High-field superconducting transition induced by correlated disorder

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The "glassy" superconducting transition at high magnetic fields can be induced by columnar disorder. A model is proposed in which the thermodynamics of Bose condensation of Cooper pairs into the lowest Landau-level eigenstate of the random potential can be solved exactly. The solution reflects a peculiar character of the high-field limit: for example, the effective dimensionality of the transition is shown to be a function of magnetic field.

The problem of a superconducting transition in the presence of strong disorder is of both practical and theoretical interest. Technologically, the goal is to introduce defects into the sample in a way that maximizes pinning of vortices and increases critical currents.[1] The theoretical challenge is to understand the mechanisms and the nature of a superconducting transition for various types of disorder. A variety of novel phases have been proposed, differing in the cases of pointlike[2,3] and linelike disorder.[4,5] In this paper we present a theory of superconducting transition at *high* magnetic fields ($\gtrsim 1$ T in high-temperature superconductors) *induced* by the presence of columnar (linelike) defects. In the absence of disorder, the high-field fluctuations of the order parameter, $\psi(\mathbf{r})$, are strongly enhanced by formation of Landau levels (LL's) for Cooper pairs.[6] Such fluctuations lead to $(D \rightarrow D-2)$-dimensional reduction in the pairing susceptibility, $\chi_{sc}(\mathbf{r},\mathbf{r}')$, and eliminate the superconducting (Abrikosov) transition for $D=2,3$.[7] The Abrikosov phase is then replaced by a different fluctuation-induced state, the density wave of Cooper pairs (SCDW), in which the thermal average $\langle|\psi(\mathbf{r})|^2\rangle$ has a weak modulation accompanied by only a short-range phase coherence.[7,8] In the presence of disorder the LL degeneracy is lifted and a possibility of superconducting transition is restored. χ_{sc} can now diverge at some finite temperature, $T_{sc}(H)$, determined by the strength of disorder.[7] At $T_{sc}(H)$ the normal state is unstable to Bose condensation of Cooper pairs into the lowest-energy eigenstate of the random potential, which we argue extends over the whole sample in situations of experimental interest. Furthermore, for experimentally relevant parameters, $T_{sc}(H)$ can be far above the SCDW transition line over much of the H-T phase diagram, allowing us to treat the correlations that produce SCDW in an approximate way.

We consider a realistic model for a superconductor in a magnetic field parallel to the columns which is exactly solvable. The model exhibits "dimensional transmutation," i.e., the effective dimensionality of the transition changes continuously as a function of magnetic field. This effect is a direct consequence of analytic properties of LL wave functions and is a signature of the high-field limit. We determine the transition line in the H-T phase diagram, the Edwards-Anderson order parameter, and the behavior of correlation length, specific heat, and magnetic susceptibility in the vicinity of the transition. There are similarities between the transition considered here and the one in the spherical model for spin glasses.[9]

We are interested in strongly anisotropic layered superconductors described by the Ginzburg-Landau (GL) Lawrence-Doniach model, with the magnetic field perpendicular to the layers. Fluctuations of the magnetic field are neglected ($\kappa \gg 1$). We focus on the *high-field limit*, where the LL structure of Cooper pairs dominates the fluctuation spectrum: this is the case for fields above $H_b \approx (\theta/16)H_{c2}(0)[T/T_c(0)]$, where θ is the Ginzburg fluctuation parameter.[10] [For instance, Bi-Sr-Ca-Cu-O (2:2:1:2), $\theta \approx 0.045$ and $H_b \approx 1$ T.] In this regime, the essential features of the physics are captured by retaining only the lowest Landau level (LLL) modes. This is the renormalized GL-LLL theory.[7,10] The partition function is $Z = \int D[\psi^*\psi]\exp(-S)$, and

$$S = \frac{d}{T}\sum_{n=1}^{N_L} \int d^2r \left\{ \eta|\psi_n(\mathbf{r}) - \psi_{n+1}(\mathbf{r})|^2 + \left[\alpha'(T,H) + \lambda\sum_i \delta(\mathbf{r}-\mathbf{r}_i)\right]|\psi_n(\mathbf{r})|^2 + \frac{\beta}{2}|\psi_n(\mathbf{r})|^4 \right\},$$

(1)

where $\alpha'(T,H) = a[T - T_{c2}(H)]$, d is the effective layer separation, n is the layer index, $\lambda > 0$ is the effective strength of the defects, and a, β, and η are phenomenological parameters. The magnetic field is assumed to be parallel to columnar defects, the effective potentials of all defects the same and well represented by δ functions. Random variables in the problem are two-dimensional coordinates of defects, $\{\mathbf{r}_i\}$. We assume that columns of damaged superconductor are distributed according to the Poisson distribution $P_N(\mathbf{r}_1, \ldots, \mathbf{r}_N) = (e^{-\rho A}\rho^N)/N!$ where P_N is the probability for finding N impurities at the positions $\mathbf{r}_1, \ldots, \mathbf{r}_N$, A is the area of the system, and ρ is the concentration of impurities.[11] After rescaling the fields and the lengths as $(2d\beta 2\pi l^2/T)^{1/4}\psi \rightarrow \psi$, $r/(l\sqrt{2\pi}) \rightarrow r$, where l is the magnetic length for charge $2e$, the quartic term can be rewritten,

$$\frac{1}{4}\sum_n \int d^2r|\psi_n|^4 = \frac{1}{4N}\sum_n \beta_A(n)\left(\int d^2r|\psi_n|^2\right)^2, \quad (2)$$

where $N = A/2\pi l^2$ is the degeneracy of the LLL and $\beta_A(n) = (N\int|\psi_n|^4)/(\int|\psi_n|^2)^2$ is the generalized Abrikosov ratio corresponding to configuration $\psi_n(\mathbf{r})$. We now observe that $\beta_A(n)$ is only weakly dependent on the actual configu-
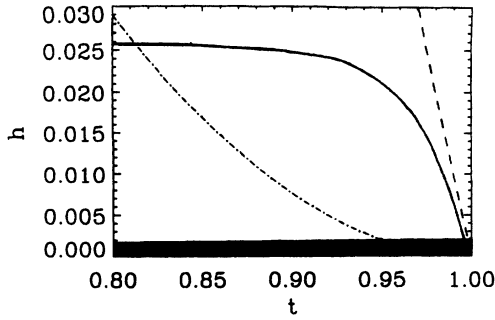
FIG. 1. The H-T phase diagram for a strongly type-II superconductor with columnar disorder [$h \equiv H/H_{c2}(0)$, $t \equiv T/T_c(0)$]. The full line represents the second-order phase-transition boundary between normal and "glassy" superconducting states for the set of parameters given in the text. The dashed-dotted line is the SCDW transition in a clean system. The dashed line is the mean field $h_{c2}(t)$. The LL approximation breaks down in the shaded region at the bottom.

ration, the well-known example being the small difference in β_A between a triangular and square lattice of zeros.[10] Thus we may substitute $\beta_A(n)$ in the quartic term by its thermal average, $\langle \beta_A \rangle$, and treat this as an input to the theory. This approximation neglects the nonperturbative lateral correlations that produce the SCDW transition.[7] It is justified if $T_{sc}(H)$ is far above the SCDW transition line. In that case the SCDW correlations enter only very close to the transition and can be ignored in most realistic situations. Since superconducting and SCDW transitions arise from two distinct mechanisms, the respective transition lines scale differently in the H-T phase diagram and, for moderate disorder, we are assured of a wide region near $H_{c2}(T)$ where the neglect of SCDW correlations should be justified (see Fig. 1).[12]

After the $\beta_A(n) \rightarrow \langle \beta_A \rangle$ substitution the thermodynamics of the model becomes exactly solvable. We first introduce variables $\{x_n\}$ to decouple the quartic term and integrate over the fields (ψ^*, ψ). This leads to $Z = \int \Pi_n dx_n \exp(-NS')$, where

$$S' = -\sum_n \frac{x_n^2}{\langle \beta_A \rangle} + \int_0^\infty dV \rho_f(V) \mathrm{Tr}_{(n,m)} \ln[g_\eta(2\delta_{n,m} - \delta_{n,m-1}$$

$$- \delta_{n,m+1}) + (g_\alpha + x_n + g_\lambda V)\delta_{n,m}]. \quad (3)$$

We drop terms coming from the rescaling of $\psi_n(\mathbf{r})$ and introduce dimensionless combinations of GL parameters $g_{\eta,\alpha,\lambda} = \{\eta, \alpha', \lambda/2\pi l^2\} \times \sqrt{(d\pi l^2)/(T\beta)}$. The density of states of a disordered system in the LLL can be found exactly by using the supersymmetric formalism.[13] For the Poisson short-range scatters it is given by

$$\rho_f(V) = \frac{1}{\pi} \mathrm{Im} \frac{d}{dV} \ln \int_0^\infty dt \exp\left(iVt - f \int_0^t \frac{dy}{y}(1 - e^{-iy}) \right), \quad (4)$$

where $f = \rho 2\pi l^2$. In the thermodynamic limit $N \rightarrow \infty$, the partition function is completely determined by the saddle point of S'. Assuming that the saddle point is at x independent of the layer index, we finally write the free energy above the critical temperature,

$$\frac{F}{NN_LT} = -\frac{x^2}{\langle \beta_A \rangle} + \frac{1}{2}\int_{-1}^{1} dk \int_0^\infty dV \rho_f(V) \ln[g_\eta e(k)$$

$$+ g_\alpha + x + g_\lambda V], \quad (5)$$

where $e(k) = 2[1 - \cos(\pi k)]$ and x is determined by the solution of

$$x = \frac{\langle \beta_A \rangle}{4} \int_{-1}^{1} dk \int_0^\infty \frac{\rho_f(V)dV}{g_\eta e(k) + g_\alpha + x + g_\lambda V}. \quad (6)$$

In Eq. (6) it is important to know the behavior of the density of states at low energies. For $f < 1$, the density of states has a δ-function singularity at $V = 0$, while for $f > 1$, $\rho_f(V) \sim V^{f-2}$ when $V \rightarrow 0$.[13] When $V < 0$, $\rho(V) \equiv 0$, as also can be inferred from Eq. (4).[13] The transition line, $T_{sc}(H)$, in the H-T diagram is determined by Eq. (6) and $x + g_\alpha = 0$, which corresponds to condensation of Cooper pairs into $k = 0$ and $V = 0$ eigenstate of the random potential. It is easily seen that there will be a nonzero transition temperature only if the concentration of impurities and magnetic field are such that $f > \frac{3}{2}$. Below this value of f LLL degeneracy is not sufficiently lifted by the random potential and thermal fluctuations prevent a finite temperature phase transition in our model.[14] Experimentally, this should manifest itself as a drop in transition temperature when the field exceeds a certain value. $f = \frac{3}{2}$ determines the effective lower critical dimension for our model. After introducing dimensionless quantities $t = T/T_c(0)$, $h = H/H_{c2}(0)$, $\theta = 2\beta H_{c2}(0)/da^2 T_c(0)\phi_0$, and $\lambda' = \lambda H_{c2}(0)/\phi_0 a T_c(0)$, where ϕ_0 is the flux quantum, we perform the integration over wave vector k in Eq. (6) to obtain the expression for transition temperature,

$$t_{sc}(h) = (1 - h)$$

$$\times \left[1 + \frac{\theta\langle \beta_A \rangle}{2\lambda'} \int_0^\infty \frac{\rho_f(V)dV}{\sqrt{V^2 + (4\eta V)/[h\lambda' aT_c(0)]}} \right]^{-1}. \quad (7)$$

Notice that when $\lambda' \rightarrow 0$ we have $t_{sc}(h) \rightarrow 0$, while for $\lambda' \rightarrow \infty$, $t_{sc}(h) \rightarrow 1 - h$.[15] Also, with increasing parameter f, $t_{sc}(h)$ increases. This is related to the observation in Ref. 1 that the irreversibility line shifts to higher temperatures with increasing doses of irradiation with heavy ions. The numerical solution for $t_{sc}(h)$ is displayed in Fig. 1 for $\lambda' = 1$, $2\eta/aT_0 = 0.01$, $\theta = 0.03$, $\langle \beta_A \rangle = 1.3$, and $f = 0.04/h$. We have set $\langle \beta_A \rangle$ to a constant for simplicity. If $H_{c2}(0) \approx 100$ T, $f = 0.04/h$ corresponds to the average distance between defects of 225 Å (at 1 T, $l \cong 180$ Å). The diameter of the columns depends on the size and energy of particles used for irradiation but it is about 50 Å and hence much smaller than the magnetic length for the fields of interest. Thus representing defects by δ functions is appropriate.

As the temperature drops below $t_{sc}(h)$, x remains at the value it had at the transition. There is now macroscopic occupancy of the lowest energy state at $V = 0$ and $k = 0$. As is well known, condensation into this state is possible only if the state is *extended*. It is a special feature of this problem that this indeed is the case for a certain range of impurity concentration. The density of states, Eq. (4), changes at $V = 0$ from being infinite when $f < 2$, to being zero when $f > 2$. Thus, for fields and impurity concentrations such that param-

eter $f<2$, true extended states [which always exist in the LLL (Ref. 16)] must lie at the bottom of the impurity band, since the number of states there diverges. The change of behavior in the density of states at $f=2$ could be caused by the fact that the mobility edge shifts to positive energies at some $f_0>2$, leaving spread out but localized states at $V=0$, which now becomes the tail of the distribution. Numerical diagonalization studies indicate that the mobility edge is indeed located near the band center for $f>4$.[17] Thus, strictly speaking, our model is appropriate for $f<f_0$. However, even for f above but close to f_0, which is often the case for fields and concentrations of experimental interest, the states at $V=0$ are still near the mobility edge and will appear extended in a finite-size sample. On this basis, we expect that useful information about the transition can still be obtained within our model.

With these cautionary remarks in mind, the natural order parameter is the thermal average of the component of $\psi_n(\mathbf{r})$ corresponding to the eigenvalue with $V=0$ and $k=0$. This is $\langle\psi_{0,0}\rangle=[NN_L(g_\alpha|_{t=t_{sc}(h)}-g_\alpha)/\langle\beta_A\rangle]^{1/2}$. The disorder average value of the field is $\overline{\langle\psi_n(\mathbf{r})\rangle}$

$=\Sigma_{V,k}\overline{\phi_V(\mathbf{r})}\exp(ikn)\langle\psi_{V,k}\rangle=0$, due to random phases of the state $\phi_{V=0}(\mathbf{r})$. Under the assumption that the lowest state is extended through the sample, $\overline{|\phi_{V=0}(\mathbf{r})|^2}\approx1/N$; the Edwards-Anderson order parameter[18] $q_{EA}=|\langle\psi_n(\mathbf{r})\rangle|^2$ then equals

$$q_{EA}=\frac{2}{\langle\beta_A\rangle}(g_\alpha|_{t=t_{sc}(h)}-g_\alpha) \tag{8}$$

below $t_{sc}(H)$, and is zero above. Thus $q_{EA}\propto[t_{sc}(h)-t]^{2\beta}$, with the exponent $\beta=1/2$. The free energy below $t_{sc}(h)$ is

$$\frac{F}{NN_LT}=-\frac{g_\alpha^2}{\langle\beta_A\rangle}+\frac{1}{2}\int_{-1}^1 dk\int_0^\infty\rho_f(V)dV\ln[g_\eta e(k)+g_\lambda V]. \tag{9}$$

To calculate the exponents that determine the divergence of correlation lengths parallel and perpendicular to the field we first note that from Eq. (6) and the definition of the critical line it follows that

$$(g_\alpha+x)\left[1+\frac{\langle\beta_A\rangle}{4}\int_{-1}^1 dk\int_0^\infty\frac{\rho_f(V)dV}{[g_\eta e(k)+g_\lambda V][g_\eta e(k)+g_\lambda V+g_\alpha+x]}\right]=g_\alpha-g_\alpha|_{t=t_{sc}(h)}. \tag{10}$$

The integral in the last equation diverges for $f<\frac{5}{2}$ as $(g_\alpha+x)^{f-5/2}$ when the transition line is approached from above, and it is finite for $f>\frac{5}{2}$. Thus we obtain $(g_\alpha+x)\propto(t-t_{sc})^{1/(f-3/2)}$ for $f<\frac{5}{2}$ and $(g_\alpha+x)\propto(t-t_{sc})$ for $f\geq\frac{5}{2}$. The same behavior follows if the transition line is approached along the line of constant temperature. This determines the value of the exponent $\nu_\parallel=1/(2f-3)$ for $f<\frac{5}{2}$ and the classical value $\nu_\parallel=\frac{1}{2}$ for $f>\frac{5}{2}$, where the correlation length parallel to the field is $\xi_\parallel\propto[t-t_{sc}(h)]^{-\nu_\parallel}$. The concentration corresponding to $f=\frac{5}{2}$ determines the effective upper critical dimension in the problem. We now turn to the correlation length perpendicular to the field, $\xi_\perp\propto[t-t_{sc}(h)]^{-\nu_\perp}$, and study the susceptibility associated with the Edwards-Anderson order parameter,

$$\chi_{EA}(\mathbf{r}-\mathbf{r}')\equiv\overline{\langle\psi_n^*(\mathbf{r})\psi_n(\mathbf{r}')\rangle\langle\psi_n(\mathbf{r})\psi_n^*(\mathbf{r}')\rangle}. \tag{11}$$

After expanding the field operators in the eigenbasis of random potential we obtain

$$\chi_{EA}(\mathbf{r}-\mathbf{r}')=\int\frac{dV_1dV_2dk_1dk_2F(\mathbf{r}-\mathbf{r}',V_1,V_2)}{[g_\eta e(k_1)+g_\lambda V_1+g_\alpha+x][g_\eta e(k_2)+g_\lambda V_2+g_\alpha+x]}, \tag{12}$$

where the function F is the two-particle spectral density[19]

$$F(\mathbf{r}-\mathbf{r}',V_1,V_2)=\sum_{i,j}\delta(V_1-V_i)\delta(V_2-V_j)\phi_i^*(\mathbf{r})\phi_i(\mathbf{r}')\phi_j(\mathbf{r})\phi_j^*(\mathbf{r}') \tag{13}$$

and $\phi_i(\mathbf{r})$ are the eigenstates of the random potential. If we now introduce $V=(V_1+V_2)/2$ and $\omega=(V_1-V_2)/2$, for V close to the mobility edge and small (q,ω), the Fourier transform of F has a diffusive form,[19,20]

$$F(\mathbf{q},V,\omega)=\frac{\rho_f(V)q^2D(q^2/\omega)}{\pi[\omega^2+q^4D^2(q^2/\omega)]}, \tag{14}$$

where $D(q^2/\omega)$ is the generalized "diffusion constant." Assuming this form for $F(\mathbf{q},V,\omega)$ and rescaling everything by the appropriate power of temperature in Eq. (12), we obtain $\nu_\perp=\nu_\parallel$. Note that the density of states $\rho_f(V)$ is roughly constant except in a narrow region, typically 1% of total bandwidth, around $V=0$, where it either diverges or vanishes. Hence, unless one experimentally probes the system very close to the transition, the observed correlation length exponent would be the one corresponding to $f=2$, i.e., $\nu_\perp=\nu_\parallel=1$. This agrees well with the experimental results of Ref. 5.

Magnetization per unit volume equals

$$\frac{M}{AN_Ld}=-\frac{2T\sqrt{th}}{d\phi_0\sqrt{\theta}}\left(q_{EA}+\frac{2x}{\langle\beta_A\rangle}\right). \tag{15}$$

Below the transition line this coincides with the usual mean-field result. Above the transition line $q_{EA}=0$ and from Eq. (10) it follows that at constant temperature close to the transition $(g_\alpha+x)\propto[h-h_{sc}(t)]^{1/(f-3/2)}$ when $f<\frac{5}{2}$ and $(g_\alpha+x)\propto[h-h_{sc}(t)]$ otherwise. Thus the magnetic susceptibility is a smooth function of the field at the transition for $f<2$, but has an upward cusp for $2<f<\frac{5}{2}$ and a discontinuity for $f>\frac{5}{2}$. The size of this discontinuity depends on the location of the transition in the H-T diagram. Differentiating the free energy twice with respect to temperature one obtains the specific heat. It is straightforward to show that at the transition it behaves the same way as susceptibility; smooth for $f<2$, a cusp for $2<f<\frac{5}{2}$, and the usual discontinuity for $f>\frac{5}{2}$. More precisely, both magnetic susceptibility and specific heat behave as $[t-t_{sc}(h)]^{-\alpha}$ for $\frac{3}{2}<f<\frac{5}{2}$, where $\alpha=(f-\frac{5}{2})/(f-\frac{3}{2})$. The behavior of the specific heat, order parameter, and correlation length in our model is related to the one obtained from the $O(2N)$ vector model in the limit $N\to\infty$ and in the *effective* dimension $D_{eff}=2f-1$.[21] The magnetic susceptibility, however, behaves differently at the transition; while it diverges in the $O(2N)$ vector model with the exponent $\gamma=[(D_{eff}/2)-1]^{-1}$, it is finite in our case even below the effective upper critical dimension. This is a consequence of a diamagnetic nature of magnetization in our problem.

We should stress again that the critical behavior of our model does not describe "true" critical properties of the GL-LLL theory with disorder, since we have ignored lateral SCDW correlations. Such correlations will always become important sufficiently close to the transition. However, as it is clear from Fig. 1, there is a wide region in the H-T phase diagram where the superconducting transition lies far above the SCDW transition line for clean systems. In this region, the "true" critical behavior will set in only very near the $T_{sc}(H)$ line and our model should be appropriate in most experimental situations.

In summary, we have studied the high-field superconducting glassy transition induced by columnar disorder. Using the exact density of states for a random array of short-range scatterers in the LLL level and the assumption that the lowest eigenstate of such a potential is extended over a finite-size sample under certain conditions, we have obtained the Edwards-Anderson order parameter, correlation length, magnetization, and specific heat close to the transition. The transition line in the H-T phase diagram has also been calculated. The critical exponents are found to depend on magnetic field.

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[15] Of course, in the limit of very strong disorder the whole LL structure may eventually collapse. Thus, while we obtain sensible results in the limit of very strong disorder, our model is best suited for weak to moderate disorder: Just when the disorder becomes too strong is the question which should be settled empirically, by comparing our results to experiments and other theoretical models.

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