these states define the extended spin space. Note that in contrast to Eqs. (15) and (16) we define here the boundary conditions via the relations $D_{\mathbf{k}\alpha}(\{N_{\mathbf{k}\beta} = \delta_{\alpha\beta}\delta_{\mathbf{k}\mathbf{k}'}\}) = G^0_{\mathbf{k}}$, i.e., the maximal dimension of the single-particle Hilbert space is defined for an occupied configuration in each category, not for an empty one. These new conditions are equivalent to the form appearing in Eqs. (15) and (16), in the thermodynamic limit. Since the Hamiltonian (25) does not mix different momenta, we find the general solution for $g_{\alpha\beta}(\mathbf{k}, \mathbf{k}')$ in the form

$$g_{\alpha\beta}(\mathbf{k}, \mathbf{k}') = \delta_{\mathbf{k}\mathbf{k}'} \otimes g_{\alpha\beta}. \tag{28}$$

Hence, the statistical interaction is diagonal in $\mathbf{k}$ space for this model. Next, to get the exact solution of the Hamiltonian (25) we choose

$$g_{\alpha\beta} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{29}$$

and substitute (29) and (28) into (27) to find that the total size of the many-particle Hilbert space is given by

$$\Omega = \prod_{\mathbf{k}} \frac{(G^0_{\mathbf{k}})!}{(N_{\mathbf{k}\uparrow} - N_{\mathbf{k}d})!(N_{\mathbf{k}\downarrow} - N_{\mathbf{k}d})!(N_{\mathbf{k}d})!(G^0_{\mathbf{k}} - N_{\mathbf{k}\uparrow} - N_{\mathbf{k}\downarrow} + N_{\mathbf{k}d})!}. \tag{30}$$

This result is exactly the same as that obtained in Ref. 7 (cf. Appendix B). Therefore, we conclude that this model also belongs to the class of models with Haldane's statistics. In this case, the changes in the distribution functions are not due to the phase shift between different momenta but rather due to the mutual (dynamic) interactions between quasiparticles with the same $\mathbf{k}$ but different spin. The interaction pushes some of the states upward in energy, leading to the following form of momentum distribution functions

$$\frac{N_{\mathbf{k}\sigma} - N_{\mathbf{k}d}}{G^0_{\mathbf{k}}} = \frac{e^{\beta U_s} e^{\beta(\epsilon_{\mathbf{k}}-\mu)} \cosh(\beta h)}{1 + e^{\beta U_s} e^{\beta(\epsilon_{\mathbf{k}}-\mu)}[e^{\beta(\epsilon_{\mathbf{k}}-\mu)} + 2\cosh(\beta h)]} \times [1 + \sigma \tanh(\beta h)], \tag{31}$$

$$\frac{N_{\mathbf{k}d}}{G^0_{\mathbf{k}}} = \frac{1}{1 + e^{\beta U_s} e^{\beta(\epsilon_{\mathbf{k}}-\mu)}[e^{\beta(\epsilon_{\mathbf{k}}-\mu)} + 2\cosh(\beta h)]}, \tag{32}$$

which are easily obtained by minimizing the thermodynamic potential with respect to $N_{\mathbf{k}\sigma}$ and $N_{\mathbf{k}d}$ separately.[18] It is easy to show that those distributions evolve from the Fermi-Dirac function to the statistical spin liquid distribution when the U_s changes from zero to infinity.[7]

The limit $U_s \to \infty$ represents the physical situation in which $N_{\mathbf{k}d} \equiv 0$. In other words, there are no double occupancies in $\mathbf{k}$ space. All states are singly occupied by the quasiparticles with either spin up or spin down, or empty. In this limit, the statistical interaction (28) reduces to the 2×2 matrix form

$$g_{\sigma\sigma'}(\mathbf{k}, \mathbf{k}') = \delta_{\mathbf{k},\mathbf{k}'} \otimes \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}. \tag{33}$$

Then, the statistical weight is[7]

$$\Omega = \prod_{\mathbf{k}} \frac{(G^0_{\mathbf{k}})!}{(N_{\mathbf{k}\sigma})!(G^0_{\mathbf{k}} - N_{\mathbf{k}\sigma} - N_{\mathbf{k}\bar{\sigma}})!}. \tag{34}$$

Such a liquid is called the statistical spin liquid. This class of quantum liquids is similar, in some respects, to the Bethe-Luttinger liquid discussed above. For example, because of the mutual interaction between spin up and down particles, one half of the total number of states ($2N_a$) are pushed out of the physical space in the $U_s \to \infty$ limit. Therefore, the entropy of the statistical spin liquid in the high-temperature limit is the same as in the case of the Hubbard chain with the infinite interaction because the entropy of the system in this temperature limit is given in terms of the degeneracy of the state only.[21] In particular, the entropy in the statistical spin liquid for $N = N_a$ equals $k_B N \ln 2$, which is the correct value for a Mott insulator. Also, the high-temperature value of a thermopower is the same for both liquids.[21] It was also shown[19] that the magnetization of statistical spin liquid has the same form as that of the Hubbard chain with the infinite repulsion, i.e., that for localized moments.[11] However, the direct consequence of the statistical interaction is also a breakdown of the Luttinger theorem: the volume enclosed by the Fermi surface is twice that for the Fermi liquid. This arises because of the differences in the microscopic character of the single-particle excitations in these two liquids.

IV. CONCLUSIONS

In this paper, we considered statistical properties of the two model system: the Hubbard chain with infinite repulsion and the statistical spin liquid. We determined the form of the Haldane statistical interaction in each case. In the one-dimensional Hubbard model a distribution function emerges due to the presence of the phase shift between pairs of states with different rapidities $\{\Lambda_\alpha\}$ and $\{k_i\}$. In the $U \to \infty$ limit, when all bound states are excluded, the charge excitations (holons) behave statistically as fermions, and the spin excitations (spinons) behave as bosons. The holons have a simple energy dispersion ϵ_k coinciding with the bare band energy, whereas the spinons are dispersionless. An equivalent alternative approach[20] is based on the fermionic representation of both the holon and the spinon degrees of freedom; in that approach the spinons and the holons acquire complicated forms of the effective dispersion relation. The latter approach allows for a generalization of the treatment of the Hubbard chain for finite U, the limit unavailable with our present analysis. Nonetheless, our analysis is also applicable to other models when the separation into the charge and the spin degrees of freedom occurs (cf. the Kondo problem[22]).

In the statistical spin liquid case the mutual interaction between quasiparticles with the same **k** leads to the exclusion of the double occupied configurations in reciprocal space. In that case the statistical interaction is diagonal in **k** but has a nondiagonal structure in spin space. In that case the statistical distribution changes with growing interaction from the Fermi-Dirac form to the spin liquid form.[7]

It is interesting that those two models of particles with the internal symmetry can be classified as the models with the fractional statistics in the Haldane sense. In contrast to the case of scalar particles,[6] the distribution functions in the present situation take the form of either holon-spinon or the spin liquid distributions. Those possibilities arise only when the particles have some internal symmetry (spin,color). One may also say that in those cases the statistical interactions have a tensorial character in space in which the Hamiltonian is diagonal.

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