Correlations in coupled layers of electrons and holes

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We investigate interparticle correlations for two layers of charge-carrier liquid trapped in adjacent wells of a gallium arsenide heterostructure. We determine in the absence of appreciable tunneling the static polarizability, the pair correlation functions, and the local-field corrections both within a layer and between the layers. The liquid ground state has points of instability to states that are inhomogeneous in the density. An important result is that, in contrast with electron-electron layers, for electron-hole layers the interlayer correlations favor the formation of the inhomogeneous states. The pair correlation functions display unusual behavior near the points of instability.

I. INTRODUCTION

Recently, there has been considerable interest in systems consisting of two parallel layers of electron liquid. These can be fabricated by trapping electrons in adjacent quantum wells of a semiconductor heterostructure. Two layer systems have some interesting properties not the least of which is that the Coulomb coupling of the carriers in one layer to the carriers in the other make the layers act as compressible polarizable backgrounds for each other. A lot of work has been done in the presence of a magnetic field but we concentrate here on the zero-magnetic-field case.

There have been many studies of these systems within the random-phase approximation (RPA). The RPA dielectric function, electronic properties, and the collective excitations have been calculated for two layers[1–7] and also for superlattice systems.[8–12] In the study of light scattering, the RPA density-density correlations have been calculated and optical properties investigated.[13–16] We have discussed corrections to the RPA for two layer systems[17,18] and the corrections have also been investigated by Lu and Golden,[19,20] by Kalman, Ren, and Golden,[21] by Kalman and Golden,[22] and by Zheng and MacDonald.[23]

The presence of additional mobile charge in the second layer accentuates correlation effects compared with the case of a single layer. In Ref. 17 we showed that at relatively high densities the corrections to the RPA could change the ground state for two electron layers from the homogeneous liquid to inhomogeneous states consisting of a charge density wave or a Wigner crystal in each layer.

Here we are particularly interested in the effect of interlayer correlations on the static properties of two layer systems. The interlayer correlations take into account the effect a charge carrier in one layer has on the charge density distribution in the second, layer. Correlations within a layer and perpendicular to the layer are quite different in nature, first because the carriers can only move parallel to the layer and second, because in the absence of tunneling there is only exchange parallel to the layer. The correlations between layers become more pronounced the smaller the separation between the layers becomes. The length scale for this is given by the layer spacing for which Coulomb interactions between layers are comparable in strength to interactions within a layer. The increase in correlation strength does not continue indefinitely since if the layers are too close together the carrier envelope functions from opposite layers overlap, producing appreciable tunneling between layers which in turn would reduce the interlayer correlations.

We will restrict ourselves to quite large layer spacings for which the intralayer correlations are much larger than the interlayer correlations and for which there is negligible tunneling between the layers. In gallium arsenide the envelope functions in quantum wells are at least 10 nm in width, so to avoid overlap we restrict the layer spacings $a \gtrsim 30$ nm.

Suppose we start with two electron layers each at a density $r_s \gtrsim 37$. If the layers are far apart interactions between electrons in opposite layers will be small and the interlayer correlations negligible. Nevertheless for $r_s \gtrsim 37$ there are such strong correlations within the layers that the electrons will form a Wigner crystal in each layer[23] no matter how widely they are separated. If we now decrease the layer spacing, the interlayer correlation effects will increase. What we find is that small correlations between layers can sustain the effect of the strong correlations within a layer resulting in the system remaining in an inhomogeneous ground state for densities higher than $r_s = 37$.

We will primarily be concerned here with the properties of a layer of holes coupled to a layer of electrons, although we do present some results for electron-electron layers. The attractive interactions between electrons and holes have a larger effect than the repulsive interactions acting between electron-electron layers. We calculate the pair correlation functions and the local-field corrections for carriers in the same layer and in different layers. We also look at the effect that interlayer correlations have on

the positions of points of instability in the liquid ground state.[17]

Lozovik and Yudson[24] considered an electron-hole layer system for configurations where the interlayer correlations dominate, that is in the opposite extreme to ours. If the interlayer correlations are very strong the electrons and holes will tend to form excitonic bound pairs which interact only weakly. They predict a weakly interacting exciton gas with superfluidity resulting from Bose-Einstein condensation of the excitons. This result applies for small spacings. When, as in our case, the layer spacing exceeds the effective Bohr radius the excitonic binding energy decreases exponentially with the square of the layer spacing. For gallium arsenide the effective Bohr radius is $a_B^* = 9.8$ nm, which is much smaller than our minimum layer spacing so excitonic formation is unlikely.

Two layers of holes would have similar properties to two electron layers at the same r_s. However, for gallium arsenide, because of the larger effective mass of holes, the same value of r_s corresponds to a much higher hole density than electron density. An areal density of 2×10^{10} cm^{-1} corresponds for example to $r_s \approx 3$ for electrons but to $r_s \approx 19$ for holes. While at $r_s = 3$ the system is relatively weakly correlated, for $r_s = 19$ the correlations are strong.

In Sec. II, we describe the method for calculating the static response and pair correlation functions for the two layer system. Section III discusses the results and in Sec. IV we make some concluding remarks.

II. THEORY

We consider a quantum well structure which produces two spatially separated, parallel layers of charge carriers. These are free to move in the x-y plane parallel to the layer but are confined in the z direction within layers of finite thickness. Each layer has only one species of charge carrier (electrons or holes). We assume that only one subband in each quantum well is occupied and that the wave functions from different wells do not appreciably overlap. Although we confine discussion to the two layer case, extension to an arbitrary number of layers or to a finite number of occupied subbands is quite straightforward.

The single-particle wave functions can be written

$$\Psi_{\boldsymbol{K},l}(\boldsymbol{R},z) = \frac{1}{\sqrt{A}} e^{i\boldsymbol{K}\cdot\boldsymbol{R}} \zeta_l(z), \tag{1}$$

where A is the area of the sample and l labels the layer. Capitalized vectors such as $\boldsymbol{R}$ or $\boldsymbol{K}$ lie in the x-y plane parallel to the layers.

The envelope function $\zeta_l(z)$ is calculated from the one-dimensional Schrödinger equation,

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + V(z)\right]\zeta_l(z) = E_l\zeta_l(z). \tag{2}$$

The potential $V(z)$ includes Hartree and correlation terms in the spirit of the local density functional approach (see, e.g., Ref. 25).

Both the Coulomb interaction between carriers in layers l and l', $V_{ll'}(\boldsymbol{Q})$, and any external potential acting on the carriers in layer l, $V_l^{\mathrm{ext}}(\boldsymbol{Q},\omega)$, contain the form factors $|\zeta_l(z)|^2$,

$$V_{ll'}(\boldsymbol{Q}) = v_{\boldsymbol{Q}} \int dz \int dz'\; e^{-q|z-z'|} |\zeta_l(z)|^2\; |\zeta_{l'}(z')|^2,$$
$$V_l^{\mathrm{ext}}(\boldsymbol{Q},\omega) = \int d\boldsymbol{R} \int dz\; e^{-i\boldsymbol{Q}\cdot\boldsymbol{R}}\; |\zeta_l(z)|^2\; V^{\mathrm{ext}}(\boldsymbol{R},z;\omega), \tag{3}$$

where $v_{\boldsymbol{Q}} = [2\pi e^2/\boldsymbol{Q}\epsilon_0]$ is the Coulomb interaction inside the semiconductor and ϵ_0 is the background dielectric constant of the semiconductor. $V^{\mathrm{ext}}(\boldsymbol{R},z;\omega) \equiv V^{\mathrm{ext}}(\boldsymbol{r},\omega)$ is the time Fourier transform of the external potential.

The instantaneous pair correlation functions $g_{ll'}(\boldsymbol{R})$ give the probability of finding a carrier at a parallel distance $\boldsymbol{R}$ in layer l' when there is a carrier at the origin in layer l. These functions can be obtained from the Fourier transform of the static structure factor $S_{ll'}(\boldsymbol{Q})$,

$$g_{ll'}(\boldsymbol{R}) = 1 + \frac{1}{\sqrt{n_l n_{l'}}} \int \frac{d\boldsymbol{Q}}{(2\pi)^2} e^{i\boldsymbol{Q}\cdot\boldsymbol{R}} [S_{ll'}(\boldsymbol{Q}) - \delta_{ll'}]. \tag{4}$$

$\delta_{ll'}$ is the Kronecker delta and n_l is the equilibrium density in the lth layer.

The structure factor is related to $\chi_{ll'}(\boldsymbol{Q},\omega)$, the density response in layer l to a unit stimulus in layer l', through the fluctuation-dissipation theorem,

$$S_{ll'}(\boldsymbol{Q}) = \frac{1}{\sqrt{n_l n_{l'}}} \frac{\hbar}{\pi} \int_0^\infty \mathrm{Im}\chi_{ll'}(\boldsymbol{Q},\omega) d\omega. \tag{5}$$

By definition $\chi_{ll'}(\boldsymbol{Q},\omega)$ gives the overall density response of the layers $\delta n_l(\boldsymbol{Q},\omega)$ to an external potential $V_{l'}^{\mathrm{ext}}(\boldsymbol{Q},\omega)$,

$$\delta n_l(\boldsymbol{Q},\omega) = -\sum_{l'} \chi_{ll'}(\boldsymbol{Q},\omega) V_{l'}^{\mathrm{ext}}(\boldsymbol{Q},\omega). \tag{6}$$

$\chi_{ll'}(\boldsymbol{Q},\omega)$ takes into account both the direct effect of $V_{l'}^{\mathrm{ext}}(\boldsymbol{Q},\omega)$ and the effect of interactions induced by changes in electron density in the other layer.

Within the static local-field approach[26] $\delta n_l(\boldsymbol{Q},\omega)$ can be written

$$\delta n_l(\boldsymbol{Q},\omega) = -\chi_l(\boldsymbol{Q},\omega)\Big[V_l^{\mathrm{ext}}(\boldsymbol{Q},\omega) + \sum_{l'\neq l}[1 - G_{ll'}(\boldsymbol{Q})] V_{ll'}(\boldsymbol{Q}) \delta n_{l'}(\boldsymbol{Q},\omega)\Big], \tag{7}$$

where the static local fields $G_{ll'}(\boldsymbol{Q})$ modify the effective interaction between particles in layers l and l'. $\chi_l(\boldsymbol{Q},\omega)$ is the response function for a single isolated layer l, which we write in the form,[27]

$$\chi_l(\boldsymbol{Q},\omega) = \frac{\chi_l^{(0)}(\boldsymbol{Q},\omega)}{1 + V_{ll}(\boldsymbol{Q})\,[1 - G_{ll}(\boldsymbol{Q})]\,\chi_l^{(0)}(\boldsymbol{Q},\omega)}, \tag{8}$$

where $\chi_l^{(0)}(\boldsymbol{Q},\omega)$ is the Lindhard function for the two-dimensional system.[28]

Combining Eqs. (6) and (7) we can write

$$[\chi^{-1}(\boldsymbol{Q},\omega)]_{ll'} = \frac{1}{\chi_l(\boldsymbol{Q},\omega)}\delta_{ll'} + [1 - G_{ll'}(\boldsymbol{Q})]V_{ll'}(\boldsymbol{Q})(1-\delta_{ll'}). \quad (9)$$

Inverting this matrix gives us $\chi_{ll'}(\boldsymbol{Q},\omega)$.

We now turn to the problem of determining the local fields $G_{ll'}(\boldsymbol{Q})$. The two layer system can be formally treated as a two component system with interactions given by $V_{ll'}(\boldsymbol{Q})$.

If the correlations were not too large the local fields could be calculated using the method of Singwi, Tosi, Land, and Sjölander (STLS)[27] extended to the case of a two component plasma.[29] Central to the STLS approach is an *ansatz* that the density-density correlation function can be approximated by the product of two densities times the static equilibrium pair correlation function (see, e.g., Ref. 26),

$$\langle \delta\hat{n}_l(\boldsymbol{R},t)\ \delta\hat{n}_{l'}(\boldsymbol{R}',t)\rangle \approx \delta n_l(\boldsymbol{R},t)\ g_{ll'}(\boldsymbol{R}-\boldsymbol{R}') \times \delta n_{l'}(\boldsymbol{R}',t). \quad (10)$$

The symbol $\hat{}$ distinguishes operators from expectation values. Using Eqs. (4) and (10) an expression can be obtained for the local field,[26,27,29]

$$G_{ll'}(\boldsymbol{Q}) = -\frac{1}{\sqrt{n_l n_{l'}}}\int \frac{d\boldsymbol{K}}{(2\pi)^2}\frac{\boldsymbol{Q}\cdot\boldsymbol{K}}{Q^2}\frac{V_{ll'}(\boldsymbol{K})}{V_{ll'}(\boldsymbol{Q})} \times [S_{ll'}(|\boldsymbol{Q}-\boldsymbol{K}|) - \delta_{ll'}]. \quad (11)$$

Equations (5), (9), and (11) form a closed set of equations for the $G_{ll'}(\boldsymbol{Q})$. In the STLS approach Eq. (8) is substituted into Eq. (9), and Eqs. (5), (9) and (11) are then solved self-consistently for the local fields $G_{ll'}(\boldsymbol{Q})$.

We are interested in relatively low densities, $r_s \gtrsim 5$, where the results of numerical simulations show that in the single layer case the correlations have become too strong for the STLS pair correlation functions to be accurate. Thus we cannot directly use the STLS approach. However as we have noted, even though correlations within each layer might be strong, it does not necessarily follow that correlations between layers must likewise be strong. The interlayer correlations only become strong when the layers are close together. Provided the layers are not too close, the STLS approach can still be used to determine the correlations between layers, even though a different approach must be introduced to determine the interlayer correlations. For relatively small interlayer correlations the local field $G_{ll}(\boldsymbol{Q})$ within each layer can be determined from the numerical simulation data for a single layer.[23] The size of the interlayer correlations can be checked at the end of the calculation for consistency.

The method we use is as follows. We start from the results of numerical simulations for the pair correlation function for a single layer $g(\boldsymbol{R})$, or equivalently the static structure factor $S(\boldsymbol{Q})$.[23] By assuming the parametrized form for $\chi_l(\boldsymbol{Q},\omega)$ given in Eq. (8), the fluctuation-dissipation theorem for a single layer can then be used to numerically deduce for every value of $\boldsymbol{Q}$ an intralayer local field $G_{ll}(\boldsymbol{Q})$.[17] As discussed above, provided the interlayer correlations are not too large we may neglect any feedback of $G_{ll'}(\boldsymbol{Q})$ on $G_{ll}(\boldsymbol{Q})$ and use $G_{ll}(\boldsymbol{Q})$ as a fixed input to solve Eqs. (5), (9), and (11) self-consistently for the interlayer local field $G_{ll'}(\boldsymbol{Q})$, with $l \neq l'$.

III. RESULTS AND DISCUSSION

The existence of points of instability in the liquid phase significantly affects the behavior of the functions $g_{ll'}(\boldsymbol{R})$ and $G_{ll'}(\boldsymbol{Q})$ in regions of the liquid phase space close to the instability points. The instabilities themselves show up as divergences in the matrix elements of the static polarizability matrix $\chi_{ll'}(\boldsymbol{Q})$.[17] When the densities and effective masses in the two layers are the same the diagonalized elements are simply $\chi_\pm(\boldsymbol{Q}) = \chi_{11}(\boldsymbol{Q}) \pm \chi_{12}(\boldsymbol{Q})$, the $\chi_+(\boldsymbol{Q})$ giving the density response for in-phase external stimuli and the $\chi_-(\boldsymbol{Q})$ the response for stimuli which are π out of phase. For a layer of holes coupled to a layer of electrons the most energetically favorable inhomogeneous state has the same density modulations in both layers so the in-phase polarizability $\chi_+(\boldsymbol{Q})$ is the diagonalized matrix element which diverges. Similarly, for two layers of electrons the divergent element will be the $\chi_-(\boldsymbol{Q})$.

As an example of this effect, Fig. 1 shows the $\chi_+(\boldsymbol{Q})$ for coupled electron-hole layers as a function of layer separation a. For $r_s = 5$, the critical spacing $a_c = 2.21a_B^\star$ and the peak in $\chi_+(\boldsymbol{Q})$ is centered at $(|\boldsymbol{Q}|/k_F) \approx 2$. For $r_s = 20$ the critical spacing $a_c = 13.8a_B^\star$ and $\chi_+(\boldsymbol{Q})$ has two peaks, one centered at $(|\boldsymbol{Q}|/k_F) \approx 2$, the other at $(|\boldsymbol{Q}|/k_F) \approx 2.5$. $r_s \approx 20$ is a crossover density in the sense that for $r_s \gtrsim 20$ the $[(|\boldsymbol{Q}|/k_F) \approx 2.5]$ peak is the one which actually diverges while for $r_s \lesssim 20$ it is the $[(|\boldsymbol{Q}|/k_F) \approx 2]$ peak which diverges.

Since our formalism only applies for liquids we cannot use it to determine the nature of the state beyond a point of instability. However, a value $(|\boldsymbol{Q}|/k_F) = 2.5$ is so close to the reciprocal lattice vector for a triangular Wigner lattice, $(|\boldsymbol{G}|/k_F) = 2.6$, that we can speculate for $r_s \gtrsim 20$ the inhomogeneous state beyond the instability

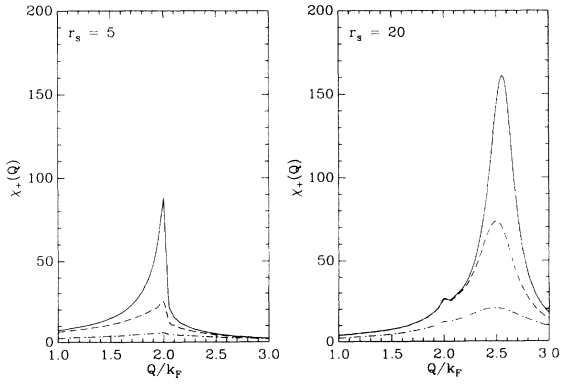

FIG. 1. The in-phase static polarizability $\chi_+(\boldsymbol{Q})$ for a layer of electrons separated from a layer of holes by a distance a close to a point of instability either to a charge density wave ($r_s = 5$) or to a coupled Wigner crystal ($r_s = 20$). Solid line: $a = 1.01a_c$, broken line: $a = 1.08a_c$, and chain line: $a = 2a_c$. The critical spacing for $r_s = 5$ is $a_c \approx 2.21a_B^\star$ and for $r_s = 20$ it is $a_c \approx 13.8a_B^\star$.

is probably a Wigner crystal in each layer. Because the instability appears in $\chi_+(\boldsymbol{Q})$ the coupled lattices would match across the interface. For $r_s \lesssim 20$ the instability occurs for $(|\boldsymbol{Q}|/k_F) \approx 2$ and the most likely inhomogeneous state would be a charge density wave of wave number $|\boldsymbol{Q}|$.

We should note that the points of instability do not necessarily correspond to any actual second-order phase transition. A first-order transition in the liquid to an inhomogeneous state could occur before the instability point, in which case there would be no subsequent second-order phase transition.

Figure 2 gives an overview of the position of the instabilities as a function of both the carrier density and the layer spacing. The points of instability are shown for electron-hole coupled layers. If for fixed r_s we consider a decrease in the spacing between the layers there will be a corresponding increase in the relative importance of the potential energy associated with interactions between layers. This increase in the potential energy contribution eventually leads to the instability in $\chi_+(\boldsymbol{Q})$. For smaller values of r_s the instability occurs for $(|\boldsymbol{Q}|/k_F) \approx 2$ and we have labeled it a charge density wave. At larger values of r_s the instability occurs for $(|\boldsymbol{Q}|/k_F) \approx 2.5$ and we have identified it with a coupled Wigner crystal. For the electron-hole system the coupled Wigner crystal instability occurs at densities as high as $r_s = 15$ but for the electron-electron system the coupled Wigner crystal instability only occurs for $r_s \gtrsim 30$[30].

We have also determined the position of the points of instability when the correlations between the layers are neglected, which is equivalent to replacing the interlayer local field $G_{ll'}(\boldsymbol{Q})$ by zero. Figure 2 shows that the positions of the instability points are not greatly affected although the correlations do noticeably increase the values of the critical spacing. The main effect of including the interlayer correlations is to favor the coupled Wigner crystal over charge density waves. (It is interesting to note that for electron-electron layers the correlations work in the opposite direction. The correlations decrease the critical spacing and tend to favor charge density waves.)

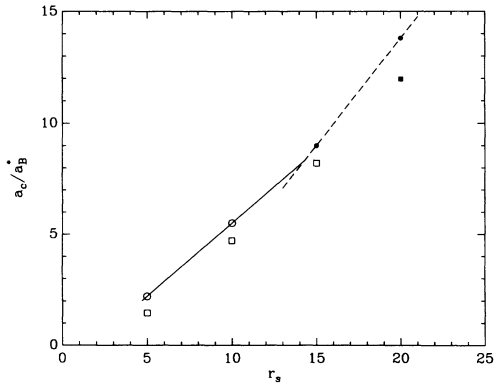

FIG. 2. Points of instability of the liquid for the electron-hole coupled layer system. Instability is to a charge density wave (open symbols) or to a coupled Wigner crystal (shaded symbols). The circles are our final results including the effect of correlations between the layers. The squares show the corresponding results when interlayer correlations are neglected. The lines are a guide for the eye.

We now turn to the pair correlation functions. For the two layer system there are two functions, the pair correlation function within a layer $g_{11}(\boldsymbol{R})$ and the intralayer pair correlation function for carriers in opposite layers $g_{12}(\boldsymbol{R})$. As with the single layer, the correlations must satisfy perfect screening sum rules although these take a slightly different form for two layer systems since for correlations between layers there is no "self" electron. While within a layer the form is the familiar $\int d^2R[1 - g_{11}(\boldsymbol{R})] = 1$, for the interlayer pair correlation function the perfect screening sum rule is null, that is $\int d^2R[1 - g_{12}(\boldsymbol{R})] = 0$.

The overall shape of $g_{11}(\boldsymbol{R})$ is largely determined by the same considerations as for the pair correlation function of a single layer (see Ref. 23). For $r_s \gtrsim 10$ the $g_{11}(\boldsymbol{R})$ is almost zero from $\boldsymbol{R} = \boldsymbol{0}$ out to a value $|\boldsymbol{R}| = R_c$ which can be comparable to the average interparticle spacing r_0. The appearance of $g_{11}(\boldsymbol{R})$ is not unlike that for a system with a hard core of radius R_c. For $|\boldsymbol{R}| > R_c$ there is an increase in $g_{11}(\boldsymbol{R})$ to a peak which is greater than unity and centered near $|\boldsymbol{R}| = r_0$. For still larger values of $\boldsymbol{R}$ the $g_{11}(\boldsymbol{R})$ approaches unity in a weak oscillatory fashion. The peak in $g_{11}(\boldsymbol{R})$ is a consequence of the conservation of particle number. The greater the fraction of available volume from which electrons are excluded, the higher the peak must be.

In Fig. 3 the intralayer $g_{11}(\boldsymbol{R})$ and the interlayer $g_{12}(\boldsymbol{R})$ are shown for coupled electron-hole layers at the two densities $r_s = 5$ and 20. Layer separations $a > 2a_c$ are not shown since both $g_{11}(\boldsymbol{R})$ and $g_{12}(\boldsymbol{R})$ behave as would be expected for weak coupling between layers. Even for $a = 2a_c$ the $g_{11}(\boldsymbol{R})$ is similar to the pair correlation function for a single layer at the same density and $g_{12}(\boldsymbol{R})$ is close to unity for all $\boldsymbol{R}$.

By $a = 1.08a_c$ the value of $g_{12}(\boldsymbol{R})$ at $\boldsymbol{R} = \boldsymbol{0}$ exceeds 2 and there is an interesting compensatory increase in $g_{11}(\boldsymbol{R})$ near the origin. The $g_{11}(\boldsymbol{R})$ initially decreases in value as $\boldsymbol{R}$ moves away from the origin, passing through a minimum at $|\boldsymbol{R}| \approx 0.8r_0$ and then turning up towards the asymptotic value of unity. The cause of the negative gradient at small $\boldsymbol{R}$ is an attractive region which develops in the hole layer at the point immediately opposite any electron in the first layer, and *vice versa*. The attractive region compensates to some extent the repulsive effect of the electron within its own layer and this permits some of the electrons in the first layer to flow back towards $\boldsymbol{R} = \boldsymbol{0}$.

It is not only the small $\boldsymbol{R} = \boldsymbol{0}$ behavior of the two functions which compensate each other. The oscillations for $|\boldsymbol{R}| > r_0$ are also in step due to the attraction between electron and hole density fluctuations. In Fig. 3 the period of the oscillations approximately agrees with the wave number of a charge density wave for $r_s = 5$ and with the reciprocal lattice vector of a Wigner crystal for $r_s = 20$.

Analogous effects occur for two coupled layers of electrons. Figure 4 shows $g_{11}(\boldsymbol{R})$ and $g_{12}(\boldsymbol{R})$ for two electron layers at $r_s = 20$. The $g_{12}(\boldsymbol{R})$ is depressed for small $\boldsymbol{R}$ because electrons from different layers do not want to sit

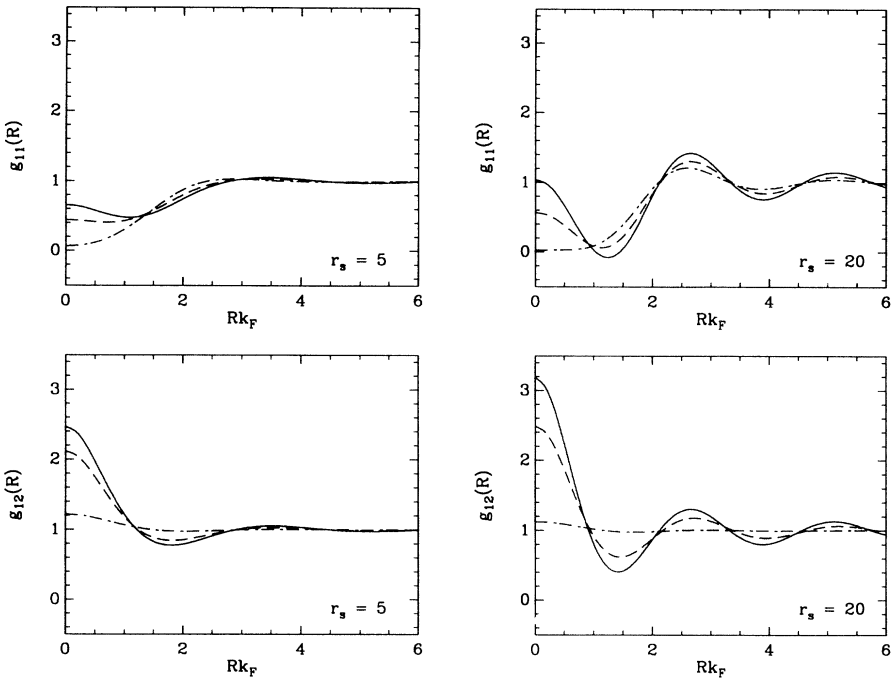

FIG. 3. The intralayer and the interlayer pair correlation functions for a layer of electrons separated from a layer of holes by a distance a. Solid line: $a = 1.01a_c$, broken line: $a = 1.08a_c$, and chain line: $a = 2a_c$, where a_c is the critical spacing.

directly opposite each other. This again has the effect of generating an attractive region opposite an electron so that once more $g_{11}(\boldsymbol{R})$ initially decreases as $\boldsymbol{R}$ moves away from zero.

In Fig. 5 we compare the intralayer pair correlation function $g_{11}(\boldsymbol{R})$ obtained from our calculation with the $g_{11}(\boldsymbol{R})$ determined using the STLS approximation. We have taken $r_s = 5$ and a layer spacing close to a_c. Since the system is strongly interacting it is to be expected that the $g_{11}(\boldsymbol{R})$ calculated within the STLS approximation will differ significantly from the $g_{11}(\boldsymbol{R})$ in the present calculation.

Figure 5 also compares our $g_{11}(\boldsymbol{R})$ with the pair correlation function for a single layer $g(\boldsymbol{R})$ taken from numerical simulation data.[23] The comparison highlights the significant effect the presence of a second layer has on the small $\boldsymbol{R}$ behavior of $g_{11}(\boldsymbol{R})$. In proximity to the instability point the large $\boldsymbol{R}$ behavior of the two functions also differs, with the $g_{11}(\boldsymbol{R})$ displaying oscillations of larger amplitude.

We have also determined the intralayer local field $G_{12}(\boldsymbol{Q})$. This modifies the effective interaction between carriers in the two layers replacing the bare Coulomb interaction $V_{12}(\boldsymbol{Q})$ by an effective interaction $V_{12}(\boldsymbol{Q})[1 - G_{12}(\boldsymbol{Q})]$. Figure 6 shows the $G_{12}(\boldsymbol{Q})$ both for coupled electron-hole layers and coupled electron-electron layers.

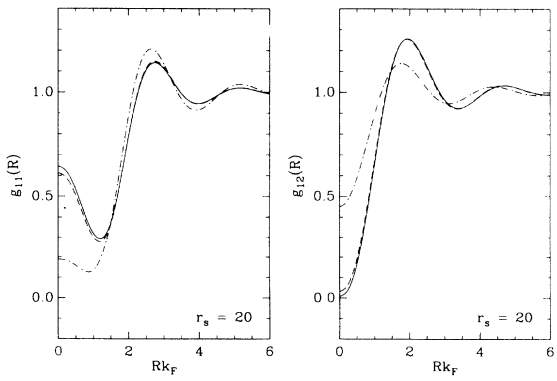

FIG. 4. Pair correlation functions for the system of two electron layers at $r_s = 20$ close to the instability to the charge density wave. The instability occurs at a critical spacing of $a_c \approx 7.3a_B^*$. Solid line: $a = 1.01a_c$, broken line: $a = 1.08a_c$, and chain line: $a = 2a_c$.

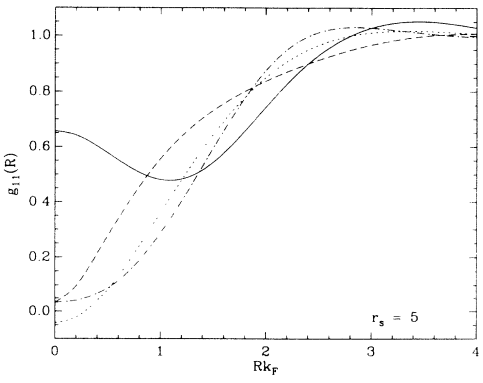

FIG. 5. Intralayer pair correlation function calculated within different approximations for the electron-hole coupled layer system. Also shown are results for a single layer. $r_s = 5$ in all cases. Solid line: our approach for $a = 1.01a_c$, broken line STLS for $a = 1.01a_c$, where a_c is the critical spacing. Chain line: Monte Carlo results for a single layer. Dotted line: STLS for the single layer.

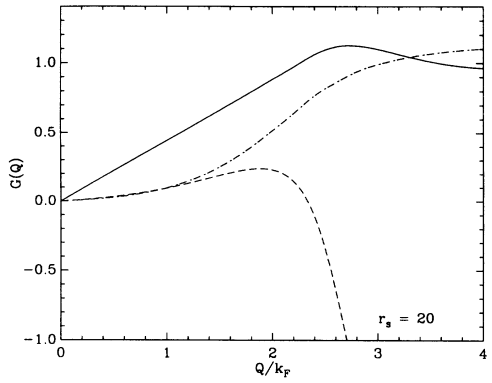

FIG. 6. Local fields for $r_s = 20$ at layer spacing $a = 1.01a_c$, where a_c is the critical spacing. The broken line is the interlayer local field for the electron-hole system and the chain line the interlayer field for the electron-electron system. The local field for a single layer deduced (Ref. 17) from data in Ref. 23 is shown as a solid line.

The density is $r_s = 20$ and the layer spacing is close to a_c. The $G_{12}(\boldsymbol{Q})$ has significant structure for $2 < |\boldsymbol{Q}|/k_F < 3$. This reflects the existence of a low-lying inhomogeneous excited state with a wave number in this range. As expected the $G_{12}(\boldsymbol{Q})$ for electron-hole layers is negative, implying an enhancement of the effective interaction by the attractive correlations. Also shown for comparison is the local field $G(\boldsymbol{Q})$ for a single layer. In contrast with $G_{12}(\boldsymbol{Q})$ the important structure in $G(\boldsymbol{Q})$ appears at smaller $\boldsymbol{Q}$ values, for $|\boldsymbol{Q}|/k_F < 2$. This reflects the fact that for a single layer the local field is determined by short-range correlations.

IV. CONCLUSIONS

For both electron-hole and electron-electron coupled layer systems we find that the behavior of static correlations is quite different for pairs of carriers within the same layer and for pairs from opposite layers. This is basically because carriers are free to move only parallel to the layers and because in the absence of tunneling exchange only acts within layers.

For electron-hole layers the interlayer correlations tend to make the liquid marginally more susceptible to the instabilities. When an instability does occur the interlayer correlations appear to significantly favor the Wigner crystal state over a charge density wave state. By contrast for electron-electron layers the interlayer correlations make the liquid slightly more resistant to instabilities and the charge density wave state is the one which is favored.

The formalism we have used applies to liquid phases only and it is difficult to draw firm conclusions about the nature of the inhomogeneous ground states associated with points of instability in the liquid. It would be interesting to learn more about these inhomogeneous phases and also to resolve the issue of whether a first-order phase transition might preempt some of the instabilities. A density functional approach might be used for this purpose. It would be useful to have results from ground state numerical simulations analogous to Ref. 23 for the single layer.

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