Bosonic high-T_c superconductivity in two dimensions

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The mixed boson-fermion model of superconductivity of Friedberg and Lee is adapted to two dimensions. Owing to the finite correlation length $l(T)$, Bose-Einstein (BE) condensation can prevail only for a finite, but still macroscopic system. It is shown that for $T < T_c \sim 40$ K, BE condensation of charged bosons that are converted from fermion (electron or hole) pairs leads not only to a perfect Meissner effect but also an energy gap in the fermion excitation spectrum. For the temperature range $T_c < T < T_{\mathrm{May}}$, where T_{May} depends, in part, on the thickness of the two-dimensional layer, although the system shows no vestige of BE condensation, a near-perfect Meissner effect would yet persist until $T \simeq T_{\mathrm{May}} \sim 150$ K, based on the adaptation of the theory of May to the present model.

I. INTRODUCTION

An important common feature of the current high-T_c superconductors is the very small "coherence length" $\xi(\sim 10$ Å).[1,2] This implies that the pair state whose existence has been confirmed experimentally can be looked upon as a local boson field $\phi(\mathbf{r})$. Based on this, several boson models have been proposed to explain the mechanism of high-T_c superconductivity.[3–5] The motivation for these bosonic models is the well-known fact that a charged Bose gas will exhibit the Meissner effect when it is in a state of Bose-Einstein (BE) condensation. Due to the small mass of the charge carriers, the critical temperature for the BE condensation will be very high which might then lead to high critical temperatures for superconductivity. However, many experiments show that for these high-T_c materials, the CuO_2 layers play an essential role for the superconducting behavior. The other layers of atoms between the CuO_2 layers serve as reservoirs to provide the charge carriers, either electrons or holes. On the other hand, BE condensation does not exist in two dimensions (2D) and, as a consequence, no superconducting transitions can be expected for 2D boson systems. One might then invoke the weak layer-layer coupling and consider instead a very anisotropic Bose gas, retaining thereby the BE condensation. However, evidence pointing to the importance of interlayer coupling is not conclusive.[6]

In the following we will propose a bosonic theory for high-T_c superconductivity without interlayer coupling. This is adapted, on the one hand, from the Friedberg-Lee (FL) boson-fermion model of superconductivity to 2D (Ref. 3) and, on the other hand, from May's work[7] on superconductivity for two-dimensional free charged bosons. According to May, although there is no sharp BE condensation in 2D, there remains a nearly perfect Meissner effect at temperatures below T_{May}, which is indistinguishable practically from the perfect London-type Meissner effect. When May's work is extended to the FL boson-fermion model in two dimensions, we shall find the following results. As the temperature T decreases from above, the two-dimensional FL model would first exhibit a near-perfect Meissner effect at $T_{\mathrm{May}} \sim 150$ K. For a layer of finite area A, as the temperature T further decreases to $T_c \sim 40$ K, an energy gap $\Delta(T)$ in the fermion excitation spectrum begins to appear as a result of the mean-field-type interaction between the boson field representing pair states and the broken pairs of free fermions. The above scenario is somewhat different from that of the original FL model in the case of 2D which will be discussed in a later section.

It should be emphasized that no attempt is made to theoretically justify the smallness of the coherence length which renders a phenomenological boson model possible. Qualitative pictures for the formations of bosons can be found elsewhere.[8–10]

In the following, we shall briefly review May's theory of the magnetic susceptibility kernel for a two-dimensional free charged boson gas in Sec. II. The Friedberg-Lee (FL) mixed boson-fermion model originally designed for three dimensions will be adopted but extended to two dimensions in Sec. III. The bosons of a definite density in this model depending essentially on the excitation energy parameter 2ν of the boson field will be the key quantity in determining the relevant transition temperature to superconductivity with near-perfect Meissner effect. On the other hand, the requirement for the existence of an energy gap in the corresponding fermion excitation spectrum is more stringent. If this requirement for a system of macroscopic but finite area is satisfied, a perfect Meissner effect usually associated with Bose-Einstein (BE) condensation will occur. The results will be discussed and a brief conclusion will be given in the final section.

II. SUSCEPTIBILITY KERNEL $K(\mathbf{Q})$ FOR FREE CHARGED BOSON SYSTEMS

The susceptibility kernel $K(\mathbf{q})$ is defined by

$$M(\mathbf{q}) = K(\mathbf{q})B(\mathbf{q}), \qquad (1)$$

where $M(\mathbf{q})$ is the $\mathbf{q}$ Fourier component of magnetization, and $B(\mathbf{q})$ the $\mathbf{q}$ component of the magnetic induction. Generally, it can be shown that $K(\mathbf{q})$ is of the form

$$K(q) = -\frac{\Lambda(q)}{cq^2}. \tag{2}$$

In the London theory for 3D, $\Lambda(q)$ becomes independent of q after the system has gone into a BE condensate. The resulting $\frac{1}{q^2}$ singularity as $q \to 0$ gives rise to the London-type Meissner effect. In fact, $K(q)$ can then be shown to be given by[7]

$$K(q) = -\frac{e^2}{mc^2}\frac{1}{q^2}\left(\frac{N_0(T)}{V}\right), \tag{3}$$

where $N_0(T)$ is the temperature-dependent population of the $\mathbf{p} = 0$ state.

In two dimensions, on the other hand, there is no sharp BE condensation in the thermodynamic limit. It can be shown[7] that the susceptibility kernel is given by

$$K^{2\mathrm{D}}(q) = -\frac{e^2}{mc^2}\frac{1}{q^2}\left(\frac{N_q(T)}{\delta_z A}\right), \tag{4}$$

where δ_z is the thickness of the quasi-two-dimensional layer and A is its area. The quantity N_q given by

$$N_q = \frac{A}{4\pi^2}\pi\left(\frac{q}{2}\right)^2\frac{2}{3}[N_0(T)] \tag{5}$$

plays the role of $N_0(T)$ of Eq. (3). This is as if the group of states with momentum $p < \frac{q}{2}$, each with a weighted average population of $\frac{2}{3}N_0(T)$, acted together as a whole in contributing to $K^{2\mathrm{D}}(q \sim 0)$, somewhat analogous to BE condensation into a single $\mathbf{p} = 0$ state in 3D. However, unlike the 3D case, the q^2 factor in N_q cancels the $\frac{1}{q^2}$ singularity, eliminating thereby the perfect London-type Meissner effect. While $\frac{N_0(T)}{V} \longrightarrow 0$ as $T \to T_c$ for 3D, our present $\frac{N_q(T)}{A}$ does not become zero at a single sharp temperature. Nevertheless, it can be straightforwardly shown that the resulting susceptibility kernel is given by

$$K^{2\mathrm{D}}(q \sim 0) = -\frac{1}{2\pi}\left[\exp\left(\frac{T_0}{T} - \frac{T_0}{T_{\mathrm{May}}}\right)\right], \tag{6}$$

where T_0 and T_{May} are defined by

$$\sigma \equiv \frac{N}{A} \equiv \frac{2mT_0}{4\pi\hbar^2} \equiv \frac{1}{\lambda_0^2}, \tag{7a}$$

$$\exp\left(-\frac{T_0}{T_{\mathrm{May}}}\right) \equiv \frac{e^2}{12mc^2\delta_z} \ll 1. \tag{7b}$$

If $m = 2m_e$, $\delta_z \simeq 10^{-7}$ cm, $\frac{T_0}{T_{\mathrm{May}}} \simeq 12.8$. Athough there is no $\frac{1}{q^2}$ singularity in $K^{2\mathrm{D}}(q \sim 0)$, we see that

$$K^{2\mathrm{D}}(q \sim 0) = \begin{cases} \text{exponentially large}, & T < T_{\mathrm{May}}, \\ \text{exponentially small}, & T > T_{\mathrm{May}}. \end{cases} \tag{8}$$

As shown by May, Eq. (6) and Eq. (7) lead to a near-perfect Meissner effect below T_{May} which may be considered as the transition temperature to superconductivity. As seen in Eq. (5), $K^{2\mathrm{D}}(q \sim 0)$ is given in terms of the ground level population $N_0(T)$. In the mixed boson-fermion model of FL, as we shall see, this ground level population will be expressible in terms of the parameter of the model.

III. MIXED BOSON-FERMION MODEL

A. Friedberg-Lee model in three dimensions

In this model, Friedberg and Lee reasoned that since the pairing between electrons, or holes, is well localized in the coordinate space, the pair state could be well approximated by a phenomenological local boson field $\phi(\mathbf{r})$, whose mass M is $2m_e$ and whose elementary charge is $2e$, where m_e and e are the mass and charge of an electron. It follows then that the transition

$$2e \longrightarrow \phi(\mathbf{r}) \longrightarrow 2e \tag{9}$$

must occur via the "s-channel" reaction. This is what leads to a mixed boson-fermion model. Each ϕ quantum carrying $2e$ is assumed to be unstable, with 2ν as its excitation energy. In the rest frame of a single ϕ quantum, the decay $\phi \to 2e$ occurs, in which each e carries an energy $\frac{k^2}{2m} = \nu$. It follows then, in a large system, there are macroscopic numbers of both bosons (ϕ quantum) and fermions (electrons or holes), distributed according to the principles of statistical mechanics. These charged bosons would then Bose condense below a critical temperature, leading to superconductivity, while the fermions would acquire an energy gap in the excitation spectrum.

This model is partly motivated by the experimental observation[11] that in all high-T_c superconductors, there is a universal law: $T_c \propto \frac{\rho}{m^*}$. If one interprets ρ as the number density of bosons of charge $2e$ and $m^* = 2m_e$, the proportionality constant could become $40K$ to 10^{20} cm$^{-3}/m_e$, which can be easily shown to be equivalent to the condition of $\sigma\lambda_{T_c}^2 \simeq 8$, or

$$\frac{\lambda_{T_c}}{d} \simeq 2\sqrt{2} \tag{10}$$

for all cupric superconductors, where $d \equiv \sigma^{-\frac{1}{2}}$, σ being the two-dimensional boson number density. Such a universal law is indeed suggestive of some sort of Bose-Einstein condensation in which the interparticle distance d and the thermal wavelength λ_{T_c} should be of comparable magnitude.

Following FL, we write the Hamiltonian as ($\hbar = 1$)

$$H = H_0 + H_1, \tag{11}$$

where

$$\begin{aligned} H_0 &= \int \hat{\phi}^\dagger(\mathbf{r})\left(2\nu - \frac{\nabla^2}{2M}\right)\hat{\phi}(\mathbf{r})d^3r \\ &\quad + \sum_\sigma \int \hat{\psi}_\sigma^\dagger(\mathbf{r})\left(-\frac{\nabla^2}{2m}\right)\hat{\psi}_\sigma(\mathbf{r})d^3r \\ &= \sum_{\mathbf{p}}\left(2\nu + \frac{p^2}{2M}\right)b_{\mathbf{p}}^\dagger b_{\mathbf{p}} + \sum_{\mathbf{k}\sigma}\frac{k^2}{2m}a_{\mathbf{k}\sigma}^\dagger a_{\mathbf{k}\sigma} \end{aligned} \tag{12}$$

and

$$H_1 = g\int[\hat{\phi}^\dagger(\mathbf{r})\hat{\psi}_\uparrow(\mathbf{r})\hat{\psi}_\downarrow(\mathbf{r}) + \text{H.c.}]d^3r$$
$$= g\frac{1}{\sqrt{\Omega}}\sum_{\mathbf{p}}\sum_{\mathbf{k}} b^\dagger_{\mathbf{p}} a_{\frac{\mathbf{p}}{2}+\mathbf{k},\uparrow} a_{\frac{\mathbf{p}}{2}-\mathbf{k},\downarrow} + \text{H.c.} \quad (13)$$

We see that H_1 couples the bosonic $\hat{\phi}(\mathbf{r})$ to a pair of fermions $\hat{\psi}_\uparrow(\mathbf{r})\hat{\psi}_\downarrow(\mathbf{r})$.

To proceed from Eqs. (11) and (12) we shall make a mean-field approximation. Anticipating some degree of Bose condensation that will become clear later we replace the boson field operators $\hat{\phi}(\mathbf{r})$ and $\hat{\phi}^\dagger(\mathbf{r})$ in Eq. (12) by their thermal averages, which are expected to be independent of $\mathbf{r}$ in a homogeneous system, i.e.,

$$H_1 = g\langle\hat{\phi}\rangle\int \hat{\psi}_\uparrow(\mathbf{r})\hat{\psi}_\downarrow(\mathbf{r})d^3r + \text{H.c.} \quad (14)$$

The well-known procedure of the Bogoliubov-Valatin transformation from the bare fermion operators to the quasiparticle operators can then be followed[3] to yield the quasiparticle excitation spectrum

$$E_k = \sqrt{(\varepsilon_k - \nu)^2 + |\bigtriangleup_k|^2}, \quad (15)$$

where the energy gap $|\bigtriangleup_k|$ is given by

$$|\bigtriangleup_k|^2 = g^2|\langle\hat{\phi}\rangle|^2. \quad (16)$$

The transformed Hamiltonian would then yield the spectrum that, aside from a constant ε_0, represents the kinetic energy of a free boson (mass $2m_e$) gas and the collection of fermionic quasiparticles of energies E_k.

For a system in three dimensions, $|\langle\hat{\phi}\rangle|^2 = \frac{N_0}{\Omega}$, which is the number of bosons condensed into the $\mathbf{p}=0$ state per unit volume for temperatures below the critical temperature. Consider the case $\rho > \rho_\nu$, $\rho_\nu = (3\pi^2)^{-1}(2m\nu)^{\frac{3}{2}}$. At $T=0$, the system is in its lowest-energy state, which (for g sufficiently small) consists of a degenerate Fermi distribution of density ρ_ν and Fermi energy $\epsilon_F = \nu$, while the remaining particles are all condensed into the $\mathbf{p}=0$ state with $N_0 = \frac{1}{2}(N - \Omega\rho_\nu)$, where N is the total number of particles, $N = 2N_b + N_f$, $N_f = \rho_\nu\Omega$ being the number of fermions and N_b the number of bosons. As T increases, N_0 decreases and becomes zero at the critical temperature while $N_b = \frac{1}{2}(N - N_f)$ remains practically unchanged except for small Sommerfeld correction $\sim(\frac{kT}{\nu})^2$ to N_f.

B. Mixed boson-fermion model in two dimensions

In two dimensions, BE condensation does not exist in the thermodynamic limit. This immediately throws some doubt on the validity of the mean-field approximation itself, as expressed in Eq. (14). This can best be seen by examining the correlation function

$$C(\mathbf{x}_1 - \mathbf{x}_2) \equiv \langle\hat{\phi}^\dagger(\mathbf{x}_1)\hat{\phi}(\mathbf{x}_2)\rangle \quad (17)$$

and the associated coherence length l. In terms of $\hat{a}_{\mathbf{k}}$ and $\hat{a}^\dagger_{\mathbf{k}}$, we obtain, for a homogeneous system,

$$C(\mathbf{x}_1 - \mathbf{x}_2) = \frac{1}{A}\left[\langle\hat{a}^\dagger_0\hat{a}_0\rangle + \sum_{\mathbf{k}\neq 0}\langle\hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{k}}\rangle\exp(-i\mathbf{k}\cdot\mathbf{x})\right], \quad (18)$$

where $\mathbf{x} \equiv \mathbf{x}_1 - \mathbf{x}_2$, and A is the two-dimensional area of the system. Since $\frac{N_0(T)}{A}$ is microscopically small as $A \longrightarrow \infty$, we have

$$C(\mathbf{x}) = \frac{1}{(2\pi)^2}\int_0^\infty kdk\int_0^{2\pi}d\theta\,\frac{\exp(-ikx\cos\theta)}{z\exp(\beta\varepsilon_k) - 1}, \quad (19)$$

where $z \equiv \exp(-\beta\mu) = \exp(\eta) > 1, \eta \equiv -\beta\mu > 0$. Explicit evaluation of the integral yields

$$C(\mathbf{x}) = \frac{\sqrt{2\pi}}{\lambda_T^2}\frac{1}{\sqrt{x/l}}\exp\left(-\frac{x}{l}\right), \quad (20)$$

where

$$l \equiv \frac{1}{\sqrt{4\pi}}\lambda_T\exp\left(\frac{\sigma\lambda_T^2}{2}\right) \quad (21)$$

is the temperature-dependent correlation length, expressed here in terms of the density $\sigma \equiv \frac{N}{A}$ and the thermal wavelength λ_T.

As we see from Eqs. (18)–(21), the finite range of coherence l is not due to the BE condensation into the $\mathbf{k}=0$ state, but rather to the phase coherence of the particles in states of various $\mathbf{k}$'s within an interval $\Delta k \le \frac{1}{l}$. This is why $\langle\hat{\phi}^\dagger(\mathbf{x}_1)\hat{\phi}(\mathbf{x}_2)\rangle$ = finite as long as $|\mathbf{x}_1 - \mathbf{x}_2| \le l$ as a consequence of phase coherence maintained over a range of order l.

If the mean-field approximation were valid, it would mean the factorization, as $|\mathbf{x}_1 - \mathbf{x}_2| \to \infty$,

$$C(\mathbf{x}) \approx \langle\hat{\phi}^\dagger(\mathbf{x}_1)\rangle\langle\hat{\phi}(\mathbf{x}_2)\rangle = |\langle\hat{\phi}\rangle|^2 e^{i[\theta(\mathbf{x}_1)-\theta(\mathbf{x}_2)]}, \quad (22)$$

where $|\langle\phi\rangle|^2$ is then expected to be independent of $\mathbf{x}_1$ or $\mathbf{x}_2$ for a homogeneous system. However, the obvious contradiction between Eq. (22) and Eq. (20) renders Eq. (22) invalid for a system with an area $A \gg l^2$. Only when $A \ll l^2$ would phase coherence be maintainable over the entire area in the sense $\theta(\mathbf{x}_1) \simeq \theta(\mathbf{x}_2) = \theta$ so that Eq. (20) becomes consistent with Eq. (22); the order parameter $\langle\hat{\phi}\rangle$ would then also acquire a finite magnitude commensurate with A, i.e., $|\langle\hat{\phi}\rangle|^2 \sim \frac{N_0}{A}$ = finite.

With the above in mind we now reexamine Eqs. (14)–(16). It might be argued that, although $\langle\hat{\phi}(\mathbf{x})\rangle = |\langle\hat{\phi}\rangle|e^{i\theta(\mathbf{x})}$ cannot maintain phase coherence over distances $\Delta x > l$, we could first divide an infinite two-dimensional system ($A \to \infty$) into regions of size $\ll l$. Within each region, at sufficiently low temperature the parameter $\langle\hat{\phi}\rangle$ exists in the sense $|\langle\hat{\phi}\rangle| \neq 0$. The phase $\theta(\mathbf{x})$ of $\langle\hat{\phi}(\mathbf{x})\rangle$ wanders from region to region, but its magnitude $|\langle\hat{\phi}\rangle|$ remains the same. Since the gap energy Δ, according to Eq. (16), depends only on the constancy of $|\langle\hat{\phi}(\mathbf{x})\rangle|^2$, we have the same Δ for the entire infinite two-dimensional system.

An important point to consider in the above context, however, is how the constant value $|\langle\hat{\phi}(\mathbf{x})\rangle|^2$ that remains the same from region to region changes as the number of such regions, each of size $< l$, increases. The above scenario would be meaningful only if $|\langle\hat{\phi}(\mathbf{x})\rangle|^2$ remains undiminished as $A \to \infty$. Since $|\langle\hat{\phi}\rangle| = \sqrt{\frac{N_0}{A}}$, we consider the calculation of $N_0 = \langle\hat{A}^\dagger_0\hat{A}_0\rangle$ from the above point of view, where $\hat{A}_0$ denotes the annihilation operator for particles in the $\mathbf{k} = 0$ state in a system of area A.

Let us divide then A into m regions, each of area $s = \frac{A}{m} = s_1 = s_2 = \cdots = s_m \ll l^2$. The boson field operator $\hat{\phi}(\mathbf{x}) = \frac{1}{\sqrt{A}} \sum_{\mathbf{k}} \hat{A}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}$ or $\hat{A}_{\mathbf{k}} = \frac{1}{\sqrt{A}} \int \hat{\phi}(\mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}} d^2x$. Hence,

$$\hat{A}_0 = \frac{1}{\sqrt{A}} \int \hat{\phi}(\mathbf{x}) d^2x = \frac{1}{\sqrt{m}} \frac{1}{\sqrt{s}} \left\{ \int_{s_1} \hat{\phi}(\mathbf{x}) d^2x + \int_{s_2} \hat{\phi}(\mathbf{x}) d^2x + \cdots \int_{s_m} \hat{\phi}(\mathbf{x}) d^2x \right\}. \tag{23}$$

Let $\hat{a}_{i,0}$ denote the boson amplitude in the ith region. Obviously $\hat{a}_{i,0} = \frac{1}{\sqrt{s_i}} \int_{s_i} \hat{\phi}(\mathbf{x}) d^2x$, just like $\hat{A}_0 = \frac{1}{\sqrt{A}} \int \hat{\phi}(\mathbf{x}) d^2x$. It then follows that $\hat{A}_0 = \frac{1}{\sqrt{m}} \sum_{i=1}^{m} \hat{a}_{i,0}$ and

$$N_0 = \langle \hat{A}_0^\dagger \hat{A}_0 \rangle = \frac{1}{m} \sum_{i=1}^{m} \langle \hat{n}_{i,0} \rangle + \frac{1}{m} \sum_{i\neq}^{m} \sum_{j}^{m} \langle \hat{a}_{i,0}^\dagger \hat{a}_{j,0} \rangle. \tag{24}$$

In Eq. (24), the first term on the right hand side[12] is $\frac{s}{\lambda_T^2}$; the second term involving the double sum would give a much greater contribution to N_0 for $m \gg 1$. Recognizing that

$$\langle \hat{a}_{i,0}^\dagger \hat{a}_{j,0} \rangle \approx \begin{cases} \langle \hat{a}_{i,0}^\dagger \hat{a}_{i,0} \rangle & \text{if } |\mathbf{x}_i - \mathbf{x}_j| < l, \\ 0 & \text{if } |\mathbf{x}_i - \mathbf{x}_j| > l, \end{cases} \tag{25}$$

we can immediately evaluate the double sum to give

$$\sum_{i\neq}^{m} \sum_{j=1}^{m} \langle \hat{a}_{i,0}^\dagger \hat{a}_{j,0} \rangle \cong \left(m \frac{l^2}{s} \right) \left(\frac{1}{2} \right) \langle \hat{a}_{i,0}^\dagger \hat{a}_{i,0} \rangle. \tag{26}$$

In the above, the factor m represents the number of ways to choose the region i. Since l is finite, for a given i there are $\frac{l^2}{s}$ ways of choosing regions j of area $s_j \ll l^2$ within sample area $A \gg l^2$. The factor $\frac{1}{2}$ is to correct for double counting. Since $\langle \hat{n}_{i,0} \rangle = \langle \hat{a}_{i,0}^\dagger \hat{a}_{i,0} \rangle \cong \frac{s}{\lambda_T^2}$, we see that $\frac{1}{m} \sum_{i\neq}^{m} \sum_{j=1}^{m} \langle \hat{a}_{i,0}^\dagger \hat{a}_{j,0} \rangle \cong \frac{1}{m} (m \frac{l^2}{s}) (\frac{1}{2}) n_{i,0} \cong \frac{1}{2} \frac{l^2}{\lambda_T^2}$, which is seen to be independent of the area A. It follows from Eq. (24) that $N_0(T)$ is independent of the area A. In fact, with the use of Eq. (21), $N_0(T) \sim e^{\sigma \lambda_T^2}$. As a result, $|\langle \hat{\phi} \rangle|^2 = \frac{N_0}{A} \simeq \frac{e^{\sigma \lambda_T^2}}{A}$ which decreases as $\frac{1}{A}$ to zero as the area A increases to infinity.[13]

Consider now a 2D system with finite but macroscopic area $A = L^2 \leq l^2(T)$. We calculate $|\langle \hat{\phi}(\mathbf{r}) \rangle| = \sqrt{\frac{N_0}{A}}$. Since the two-dimensional N_0 is given by

$$N_0 = \frac{1}{e^\eta - 1}, \tag{27}$$

where $\eta = -\beta\mu$ is determined by the condition $\sigma = \frac{N}{A} = \frac{1}{\lambda_T^2} \int_0^\infty \frac{dx}{\exp(x+\eta) - 1}$, this yields $\exp(-\eta) = 1 - \exp(-\sigma\lambda^2)$ or

$$N_0 = e^{\sigma \lambda_T^2} - 1. \tag{28}$$

Correspondingly $|\langle \hat{\phi}(\mathbf{r}) \rangle| = \sqrt{\frac{N_0}{A}} = \sqrt{\frac{4\pi l^2}{\lambda_T^2 A}} \geq \sqrt{\frac{4\pi}{\lambda_T^2}}$ and the energy gap, according to Eq. (16), is given by

$$|\Delta_{\mathbf{k}}|^2 = g^2 \frac{4\pi l^2}{\lambda_T^2 A}, \tag{29}$$

which is now finite since $A \leq l^2(T)$.

The condition that $A \leq l^2(T)$ defines a critical temperature T_c for the existence of the energy gap in the fermionic excitation spectrum, i.e.,

$$A = l^2(T_c). \tag{30}$$

This T_c can be related to the previously introduced T_0 in Eq. (7a) as follows: $\frac{4\pi A}{\lambda_c^2} = \frac{4\pi l^2(T_c)}{\lambda_c^2} = e^{\sigma_b \lambda_c^2}$. Thus, $\sigma_b \lambda_c^2 = \ln \frac{4\pi A}{\lambda_c^2} = \ln \frac{4\pi A \sigma_b}{\lambda_c^2 \sigma_b} = \ln 4\pi N_b - \ln \lambda_c^2 \sigma_b \simeq \ln 4\pi N_b$, or $\sigma_b \lambda_c^2 = \sigma_b \lambda_0^2 (\frac{\lambda_c}{\lambda_0})^2 = (\frac{\lambda_c}{\lambda_0})^2 \simeq \ln 4\pi N_b$ so that $\frac{T_0}{T_c} = \ln 4\pi N_b$. For example, if $N_b = 10^{22}$, $\frac{T_0}{T_c} \simeq 53$. For $m = 2m_e$, we find $T_0 \sim 2000$ K, so that $T_c \simeq 40$ K. Thus in this weak coupling limit of the FL theory as adapted to two dimensions, there will be BE condensation $\langle \hat{\phi} \rangle$ =finite for $T < T_c$. Correspondingly, a perfect Meissner effect will persist from $T = 0$ until $T = T_c$. Note that the existence of T_c related to A through Eq. (30) is arrived at independently of the presence of fermions. It is a modification of May's result for 2D bosonic systems in general. As a consequence, one may conjure up a scenario that, at a given T, magnetic flux that was originally repelled from a system of finite area A could now penetrate the same system but with an enlarged area. This is in contrast to the 3D case, but it is not as surprising as it might seem. We recall that it costs free energy for the system to repel magnetic flux. While it may be energetically favorable overall for a system of smaller area A to maintain the coherent superconducting state and repel the flux, to repel the prevailing flux from a larger area may proportionally be too costly in energy. This is in view of the fact that the coherence of the (Bose-condensed) superconducting state can only be maintained up to a finite distance $l(T)$. The latter means that the corresponding energy lowering due to the maintenance of the superconducting state relative to the normal state is proportional to A only when $A < l^2(T)$. When $A > l^2(T)$, this energy lowering per unit area falls off as A increases, and may not be able to compensate for the energy cost in repelling the flux from a larger area.

In the temperature range $T_c < T < T_{\text{May}}$, since $\langle \hat{\phi} \rangle = 0$, the energy gap Δ also vanishes. The usual perfect Meissner effect associated with BE condensation of a charged boson gas now gives way to the nearly perfect Meissner effect described by the susceptibility kernel of Eq. (8). As discussed in Sec. II, a group of states with momentum $p < \frac{q}{2}$ rather than a single $p = 0$ state contributes collectively to $K^{2D}(q \sim 0)$ of Eq. (8) in which T_{May} is expressed in terms of T_0 in Eq. (7b). In turn, T_0 of Eq. (7a) is now determined by the two-dimensional boson density $\sigma_b = \frac{N_b}{A} = \frac{\frac{1}{2}(N - N_f)}{A} = \frac{\frac{1}{2}(N - \sigma_\nu A)}{A} = \frac{1}{2}(\sigma - \sigma_\nu)$, where $\sigma_\nu \equiv \frac{m\nu}{\pi\hbar^2}$ is the density corresponding to a Fermi energy $\epsilon_F = \nu$.

For $T > T_{\text{May}}$ the system becomes nonsuperconducting, exhibiting no Meissner effect at all.

IV. SUMMARY AND CONCLUSION

We have adapted the mixed boson-fermion model of Friedberg and Lee to two dimensions. For illustration purpose, only the weak coupling limit has been considered. Owing to the nonexistence of BE condensation in two dimensions as the area $A \to \infty$, we are led to consider a finite but macroscopically large area A, whose linear dimension L is not larger than the correlation length $l(T)$.

An analogous case is that of a two-dimensional crystal. Although the correlation length for long-ranged order is not infinite, as required of a theoretical crystal, it is large enough to be of macroscopic size. This means that, in practice, we should be able to make two-dimensional crystals of finite but macroscopic size.[14,15]

For the case of $\rho > \rho_\nu$ in the FL model, the condition that $A = l^2(T_c)$ defines the critical temperature T_c below which the usual BE condensation prevails for this two-dimensional Bose system of finite size. In turn, this leads to a finite energy gap $\Delta(T)$ for $T < T_c$ in the fermion excitation spectrum through the mean-field approximation in the FL model. The system then also exhibits a perfect Meissner effect as for the usual charged boson gas in three dimensions. Although this T_c depends on the number of bosons N_b which,[16] for $\rho > \rho_\nu$, depends very simply on the boson formation energy parameter 2ν of Eq. (12), T_c has been estimated to be $\sim$ 40 K for $N_b \sim 10^{22}$. On the other hand, for the temperature range $T_c < T < T_{\rm May}$, there is no longer any trace of BE condensation. However, a nearly perfect Meissner effect as first suggested by May[7] would persist in the absence of any energy gap in the fermion excitation spectrum. $T_{\rm May}$ has been estimated as $\sim$ 150 K. The statements above refer to a strictly two-dimensional layer. Any interlayer coupling tends to enhance the long-ranged order or increase $l(T)$ which, in turn, would enhance the superconductivity transition temperature.[6] For $T > T_{\rm May}$, there is no longer any remnant of long-ranged order and superconductivity disappears. It would be interesting to generalize the present theory to multiple layers and see explicitly the effect of interlayer coupling.

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[12] Since $\langle \hat{n}_{i,0} \rangle = \langle \hat{a}^\dagger_{i,0} \hat{a}_{i,0} \rangle = \frac{1}{s_i} \int_{s_i} \int_{s_i} \hat{\phi}^\dagger(\mathbf{x}_1) \hat{\phi}(\mathbf{x}_2) d^2x_1 d^2x_2 = \frac{1}{s_i} \int_{s_i} \int_{s_i} C(\mathbf{x}_1 - \mathbf{x}_2) d^2x_1 d^2x_2 \simeq \frac{1}{s} \frac{1}{\lambda_T^2} s_i s_i = \frac{s}{\lambda_T^2}$, it is striking to observe that $\sum_{i=1}^m \langle \hat{n}_{i,0} \rangle \cong m \frac{s}{\lambda_T^2} = \frac{A}{\lambda_T^2}$ and $\frac{\sum_{i=1}^m \langle \hat{n}_{i,0} \rangle}{N_0} = \frac{A}{\lambda_T^2 e^{\sigma \lambda_T^2}} = \frac{A}{l^2} \geq 1$. On the other hand, in three dimensions, the above ratio becomes $\frac{V}{l^3} = 1$ for $T < T_c$ because the coherence length extends over the whole volume for $T < T_c$.

[13] Although this result is consistent with and obtainable from the Bose distribution function, it is instructive to have derived it by the division of A into subregions and the use of the correlation $C(\mathbf{x}_1, \mathbf{x}_2)$ with a finite coherence length l over these subregions, arriving thus at the conclusion that $|\langle \hat{\phi}(\mathbf{x}) \rangle|^2$ is not an intensive quantity independent of A, as one might have expected intuitively.

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[16] The case of $\rho < \rho_\nu$ has been considered in Ref. 3. The theory is then reminiscent of the familiar BCS theory.