Speed of Magnetic-Flux Penetration and Eddy-Current Damping in Thin-Film Superconductors*

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The resistive-voltage enhancement in the intermediate state of superconducting lead films in the presence of an oscillatory-magnetic-field component has been investigated together with the penetration into the specimen of the magnetic field oscillation. The flux penetration was measured using small flat pickup coils sandwiched between two identical Pb films. The oscillatory field H_1 was parallel to the static field H_0, both being oriented perpendicular to the film plane. The ratio H_1/H_0 was 10^{-2}–10^{-3}. The experiments were carried out varying the specimen film thickness, the static field, and the frequency and amplitude of the oscillatory field. Both the resistive voltage and the voltage generated in the pickup coil were found to pass through a maximum at approximately the same frequency of H_1. The results indicate that the resistive-voltage enhancement is caused by the oscillatory eddy currents in the specimen associated with the magnetic field oscillation. These eddy currents assist in overcoming the pinning forces thus reducing the influence of flux pinning. The frequency dependence of the effect can readily be understood from the relaxation time τ for magnetic-flux penetration yielding a maximum for $2\pi\nu\tau = 1$. A simplified treatment of the magnetic-flux penetration for the case of a demagnetizing factor close to 1 is given using an energy-balance argument due to Faber. The theoretical values of τ agree reasonably with the experimental values, if the electrical conductivity in the expression for τ is obtained from the slope of the voltage-current curves and if the dependence of this slope upon magnetic field and current is taken into account.

I. INTRODUCTION

In a series of recent experiments[1-6] it has been shown that the influence of flux pinning on the flux-flow phenomena in superconducting films or foils can be reduced by the application of small oscillatory magnetic fields. In these experiments the static magnetic field was oriented perpendicular to the plane of the film, whereas the oscillatory field was either perpendicular to the static field and parallel to the axis of the film[1,2,5] or parallel to the static field and perpendicular to the film.[3-6] The oscillatory field was always found to reduce the influence of flux pinning and to cause an enhancement of the resistive voltage. This voltage enhancement has also been observed for the Nernst effect.[4] Studies of this resistive-voltage enhancement have shown[3,6] that the effect depends sensitively on the frequency of the oscillatory field and passes through a maximum at a certain frequency ν^*. The latter experiments[3,6] on the variation of the effect with the frequency of the oscillatory field have all been performed with the oscillatory field oriented *parallel* to the static field and perpendicular to the film. It has been suggested[6] that the flux-flow-voltage enhancement is caused by the oscillatory eddy currents in the sample and that the variation with frequency can be understood from the eddy-current damping of the external-magnetic-field variation.

In the present paper we report the results of an extension of our earlier experiments[3] on the flux-flow-voltage enhancement of superconducting lead films in small oscillatory magnetic fields. In order to verify the close connection between these phenomena and the manner in which an oscillatory magnetic field penetrates into a thin-film superconductor, we have extended the experiments by measuring in addition the penetration into the specimens of the external-magnetic-field variation with pickup coils. The experiments were carried out varying the thickness of the specimen films, the static magnetic field, and the frequency and amplitude of the oscillatory field. The frequency ν^*, at which the peak in the resistive-voltage enhancement occurs, was found to increase with increasing static field for fields below the critical field H_c and to decrease with increasing film thickness. We note that for the films of the present investigation the critical field is less than the bulk value. A detailed study of the variation of H_c with film thickness for lead has been reported by Cody and Miller.[7] As we had expected, the rate of change of magnetic flux within the film as detected by the pickup coils was also found to pass through a maximum at a frequency equal or very close to the value ν^* at which the peak in the resistive voltage of the specimens had been observed. Thus the experiments confirm that the resistive-voltage enhancement in the films in the presence of an oscillatory-magnetic-field component is caused by the oscillatory eddy currents in the sample and that the variation of this effect with the frequency of the oscillatory field is determined by the relaxation time for the penetration into the film of an external-magnetic-field variation.

A simplified treatment is given of the relaxation time τ for magnetic-flux penetration in the case of a thin-film geometry with a demagnetizing factor close tc 1, using an energy-balance argument due to Faber. Very reasonable agreement is obtained between the experimental and the calculated values of τ.

A detailed investigation of the complex magnetic susceptibility in thin-film type-I superconductors in perpendicular and parallel magnetic fields has been reported recently by Cody and Miller.[7,8] The speed of magnetic-flux penetration into type-I superconductors of nonzero demagnetizing coefficient has been studied recently by Rinderer *et al.*[9]

II. SIMPLIFIED TREATMENT OF MAGNETIC-FLUX PENETRATION FOR THE CASE OF A DEMAGNETIZING FACTOR CLOSE TO 1

For an understanding of the magnetic behavior of a thin-film superconductor in an oscillatory-magnetic-field component it is necessary to evaluate the speed with which the external-magnetic-field variation penetrates into the specimen. In the following we consider the velocity of magnetic-flux penetration into an electrical conductor for the case where demagnetizing effects play a dominant role, the demagnetizing factor being close to 1. In this section the discussion refers to an electrical conductor in general and need not be restricted to the case of a superconductor. Without demagnetization, i.e., in the one-dimensional case, the speed of magnetic-flux penetration into a conductor or the related phenomenon of the skin effect presents a rather simple problem which can be treated from Maxwell's equations with very little algebra. However, for nonzero demagnetizing coefficients, i.e., for the case of two and three dimensions, a treatment of the speed of flux penetration into a conductor and the skin effect leads to considerable mathematical difficulties. The complexity of the problem arises from the coupled differential equations for the different vector components which constitute Maxwell's equations. It appears that this problem has not yet been treated rigorously. Preliminary results of a numerical treatment have been reported.[10]

Instead of solving the problem of magnetic-flux penetration into a thin-film conductor rigorously, we give in the following a strongly simplified treatment. Our arguments follow from an energy-balance consideration[11] without incorporating the detailed mechanism of the flux penetration. They are very similar to those recently used by Rinderer *et al.*[9] for calculating the speed of penetration of magnetic flux tubes into flux-free superconducting specimens with a finite demagnetizing coefficient. We consider an electrical conductor in the form of a flat circular disk. An external magnetic field perpendicular to the disk is turned on abruptly. We wish to calculate the relaxation time τ required for the magnetic flux to penetrate into the center of the disk. Immediately after the magnetic field is turned on, flux will be completely expelled from the disk because of eddy-current screening, whereas at some distance above and below the disk the field is propagating through space with the speed of light. This results in a distortion of the magnetic field similar to the Meissner effect in a superconductor. Our basic argument[9] states that the excess of the Gibbs free energy ΔG associated with the unstable state assumed by the system immediately following the abrupt application of the magnetic field is dissipated through the Joule heating of the eddy currents generated during the flux penetration.

The Joule heat Q produced in the disk during the flux penetration is given by

$$Q=\int_0^\tau dt\int \sigma E^2(r,t)dV, \qquad (1)$$

where σ is the electrical conductivity and $E(r,t)$ the electric field at radius r and time t. The second integral is performed over the volume of the disk and is, in principle, a function of t. For the line integral along a circle of radius r, we have

$$\oint E(r,t)\,ds=2r\pi E(r,t)=-(1/c)\,\dot{\Phi}(r,t), \qquad (2)$$

c being the velocity of light and $\dot{\Phi}(r,t)$ the time derivative of the flux contained within the circle of radius r. If we approximate

$$\dot{\Phi}(r,t)=r^2\pi H_a/\tau \quad , \qquad (3)$$

we obtain

$$E(r,t)=E(r)=-H_a r/2c\tau \quad , \qquad (4)$$

where H_a is the applied magnetic field. We note that by assuming the approximation of Eq. (3), we have eliminated the time dependence of the second integral in Eq. (1), thus simplifying the calculation considerably. In the following we further neglect the fact that the relaxation time τ, as defined in Eq. (3), depends in principle on the coordinate r. In Eqs. (3) and (4) we assume that the disk is sufficiently thin such that we can neglect any dependence of $\dot{\Phi}$ and E on the coordinate in axial direction. Inserting (4) into (1), we find

$$Q=(\sigma/8c^2\tau)\,H_a^2R^2V \quad , \qquad (5)$$

R and V being the radius and the volume of the disk, respectively. Equating Q with the excess Gibbs free energy ΔG, given by

$$\Delta G=(1/8\pi)[H_a^2/(1-D)]\,V \quad , \qquad (6)$$

we obtain from (5) and (6)

$$\frac{1}{\tau}=\frac{c^2}{\pi(1-D)}\,\frac{1}{R^2\sigma} \quad . \tag{7}$$

Here D is the demagnetizing factor. Equation (7) has been empirically obtained previously[7] (except for a factor of $\frac{1}{2}\pi$) from the maximum in the resistive loss per cycle of circular copper foils.

For an oblate spheroid, we have

$$1-D=m(\tfrac{1}{2}\pi), \qquad m\ll 1 \tag{8}$$

where m is the ratio of the polar to the equatorial semiaxis.[12] Hence, for constant thickness of the disk, $1-D$ is proportional to R^{-1} and

$$1/\tau \sim 1/R \quad . \tag{9}$$

In thin films σ is proportional to the film thickness d, if the electrons are scattered predominantly at the two film surfaces. For constant radius R, $1-D$ is proportional to d, and we have from Eq. (7)

$$1/\tau \sim 1/d^2 \, . \tag{10}$$

The skin effect in the one-dimensional case can also be treated, of course, using arguments almost identical to those given above. It is very easy to show that from the energy-balance argument one finds the same result as from Maxwell's equations. The main difference in the expression for τ for the one-dimensional case and for the case with D close to 1 is the appearance of the factor $1-D$ in the denominator of Eq. (7). This factor then reduces the variation of τ with R^2 of the one-dimensional case to a *linear* dependence between τ and R, as indicated in relation (9).

The case of an infinite strip of width $2L$ placed perpendicular in a magnetic field, with the same arguments given above, yields the result

$$\frac{1}{\tau}=\frac{3c^2}{8\pi(1-D)}\,\frac{1}{L^2\sigma} \quad . \tag{11}$$

As mentioned above, the main approximation of our treatment is contained in Eq. (3). It is expected that because of this approximation the result expressed in Eqs. (7) and (11) is affected only by a numerical factor of order unity.

The results from this section can be applied directly to a superconductor which further may be in the mixed or the intermediate state. However, it is not a trivial question what to take for the electrical conductivity σ in this case. We find that Eqs. (7) and (11) are qualitatively in agreement with our experimental results if σ is taken from the slope of the curve of the resistive voltage plotted versus the transport current. The conductivity σ is then, of course, a function of magnetic field and current.

III. EXPERIMENTAL ARRANGEMENT

The experiments were carried out with lead films prepared by vacuum deposition of 99.999%-pure Pb from a Joule-heated tantalum boat on a glass substrate. The starting pressure in the bell jar was $(1-2)\times 10^{-6}$ Torr. The film thickness was measured gravimetrically and ranged between 1 and 12 μm.

Measurements were made at 4.2 °K of the resistive voltage and of the rate of change of magnetic flux in the specimen films when a small sinusoidal oscillatory magnetic field was superimposed on the static field. During all experiments both the static and the oscillatory magnetic field were oriented perpendicular to the film plane. The amplitude of the oscillatory field ranged between 0 and 1.8 G rms. The frequency was varied between 10 and 2×10^5 Hz. The specimens were placed in the center of a coil made from copper wire, which was wound on a Lucite body and which supplied the oscillatory magnetic field. The diameter and the length of this coil were 2.75 and 2.5 cm. The coil was mounted in the center of a copper can with a 5.2-cm diameter. A superconducting magnet coil of 17.5-cm length fitting around the copper can provided the static magnetic field. The copper can was part of a general purpose cryostat and contained a series of holes to establish direct contact between the liquid-helium bath and the specimens. The complete coil arrangement was checked both at room temperature and at 4.2 °K using a pickup coil without a specimen yielding a pickup voltage of the correct magnitude which increased linearly with frequency over the whole frequency range investigated.

The specimens were deposited on glass disks of 1.0-mm thickness and 22-mm diameter. For the *electrical-resistance measurements* the Pb films were shaped as indicated on the inset of Fig. 1. Voltage and current leads were attached with indium dots on the wide section at both sample ends before the deposition. The narrow section between the wide sections had a length of 13 mm and a width of either 2.0 or 3.0 mm. The characteristics of the different specimens are listed in Table I. The table also contains the electrical-resistance ratio $R(295\ ^\circ\text{K})/R(4.2\ ^\circ\text{K})$ of the specimens, where $R(4.2\ ^\circ\text{K})$ was measured in a perpendicular magnetic field of about 750 G. The resistive-voltage data were taken by plotting the voltage vs current using a Keithley model 140 nanovolt dc amplifier and a Moseley model 7000 AM x-y recorder. To eliminate hysteresis effects the data were obtained using the following sequence. First, the static magnetic field H_0 was slowly raised to the desired value without overshooting. Next the frequency of the oscillatory field was set and the amplitude H_1 of the oscillatory field was raised without overshooting to the desired value. Then the voltage was measured as a function of the sample current, the current being increased monotonically. Before taking a

TABLE I. Specimen characteristics, including the film thickness, the electrical-resistance ratio, and the arrangement of the pickup coil.

Sample No.	Experiment	Configuration	Sample shape	Film thickness (μm)	$R(295\,°K)/R(4.2\,°K)$	Pickup coil
68	resistive voltage	single film	2-mm-wide and 13-mm-long strip (see inset of Fig. 1)	3.0	560	...
69	resistive voltage	single film	2-mm-wide and 13-mm-long strip (see inset of Fig. 1)	11.6	1300	...
Mi-6	resistive voltage	single film	300-μm-wide and 3.0-mm-long strip	3.8	575	...
70B	flux penetration	sandwich from two films	disk with 16-mm diameter	6.85	...	center coil of 20 turns with 2.5-mm diameter and identical coil 4.5-mm off-center
71B	flux penetration	sandwich from two films	ring with 16-mm o.d. and 5.5-mm i.d.	6.45	...	center coil of 30 turns with 7-mm diameter
71A	resistive voltage and flux penetration	sandwich from two films	3-mm-wide and 13-mm-long strip (see inset of Fig. 1)	6.45	1310	single loop 1 mm wide and 12.5 mm long
72	resistive voltage and flux penetration	sandwich from two films	3-mm-wide and 13-mm-long strip (see inset of Fig. 1)	1.9	509	single loop 1 mm wide and 12 mm long

voltage-current curve for a different frequency of the oscillatory field, the sample current, H_1, and H_0 were returned to zero.

The *rate of change of magnetic flux* within the specimen films in the presence of an oscillatory-magnetic-field component was observed with pickup coils. At first these experiments were performed using circular lead films of 16-mm diameter and with a 8-mm-diameter hole in the center. The hole in the center contained a single loop with voltage leads consisting of a properly shaped gold film, which had been deposited before the Pb film. In the region of the voltage leads, where the Au and Pb films overlapped, the Au film had been covered

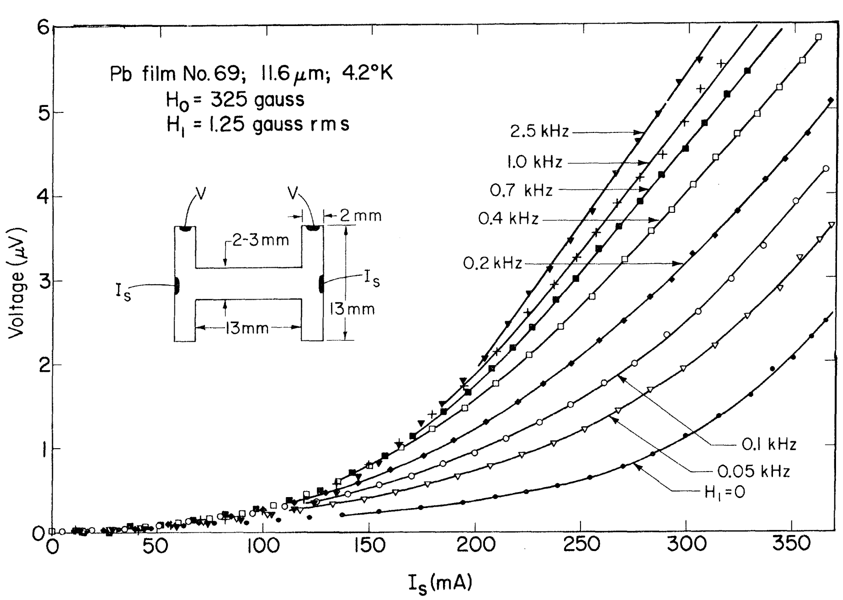

FIG. 1. Resistive voltage vs electrical current of sample No. 69 for different frequencies of the oscillatory magnetic field ($H_0 = 325$ G; $H_1 = 1.25$ G rms).

with a film of Al_2O_3 to provide electrical insulation between the Pb film and the pickup loop. With this configuration the main features of the penetration of the oscillatory-field component into the Pb films, described in Sec. IV, could be seen. However, because of the relatively large stray-field pickup of this configuration, the pickup experiments were finally performed using the following procedure. Small flat pickup coils with 1–30 turns were made from 0.07-mm-diameter copper wire and placed between two identical Pb films deposited simultaneously on two separate substrates. In this sandwich configuration both Pb films faced the pickup coil, the glass substrate being at the outside. Mechanical stability and electrical insulation were achieved using plastic insulating tape and cigarette paper.

At first, the pickup measurements using the sandwich configuration were made with circular lead films (sample Nos. 70B and 71B). The characteristics of the specimens, in which the pickup coil was sandwiched between the two Pb films, are given in Table I. Finally a geometry was adopted in which the resistive-voltage enhancement and the flux penetration in an oscillatory-magnetic-field component could be measured using the *same* specimen (sample Nos. 71A and 72). In this configuration the pickup coil was sandwiched between two Pb films shaped as shown in the inset of Fig. 1, with current and voltage leads attached to one Pb film. To the other Pb film, current leads were attached such that the sample current could be passed in the same direction through both films simultaneously. In these latter specimens the pickup coil consisted of a single loop 1.0 mm wide and 12.5 mm long placed along the axis of both Pb films.

The pickup voltages were measured with a Princeton Applied Research model HR-8 lock-in amplifier and occasionally also with a Tektronix model 7504 oscilloscope yielding good agreement with the data taken with the lock-in amplifier. As seen from the oscilloscope, the waveform of the pickup voltages was sinusoidal. In order to eliminate hysteresis effects the data were taken using the following sequence. First H_0 was raised slowly to its desired value. If a sample current I_S was present during the measurements, I_S was then raised to its desired value. Finally the frequency was set and H_1 was raised. In setting H_0, I_S, and H_1, care was taken again that no overshooting occurred. Before taking a measurement for a different frequency or a different value of H_0, the quantities H_0, I_S, and H_1 were returned to zero. For sample Nos. 70B and 71B a series of frequencies were run at constant value H_0 without turning H_0 back to zero between each two frequencies.

IV. EXPERIMENTAL RESULTS

A. Resistive-Voltage Measurements

The influence of the oscillatory-magnetic-field component H_1 on the resistive voltage of the Pb films is shown in Fig. 1 for different frequencies of the oscillatory field. In the presence of H_1 a voltage enhancement is generally observed which varies with the frequency of H_1. As a function of the frequency of H_1 the resistive voltage passes through a maximum at a distinct frequency ν^*, as seen in Fig. 2, for different amplitudes of H_1. The frequency ν^*, at which the maximum in the voltage

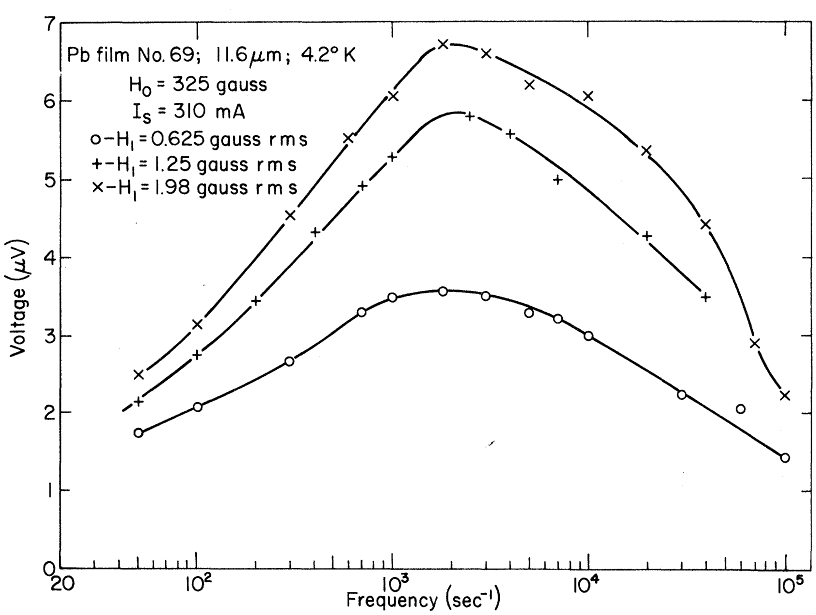

FIG. 2. Resistive voltage vs the frequency of the oscillatory magnetic field of sample No. 69 for different amplitudes of H_1 (H_0 = 325 G; I_s = 310 mA).

occurs, depends somewhat on the sample current I_S and generally increases with increasing I_S, as can be seen from Fig. 3. This dependence of ν^* on I_S generally becomes weaker at higher sample currents. It was observed that the frequency ν^* shifts slightly to higher values with increasing amplitude of H_1. The frequency ν^* increases strongly with increasing value of the static magnetic field H_0 as shown in Fig 4 for two Pb films of different thickness. We note that the data in Fig. 4 refer to a magnetic field range below the critical field for these films[7,13,14] where the electric resistance increases strongly with H_0.[14] We further note from Fig. 4 that ν^* decreases strongly with increasing film thickness. The data in Fig. 4k were taken at rather high values of I_S, where the dependence of ν^* on I_S is not so pronounced.

The results on the resistive-voltage enhancement in the presence of an oscillatory-magnetic-field component suggest that this effect is caused by the oscillatory eddy currents in the specimen associated with the penetration of the oscillatory field. The oscillatory eddy currents assist in overcoming the pinning forces thereby reducing the influence of flux pinning. At low frequencies the oscillatory eddy currents in the sample increase with frequency until the skin depth of the oscillatory-field component becomes roughly equal to the specimen width. At higher frequencies we have the reverse behavior since H_1 penetrates less and less into the sample. In this way the variation of the resistive-voltage enhancement with the frequency of H_1 can be readily understood from the relaxation time for the penetration of magnetic flux into the specimen. If we assume heuristically that the maximum in the resistive voltage occurs when the frequency of the oscillatory field is roughly equal to the inverse relaxation time for flux penetration, the variations of ν^* with H_0, film thickness, I_S, and amplitude of H_1 can all be understood qualitatively from the variation of the electrical resistance of the samples with these quantities. With increasing electrical resistance the eddy-current damping and thus the relaxation time for flux penetration become smaller as indicated in Eqs. (7) and (11). Here we have in mind, of course, the strong increase of the electrical resistance of the films at increasing perpendicular magnetic field[13,14] below the critical-field value. The dependence of ν^* on I_S and on the amplitude of H_1 is related to the current dependence of the electrical resistance in the nonlinear regime of the voltage-current behavior. We note that the enhancement of the resistive voltage is intimately connected with flux pinning and the nonlinear voltage-current relation and does not appear in a normal conductor.

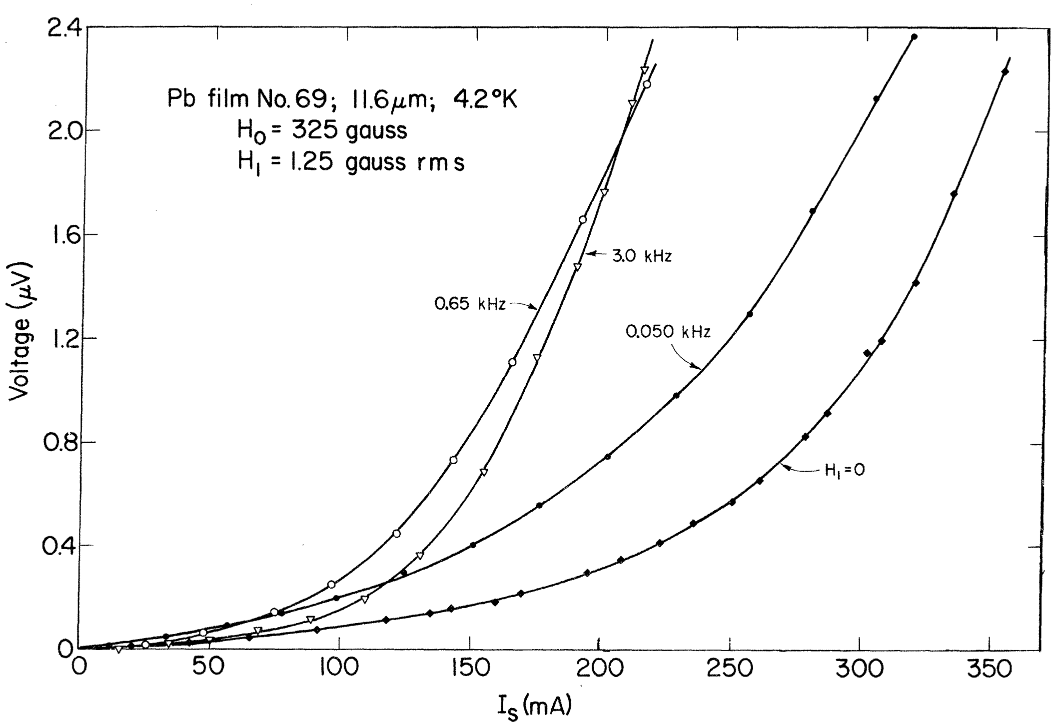

FIG. 3. Resistive voltage (on an expanded scale) vs electrical current of sample No. 69 for three frequencies of the oscillatory-field component (H_0=325 G; H_1=1.25 G rms).

When H_0 and H_1 had been set, the resistive-voltage enhancement observed in the presence of the oscillatory-field component did not disappear completely after H_1 was turned back to zero. The fraction of the voltage increase which remained after H_1 was turned off decreased with increasing value of H_0. However, no voltage increase was observed when H_0 was also turned to zero and then raised again without raising H_1. Turning the oscillatory field on and off before the electrical current was passed through the specimen did not influence the voltage reading. These results indicate that the oscillatory-field component can cause an irreversible change in the intermediate-state structure of the films when an electrical current is flowing through the specimens. Such changes in the magnetic structure of thin-film type-I superconductors in the presence of an electrical-transport current have been observed previously,[15,16] and current-hysteresis effects in the electrical resistance associated with the rearrangement of the flux structure have been reported.[14,15] It appears that an irreversible change of the magnetic-flux structure plays some role in the resistive-voltage enhancement which we observe, this effect becoming more important at lower values of H_0.

B. Measurements of Flux Penetration

In order to obtain direct evidence that the variation of the resistive-voltage enhancement with the frequency of H_1 can be explained from the relaxation time for magnetic-flux penetration into the specimens (skin effect), we have measured the flux penetration directly using flat pickup coils sandwiched between two identical Pb films.

Figure 5 shows the voltage detected with the

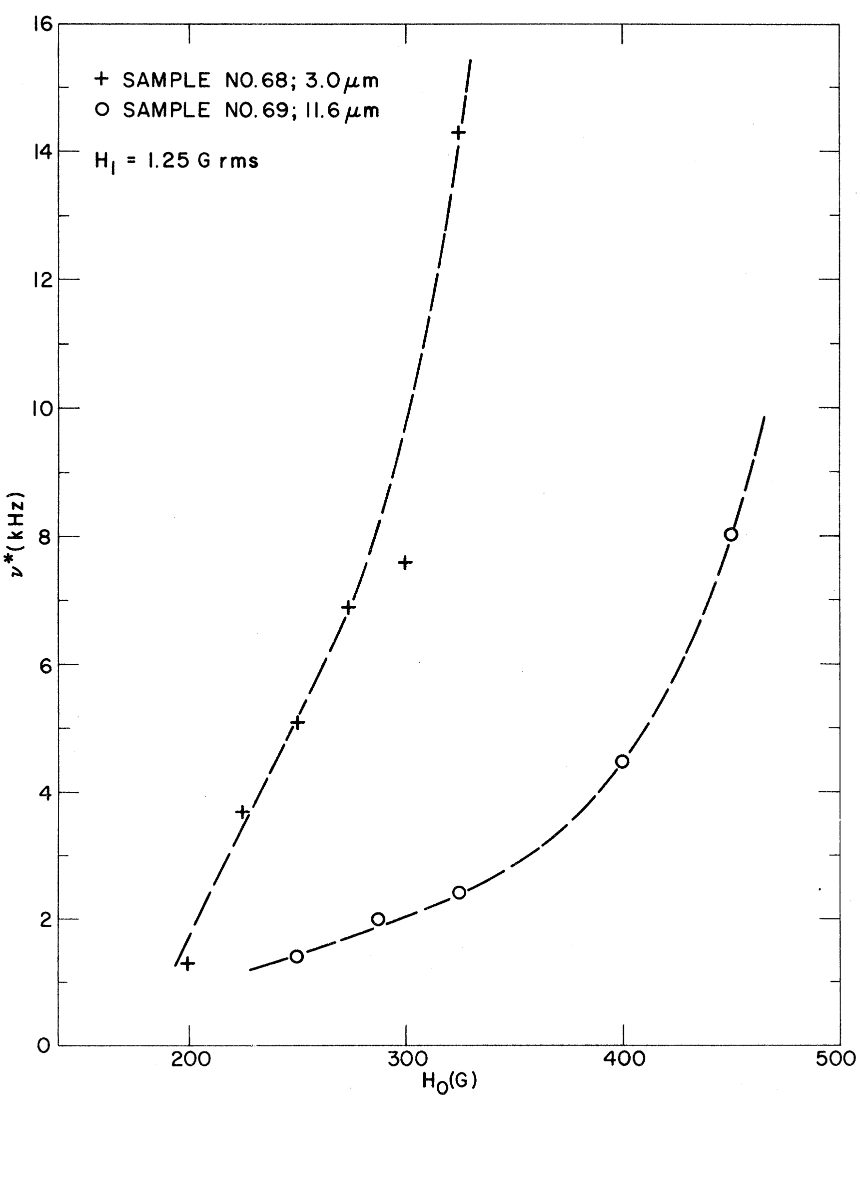

FIG. 4. Frequency ν^* vs the static field H_0 for sample Nos. 68 and 69 ($H_1 = 1.25$ G rms).

pickup coil in sample No. 71B, i.e., for two ring-shaped Pb films with 16-mm o.d., each having a film thickness of 6.45 μm. The data at room temperature for $H_0 = 0$ show a linear increase of the pickup voltage with the frequency of the oscillatory field, indicating that here the magnetic field oscillation penetrates completely into the center of the specimen at all frequencies. At 4.2 °K the pickup voltages are considerably reduced below the values at room temperature, indicating that flux penetration into the pickup coil is hindered because of eddy-current damping within the specimen film. At 4.2 °K the pickup voltage is seen to pass through a maximum at a distinct frequency ν'. This frequency ν' increases with increasing value of H_0 for fields below H_c. In Fig. 6 we show the variation of the frequency ν', at which the maximum in the pickup voltage appears, with the static field H_0 for sample Nos. 70B and 71B. Above the critical field of the films ν' changed very little with the value of H_0. The location of the arrows on the right-hand side of the curves in Fig. 6 indicates the ν' values obtained at $H_0 = 750$ G. Apparently, the maximum in the pickup voltage at the frequency ν' occurs, since for frequencies above ν' with increasing frequency the field oscillation penetrates less and less into the interior of the sample because of eddy-current damping. Again, we expect the frequency ν' to be roughly equal to the inverse relaxation time for magnetic-flux penetration into the specimen. The variation of ν' with H_0, shown in Fig. 6, can then be qualitatively understood from the corresponding variation of the electrical resistance of the specimens together with Eqs. (7) and (11). Many experiments have shown[13,14,17] that the electrical resistance in superconducting Pb films varies with H_0 similarly as shown by the curves in Fig. 6, increasing strongly as H_0 approaches the critical field and being nearly field independent above H_c. The increase of the pickup voltage at the high-frequency side, seen in Fig. 5 for the lower values of H_0, may be caused by oscillatory stray fields around the circuit of the pickup coil. We see from Fig. 6 that in sample No. 70B the ν' values for the coil located 4.5 mm off-center are appreciably higher than those for the center coil. This indicates that for the outer coil flux penetration occurs up to higher frequencies than for the center coil, as one would expect.

The magnitude of the pickup voltage and the frequency ν' shifted usually somewhat towards higher values when an external electrical current was

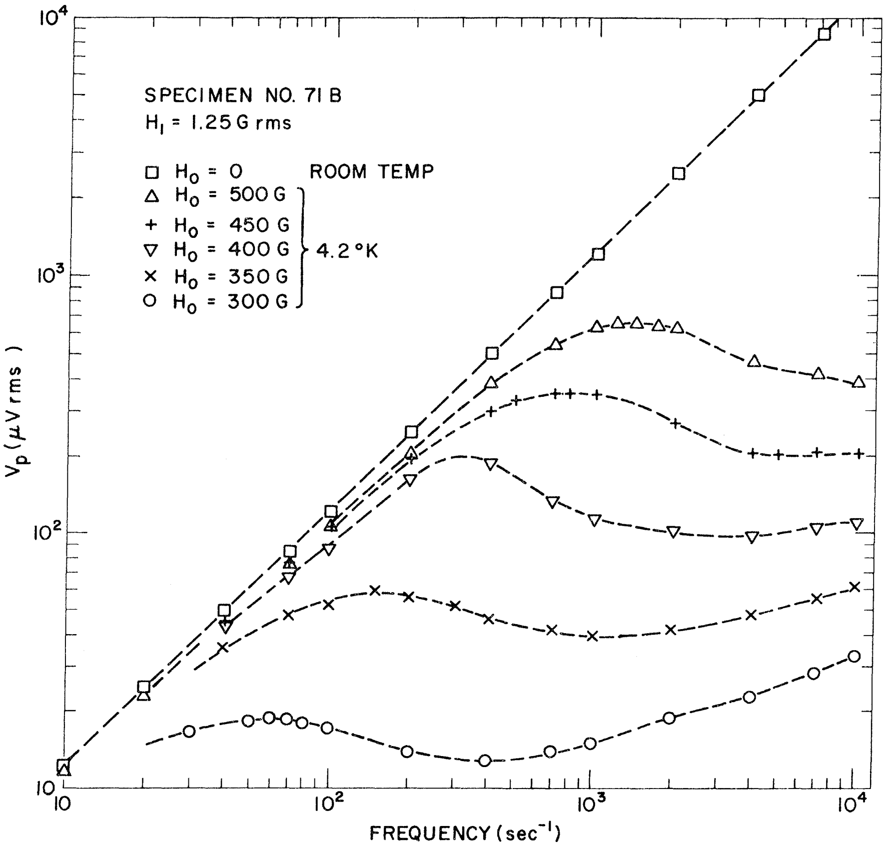

FIG. 5. Pickup voltage for sample No. 71B vs the frequency of the oscillatory field for different values of H_0 ($H_1 = 1.25$ G rms).

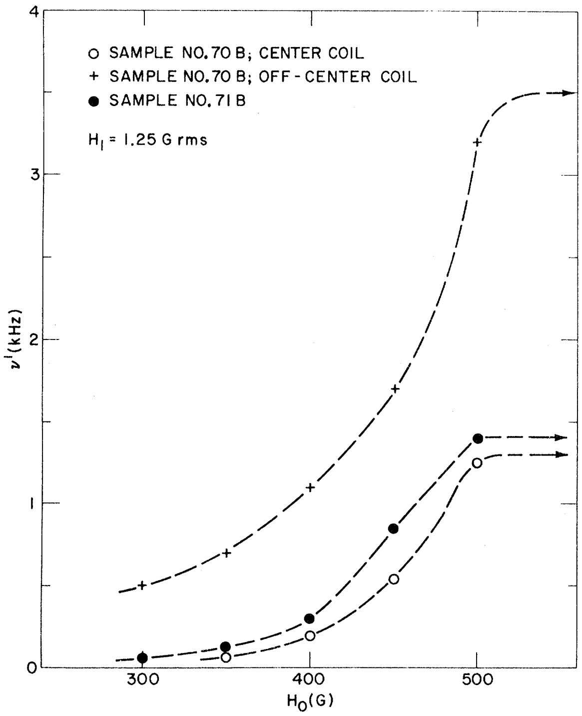

FIG. 6. Frequency ν' vs the static field H_0 for sample Nos. 70B (center and off-center coil) and 71B. The arrows on the right-hand side indicate the ν' values for $H_0 = 750$ G ($H_1 = 1.25$ G rms).

passed through the specimen during the experiment. A small shift of ν' towards higher values was also observed when the amplitude of H_1 was increased. These effects can be readily understood through the current dependence of the electrical resistance of the samples, the eddy-current damping becoming smaller with increasing resistance.

The enhancement of the pickup voltage by an external electrical current passing through the sample showed strong hysteresis, the pickup voltage remaining enhanced after the current had been turned to zero. Figure 7 shows the pickup voltage measured in sample No. 70B, after a different number of rectangular *current pulses* with a 1.5-A amplitude and a 10^{-2}-sec width had been passed through the two films of the sandwich in the same direction. These results again can be understood from a current-induced irreversible rearrangement of the magnetic-flux structure leading to an irreversible resistance enhancement. Such rearrangements of the flux structure have been noted previously.[14–16]

C. Measurements of Resistive Voltage and Flux Penetration in Same Specimen

The similarity of the results obtained for the resistive-voltage enhancement in the presence of an oscillatory-magnetic-field component and for the magnetic-flux penetration indicates that both phenomena are closely related to each other. This was demonstrated more clearly in a series of experiments where both types of measurements were performed in the same specimen (sample Nos. 71A and 72). In particular, both the resistive-voltage enhancement and the flux penetration were measured for these samples with an *electrical current of identical value* passing through the two films of the sandwich in the same direction. Under these conditions the two frequencies ν^* and ν' for the maxima were found to be approximately equal, as shown for sample No. 71A in Fig. 8. The frequencies ν^* and ν' for the maxima are listed in Table II for the two samples and for different values of H_0. The table also includes the electrical-current values at which these data were obtained. We note that in sample No. 72 with 1.9-μm film thickness both frequencies ν^* and ν' are appreciably higher than in sample No. 71A with a 6.45-μm thickness at a constant value of H_0. The ν' values in Table II were obtained after slightly correcting the pickup voltages by subtracting the background pickup found for zero static magnetic field.

V. DISCUSSION

Our experiments indicate that the presence of an oscillatory magnetic field component results in an enhancement of the resistive voltage in a superconducting strip. This enhancement can be understood from the oscillatory eddy currents in the specimen which reduce the influence of flux pinning. In this way the effect is associated with the nonlinear dependence of the resistive voltage on the current in the intermediate or the mixed state of

TABLE II. Frequencies ν^* and ν' at which the maximum in the resistive voltage and in the pickup voltage, respectively, occurs for sample Nos. 71A and 72 and for different values of H_0. I_S is the current passing through the samples during the measurements. Amplitude of H_1 is 1.25 G rms.

H_0 (G)	ν^* (resistive voltage) (sec^{-1})	ν' (flux penetration) (sec^{-1})	I_S (mA)
Sample No. 71A			
300	700	700	300
350	1000	1200	280
400	1600	2000	200
450	3400	3700	170
Sample No. 72			
200	230	240	350
250	1800	1300	160
300	7000	6000	100
350	25×10^3	18×10^3	60
400	40×10^3	50×10^3	50

a superconductor. This voltage enhancement does not appear in a normal conductor and vanishes in a superconductor as one approaches the critical field. As indicated by the combined measurements of the resistive-voltage enhancement and of the magnetic-flux penetration into the specimens with pickup coils, the variation of the voltage enhancement with the frequency of the oscillatory-field component can be explained from the relaxation time of the flux penetration. The effect passes through a maximum when the skin depth of the oscillatory field is approximately equal to the width of the specimen. Above the frequency ν^* for this maximum the oscillatory eddy currents penetrate less and less into the material thus reducing the influence of flux pinning in a smaller and smaller portion of the sample. The enhancement of current-induced flux motion in superconducting Pb films in the presence of a small oscillatory-magnetic-field component has recently been observed directly with a high-resolution magneto-optical method.[16]

The variation of the resistive-voltage enhancement with the frequency of H_1 was always found to be quite similar to the frequency dependence of the magnetic-flux penetration, with the frequencies ν^* and ν', at which the maximum in the resistive voltage and in the flux penetration, respectively, appeared being approximately equal (Table II). Our data indicate that the variation of both frequencies ν^* and ν' with the static field H_0 and with the transport current I_S follows qualitatively the corresponding variation of the electrical conductivity with both quantities as expected from Eqs. (7) and (11). Here the electrical conductivity is taken from the slope of the $V(I_S)$ curve, which depends, in general, upon the magnetic field and the current. The maximum in both the resistive voltage and in the flux penetration is attained when the frequency is about equal to the inverse relaxation time for flux penetration or more accurately for

$$2\pi\nu\tau = 1 \quad . \qquad (12)$$

In Table III we list the experimental values of τ^{-1} obtained from measurements of the frequency ν' or ν^* together with Eq. (12) for four specimens and compare them with theoretical values calculated from Eq. (7) or (11). For sample Nos. 70B, 71A, and 72 the experimental data refer to pickup experiments. For specimen Mi-6 we show the experimental results from resistive-voltage measurements at different magnetic fields. The conductivity σ was obtained from resistance measurements at the

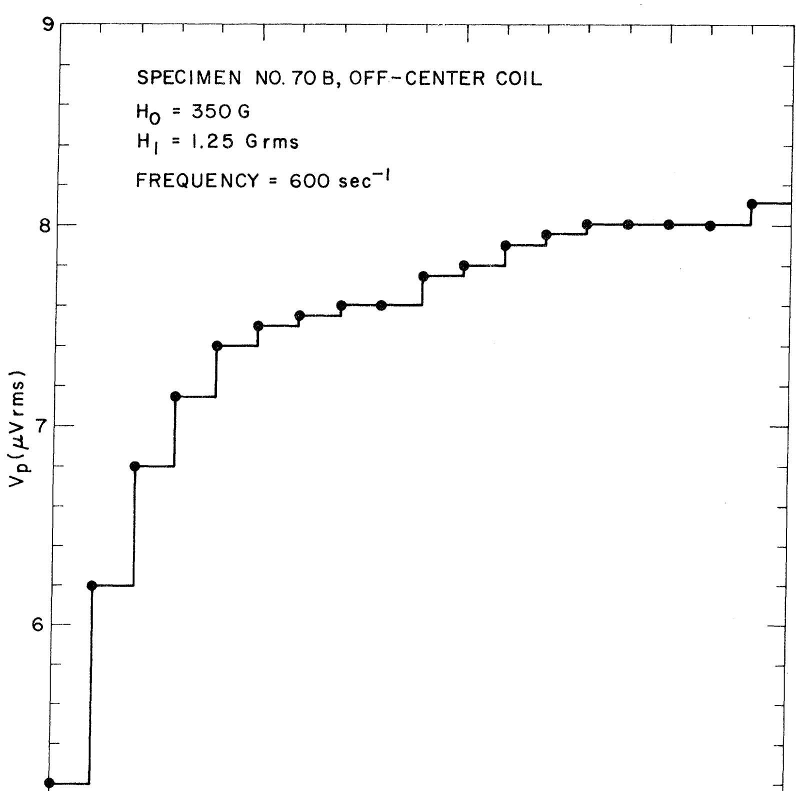

FIG. 7. Pickup voltage measured after a number of current pulses with 1.5-A amplitude and 10^{-2}-sec width had been passed in the same direction through both specimen films vs the number of pulses (H_0=350 G; H_1=1.25 G rms, frequency =600 $\sec^{-1}$; sample No. 70B).

TABLE III. Experimental and theoretical values of the inverse relaxation time τ^{-1} for flux penetration at the indicated values of the static magnetic field and 1.25 G rms amplitude of H_1.

			$\tau^{-1} 10^3 (\text{sec}^{-1})$	
Specimen	H_0 (G)	σ $10^8 (\Omega\,\text{cm})^{-1}$	From experiment using Eq. (12)	Calculated from Eq. (7) or (11)
70B, center coil	925	...	8.2	5.1
71A	750	0.60	35	21
72	750	0.23	440	182
Mi-6	160	4.3	190	93
Mi-6	250	2.71	250	146
Mi-6	325	1.33	380	300
Mi-6	400	0.75	640	530

indicated value of H_0. The σ values for sample Mi-6 were found from the slope of the $V(I_S)$ curves at the current value at which the frequency ν^* was measured. For sample No. 70B σ was estimated using previous results for Pb films of similar thickness. For sample No. 70B the demagnetizing factor was obtained from Eq. (8). For sample Nos. 71A, 72, and Mi-6, D was found from the relation[18]

$$D = 1/(1+p) \;, \tag{13}$$

where p is the ratio of thickness to width of the Pb strip. For sample Nos. 70B, 71A, and 72 the film thickness was taken as twice the thickness of the individual Pb films constituting the sandwich configuration. Although the two films were somewhat separated from each other in the sandwich we expect no appreciable error from ignoring this separation.

The agreement between the experimental and theoretical values of the relaxation time for flux penetration in Table III is very reasonable in view of the approximations in the theory. In particular the results for sample Mi-6 in Table III indicate that Eqs. (7) and (11) remain valid in good approximation below the critical field if σ is obtained from the slope of the $V(I_S)$ curve taking into account the dependence of this slope upon magnetic field and current. Further, we note for sample Nos. 71A and 72 that the experiments confirm the proportionality between τ and the square of the film thickness,

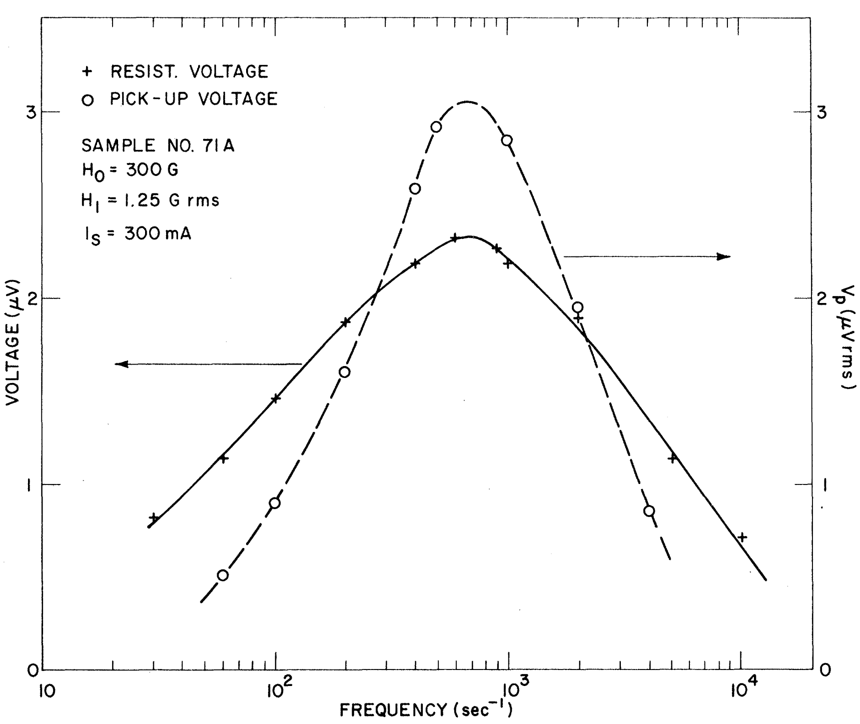

FIG. 8. Resistive voltage and pic up voltage vs the frequency of the os cillatory field for sample No. 71A. During all measurements a current of 300 mA was passing through both films of the sandwich in the same directions ($H_0 = 300$ G; $H_1 = 1.25$ G rms).

as expected from Eq. (10).

The present investigation is somewhat different from the situation investigated by Rinderer *et al.*[9] These authors studied the speed of the penetration of magnetic flux into the purely superconducting flux-free phase. On the other hand, our experiments deal with the penetration of a magnetic field component which is small compared to the field already in existence throughout the specimen.

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