PHYSICAL REVIEW B VOLUME 50, NUMBER 15 15 OCTOBER 1994-I

Resistance noise near the Anderson transition

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(Received 25 February 1994; revised manuscript received 29 June 1994)

We discuss the results of $1/f$ noise measurements made on films of polycrystalline indium oxide and ZnO accumulation layers. In these systems, the amount of static disorder can be readily and reversibly changed by fine-tuning the stoichiometry, which gives one a convenient and unique method to study the sensitivity of the noise to changes in some transport parameters. We present detailed experimental evidence that rules out classical percolation phenomena as an explantation for the high noise level observed in these materials. We elaborate on a qualitative model given by Cohen *et al.* that ascribes the noise characteristics of these systems to an impending metal-insulator transition and dwell on some of its implications.

I. INTRODUCTION

The resistance of almost all solid-state systems, whether metallic, semimetallic, or semiconducing, fluctuates with a normalized power spectrum S_R/R^2 that often exhibits $1/f^\gamma$ dependence with $\gamma \approx 1$. The ubiquity of this power-law spectral dependence, suggesting the absence of a characteristic time scale, has been a source of fascination and continued debate over the last two decades. Tempted by the near universality of the phenomenon, many attempts were made to find a unified model that could account for it. No single model has yet been recognized as fully explaining the details of this phenomenon. For diffusive systems, however, there is a growing consensus that the resistance fluctuations reflect motion of defects that couple to the measured resistance via potential variations.[1,2] Also, some form of static disorder appears to be a common ingredient in all models of $1/f$ noise. It is natural, then, to expect that static disorder plays a significant role in determining the noise characteristics of metals and particularly its magnitude. Hooge[3] has suggested the following formula to characterize noise levels measured in many metallic and semiconducting systems:

$$S_R/R^2 = \alpha_H/(fN_c) , \tag{1}$$

where N_c is the total number of carriers in the sample and α_H is the "Hooge parameter," a dimensionless number claimed to be of the order $\approx 10^{-3}$ for a great number of different systems. Equation (1) automatically accounts for the well-established, inverse proportionality between noise magnitude and the sample physical volume.[1] But in addition, it suggests that all other things being equal, a system with lower carrier density exhibits bigger noise. This relation has sometimes been interpreted as implying that each carrier "fluctuates individually" which can be shown to be theoretically inconsistent with the estimated time a carrier spends in a sample, on the one hand, and the low frequency at which $1/f$ noise still exists, on the other hand.[2] We argue below that a low-density metallic system may indeed show bigger noise than a similar one with a higher carrier density for reasons that have nothing to do with individual fluctuations of carriers. For the time being though, we adopt the more commonly employed[2] procedure of characterizing $1/f$ noise magnitude through the empirical relation:

$$S_R/R^2 = \alpha/(f^\gamma \Omega) , \tag{2}$$

where Ω is the sample volume. A useful measure of Ω is the number of atoms in the sample. With this definition of Ω and for $\gamma = 1$, α in (2) is a dimensionless parameter α_a, which will be used in this paper as signifying noise levels. For typical metals such as Cu, Ag, etc., and at room temperatures, α_a has a value of the order of 10^{-3}–10^{-2}.

As alluded to above, it is natural to expect that static disorder plays a role in giving rise to $1/f$ noise in general and in controlling α_a in particular. Yet, only a few attempts were made to study noise levels in systems where the degree of disorder could be varied systematically. Eberhard and Horn[4] and Fleetwood and Giordano[5] studied the effects of thermally annealing metallic samples on their noise levels. Both groups found some correlation between the alleged decrease of defects concentration (judged by the decrease in resistivity) and the diminished $1/f$ noise magnitude. Pelz and Clarke[6] reported on changes of noise levels in thin Cu films with resistivity. These authors used high-energy electron-beam bombardment to increase the Cu film resistivity ρ and several stages of thermal annealing to reduce it back to its original value. Again, a correlation between ρ and α_a was established in the sense that α_a tended to increase with ρ. Both groups stressed that the main implication of their results is that ρ correlates with the density of defects in a metal and thus the observed increase of α_a suggests the relevance of defect motion to $1/f$ noise.

Recently, Cohen, Ovadyahu, and Rokni[7] reported on a noise versus disorder study in thin films of polycrystalline In_2O_{3-x} and ZnO accumulation layers where the degree of disorder could be varied over a wide range. For metallic samples it was found that α_a increases very rapidly with static disorder and reached values as high as 10^8 for

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$K_F l \approx 1$ (K_F is the Fermi wave vector and l the elastic mean-free path). For both systems, the material resistivity was changed by varying the stoichiometry by taking out (or letting in) oxygen atoms. The full details of such a procedure, which turns out to be *reproducible and reversible,* will be discussed in the next section. A typical dependence of the noise level of In_2O_{3-x} samples on stoichiometry (reflected by the different resistances) is depicted in Fig. 1. At first glance, these results seem to show a similar trend as in the aforementioned experiments in the sense that α_a increases with ρ. However, we believe that the main reason for that is not due to an increased *number* of scatterers. In the first place, the assumption that ρ reflects the *number of structural defects* is not always true. For instance, in the process of increasing the number of defects (as in doping a semiconductor), the mobility may actually *improve.* This may be the situation in the present case: taking away oxygen from the sample causes a larger deviation from stoichiometry. Oxygen vacancies are, in the first place, a source of scattering,[8] i.e., they are *defects.* At the same time, the number of carriers increases as well, i.e., N (as measured by the Hall effect) becomes larger. This is analogous to the process of doping a semiconductor. Yet, for both In_2O_{3-x} and ZnO a higher concentration of O vacancies usually results in a higher average sample conductivity, and in the resistivity range $\rho > 10^{-3}$ Ω cm, to a higher mobility as well.[8,9]

Similarly, an increased α_a may *not* be taken as an unambiguous indication for an increased number of fluctuators: Cohen, Ovadyahu, and Rokni have ascribed the sharp dependence of α_a on ρ to an impending Anderson transition rather than to a change in the strength of the fluctuating potential. Basically, the idea is that when $K_F l \rightarrow 1$ from above, time-dependent potential fluctuations might be sufficient to induce insulating behavior in a system whose averaged behavior is still metallic. This leads to an exponential dependence of α_a on $K_F l$ when one is sufficiently close to the metal-insulator transition. Thus, even if the magnitude of the fluctuating potential is somewhat decreased in the process of reducing $K_F l$, the noise level may still *increase.* In other words, the noise *level* is not always simply related to the number of "fluctuators." We further comment on the need to view the noise as being a convoluted result of more than one parameter in the discussion section below.

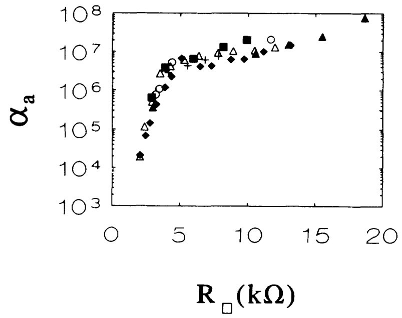

FIG. 1. Noise magnitude vs $R_\square$ for a single batch of In_2O_{3-x} samples. The different symbols correspond to different UV-treatment cycles (see text). Film thickness is 130 Å.

In the next section we describe sample preparation and characterization as well as measurements techniques. We also present experimental evidence to support the claim that the studied In_2O_{3-x} films are not heterogeneous and that the metal-insulator transition approached when their $K_F l \rightarrow 1$ is not due to classical percolation. The distinction between the two types of metal-insulator transitions is elaborated upon in Sec. III.

II. EXPERIMENT AND SUMMARY OF THE MAIN RESULTS

The In_2O_{3-x} films used in this study were prepared by *e*-gun evaporation of 99.999% pure (CERAC) In_2O_3 in the presence of $\approx 10^{-4}$ mbar oxygen pressure onto glass substrates held at room temperature. The base pressure of the vacuum system was $\approx 2\times 10^{-6}$ mbar. Typical evaporation rates were 0.2–0.6 Å/s as monitored, *in situ* by a quartz oscillator calibrated against a Tolansky interferometer and x-ray interferometry. The as-prepared films were amorphous and their geometrical, in-plane shape was controlled by either using stainless-steel masks or (for samples with lateral dimensions smaller than 2 mm) by optical lithography and lift-off techniques. To obtain the polycrystalline In_2O_{3-x} samples the prepatterned amorphous films were crystallized by subjecting them to temperatures of 250–350 °C in an oxygen-rich ambience. All In_2O_{3-x} samples used in this study had a thickness d between 110 and 160 Å. A transmission electron micrograph of a typical film is shown in Fig. 2. It can be seen that the sample is fully continuous without any holes or cracks and is composed of rather big crystals (0.1–0.3 μm).

Following preparation, the samples were mounted in a small vacuum chamber that had electrical feedthroughs for resistance, and noise and Hall effect measurements all employing a standard "Hall bar," six-terminal geometry. The Hall effect measurements employed a fixed magnet with field $H = 0.34$ T and a $\mp 180°$-rotatable probe that was used to eliminate spurious longitudinal resistance from the Hall signal. In addition to the electrical feedthroughs, the sample cell was equipped with a quartz window that enabled an *in situ* exposure to UV radiation. The resistance of both ZnO and In_2O_{3-x} samples decreases upon exposure to UV radiation due to a change in stoichiometry associated with removal of oxygen atoms from the lattice. As long as a sample was kept under vacuum, its lower resistance state could be maintained for many hours. The original resistance could be restored by bleeding a controlled amount of oxygen back into the measurement chamber using a needle valve. This process could be repeated many times over with a fair degree of reproducibility in the sense that the transport parameters pertinent to this work are found to be essentially independent of "history" (cf. Fig. 1 for the noise and Fig. 3 for the respective Hall coefficient R_H).

The noise measurement rig employed a stack of NiCd batteries and an array of interchangeable metal films series resistors that formed the current source. The volt-

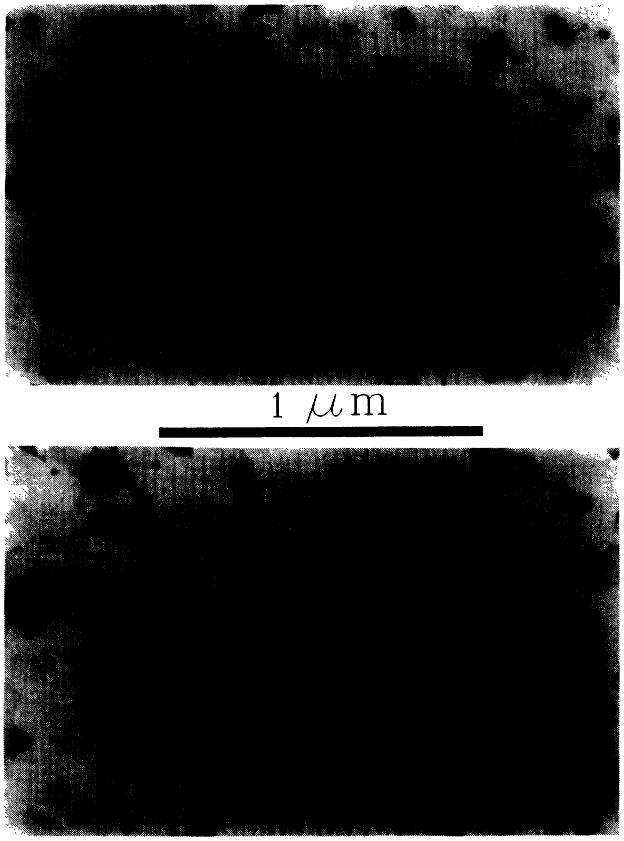

FIG. 2. Transmission electron micrograph of a In_2O_{3-x} film for two grossly different states of stoichiometry (see text for details). The two bright-field images were taken of the same area, and one can identify and compare the same crystallites before and after UV exposure. Note that apart from some contrast variations (resulting presumably from slight changes in local orientation), the samples look identical.

age across the sample was fed to the differential input of a low noise ac preamplifier (PAR-113) whose output was connected to a spectrum analyzer (HP-35660A). The noise level measurement rig was calibrated by using the Johnson noise generated by standard metal-film resistors.

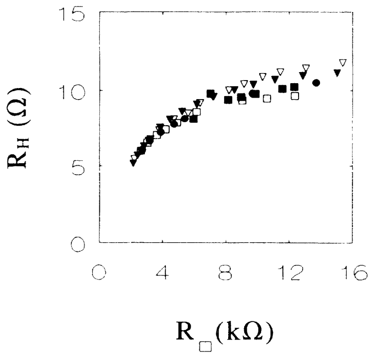

FIG. 3. The Hall coefficient as a function of $R_\square$ for the same batch as in Fig. 1.

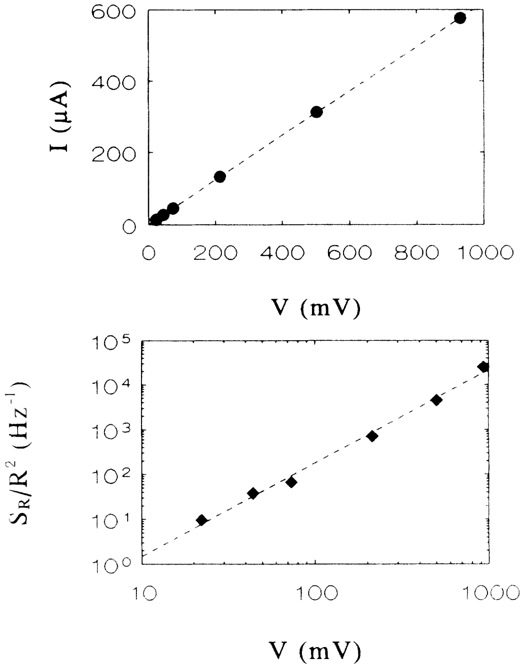

FIG. 4. Current-voltage characteristics (top) and normalized noise (bottom) as a function of the dc bias applied for a typical In_2O_{3-x} sample. The sample is 130 Å thick and has lateral dimensions of 0.5×0.5 mm^2. The noise level is based on the reading at $f = 10$ Hz.

The electric fields used were always smaller than 3 V/cm. As shown in Fig. 4, in this range of fields both $R_\square$ and S_R/R^2 are in the linear regime. To determine the noise parameters of given sample (α_a and γ) it was usually enough to record spectra over the range 2–202 Hz. Noise parameters were calculated for each sample by averaging over 500 time scans in this range. In a few cases we have extended the measurements over a much wider range as in Fig. 5 which confirmed that both α_a and γ are sufficiently well defined by restricting the measurements to the 2–202-Hz range. For all measurements reported below, the exponent γ varied between 1.1 and

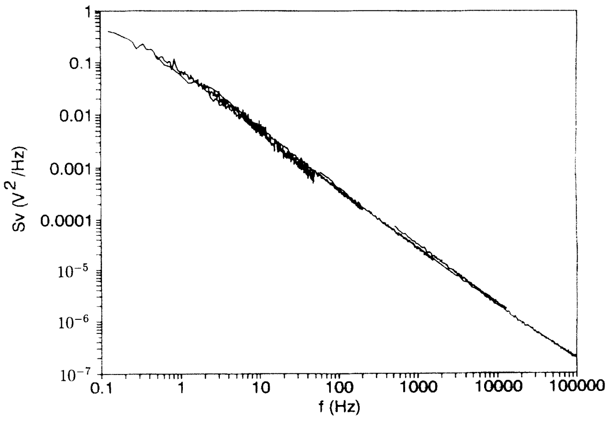

FIG. 5. The power spectrum of a typical In_2O_{3-x}, 130 Å-thick film. To obtain this extended frequency range, data were collected over six separate frequency intervals which were then spliced together.

1.25 without any clear systematic.

We have also made a series of measurements on accumulation layers created by UV radiation on the prismatic plane of a ZnO single crystal (1 mm width and 3 mm length). A typical dependence of the noise level on $R_\square$ is shown in Fig. 6 which clearly resembles the behavior observed in the In_2O_{3-x} films. Note that the noise magnitude in ZnO is comparable to that of the In_2O_{3-x} films despite the fact that the former is a single crystal and has no grain boundaries. Grain boundaries are thus not a prerequisite for having high noise levels.

The results for the noise versus $R_\square$ portrayed in Fig. 1 are typical of many batches of In_2O_{3-x} films. By a batch we refer to a set of samples created from a *single* evaporated and patterned film from which "new" samples were generated via the process of a UV-treatment.

Since the UV treatment was our major material-manipulation technique and its use enables such a profound change of parameters, we took some steps to inquire into what the microscopic changes that accompany its application are. The following facts were observed. Rutherford backscattering reveals an appreciable change in the sample stoichiometry with UV treatment. The sample shown in Fig. 2, for example, had a In/O ratio of 1.45 initially (bottom), and 1.40 after 15-min exposure to UV (top). For these extremes, the sheet resistance of the sample, $R_\square$, changed from 20 to 4.5 kΩ, respectively, and its carrier concentration changed from 3×10^{-19} to 7×10^{-19} cm^{-3}, respectively. At the same time, no significant change in the sample TEM micrograph is observable due to the UV treatment (cf. Fig. 2), in particular, there is evidently no change of the grain size. This is consistent with previous reports that ascribe the main scattering source in this material to the oxygen voids themselves rather than to grain boundaries.[8]

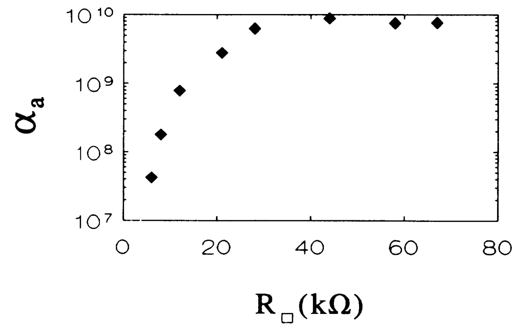

FIG. 6. $1/f$ noise magnitude as a function of $R_\square$ for a ZnO sample. α_a in this case was calculated from Eq. (1) using 50 Å for the effective thickness (see Ref. 9).

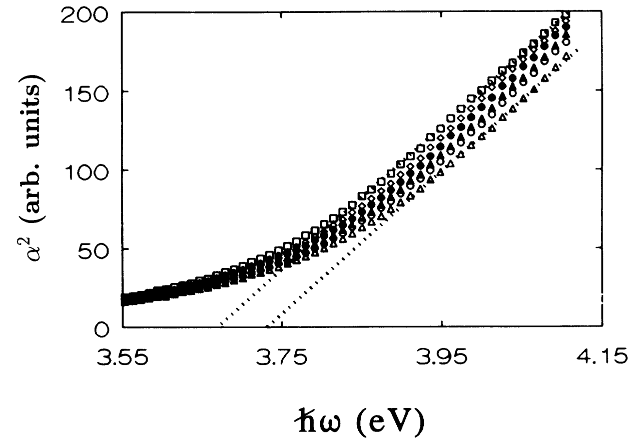

FIG. 7. Absorption coefficient as a function of photon energy for six In_2O_{3-x} films generated from a single batch by UV treatment. The batch thickness is 110 Å and is deposited on a fused quartz substrate. The dashed lines (for clarity, shown only for the two extremal cases) are used to extract E_g from the data through Eq. (3).

Finally, we have measured, using again an *in situ* cell in which two quartz windows were incorporated, the changes in the optical transmission through the In_2O_{3-x} film as a function of the sheet resistance $R_\square$ that was affected by the UV treatment. Figure 7 depicts the absorption coefficient α as a function of photon energy $\hbar\omega$ in the near UV range for several $R_\square$ values (i.e., different stoichiometries of the *same* film). From the linear part of the $\alpha^2(\omega)$ versus $\hbar\omega$ plot one can obtain an effective energy gap E_g by the equation

$$\alpha(\omega)\approx\alpha_0(\hbar\omega-E_g)^{1/2}\,, \tag{3}$$

which is the standard formula[10] for (allowed) interband transitions in a crystalline material. Clearly, E_g increases as the material is driven further away from stoichiometry. We interpret these results as a Burstein shift[11] caused by the increase of the Fermi energy E_F due to the removal of oxygen from the lattice and the concomitant establishment of states in the conduction band. In Fig. 8 one can compare the shift of E_g with the values of E_F calculated on the basis of Hall effect measurements performed on the very same film. The fact that the two

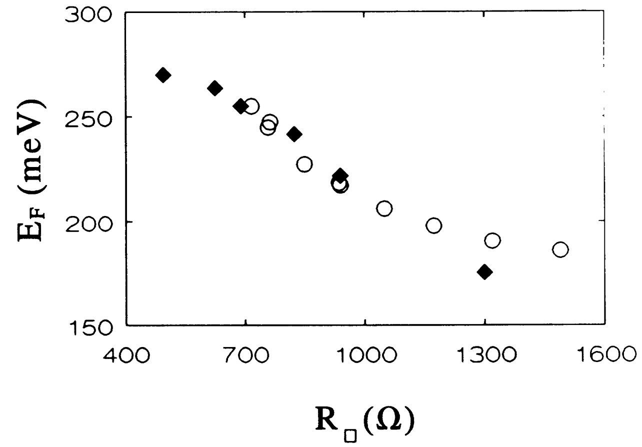

FIG. 8. The Fermi energies plotted as a function of $R_\square$ for the batch of samples used in Fig. 7 (empty circle). These values were calculated using free-electron formulas on the basis of the measured Hall coefficient. The diamonds stand for the respective values of E_g's for these samples (from the data in Fig. 7 and adjusting E_F to coincide with the Hall-effect-derived value for the sample with $R_\square\approx700$ Ω).

independent measurements yield similar *changes* in E_F with UV treatment would be natural if the film is changed homogeneously with this procedure which we believe to be the case. If the process is spatially inhomogeneous (say, that some grains are much more depleted of oxygen than others, and thus are also more conducting—a situation that would resemble a classical percolation scenario), one would expect quite different results. This is so since electronic transport and, in this case, the Hall effect, would then probe *only part* of the material while the optics are sensitive to the *whole* film. The latter should then reflect some inhomogeneous broadening and a different value of ΔE_F versus $R_\square$ than that inferred from the Hall coefficient R_H. This is not observed. We therefore conclude that the UV treatment does not significantly alter the homogeneity of our samples.

We note again that the noise magnitude α_a and the Hall coefficient R_H are essentially independent of "history" and therefore α_a can be meaningfully presented as a function of a sample transport parameter such as its sheet resistance $R_\square$. To the best of our knowledge, this is the first system in which such a reversible process has been observed. By considering more than 20 different batches of $\mathrm{In_2O_{3-x}}$ the following general features are found (as previously reported by Cohen, Ovadyahu, and Rokni).[7]

(1) The noise magnitude is unusually high. Even the smallest α_a observed was at least five orders of magnitude higher than that characteristic of metals. For samples with $R_\square \leq 4$ kΩ (cf. Fig. 1), α_a increases sharply with $R_\square$.

(2) For samples with $R_\square \geq 10$ kΩ, α_a saturates at a high level but is much less sensitive to a further increase in disorder.

(3) The fastest increase of α_a with $R_\square$ was shown to occur near the point where the temperature coefficient of resistance (TCR) of the films changed sign.[7] This regime is near (but on the metallic side of) the metal-insulator transition. Our noise and transport data for the ZnO accumulation layers, though much less extensive, revealed a remarkably similar behavior except that the fast increase of the noise persisted up to $R_\square \approx 20$ kΩ (Fig. 6).

Led by these observations, Cohen, Ovadyahu, and Rokni[7] hypothesized that the increase of α_a with disorder is a precursor to the Anderson transition in a close analogy with the scenario that leads to the well-known TCR sign change.[12]

III. DISCUSSION

A. Anderson localization versus classical percolation

Before exploring some of the consequences of the Cohen, Ovadyahu, and Rokni conjecture, some distinction between the Anderson transition and other metal-insulator transitions seems to be in order. It must be realized from the outset that near *any* metal-insulator transition, fluctuation phenomena will cause enhanced resistance noise. Therefore, an experimental distinction between different scenarios must address additional aspects of resistance noise than just α_a. In the following we discuss the detailed transport and noise characteristics expected from classical percolation versus those of the Anderson transition and confront these with our experimental findings.

"Classical percolation" in the context of electrical conductivity is typically implemented by using a mixture of two materials; one with finite conductivity, σ_1, and the other with a vanishingly small one, σ_2 (ideally zero). The effective conductivity of the composite, σ_{eff}, will depend on the fraction of the sample volume, p, occupied by the conducting material and, near the percolation threshold it assumes the form[13]

$$\sigma_{\mathrm{eff}} \approx \begin{cases} \sigma_1 (p - p_c)^t \;, & p \geq p_c \\ \sigma_2 \approx 0 \;, & p < p_c \;, \end{cases} \tag{4}$$

where p_c is the fraction of the volume at which the conducting material just percolates through the system, and t is a critical index. For p just above p_c, a very small volume is controlling both resistivity and resistance noise. Furthermore, it is easy to see that both will diverge at p_c according to Eqs. (2) and (4). Thus, for an ideal situation, the noise magnitude may be described as the power law of the sample resistance R:[14]

$$S_R / R^2 \approx R^\eta \;, \tag{5}$$

where η is dependent, among other things, on t, which, in turn, depends on the dimensionality but, in any case η is a number of the order of unity.[14] An increase in the noise levels near the percolation transition has indeed been observed by several groups.[15]

Another attribute of percolation is the behavior of L_c, the percolation radius. On the metallic side of the transition (i.e., $p \geq p_c$), which is the only region accessible to measurements in the "ideal" case, L_c is the characteristic distance between the narrow constrictions which control R and S_R. Clearly, l_c diverges at p_c. In real systems, technological imperfections will probably cutoff the divergence of L_c but values of L_c that are two orders of magnitude bigger than typical metallic islands are quite common for percolating systems[16] near a percolation transition.

The Anderson transition has several features in common with classical percolation. Namely, it is a metal-insulator transition and associated with it there is a correlation length ξ, which diverges at the transition. It differs from percolation in that the phase transition is driven by static disorder rather than by change of an effective volume. The difference between the two types of transitions is more obvious when one considers the situation *at finite temperatures* such as is the case in our experiments. (Actually, $T \neq 0$ is the *only* regime where one is justified in considering classical percolation.) Apart from some change in the values of σ_1 and σ_2, the percolation problem is essentially temperature independent. The Anderson transition, on the other hand, is profoundly affected. The modifications to the latter due to a finite temperature have been considered by Thouless[17] and by Imry.[18] Temperature introduces a new length scale into the problem which is L_ϕ, the distance over which the charge carrier preserves its quantum-mechanical phase.

On the metallic side this length is usually identified with the inelastic diffusion length L_{in}. On the insulating side, L_ϕ is of the order of the hopping length.[19] In either case, this is a temperature-dependent length which is quite small at room temperatures (about 25–30 Å for $\text{In}_2\text{O}_{3-x}$). Introducing this cutoff length has two effects on the problem at hand. In the first place, it severely limits the attainable resistivity of the sample. So, despite the fact that its ground state is *insulating,* the actual conductivity of the sample at the transition is of the order of $e^2/(\hbar L_\phi)$ which is considerably higher than a typical σ_2. Second, L_ϕ is also the coherence length for conductance fluctuations in general and noise in particular[20] and, unlike ξ (and L_c), it *does not diverge at the transition.* At finite temperatures then, neither the resistance nor the noise diverge at the Anderson transition. Nonetheless, as pointed out by Cohen, Ovadyahu, and Rokni, S_R/R^2 increases with R (or K_Fl) *much faster* than it does due to percolation [Eq. (5)]. In fact, for a system approaching the Anderson transition from the metallic side simple arguments based on the scaling theory[21] yield

$$S_R/R^2 \approx \exp[-(2L_\phi/\xi_0)K_Fl] \;, \tag{6}$$

where ξ_0 is the microscopic length, and since $K_Fl \propto R^{-1}$, a very steep increase of α_a with R is expected. The main difference in this respect between the two scenarios can be traced to the behavior of the average sample resistance which *diverges* for the percolation problem and is *bounded* in the disorder driven transition.

Comparing the above predictions with our experimental findings we note the following.

(a) The increase of α_a with R near the transition we observe could be very steep. In several individual batches we observed α_a to increase by ≈ 3 orders of magnitude for a factor of 3 change in R (cf. e.g., Fig. 1 above). This is difficult to reconcile with Eq. (5) as it would necessitate $\eta \geq 6$. Such a high value for η is clearly not consistent with a "pure," (geometrical) percolation picture. Garfunkel and Weissman,[14] using granular films with a deliberate attempt to observe percolation behavior, reported on unusually high η's near the transition and also found it difficult to reconcile it with simple percolation ideas. At the same time, such a sharp dependence is consistent with an exponential dependence [Eq. (6)] as was shown by Cohen, Ovadyahu, and Rokni[7] for the $\text{In}_2\text{O}_{3-x}$ data where the relevant parameters are known from independent measurements (unlike the heterogeneous case in Ref. 14). The plausibility of the parameters used in the fit[7] gives strong evidence for the relevance of the localization scenario.

(b) The structural geometrical aspects of the samples are evidently not that of geometrical percolation. Instead of the tenuous, fractal-like appearance of a deliberately made heterogeneous mixture usually observed (see, e.g., Ref. 13), a tight, completely space-filling film is preserved throughout the transition (cf. Fig. 2). Also, the comparison between the optical absorption and Hall effect data discussed in the previous section is consistent with a nonheterogeneous system.

(c) As noted above, near a percolation threshold the bottleneck resistors that control the noise are separated by a large distance. One would then expect a sizable part of the noise to be correlated over distances that are comparable with L_c. The latter could easily be 50–100 times the grain size[16] which, in our case, means L_c of the order of 10–20 μm. To check on this possibility, we have measured the noise level in a series of samples having a wide range of volumes, Ω. For these experiments, six batches of $\text{In}_2\text{O}_{3-x}$ films were all prepared in a common evaporation run and were subsequently patterned and crystallized simultaneously so they should have similar physical characteristics. A measurement of α_a versus $R_\square$ was then made for *each* of these batches separately in the range $R_\square \approx 3$–8 kΩ. In Fig. 9 we plot the value of α_a of each batch as a function of its volume for $R_\square = 4$ kΩ found by interpolation from the respective $\alpha_a(R_\square)$ curves. This specific value of $R_\square$ was chosen because it corresponds to the "critical" value but the result $\alpha_a \propto \Omega^{-1}$ was found to hold for *any* $R_\square$ in the above range. The inverse proportionality of α_a with Ω expected for *independent* fluctuators is thus preserved down to a few micrometers in lateral size which means that the noise sources in our samples have a coherence length that must be considerably smaller than might be expected of a percolation scenario.

On the basis of the above points we conclude that classical percolation does not play a significant role in the noise features of our samples and that the main reason for the increase of the noise in both $\text{In}_2\text{O}_{3-x}$ and ZnO is consistent with an impending Anderson transition.

B. The interplay between Anderson localization and resistance noise

The general problem of $1/f$ noise involves two aspects: The first is an *assumed* cause for fluctuations in the potential and the second is the *coupling mechanism* by which these fluctuations manifest themselves in the measured resistance. Only the second issue has been ad-

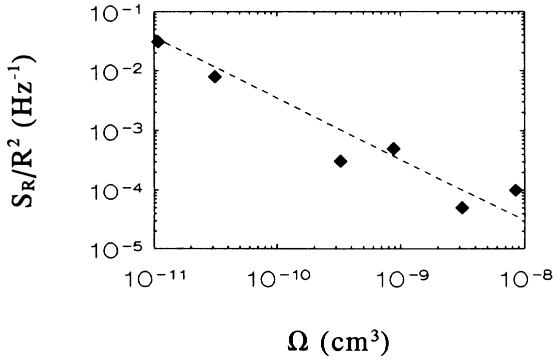

FIG. 9. Normalized noise (at $f = 10$ Hz) as a function of volume for a batch of $\text{In}_2\text{O}_{3-x}$ samples. The samples have a constant aspect ratio of length to width of about 3, and the same thickness (110 Å). The smallest lateral dimension in these series is 12 μm. The dashed line depicts the simple inverse proportionality of the noise magnitude with volume expected of independent fluctuators.

dressed by us. The same is true for the recently proposed model of Feng, Lee, and Stone[22] based on ideas of universal conductance fluctuations. The common physics in the approach taken by Feng, Lee, and Stone and ours is the notion that quantum interference effects could be dominant in *translating potential fluctuations* to *resistance noise* and the relevance of L_ϕ in such cases. The main difference in the two approaches is that we consider the $K_F l \approx 1$ case where quantum-interference effects are manifestly dominant even at room temperatures as attested, e.g., by the TCR sign change, while Feng, Lee, and Stone address the $K_F l \gg 1$ limit which is usually relevant only at very low temperatures. An interesting corollary of our considerations, for example, is that high noise levels is an *inherent* feature of all *homogeneously disordered,* room-temperature resistors whose TCR was made small by inducing sufficiently strong, short-range disorder to bring these materials near their metal-insulator transition. Figure 10 illustrates the evolution of the TCR with disorder for three of the films used in the noise data of Fig. 1. The correlation between the sharp increase of α_a and the TCR sign change (Figs. 1 and 10, respectively) is clear in this particular case. We note, however, that a small TCR could be achieved *without invoking incipient localization in the process.* For example, a composite structure can, in principle, be made from a mixture of materials with positive and negative TCR such that the effective TCR is zero over a finite range. If the constituents of such a heterogeneous structure are "clean" and the mixture is coarse enough (namely, each component having a lateral size much larger than L_ϕ), the system, as a whole, is not near the Anderson transition and thus there is no reason for its noise to be large either.

It must be realized that the Feng, Lee, and Stone picture, as well as ours, strictly applies for a *coherent* cube of a conducting material. A macroscopic sample is usually composed of a high number of such basic elements and the behavior of a large sample must ultimately involve some sort of ensemble averaging. Thus, while it seems reasonable to expect that the essential physics may be captured by our simple treatment, we can only trust its predictions qualitatively. Despite this obvious caveat, Cohen, Ovadyahu, and Rokni have shown that Eq. (6) yields reasonable parameters when compared with the experimental results.[7] This encourages us to see what else can be deduced from such a simplistic picture and by considering the noise problem from the *energetic* point of view. To bring about the Anderson transition, the effective potential disorder V has to be bigger than E_F. (By an effective V we mean the combined effect of V_0, the static disorder, and V_ω, a superimposed dynamical one that is responsible for the $1/f$ noise.[7] It is easy to see that this imposes some requirements on the energies involved by considering the scaling expression[21]

$$\Delta E \approx E_F |g - g_c| \text{ for } g \cong g_c \ , \tag{7}$$

where g is the Thouless dimensional conductance and $\Delta E \equiv E_F - E_c$ (E_c is the mobility edge). The inverse proportionality between Δg and E_F has been recently verified experimentally[24] for *static* disorder. Given the low frequencies of the $1/f$ noise phenomena, relative to the transport mean-free times, we expect that the same should hold true for the *dynamic* disorder as well. Associating g in Eq. (7) with $K_F l$, taking $g_c = 1$ and noting that the range of $K_F l$ where α_a increases fast with disorder (Fig. 4 of Ref. 7) is about 2–3, we can estimate an "effective V_ω" for the In_2O_{3-x} and ZnO samples to be 2–3 times E_F which for both systems is 2500–3000 K.[8,9] This seems like a very reasonable energy to expect from a local deformation produced by a movement of, say, an oxygen atom which may well be the main source of noise in these materials. Note also that to account for $1/f$ noise in the whole frequency range $\omega \approx 10^{-2}$–10^4 Hz at room temperature, it is enough to have an effective barrier energy covering the range $\approx$450–800 mV (using the expression $\omega \approx \nu \exp[-V/k_B T]$ with an attempt frequency $\nu = 10^{12}$). Interestingly, this range coincides with $\approx 3E_F$ of In_2O_{3-x} films (cf. Fig. 8).

It would be interesting to check on the generality of these correlations by studying the noise in other systems near their Anderson transition, preferably in materials with much smaller Fermi energies like, e.g., semiconductors. These systems are the ones most widely used for studies of the metal-insulator transition since (among other things), their relatively small E_F can be easily exceeded by quite a moderate degree of disorder, V_0. But, at the same time, the conductivity of such systems should be quite sensitive to V_ω as well. Experiments on such systems (provided they are carried out at $k_B T \ll E_F$) may be relevant for understanding the interplay between V_0, V_ω, and the dependence of the latter on temperature. An intriguing aspect of studying semiconductors near their metal-insulator transition is the possibility that the source of noise may originate from a purely electronic mechanism[25] which could be even more effective than motion of defects.

Equation (7) and the above discussion, from which V_ω/E_F emerges as the natural dimensionless parameter, suggest that, *all other things being equal,* one expects the noise to be bigger the lower E_F is. This seems to us to be true in general, even outside the range of applicability of Eq. (7). The sensitivity of the noise to the value of E_F,

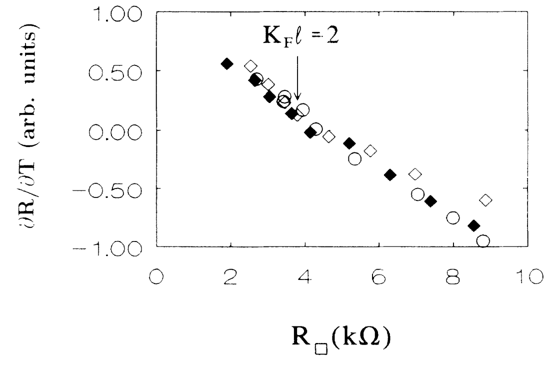

FIG. 10. The temperature coefficient of resistance TCR as a function of $R_\square$ for three of the samples used for the noise study in Fig. 1. The TCR changes sign in this material when the respective $K_F l$ of the films is about 2 (see Ref. 7).

which in turn, is proportional to $N^{2/3}$, may be the reason for the empirical observation made by Hooge[3] and discussed in Sec. I above. Near a metal-insulator transition, however, this possible dependence of α_a on carrier density is unlikely to stand out. Indeed, we can single out in our data several pairs of samples for which N happened to be *identical* to within a few percent while α_a differed by up to six orders of magnitude. At the same time, the noise magnitude for these samples correlated rather well with $K_F l$. This, again, demonstrates that near the critical point α_a is highly sensitive to the value of the disorder while other parameters, relevant as they may be, play only a secondary role.

Finally, we want to address the interplay between Anderson localization and $1/f$ noise from another point of view. The Anderson transition concerns a degenerate electronic system, driven insulating by a purely *static* disorder at $T=0$. All experiments, however, probe the system at *finite temperatures*. If, as experiments seem to suggest, a nonstatic potential is an *inherent* property of all disordered systems at nonzero temperatures, then we ought to understand the possible effect of that on the nature of the transition. The existence of a time-dependent potential is tantamount to a *fluctuation* in the parameter that drives the phase transition. Unless the fluctuation is small, the details of the transition may be considerably affected. In particular, the functional dependence of σ_0, the "zero T" conductivity, on the disorder parameter may be distorted. A close analogy is a superconductive transition, measured resistively as a function of *temperature*, while the latter fluctuates by $\Delta T \approx T_c$. Clearly, the critical temperature T_c and the width of the transition will both be "off." For a quantum coherent sample (i.e., a cube of size L comparable with L_ϕ), the fluctuation in the conductance ΔG produced by V_ω near the transition may be of the order of G itself.[26] It is far from clear that reducing the temperature may completely alleviate this problem. Recent experiments[23] suggest that significant numbers of atoms in the sample move even at very low temperatures over a time scale of minutes. Also, the metal-insulator transition may involve inherent fluctuations which are unlikely to be frozen out at any temperature.[25] It thus seems that $\sigma(T\rightarrow 0)$ will appear to approach zero *continuously* even when the $T\equiv 0$ transition is of first order. In the presence of a $1/f$-like potential fluctuation it is hard to see why this problem is not usually recognized. Unless the measured sample is *macroscopically inhomogeneous*, even time averaging (the usual experimental way to eliminate noise) may not be effective. And if one resorts to band-pass measurements, one has to show that the details of the measured transition are *independent* of the measurement frequency. These issues clearly deserve further experimental and theoretical studies.

ACKNOWLEDGMENTS

The authors gratefully acknowledge discussions with Y. Imry, M. Pollak, and M. Rokni. This research has been supported by a grant administered by the Israeli Academy for Sciences and Humanities.

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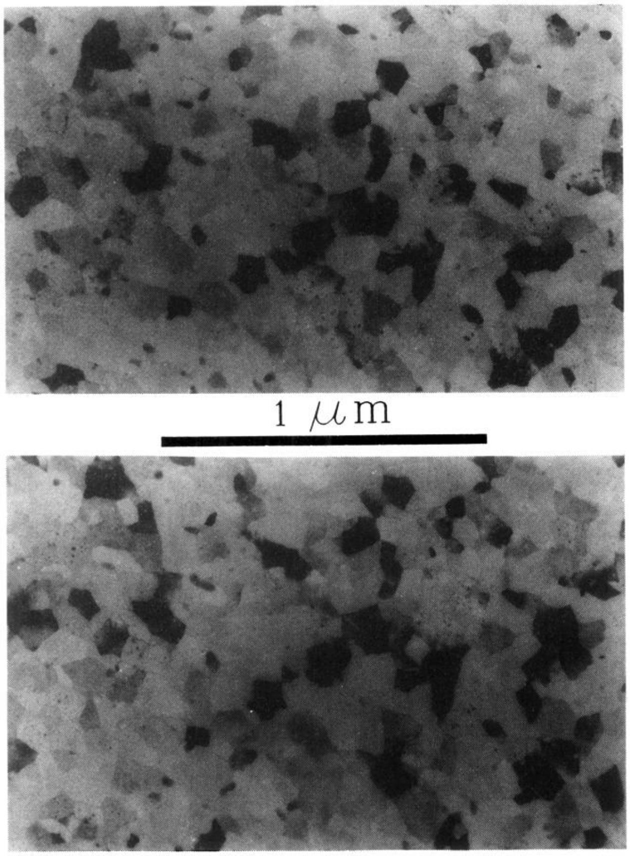

FIG. 2. Transmission electron micrograph of a In_2O_{3-x} film for two grossly different states of stoichiometry (see text for details). The two bright-field images were taken of the same area, and one can identify and compare the same crystallites before and after UV exposure. Note that apart from some contrast variations (resulting presumably from slight changes in local orientation), the samples look identical.