Edge magnetoplasmons of two-dimensional electron-gas systems

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(Received 8 June 1994)

Magnetoplasma modes localized near the edge of a two-dimensional electron gas with a nonabrupt edge density profile are investigated within a simple classical approximation. Two distinct groups of edge magnetoplasmons are found. The high-frequency modes occur only for finite values of the wave vector q along the edge; they are the higher multipole edge modes that occur when the magnetic field $\mathbf{B}=0$. The low-frequency modes have frequencies $\omega(q)$ that approach zero as q approaches zero. They exist only for finite values of $\mathbf{B}$ and propagate in only one direction along the edge for a given direction of $\mathbf{B}$.

The surface plasmons first predicted by Ritchie[1] are now commonly referred to as surface monopole modes. Additional surface plasma modes[2] had been predicted for over 20 years before they were experimentally observed on K and Na surfaces by inelastic electron scattering.[3] These modes appear only when the electron density decreases sufficiently slowly from its bulk value to zero through the surface. They are commonly called "multipole" surface plasmons because the integral normal to the surface of their electron density variation vanishes.

There has recently been considerable interest in plasma modes localized near the edge of a two-dimensional electron gas (2DEG).[4–10] For an abrupt edge density profile the dispersion relation of the edge plasmon can be obtained analytically using a simple local model for the conductivity.[4] For a nonabrupt profile in the absence of a magnetic field higher multipole modes of frequency $\omega_n(q)$ are found[5] for values of the wave vector q along the edge larger than some critical q_n. The presence of a magnetic field perpendicular to the 2D layer plays an important role in determining the allowed edge mode frequencies. One example is the interedge (i.e., the one-dimensional interface between different 2D electron gas regions) mode found by Mikhailov and Volkov.[8] Another is the sequence of low-frequency magnetoplasmon modes explored by Nazin and Shikin[9] for electrons on liquid helium.

In this paper we study the edge magnetoplasmon modes located near the edge of 2DEG systems. The integral equation, which is obtained within a local approximation, can be solved numerically for arbitrary electron density profiles. In particular, when the selvage region is modeled with a simple double-step function, the integral equation can be solved analytically by replacing the exact kernel by an approximate one with simpler analytical properties. The aim of the present paper is to investigate carefully the effect of the magnetic field on the edge modes and discuss the physics underlying the results.

Let us consider a 2DEG confined in the x-y plane with an equilibrium density profile $n(x)$ in a perpendicular magnetic field $\mathbf{B}$ along the positive z direction. The collective electronic modes of the system correspond to self-sustaining density fluctuations of the form of $\delta n(\mathbf{r},t)=\delta n(x)\exp(iqy-i\omega t)$, when the electron density profile is assumed to be homogeneous in the y direction. Three basic equations must be solved self-consistently in order to obtain the plasmon modes. These equations are the Poisson's equation $\nabla\cdot[\epsilon\mathbf{E}(\mathbf{r})]=4\pi\delta n(\mathbf{r})$, where ϵ is the dielectric constant, the equation of continuity $\nabla\cdot\mathbf{j}(\mathbf{r})=i\omega\delta n(\mathbf{r})$, and the constitutive equation $\mathbf{j}(\mathbf{r})=\sigma(\mathbf{r})\cdot\mathbf{E}(\mathbf{r})$, where $\sigma(\mathbf{r})$ is the conductivity tensor.

From the simple Drude model, one can express the conductivity tensor explicitly in terms of the electron equilibrium density distribution $n(x)$ as $\sigma_{xx}(\mathbf{r})=\sigma_{yy}(\mathbf{r})=in(x)e^2\omega/m_e(\omega^2-\omega_c^2)$ and $\sigma_{xy}(\mathbf{r})=-\sigma_{yx}(\mathbf{r})=i\sigma_{xx}(\mathbf{r})\omega_c/\omega$, where m_e is the electron mass and $\omega_c(=eB/m_ec)$ is the electron cyclotron frequency.

In the electrostatic limit the electric field can be expressed as the gradient of a scalar potential $\phi(\mathbf{r})$. By combining the three equations we can arrive at a self-consistent integral equation for the potential $\phi(x)$,

$$\phi(x)=\frac{4\pi e^2}{m_e\epsilon(\omega^2-\omega_c^2)}\times\int dx' L_q'(x-x')\left[n(x')\left[q^2-\frac{d^2}{d(x')^2}\right]-\frac{dn(x')}{dx'}\left[\frac{d}{dx'}-\frac{\omega_c}{\omega}q\right]\right]\times\phi(x') , \tag{1}$$

where the integration kernel L_q is given by

$$L_q(x)=\frac{1}{4\pi}\int dp\frac{\exp(ipx)}{\sqrt{p^2+q^2}} . \tag{2}$$

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The requirement of the existence of nontrivial solutions of Eq. (1) determines the collective modes of the electron system.

When we choose an electron density profile described by a double-step model

$$n(x)=n_B\Theta(-x-a)+n_S\Theta(-x)\Theta(x+a)\ , \tag{3}$$

where Θ is the single-step function, to account for the electron-density variation near the edge at $x=0$, Eq. (1) can be solved analytically using the Wiener-Hopf method if L_q in Eq. (2) is replaced by an approximate kernel $L_q(x)=2^{-3/2}\exp(-\sqrt{2}|qx|)$. With the condition that the potential be continuous at both the boundaries $x=0$ and $x=-a$, the plasmon modes are found to be solutions of the equation

$$(\alpha_S-\alpha_B)\cosh(\gamma_S qa)+(\alpha_S\alpha_B-1)\sinh(\gamma_S qa)=0\ , \tag{4}$$

where $\gamma_v^2=2[\omega_v^2-(\omega^2-\omega_c^2)]/[2\omega_v^2-(\omega^2-\omega_c^2)]$ with $v=B$ or S, $\alpha_B=\{\gamma_B[2\omega_B^2-(\omega^2-\omega_c^2)]-2(\omega_B^2-\omega_S^2)\omega_c/\omega\}/\gamma_S[2\omega_S^2-(\omega^2-\omega_c^2)]$, and $\alpha_S=[\sqrt{2}(\omega^2-\omega_c^2)+2\omega_S^2\omega_c/\omega]/\gamma_S[2\omega_S^2-(\omega^2-\omega_c^2)]$. In the limiting case of $n_S=n_B$ or $a=0$ where the double-step model becomes a single step, Eq. (4) reduces to the correct form

$$2\omega_B^2+2\sqrt{2}\omega\omega_c-3\omega^2=0 \tag{5}$$

as obtained in Ref. 4. On the other hand, when $a\to\infty$ and $q\neq 0$, Eq. (4) reduces to $(\alpha_B+1)(\alpha_S-1)=0$. Setting the first factor equal to zero gives the form of Eq. (5) with ω_B replaced by ω_S. This is just the edge magnetoplasmon of the lower density edge region. Setting the second factor to zero yields

$$\gamma_B(2\omega_B^2-\omega^2+\omega_c^2)+\gamma_S(2\omega_S^2-\omega^2+\omega_c^2)=2(\omega_B^2-\omega_S^2)\frac{\omega_c}{\omega}\ . \tag{6}$$

Equation (6) determines the frequency of the interedge plasmon localized near the edge $x=-a$. In other words, the system sustains two well-defined magnetoplasma modes, an edge mode of the lower density edge region and an interedge magnetoplasmon localized at the inner edge $x=-a$. This behavior is expected since for any finite wave vector q, when a becomes very large, the two localized edge modes will not interact with one another.

If one studies Eq. (6) carefully, one finds that the interedge mode propagating in the positive y direction exists only for a finite magnetic field along the positive z direction. This is consistent with the result previously obtained by Mikhailov and Volkov.[8]

In Fig. 1, we show the edge magnetoplasmon frequency ω as a function of wave vector q for an applied magnetic field along the positive z direction as calculated from Eq. (4). For plasma oscillations propagating along the positive y direction, there are clearly two groups of edge magnetoplasmon modes. One group consists of two low-frequency modes whose frequency approaches zero for small q, and the other of high-frequency modes which emerge from the bulk magnetoplasma mode with density n_B. We have identified the two modes in the low-frequency group as the monopole edge magnetoplasmon

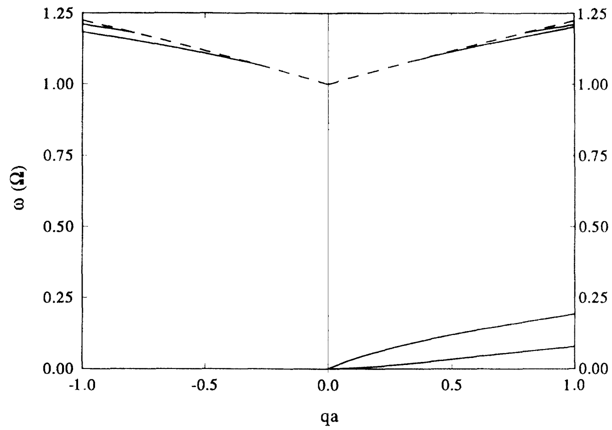

FIG. 1. Dispersion of edge magnetoplasmon modes for a double-step electron density profile [see Eq. (3)] obtained with the approximate kernel for the applied magnetic field along the positive z direction. Here $n_S/n_B=0.6$, $|\omega_c|=\Omega$, and $\Omega=(4\pi n_s e^2/\epsilon m a)^{1/2}$. The 2D bulk magnetoplasmon with an electron density n_B is also shown as the dashed line.

(with higher frequency) and the interedge magnetoplasmon, respectively. As $q\to 0$, the monopole mode is determined essentially by Eq. (5) as if the electron density variation near the edge plays no part. The reason for this is that in the limit $qa\ll 1$ the electric field associated with the edge monopole oscillation samples deep into the bulk region. The variation of the electron density near the edge makes no contribution to the restoring force. The self-consistent potential associated with the interedge mode is bound at the edge $x=-a$ and becomes more localized as q increases. In fact, as q becomes larger than $3/a$, the frequency of the interedge mode is given by Eq. (6). However, in the limit of $q\to 0$, the dispersion of this mode is quadratic. The high-frequency modes correspond to the higher multipole modes as discussed previously in Ref. 5. For plasma oscillations propagating along the negative y direction only the high-frequency group of modes exists.

The analytical solutions of Eq. (1) for simple step-function profiles provide a basic understanding of the nature of these edge magnetoplasmon modes. For an arbitrary electron density variation at the sample edge Eq. (1) cannot be solved analytically. To facilitate numerical calculations, we transform the integral equation into a matrix equation by expanding the potential $\phi(x)$ in Laguerre polynomials,

$$\phi(x)=\exp(|q|x)\sum_{n=0}^{\infty}c_n L_n(-2|q|x)\ . \tag{7}$$

We find that if the electron-density profile near the edge takes the form of a polynomial in x, the computation is simplified considerably. The plasmon dispersion relations are obtained by requiring that nontrivial solutions exist for the matrix equation and calculated by truncating the matrix at a finite order N. We find that the numerical results converge quickly with increasing N, and that choosing $N=20$ gives the desired numerical accuracy for many density profiles.

To demonstrate how a smooth selvage region affects the magnetoplasmon spectrum, we studied several models for the electron density profile in which the density drops from its bulk value to zero smoothly. Although the details may differ a little, the basic features remain the same. The edge magnetoplasmon modes form two distinct groups. The modes in the high-frequency (greater than ω_c) group exist as well-defined edge excitations only for finite values of wave vector q. We identify these modes as the higher multipole modes corresponding to the trapped bulk magnetoplasmon modes of the lower-density region. The low-frequency gapless group exists only for plasma oscillations propagating along the positive y direction, and they appear to be infinite in number if damping is ignored. The usual monopole mode is one of this group. It appears that the modes in the low-frequency group are all acoustic modes in the long-wavelength limit. Furthermore, at a given q, as the magnetic field increases, the frequency of the monopole mode decreases while that of the other modes increases.

In Fig. 2, we plot the edge magnetoplasmon dispersion for the electron profile which decreases smoothly (both the profile and its first derivative are continuous) in the selvage region, i.e.,

$$n(x)/n_0 = \begin{cases} 0 & \text{if } x > 0 \\ 3(x/a)^2 + 2(x/a)^3 & \text{if } 0 > x > -a \\ 1 & \text{if} -a > x \ . \end{cases}$$

Figure 2 clearly shows the feature of edge magnetoplasmons for a smooth electron-density profile. From our numerical study, we find that the number of nodes in the x direction for the electron-density variation associated with the high-frequency excitations increase by unity with each successive higher mode. Surprisingly, the density variations associated with the low-frequency group exhibit the same behavior with each successive lower-frequency mode. In other words, with similar density variations, we find two charge-density excitations with entirely different frequencies for a fixed q. The reason for this appears to be the highly frequency-dependent nature of the conductivity tensor. The very different response in the high- and low-frequency region appears to give rise to two modes with similar spatial variation and quite different frequency.

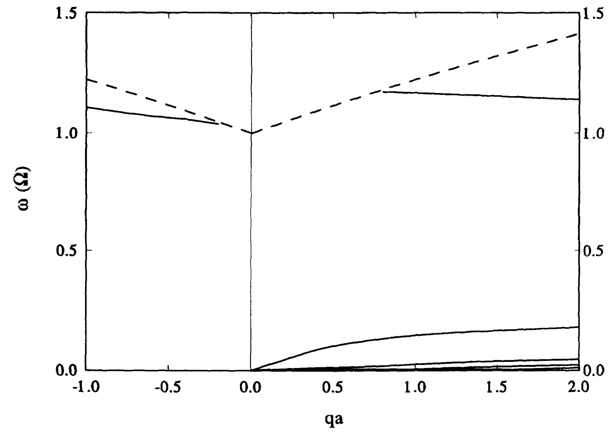

FIG. 2. Dispersion of edge plasmon modes for an electron density profile decreasing smoothly near the edge [see Eq. (8)] for the applied magnetic field along the positive z direction. Here $|\omega_c| = \Omega$, $\Omega = (4\pi n_s e^2/\epsilon m a)^{1/2}$. The 2D bulk plasmon dispersion with an electron density n_0 is shown as the dashed line.

The long-wavelength limit of a particular smooth-edge density profile for a classical 2D electron gas has been studied previously by Nazin and Shikin,[9] who found, in addition to the usual monopole mode, an infinite number of acoustic edge plasmons. Similar results were later discussed by Aleiner and Glazman[10] in the limit of a strong magnetic field. These modes for 2D electron gas systems appear to be the same as those low-frequency modes discussed here.

Although multiple-plasmon modes in 2DEG systems have yet to be observed experimentally, samples of semiconductor quantum-well structures which simulate 2DEG systems can be prepared with controllable electron density profile near the edge. Such systems provide a realistic opportunity to experimentally study the edge magnetoplasmons.

We would like to thank Ximing Chen for helpful discussions. This research was supported in part by the Division of Materials Sciences of the U.S. Department of Energy under Contract NO. DE-AC05-84OR21400 with Martin-Marietta Energy Systems, Inc.

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