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Effect of superconducting fluctuations on spin susceptibility and NMR relaxation rate

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We study the effect of superconducting fluctuations on the spin susceptibility χ_s and NMR relaxation rate $1/T_1$ just above T_c for arbitrary impurity concentrations. Fluctuations are found to reduce χ_s below the Pauli susceptibility. $1/T_1T$ is enhanced above the Korringa value for weak pair breaking, but suppressed when the pair breaking is large.

INTRODUCTION

In this paper we study the effects of superconducting (SC) fluctuations on the spin susceptibility χ_s and the NMR relaxation rate $1/T_1$ using standard diagrammatic techniques.[1,2] While the effect of fluctuations on χ_s has not been examined before, there have been previous studies[3–5] of $1/T_1$, focusing mainly on the most singular contribution, the anomalous Maki-Thompson (MT) term,[2] in the dirty limit $T\tau \ll 1$. We were led to reexamine the problem in the context of the high-T_c superconductors for several reasons. First, these systems are in an intermediate regime with $T\tau \sim 1$, (where τ is the momentum relaxation time). Second, with the unusually strong pair breaking suspected in high-T_c materials, one might expect the main MT contribution to be suppressed and $1/T_1$ to be dominated by less singular contributions. (See Refs. 6 and 7 for a similar situation for the c-axis conductivity.) Finally, it has recently been suggested that dynamic pairing correlations[8] beyond the perturbative weak coupling regime are responsible for the spin gap anomalies[9] observed well above T_c in the underdoped cuprates. The analysis presented here, which only treats static fluctuations very close to T_c, nevertheless constitutes the first correction[10] to Fermi liquid behavior, to order $\max(1/\epsilon_F\tau, T_c/\epsilon_F)$, arising from pairing correlations above T_c. At the end of the paper we discuss the prospects of experimentally studying these effects in the high-T_c cuprates and other layered superconductors.

Our main results, valid for $\varepsilon=(T-T_c)/T_c \ll 1$, can be summarized as follows.

(1) SC fluctuations lead to a suppression of the spin susceptibility χ_s, due to the combined effect of the reduction of the single-particle density of states (DOS) and of the regular part of the MT process.

(2) "Cooperon" impurity interference terms, involving impurity ladders in the particle-particle channel, are crucial for the χ_s suppression in the dirty limit.

(3) The processes contributing to χ_s are negligible in usual fluctuation calculations (e.g., conductivity σ). χ_s is unusual in that the Aslamazov-Larkin (AL), and anomalous MT terms, which dominate σ, are absent.

(4) For weak pair breaking $(1/\tau_\varphi \ll T_c)$, we find an enhancement of $1/T_1T$ coming from the anomalous MT term. We recover known results[4] in the dirty limit, and extend these to arbitrary impurity scattering.

(5) In the clean limit $(T_c\tau \gg 1)$ we find a different asymptotic behavior of $1/T_1T$ depending on whether one has $T_c\tau$ is greater or smaller than $1/\sqrt{\varepsilon}$.

(6) Finally, strong dephasing suppresses the anomalous MT contribution, and $1/T_1T$ is then dominated by the less singular DOS and regular MT terms. These contributions lead to a suppression of spectral weight and a decrease in $1/T_1T$.

We begin with the dynamic susceptibility $\chi^{(R)}_{+-}(\mathbf{k},\omega) = \chi_{+-}(\mathbf{k},i\omega_\nu \to \omega+i0^+)$ with

$$\chi_{+-}(\mathbf{k},\omega_\nu)=\int_0^{1/T} d\tau\, e^{i\omega_\nu\tau}\langle \hat{T}[\hat{S}_+(\mathbf{k},\tau)\hat{S}_-(-\mathbf{k},0)]\rangle. \quad (1)$$

$\hat{S}_\pm$ are the spin raising and lowering operators, $\hat{T}$ denotes time ordering, the brackets represent thermal and impurity averaging and $\omega_\nu = 2\pi\nu T$. The spin susceptibility $\chi_s=\chi^{(R)}_{+-}(\mathbf{k}\to 0,\omega=0)$ and the NMR relaxation rate is given by

$$1/T_1T=\lim_{\omega\to 0}\frac{A}{\omega}\int (d\mathbf{k})\, \mathrm{Im}\chi^{(R)}_{+-}(\mathbf{k},\omega) \quad (2)$$

where the constant $A>0$. We use $\int(d\mathbf{k})=\int d^d\mathbf{k}/(2\pi)^d$ in d dimensions. For later reference we note that for noninteracting electrons $\chi^0_{+-}(\mathbf{k},\omega_\nu)=-T\Sigma_n\int(d\mathbf{p})G(\mathbf{p}+\mathbf{k},\epsilon_n+\omega_\nu)G(\mathbf{p},\epsilon_n)$ with $\epsilon_n=2\pi T(n+1/2)$. This leads to the well-known results for $T\ll\epsilon_F$: $\chi^0_s=N(0)$ (Pauli susceptibility) and $(1/T_1T)^0=A\pi[N(0)]^2$ (Korringa relaxation), where $N(0)$ is the DOS at the Fermi level.

To leading order in $\max(1/\epsilon_F\tau, T_c/\epsilon_F)$ the fluctuation contributions to χ_{+-} are given by the diagrams shown in Fig. 1. The diagrams are constructed from fermion lines, SC fluctuation propagators (wavy lines) and impurity vertex corrections (shaded objects), each of which will be described in detail below. Note that the two fermion lines attached to the external vertex have opposite spin labels for χ_{+-}. Consequently, the AL diagram (1) does not exist since one cannot consistently assign a spin label to the fermion line marked with a '?' for spin-singlet pairing.

The Maki-Thompson (MT) diagram is shown in Fig. 1 (2), and MT with Cooperon impurity corrections in (3) and (4). There is an important difference in the topology, and thus the sign, of the MT graph for χ_{+-} and that for conductivity, arising from the spin structure. Drawing the fluctuation propagator explicitly as a ladder of attractive interaction lines, we see that the diagram (2) is a nonplanar graph with a single fermion loop, in contrast with the conductivity graph which is planar and has two fermion loops.

The diagrams (5) and (6) represent the effect of SC fluctuations on the self-energy, leading to a decrease in the DOS. The DOS diagrams (7) and (8) include impurity vertex corrections. (Only a single impurity scattering line is shown since additional scattering, in the form of a ladder, has no effect.) Finally (9) and (10) are DOS diagrams with Cooperon impurity corrections.

The fermion lines represent the one-electron Green function $G(\mathbf{p},\omega_n)=[i\tilde{\epsilon}_n-\xi(\mathbf{p})]^{-1}$, where $\tilde{\epsilon}_n=\epsilon_n+\mathrm{sgn}(\epsilon_n)/2\tau$. The momentum relaxation rate $1/\tau\ll\epsilon_F$, however, $T_c\tau$ is arbitrary. We will first discuss the two-dimensional (2D) case, and then turn to layered systems and 2D to 3D crossover at the end. For the isotropic case $\xi(\mathbf{p})=|\mathbf{p}|^2/2m-\epsilon_F$.

Pairing fluctuations above T_c are described in the usual way[1,11] by the fluctuation propagator (wavy line in Fig. 1) $L(\mathbf{q},\Omega_\mu)$ where $\mathbf{q}$ is the momentum and Ω_μ is the frequency of the pair. We restrict our attention to $\varepsilon=(T-T_c)/T_c\ll1$, and thus it suffices to focus on long wavelength, static ($\Omega_\mu=0$) fluctuations. In this regime we have

$$L^{-1}(\mathbf{q},\Omega_\mu=0)=-N(0)[\varepsilon+\eta_d q^2], \tag{3}$$

where in 2D, the DOS $N(0)=m/2\pi$ and

$$\eta_2=-\frac{v^2\tau^2}{2}\left[\psi\left(\frac{1}{2}+\frac{1}{4\pi\tau T}\right)-\psi\left(\frac{1}{2}\right)-\frac{1}{4\pi\tau T}\,\psi'\left(\frac{1}{2}\right)\right]$$
$$\to\begin{cases}\pi D/(8T_c) & \text{for } T_c\tau\ll1,\\ 7\zeta(3)v^2/(32\pi^2Tc^2) & \text{for } T_c\tau\gg1.\end{cases} \tag{4}$$

Here v is the Fermi velocity, $D=v^2\tau/2$ is the 2D diffusion constant, and $\psi(z)$ and $\zeta(x)$ are the digamma and Riemann zeta functions, respectively. Pair breaking will be described by a phenomenological $\tau_\phi\gg\tau$; see discussion preceding Eq. (15) below.

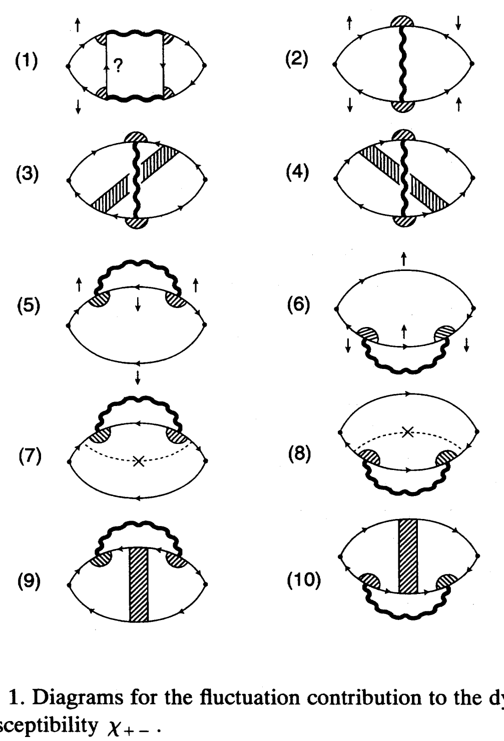

FIG. 1. Diagrams for the fluctuation contribution to the dynamic spin susceptibility χ_{+-}.

We now turn to vertex corrections due to impurity scattering. First, note that external vertices are not renormalized.[12] Next, the (three-legged) impurity vertex $\lambda(\mathbf{q},-\epsilon_n,\epsilon_{n+\nu})$ is defined as the sum of impurity ladders dressing the bare vertex consisting of two fermion lines, with frequencies $-\epsilon_n$ and $\epsilon_{n+\nu}=\epsilon_n+\omega_\nu$, and a fluctuation propagator $L(\mathbf{q},\omega_\nu)$. This is given by

$$\lambda^{-1}(\mathbf{q},-\epsilon_n,\epsilon_{n+\nu})=\left(1-\frac{\Theta(\epsilon_n\epsilon_{n+\nu})}{\tau\sqrt{(\tilde{\epsilon}_n+\tilde{\epsilon}_{n+\nu})^2+v^2q^2}}\right), \tag{5}$$

where $\Theta(x)$ is the Heaviside step function.

Finally, the Cooperon $C(\mathbf{q},-\epsilon_n,\epsilon_{n+\nu})$ is defined as the sum of impurity ladders in the particle-particle channel where $-\epsilon_n$ and $\epsilon_{n+\nu}$ are the frequencies of the two fermion lines and $\mathbf{q}$ their *total* momentum. We have

$$C(\mathbf{q},-\epsilon_n,\epsilon_{n+\nu})=\frac{1}{2\pi N(0)\tau}\left[\frac{1}{\tau}\frac{\Theta(\epsilon_n\epsilon_{n+\nu})}{|2\epsilon_n+\omega_\nu|+Dq^2}+\Theta(-\epsilon_n\epsilon_{n+\nu})\right]. \tag{6}$$

SPIN SUSCEPTIBILITY

The external frequency and momentum can be set to zero at the outset, thus simplifying the calculation. The MT diagram (2) then yields a result which is identical to the sum of the DOS diagrams (5) and (6), which are evaluated as follows:

$$\chi_{s5}+\chi_{s6}=-2T\int(d\mathbf{q})L(\mathbf{q},0)T\sum_n\lambda^2(\mathbf{q},\epsilon_n,-\epsilon_n)\times\int(d\mathbf{p})G^3(\mathbf{p},\epsilon_n)G(-\mathbf{p},-\epsilon_n). \tag{7}$$

Doing the $\mathbf{p}$ integration, and using $\lambda\simeq|\tilde{\epsilon}_n|/|\epsilon_n|$, ignoring its $\mathbf{q}$ dependence, we are left with $\Sigma_n 1/[\epsilon_n^2|\tilde{\epsilon}_n|]$, which can be evaluated in terms of digamma functions. Finally using $\int(d\mathbf{q})L(\mathbf{q},0)\simeq-[4\pi N(0)\eta_2]^{-1}\ln(1/\varepsilon)$ we get

$$(\chi_{s5}+\chi_{s6})/\chi_s^0=(\tau/2\pi\eta_2)f(T_c\tau)\ln(1/\varepsilon), \tag{8}$$

where $f(x)=x\{\psi(1/2)-\psi[1/2+1/(4\pi x)]\}+\pi/8$.

We first discuss the clean limit, where the fluctuation contribution given by $\chi_s^{\mathrm{fl}}=\chi_{s2}+\chi_{s5}+\chi_{s6}$; all other diagrams are negligible for $T_c\tau\gg1$. The final result is

$$\chi_s^{\mathrm{fl}}/\chi_s^0=-(2T_c/\epsilon_F)\ln(1/\varepsilon)\text{for } T_c\tau\gg1. \tag{9}$$

The dirty limit of (8) yields a result of $\mathscr{O}(T_c/\epsilon_F)$, which is negligible, and the dominant $\mathscr{O}(1/\epsilon_F\tau)$ contribution must come from elsewhere. Graphs (7) and (8) vanish, since $\int(d\mathbf{p})G^3(\mathbf{p},\epsilon_n)=0$. The important graphs are those with the Cooperon impurity correction: MT (3) and (4), and DOS (9) and (10). This is an interesting example where Cooperons, which play a crucial role in weak localization, are important for SC fluctuations. As an example, we evaluate

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$$\chi_{s3}+\chi_{s4}=2T\int(d\mathbf{q})L(\mathbf{q},0)T\sum_n \lambda^2(\mathbf{q},\epsilon_n,-\epsilon_n)$$

$$\times C(\mathbf{q},\epsilon_n,-\epsilon_n)\mathscr{T}(\epsilon_n)\mathscr{T}(-\epsilon_n), \quad (10)$$

$$\mathscr{T}(\epsilon_n)=\int(d\mathbf{p})G^2(\mathbf{p},\epsilon_n)G(\mathbf{q}-\mathbf{p},-\epsilon_n)$$

$$\simeq -i2\pi N(0)\tau^2\mathrm{sgn}(\epsilon_n) \quad \text{for } \mathbf{q}=0.$$

Using $\lambda\simeq(2\tau|\epsilon_n|)^{-1}$, and $Dq^2\ll|\epsilon_n|$ in the Cooperon C we obtain the sum $\Sigma_n 1/(\epsilon_n^2|\epsilon_n|)$. Doing this sum and the $\mathbf{q}$ integration we obtain one half the final result given below; diagrams (9) and (10) give the other half. The total fluctuation susceptibility $\chi_s^{\rm fl}=\chi_{s3}+\chi_{s4}+\chi_{s9}+\chi_{s10}$, is

$$\chi_s^{\rm fl}/\chi_s^0=-[7\zeta(3)/\pi^3\epsilon_F\tau]\ln(1/\varepsilon) \quad \text{for } T_c\tau\ll 1. \quad (11)$$

It is tempting to physically understand $\chi_s^{\rm fl}<0$ in Eqs. (9) and (11) as arising from a suppression[13] of the DOS at ϵ_F. However, only diagrams (5) and (6) can strictly be interpreted in this manner; the MT graphs and the coherent impurity scattering described by the Cooperons do not permit such a simple interpretation.

RELAXATION RATE

This calculation requires rather more care than χ_s because of the subtleties of analytic continuation. Let us define the local susceptibility $K(\omega_\nu)=\int(d\mathbf{k})\chi_{+-}(\mathbf{k},\omega_\nu)$. The MT contribution, with $\epsilon_{n+\nu}=\epsilon_n+\omega_\nu$, is given by

$$K_2(\omega_\nu)=T^2\int(d\mathbf{q})L(\mathbf{q},0)\sum_n \lambda(\mathbf{q},\epsilon_{n+\nu},-\epsilon_{n+\nu})$$

$$\times\lambda(\mathbf{q},\epsilon_n,-\epsilon_n)\mathscr{T}(\mathbf{q},\epsilon_{n+\nu})\mathscr{T}(\mathbf{q},\epsilon_n). \quad (12)$$

Defining $X(\mathbf{q},\epsilon_n)=\sqrt{4\tilde{\epsilon}_n^2+v^2q^2}$, we have $\mathscr{T}(\mathbf{q},\epsilon_n)=\int(d\mathbf{p})G(\mathbf{p},\epsilon_n)G(\mathbf{q}-\mathbf{p},-\epsilon_n)=2\pi N(0)/X(\mathbf{q},\epsilon_n)$ and $\lambda(\mathbf{q},\epsilon_n,-\epsilon_n)=X(\mathbf{q},\epsilon_n)/[X(\mathbf{q},\epsilon_n)-1/\tau]$. The resulting Matsubara sum is $S=\Sigma_n 1/\{[X(\mathbf{q},\epsilon_n)-1/\tau][X(\mathbf{q},\epsilon_{n+\nu})-1/\tau]\}$.

The "anomalous" MT contribution [subscript (an)] comes from that part of S which involves $-(\omega_\nu/2\pi T)=-\nu\leq n\leq -1$. The reason this piece dominates, and has to be treated separately, is that $S_{(\rm an)}$ has a singular $\mathbf{q}$ dependence. We evaluate $S_{(\rm an)}$ using contour integration and after $i\omega_\nu\to\omega+i0^+$ we obtain

$$\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}K_{2(\rm an)}^{(R)}(\omega)=-\frac{\pi[N(0)]^2}{8}\int(d\mathbf{q})L(\mathbf{q},0)\mathscr{K}(\mathbf{q}), \quad (13)$$

$$\mathscr{K}(\mathbf{q})=2\tau\int_{-\infty}^{\infty}dz\,\mathrm{sech}^2\left(\frac{z}{4T\tau}\right)F(z-i)F(z+i), \quad (14)$$

where $F(x)=1/(\sqrt{\ell^2q^2-x^2}-1)$ and $\ell=v\tau$ is the mean free path.

The first simple limiting case for (14) is $\ell q\ll 1$, for which $\mathscr{K}(\mathbf{q})=2\pi/Dq^2$. The characteristic q values are determined by L; in the dirty limit, we have $Dq^2\sim\varepsilon T_c$, leading to $\ell^2q^2\sim\varepsilon T_c\tau\ll 1$. In the clean case, on the other hand, $v^2q^2/T_c\sim\varepsilon T_c$ and $\ell^2q^2\sim\varepsilon(T_c\tau)^2\ll 1$ only when $1\ll T_c\tau\ll 1/\sqrt{\varepsilon}$.

For the above conditions (either $T_c\tau\ll 1$ or $1\ll T_c\tau\ll 1/\sqrt{\varepsilon}$) we obtain the singular MT contribution $\int d^2q[(Dq^2+1/\tau_\varphi)(\varepsilon+\eta_2q^2)]^{-1}$ where we have introduced the pair breaking rate $1/\tau_\varphi$ as an infrared cutoff. We define the dimensionless pair breaking parameter $\delta=\eta_2/D\tau_\varphi\ll 1$; in the dirty limit $\delta\sim 1/T_c\tau_\varphi$ while for the clean case $\delta\sim 1/T_c^2\tau\tau_\varphi$. The "bare" transition temperature T_{c0} is shifted by the pair breaking, so that $\varepsilon=\varepsilon_0+\delta$, with $\varepsilon_0=(T-T_{c0})/T_{c0}$, and we obtain the final result

$$\frac{(1/T_1T)^{\rm fl}}{(1/T_1T)^0}=\frac{\pi}{8\epsilon_F\tau}\frac{1}{\varepsilon-\delta}\ln(\varepsilon/\delta). \quad (15)$$

The other limiting case of interest is the "ultra-clean limit" when the characteristic q values satisfy $\ell q\gg 1$. This is obtained when $T_c\tau\gg 1/\sqrt{\varepsilon}\gg 1$. From (14) we then find $\mathscr{K}(\mathbf{q})=4\ln(\ell q)/vq$, which leads to

$$\frac{(1/T_1T)^{\rm fl}}{(1/T_1T)^0}=\frac{\pi^3}{\sqrt{14\zeta(3)}}\frac{T_c}{\epsilon_F}\frac{1}{\sqrt{\varepsilon}}\ln(T_c\tau\sqrt{\varepsilon}). \quad (16)$$

We note that in all cases the anomalous MT contribution leads to an *enhancement* of $1/T_1T$ over the normal state Korringa value. In particular, the SC fluctuations above T_c have the *opposite* sign to the effect for $T\ll T_c$ (where $1/T_1$ drops exponentially with T). One might argue that the enhancement of $1/T_1T$ is a precursor to the coherence peak just below T_c. Although the physics of the Hebel-Slichter peak (pile-up of DOS just above gap edge and coherence factors) appears to be quite different from that embodied in the MT process, we note that both effects are suppressed by strong inelastic scattering.

We now discuss the DOS and regular MT contributions which are important when strong dephasing suppresses the anomalous MT contribution discussed above. The contribution from diagrams (5) and (6) is given by

$$K_{5+6}(\omega_\nu)=-2T^2\int(d\mathbf{q})L(\mathbf{q},0)$$

$$\times\sum_n \lambda^2(\mathbf{q},\epsilon_n,-\epsilon_n)\mathscr{T}_1(\epsilon_n)\mathscr{T}_2(\epsilon_{n+\nu}) \quad (17)$$

with

$$\mathscr{T}_1(\epsilon_n)=\int(d\mathbf{p})G^2(\mathbf{p},\epsilon_n)G(\mathbf{q}-\mathbf{p},-\epsilon_n)$$

$$=-i\pi N(0)\mathrm{sgn}(\epsilon_n)/(2\tilde{\epsilon}_n^2)$$

and

$$\mathscr{T}_2(\epsilon_{n+\nu})=\int(d\mathbf{k})G(\mathbf{p}+\mathbf{k},\epsilon_{n+\nu})=-i\pi N(0)\mathrm{sgn}(\epsilon_{n+\nu})$$

where we have set $\mathbf{q}\simeq 0$ everywhere except in L. Using $\lambda(0,\epsilon_n,-\epsilon_n)\simeq|\tilde{\epsilon}_n|/|\epsilon_n|$ we are left with the frequency sum $\Sigma_n\mathrm{sgn}(\epsilon_n)\mathrm{sgn}(\epsilon_{n+\nu})/\epsilon_n^2$ and a simple $\mathbf{q}$ integration.

The other remaining contribution is from the "regular" part of the MT diagram, corresponding to terms with $n<-\nu$ and $n\geq 0$ in the Matsubara sum in (12). This contribution is exactly one half of the total DOS contribution from diagrams (5) and (6). All other diagrams either vanish [graphs (7) and (8)] or contribute at higher order in $1/\epsilon_F\tau$ (Cooperon corrections). The final results are given by

$$(1/T_1T)^{\rm fl}/(1/T_1T)^0 = -(6T_c/\epsilon_F)\ln(1/\varepsilon) \quad (18)$$

for $T_c\tau \gg 1$, and

$$(1/T_1T)^{\rm fl}/(1/T_1T)^0 = -[21\zeta(3)/\pi^3\epsilon_F\tau]\ln(1/\varepsilon) \quad (19)$$

for $T_c\tau \ll 1$. The sign of the result indicates a suppression of low-energy spectral weight as in the χ_s calculation.

LAYERED SYSTEMS

It is straightforward to extend the above analysis to layered systems with the spectrum $\xi(\mathbf{p}) = \epsilon(\mathbf{p}) - \epsilon_F = p_\parallel^2/2m - \epsilon_F + w\cos(p_\perp a)$. Here $p_\parallel$ and $p_\perp$ are the electron momenta parallel and perpendicular to the plane, respectively, a is the interlayer distance, w is the interlayer hopping. (For details on the fluctuation propagator and impurity vertex corrections in layered systems, see Refs. 11 and 7.)

The layered system results are obtained by making the replacement

$$\ln(1/\varepsilon) \rightarrow 2\ln(2/[\sqrt{\varepsilon} + \sqrt{\varepsilon + r}]), \quad (20)$$

in the 2D results given above, with $r = 4\eta_2 w^2/v^2$. The physical meaning of r is clarified by noting that $r(T_c) = 4\xi_\perp^2(0)/a^2$ where $\xi_\perp(0)$ is Ginzburg-Landau coherence length in c-direction. Thus r is the anisotropy parameter which controls the dimensional crossover from the 2D [$r \ll \varepsilon$ with $\xi_\perp(T) \ll a/2$] to the 3D regimes [$r \gg \varepsilon$ with $\xi_\perp(T) \gg a/2$] where the $\ln(1/\varepsilon)$ singularity changes to [constant $-(\varepsilon/r)^{1/2}$].

EXPERIMENTAL IMPLICATIONS

Much of the experimental work on fluctuations as probed by NMR has been restricted to small particles (zero-dimensional limit) of conventional superconductors; see Ref. 14 for a review. There has been resurgence of interest in SC fluctuations since the high-T_c cuprates show large effects above T_c due to their short coherence length and quasi-two-dimensional structure. However, in order to extract the "fluctuation contribution" from experiments one needs to know the normal state background, which in a conventional metal would simply be the Pauli susceptibility for χ_s and the Korringa law for $1/T_1T$. In the high-T_c materials the backgrounds themselves have nontrivial temperature dependences[15] above T_c: for example, the non-Korringa relaxation for the Cu(2) nuclei in $YBa_2Cu_3O_{7-\delta}$, and the spin-gap behavior[9] with $d\chi_s/dT > 0$ and the O and Y $1/T_1T \sim \chi_s(T)$ in the underdoped cuprates. In fact, as noted in the Introduction, part of the theoretical motivation for the calculation presented here came from earlier work[8] on spin gaps as precursor effects to short-coherence length superconductivity, even though the dynamics of the fluctuations and the deviation from particle-hole symmetry, which are crucial in that work, are ignored here.

From an experimental point of view, it appears that the best systems for observing the effects calculated in this paper would be conventional layered superconductors where the normal state backgrounds are well understood and small deviations from these may be reliably extracted.

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