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Magnetic dynamic hysteresis of a resistively shunted Josephson-junction array

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The magnetization curves of a slablike uniform Josephson-junction array with resistive shunts are calculated from the dc and ac Josephson equations and the gauge-invariant phase difference. The parameters are chosen in accord with actual values for high-T_c superconductors (HTSC's). With decreasing sweep rate of the applied field, the result changes from a complete shielding to one approaching the envelope of the static magnetization curve, which is oscillatory and amplitude modulated. At certain intermediate sweeping rates the curves are critical-state-like. However, since these rates are a few orders of magnitude higher than those used in conventional experiments, the intergranular critical state shown in HTSC's cannot be modeled by such a uniform array.

Soon after the discovery of high-T_c superconductors (HTSC's), it was realized that most HTSC's are granular in nature. Their magnetization has intra- and intergranular contributions.[1] The transport or induced sample-circulating currents flow through the intergranular Josephson-junction network and are therefore limited by the junction strengths. Moreover, from irreversible magnetization measurements, it was found that these currents obey the critical-state (CS) model, similar to the situation in conventional type-II superconductors (SC2's).[2–5] To explain this, Josephson-junction array (JJA) models were proposed by a number of authors. Some treated the problem mainly by emphasizing the similarity between JJA's and SC2's and concluded that the intergranular CS arises from vortex pinning.[3,6,7] A significant development was made by Majhofer *et al.*,[8] who calculated directly the hysteresis loops of resistively shunted JJA's based on dc and ac Josephson equations. From the results shown, they remarked that a broad spectrum of irreversible effects observed in HTSC's can be reproduced by their model. In spite of this, a systematic analysis of the dependence of the results on the array parameters and the conditions for the appearance of a critical state in the array is still lacking. On the other hand, we have calculated and systematically studied the continuous static magnetization curves of uniform Josephson junctions and slablike JJA's based on the dc Josephson equation and gauge invariance.[9–11] In this work, we extend this systematic work to include the ac effect, giving special emphasis on the dynamic magnetic behavior of a JJA with typical HTSC parameters.

Our central aim is then to study how a CS can arise in a resistively shunted JJA. For this, it is convenient to assume an infinitely long cylinder or slab as the overall shape of the JJA, like in Bean's treatment for the CS model.[12,13] As proved earlier,[14] the CS magnetization of a cylinder is the same as that of a square column. This latter shape was actually chosen for the JJA in Ref. 8, where two-dimensional calculations were performed. We choose in the present work a slablike JJA, so that the problem becomes one dimensional. The advantages of this choice are (1) the results can be compared with the static ones calculated in Ref. 11, which are correlated with those of uniform junctions; (2) it is easy to make systematic analyses; (3) the system is simpler so that the results can be more accurate, without loosing generality.

The slablike JJA consists of identical, infinitely long, superconducting grains parallel to the z axis, whose cross-sectional centers form a square lattice on the xy plane. The JJA is infinitely wide along the y direction, and contains N rows (layers) along the x axis; thus the thickness of the JJA is $a = Na_0$, where a_0 is the lattice constant. Magnetic field H is applied in the z direction, so that demagnetizing field is zero. We further assume that the junctions between all the adjacent grains are short, with effective area A_J, critical current per unit length at zero field I_0, and normal resistance R per unit length. For each cell, the effective void area is A_V. As explained in Refs. 10 and 11, magnetization calculations are one dimensional in such a JJA; only one column of junctions on the $y = 0$ plane and the grains on its both sides have to be treated. Similar to Ref. 8 from the dc and ac Josephson equations, we build up the following N simultaneous differential equations for the gauge-invariant phase differences θ_i, $i = 1, 2, \ldots, N$, corresponding to the sequence of the junctions:

$$\frac{d\theta_i}{dt^*} = -\frac{2\pi h}{N-1} + \theta_{i+1} - \theta_i - \beta \sin\theta_i \quad (i = 1),$$

$$\frac{d\theta_i}{dt^*} = \theta_{i-1} - 2\theta_i + \theta_{i+1} - \beta \sin\theta_i \quad (2 \le i \le N-1),$$

$$\frac{d\theta_i}{dt^*} = \frac{2\pi h}{N-1} - \theta_i + \theta_{i-1} - \beta \sin\theta_i \quad (i = N). \qquad (1)$$

In these equations, t^* is the normalized time t to the nominal time constant τ of one cell, h is the normalized H as in Ref. 11, and β has the same meaning as in Ref. 8:

$$t^* = t/\tau = tR/L, \tag{2}$$

$$L = \mu_0 A_V, \tag{3}$$

$$h = (N-1)\mu_0 A_V H/\Phi_0, \tag{4}$$

$$\beta = 2\pi L I_0/\Phi_0, \tag{5}$$

where L is the self-inductance of a void per meter length and Φ_0 the flux quantum.

In order to simulate the situation of some HTSC samples, in the computation we choose $A_J \ll A_V = 10^{-11}$ m^2 and $N = 41$. If the effective grain volume fraction f_g is typically 0.7,[15] the former condition corresponds to $a_0 \approx 6 \times 10^{-6}$ m, and the JJA thickness is thus about 0.25 mm. For comparison of our results with those given in Refs. 11 and 8, we need to define a parameter

$$\alpha = (N-1)a_0/\lambda_J = (N-1)\sqrt{\beta}, \tag{6}$$

where λ_J is the penetration depth:

$$\lambda_J = a_0(\Phi_0/2\pi\mu_0 I_0 A_V)^{1/2}. \tag{7}$$

α is an effective thickness of the JJA normalized to λ_J, and it characterizes the general features of the static solution. When $\alpha > 4$, the JJA is magnetically irreversible.[11] The curves given in Ref. 8 are for $\beta = 0.1$ and 2, and $N = 30$. This corresponds to $\alpha = 9$ and 41, calculated from Eq. (6). Therefore, we choose a case of $\alpha = 20$ ($\beta = 0.25$), which lies between the two values and whose static curve has been calculated in Ref. 11. The intergranular critical current density, J_c, is assumed equal to I_0/a_0:

$$J_c = \frac{I_0(1-f_g)^{1/2}}{A_V^{1/2}} = \frac{\Phi_0 \beta I_0 (1-f_g)^{1/2}}{2\pi\mu_0 A_V^{3/2}}. \tag{8}$$

For deriving Eq. (8), Eqs. (3) and (5) have been used. Substituting the above values for relevant parameters in Eqs. (8) and (7), we obtain $J_c \approx 10^2$ A/cm^2 and $\lambda_J \approx 12$ μm. Both values are reasonable for the intergranular matrix of usual HTSC's.

The final results for the intergranular magnetization M are normalized like the field:

$$m = (N-1)\mu_0 A_V M/\Phi_0 = \theta_N/\pi - h. \tag{9}$$

Equations (1) are numerically solved by using the Runge-Kutta method. From the calculated phase differences, $m(h)$ curves are obtained. All the curves are computed starting from the initial state, $h = 0$ and $\theta_i = 0$, with changing h in steps of 0.2 or -0.2. Each curve has a given waiting time at every h step. The $m(h)$ curves shown in Fig. 1 are for the average sweeping rate $(dh/dt^*)_{\rm av} = 0.002$, 0.02, 0.05, 0.1, 0.2, 0.5, 2, and 20, together with the static one calculated in Ref. 11. We can see that at the highest $(dh/dt^*)_{\rm av}$, the slope dm/dh is very close to -1, a complete shielding. With decreasing $(dh/dt^*)_{\rm av}$, m increases gradually and becomes oscillatory. At the lowest $(dh/dt^*)_{\rm av}$, the $m(h)$ curve finally becomes almost identical with the lower envelope of the static curve. It can be realized that between $(dh/dt^*)_{\rm av} = 0.05$ and 0.1 there is a value which gives a curve similar to the initial CS curve with a flat high h portion for constant critical-current density (Bean's model).[12–14] Therefore, we have further calculated an $m(h)$ loop for $(dh/dt^*)_{\rm av} = \pm 0.07$. The results are given in Fig. 2.

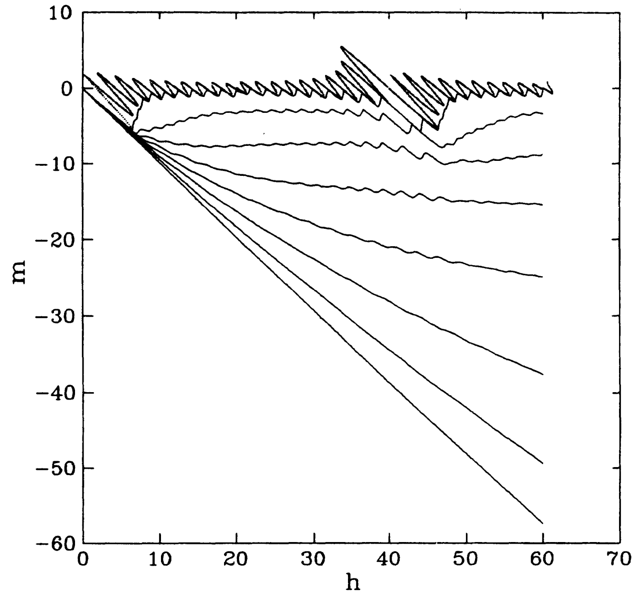

FIG. 1. Initial magnetization curves $m(h)$ for the studied Josephson-junction array. The h steps are 0.2. From the bottom curve upwards, the average field-sweeping rate $(dh/dt^*)_{\rm av} = 20$, 2, 0.5, 0.2, 0.1, 0.05, 0.02, and 0.002. The upper oscillatory curve is static.

We now explain the above results in the light of the dc and ac Josephson effects. According to the dc effect, the tunneling current $I_i^{\rm dc}$ through the ith junction is proportional to $\sin\theta_i$,[16,17]

$$I_i^{\rm dc} = I_0 \sin\theta_i, \tag{10}$$

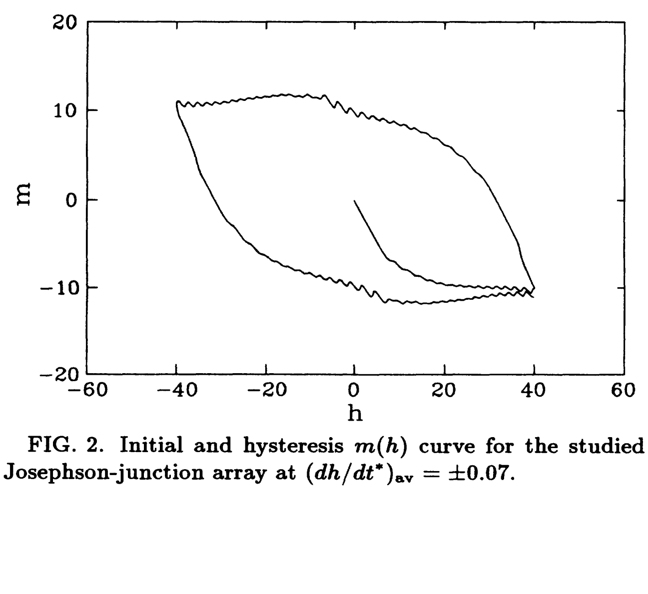

FIG. 2. Initial and hysteresis $m(h)$ curve for the studied Josephson-junction array at $(dh/dt^*)_{\rm av} = \pm 0.07$.

where the gauge invariant phase difference θ_i is proportional to the flux between the ith and the central junctions. Since the flux is cumulative, θ_i increases from the center to the surface, so that I_i^{dc} will oscillate along the x axis and the local field will have an oscillatory component. Josephson vortices can be defined by each Φ_0 increment of flux along the x axis. Such spatial oscillations cause the oscillatory nature of the static $m(h)$ curve. Also, this curve should be periodically modulated. This is because when $h = (N-1)(j+1/2)$ and $(N-1)j$, where $j = 0, 1, \ldots$, there is half-integral and integral numbers of Φ_0 in each cell, respectively, which makes $\sin\theta_i = 0$ and $I_i^{\mathrm{dc}} = 0$ for all the layers. Thus, we will have a periodic $m(h)$ curve with period of $\Delta h = N-1$, if A_J is negligible, as seen from the static curve in Fig. 1.

For the ac effect, the current I_i^{ac} is proportional to $d\theta_i/dt$,[16,17]

$$I_i^{\mathrm{ac}} = \frac{\Phi_0}{2\pi R}\frac{d\theta_i}{dt}, \tag{11}$$

i.e., proportional to the changing rate of the corresponding flux mentioned above. Thus, it is in principle the same as eddy currents in normal conductors. When changing h, these currents will tend to shield the magnetic field and have a roughly exponential decay from the surface. This ac effect is sensitive to dh/dt. Due to the interference of the dc effect, the effective time constant for the entire JJA is much larger than τ. The lowest curve in Fig. 1 corresponds to a case where the ac effect dominates.

The CS-like behavior observed at intermediate values of dh/dt is a combination of the dc and ac effects. The current I_i flowing through each junction is an addition of I_i^{dc} and I_i^{ac}. The latter decays roughly exponentially from the surface, whereas the former oscillates after first vortex entry. The internal field (h_i) profiles at several states on the $m(h)$ curve shown in Fig. 2 are given in Figs. 3(a) and 3(b). We see from Fig. 3(a) that when $h = 5$, below the first static h maximum, h_i decays from both surfaces almost exponentially. This is a consequence of two nearly exponential decays of dc and ac Josephson currents. With increasing h, a few vortices enter the JJA, and h_i shows several oscillations (dc effect) superimposed upon an exponential background (ac effect). Decreasing h from its maximum changes the sign of I_i^{ac}, and the h_i profiles become inverted [Fig. 3(b)]. Since the ac effect is similar to the eddy current effect and the latter is again similar to the case of the CS (both obey Lenz's law), the resulted $m(h)$ curve is similar to the CS curve.

However, there are some essential differences between this CS-like case and the real CS. (1) The field profile is always linear in a field-independent CS (Bean's model), but it is nearly exponential in the CS-like case, where no field dependence of the critical current for the junctions is considered. (2) In Bean's model, there is a full penetration field H_p,[14] defined as the field at which the supercurrents penetrate to the center along the initial magnetization curve. When $H > H_p$, the initial curve collapses with the hysteresis loop. If defining such an h_p between 10 and 20 as one should expect from the profiles of Fig. 3(a), there is not such a collapse for the CS-like case, as seen from Fig. 2. (3) Ideal CS is a static state, so that its magnetization curve is not field-sweeping rate dependent. In actual SC2's, the CS is a consequence of vortex pinning and creep, so that it is thermodynamically quasistatic.[18] The CS-like case itself is highly dynamic, so that its magnetic behavior can vary rather much, depending on the rate.

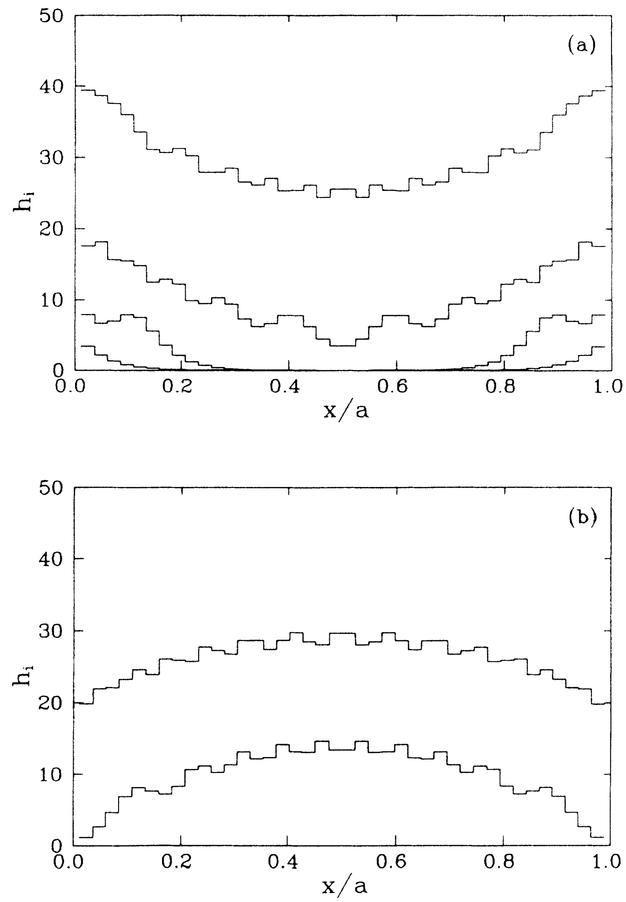

FIG. 3. The internal field profiles for the magnetic states on the curve in Fig. 2. (a) The curves from down to up correspond to $h = 5$, 10, 20, and 40 on the initial $m(h)$ curve. (b) The curves from up to down are for $h = 20$ and 0 on the reverse $m(h)$ curve.

We now consider realistic values of L and R, from which we will see that the CS-like feature cannot be used for explaining the intergranular properties of HTSC's, even if all the differences from the CS are disregarded. Since $A_V = 10^{-11}$ m^2 as explained earlier, we have $L \approx 10^{-17}$ H calculated using Eq. (3). It is well known that HTSC's are not good conductors in the normal state. A typical value of normal resistivity ρ of sintered HTSC's is 10^{-5} Ω m. R, defined as the resistance across a junction per meter length in the z direction, should be somewhat less than 10^{-5} Ω, whose value depends on the relative volume of the junctions to the grains, or, on f_g. If this makes R 10 times smaller, then $\tau = L/R$ should equal 10^{-11} s. (τ is estimated as 10^{-12} s in Ref. 19.) Using these data and Eqs. (2) and (4), we find that the lowest sweeping rate in Fig. 1 should correspond to

$(dH/dt)_{\rm av} = 8 \times 10^5$ kA/(m s) for the calculated JJA. In ac magnetization, if the maximum field is assumed to be 1 kA/m, this sweeping rate gives a frequency $f = 0.2$ MHz. The intergranular CS has been detected at much lower frequencies (1–1000 Hz in ac susceptibility measurements, even lower in dc measurements), at which $I_i^{\rm ac}$ will be practically zero. This means that if the intergranular magnetization is explained by a uniform resistively shunted JJA model, the result will be the same as the static solution (like the upper curve in Fig. 2) and not CS-like. Therefore, the CS-like behavior should only occur at frequencies of a few MHz if similar maximum fields are available.

It is worth mentioning that our calculations under other conditions show that the values of α and β do have influence on the feature of magnetic irreversibility, as mentioned in Ref. 8. When β is above a critical value around 2 so that its corresponding α is beyond the largest value chosen in Ref. 11, a kind of static CS behavior occurs for JJA. From Eq. (8), this value leads to $J_c \approx 10^3$ A/cm^2, higher than the actual values of most HTSC's at 77 K.

In conclusion, we have calculated the magnetization curves of a slablike uniform Josephson-junction array with resistive shunts from the dc and ac Josephson equations and gauge-invariant phase differences. The choice of parameters is accord with the actual situation of many HTSC samples. Since the slab thickness is 20 times the penetration depth, the magnetic irreversibility arises from both the dc and ac effect. At high field-sweeping rates, the ac effect makes the JJA be almost completely shielded. A low rate leads to an oscillatory and modulated magnetization curve, which is dominated by the dc effect. Certain intermediate rates can give rise to curves similar to those calculated from the critical-state model. However, both have essential differences. Owing to a very small time constant in HTSC's, conventional experimental conditions of measurements always correspond to the low-rate limit so that the intergranular CS in HTSC's cannot be modeled by such a resistively shunted Josephson-junction array.

However, since such a JJA does not result in a static CS, the use of Eq. (8), where a maximum junction current is assumed in the penetrated region, is questionable. Moreover, there are large distributions of grain sizes and junction strengths in HTSC's, whose electromagnetic properties involve thermally activative processes at finite temperatures. Considering all of these, more sophisticated models will finally explain the nature of the intergranular CS, which is beyond the scope of the present work.

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