Highly Regular Domain Motion in the Dynamic Intermediate State of Superconducting Tin†

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Using the Sharvin point-contact technique, the regularity of domain motion in the dynamic intermediate state of superconducting tin has been investigated as a function of temperature, applied magnetic field, and transport current. We have found that, when the magnetic field is increased so as to lie in a range closer to the critical field than has been previously investigated systematically, an extremely drastic increase in the regularity of the motion occurs, independent of temperature and sample condition. It is suggested that this drastic increase represents the onset of motion of a modified Landau domain structure, freed from the perturbing influence of the ends of the sample. A new phenomenon of "domain healing" is reported in the highly regular regime.

I. INTRODUCTION

In the presence of a sufficiently strong magnetic field, a type-I superconductor of arbitrary shape passes into the intermediate state and then consists of a mixture of normal and superconducting regions. There is now no doubt that the additional presence of a sufficiently large transport current through the sample will rather generally cause *some* type of continuous motion of these regions relative to the sample boundaries. "Flux transfer,"[1] voltage fluctuations,[2] and more recently, direct visual observations,[3] all support this conclusion. The observations reported in Ref. 3 also support the view that the induced motion is in general quite irregular.

However, in the absence of a transport current, it is further well known that a spatially very regular structure of normal and superconducting layers can be realized under certain experimental conditions, namely, with the magnetic field applied at an acute angle to a thin flat sample.[4] It is therefore of interest to inquire whether the induced motion can also be regular under the same conditions. Evidently the simplest case that might be realized in the presence of a transport current would involve the continuous displacement of the entire regular equilibrium structure with respect to the sample boundaries. The only positive evidence for regular motion in this strict sense has been offered by Sharvin[5] and Sharvin and Landau[6] using the point contact technique. In a pioneering and comprehensive study of superconducting indium,[6] evidence was obtained for regular motion over a wide range of temperature T and relative content of the normal phase C_N, viz., $1.5\,\mathrm{K} < T < 3\,\mathrm{K}$ and $0.2 < C_N < 0.8$. However, none of the raw experimental data were presented, so that it is not possible to determine exactly how regular the motion was. The only available data which allow a quantitative statement to be made concerning the actual regularity of the motion refer to tin[6] and indicate variations of at least a factor of 2 in the width of a sequence of 15 normal domains passing under a point. Despite the rather large variation, this nonetheless represents the most regular domain motion reported in the literature prior to the present results. Apart from its intrinsic interest, the question of regularity must arise in any attempt to compare theoretical predictions which assume perfect regularity, for example, of the effect of transport current on the domain width,[7] with experiment. For these reasons our main purpose in this work was simply to determine how regular a domain motion can be achieved in practice. We therefore undertook a study, following very closely the procedures outlined in Ref. 6, but focusing our attention on the regularity of the motion and systematically extending the range of variation of C_N to higher values. Tin was employed, rather than indium, for reasons purely of convenience, namely, we find good single crystals of tin somewhat easier to grow than those of indium. Although the complete theoretical description of the motion is more complicated in the case of tin than it is for indium,[6] we have reasons, which should become clear, to expect that the major features of our results on the regularity itself will hold for any superconductor in the intermediate state.

II. EXPERIMENTAL

Some time will be spent discussing the experimental method employed, since objections have previously been raised[8] to the use of point contacts as a means of probing domain motion. The results reported in Ref. 6, together with additional ones to be discussed here, have firmly convinced us that the presence of contacts, formed in the manner originally described by Sharvin,[5] introduces no distortion into the motion.

The basic setup is shown in Fig. 1. The sample is a single-crystal parallelepiped of tin cast from

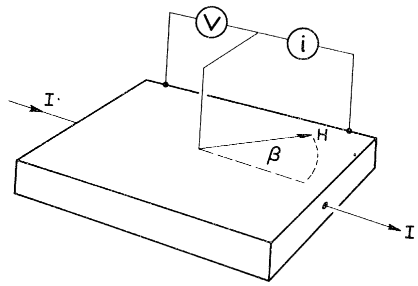

FIG. 1. Experimental setup (see text).

starting material of $5N$ purity and of typical dimensions $20\times 10\times 0.50$ mm. The crystals were grown *in vacuo* between two glass slides treated in the manner described by Sabo[9] in order to prevent the molten tin sticking to the glass. Preliminary experiments with crystals of different orientations convinced us that there was no orientation dependence to the quantities of interest, and the experimental data reported here refer to unoriented single crystals. (We note that a similar lack of any orientation dependence was reported in Ref. 6.) The domain-motion probe is a thin (30 μ diam) copper wire welded to the sample by passing a current of the order of 10^{-3} A for a few seconds. The wire is further supported mechanically in the vicinity of the contact by a diluted layer of GE 7031 varnish. Despite the dilution, it was found that the varnish must be allowed to dry for at least 12 h; otherwise the contact has a tendency to pull off when cooled. The varnish was applied by inclining the sample at a small angle to the horizontal and letting a small drop spread slowly from one edge. In this way a substantial fraction of the sample was covered with a thin layer of varnish and, whether for this or other reasons, it was found that repeated thermal cycling could eventually produce visible strain lines on the sample surface. Although the general regularity of domain motion was then certainly reduced, the drastic improvement at high C_N was completely unaffected, as will be shown.

A current i was passed between the wire and sample (Fig. 1), and the potential difference V was measured using a commercial Keithly Nanovoltmeter and recorded as a function of time on an X-Y recorder. With careful shielding, using a small solid-state magnet to supply the magnetic field H and batteries to supply both the transport current I and the measuring current i, we were able to isolate the entire experiment from extraneous ac fields and achieve a short-term resolution somewhat better than 5×10^{-10} V. For a number of contacts which increased in resistance by $\Delta R \sim 10^{-1}\ \Omega$ when a normal region passed underneath, we were then able to confirm that the form of the voltage oscillations was unchanged when the measuring current i was varied from 10^{-2} A down to 2×10^{-8} A, extending the previous limits[6] on this variation downward by about two orders of magnitude. We have thus satisfied ourselves that the current through the point has no measurable effect on the domain motion. The experiments participated in previously by one of us[8] led to a different conclusion with a marked dependence on i being observed. However, the method of forming the contact (with heavy-gauge phosphor-bronze wire sharpened to a point) used in that work may conceivably have produced a larger flaw in the sample than that produced by welding on the extremely fine wires used here. Since "pinning" of the domains at a substantial flaw would probably depend on the measuring current, we believe that the previous complex and irreproducible results observed in Ref. 8 can be attributed directly to the nature of that contact. In any case, all the results presented here, obtained using $i\sim 1$ mA, are independent of the measuring current i when that is varied over at least five orders of magnitude. The angle β which the magnetic field makes with the plane of the sample was small throughout our experiments, but no systematic change in the character of the regularity could be noticed as long as β was less than about 20°. For any value of β, the relationship between the value of C_N and H/H_c can be calculated,[4] assuming C_N is uniform across the sample, giving

$$C_N = \frac{H}{H_c}\sin\beta\,[1-(H/H_c)^2\cos^2\beta]^{-1/2} \quad . \tag{1}$$

For $\beta = 10°$, Eq. (1) gives the result shown in Fig. 2: a very rapidly varying function of H/H_c near

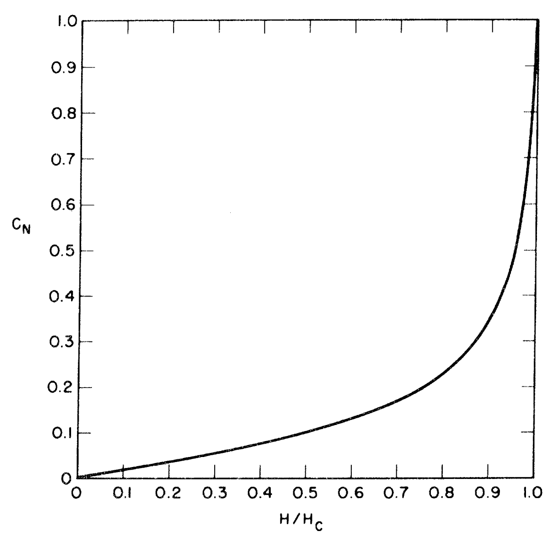

FIG. 2. Fraction of the sample in the normal state, C_N, plotted against the reduced external field H/H_c, according to Eq. (1) using $\beta = 10°$.

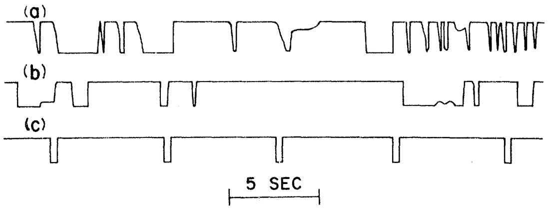

FIG. 3. Sample 1, dimensions $20\times10\times0.5$ mm, $T=1.93$ K, $\beta=17°$: (a) $I=3.0$ A, $C_N=0.66$; (b) $I=3.0$ A, $C_N=0.80$; (c) $I=1.0$ A, $C_N=0.94$.

$H/H_c=1$. For $H_c=200$ Oe, for example, the entire range of $0.8<C_N<1.0$ is covered in 2.0 Oe. A solid-state permanent magnet system was employed in these investigations and gave essentially perfect field stability together with the ability to change the field by as little as 0.1 Oe at 200 Oe, both features being essential to varying and maintaining C_N in the high-C_N region. However, the field itself is varied by changing the reluctance of the magnet system, and hysteresis difficulties prevent us knowing the absolute value of the field to much better than 2.0 Oe. One can also define a value of C_N directly from the observed voltage oscillations, viz., the time spent by the contact in the normal state divided by the total time averaged over a sequence of oscillations. Highly regular behavior has been observed over a rather narrower range of C_N, so defined, than our absolute knowledge of the magnetic field enables us to specify through Eq. (1). For this reason we have chosen to label our experimental data with the directly observed C_N.

All our data refer to points placed in the vicinity of the geometric center of the sample and at most 2 mm from it. There is little doubt that the domain structure and motion must be modified somewhat near the edges of the sample, but we have not investigated such modifications in this work. However, in all cases, again following the technique of Ref. 6, a second point was placed within the central region, the line joining the two points being at an angle of $90°\pm10°$ to the direction of current flow. The point separations employed varied between 0.1 and 0.5 mm and enabled us to probe directly the spatial aspect of the regularity of the domain motion: The two points were provided with independent current supplies and recording circuits. Two Hewlett-Packard X-Y recorders were used, the internal X-axis time sweeps being initiated simultaneously by hand. Over a sweep occupying 100 sec they maintained synchronization to better than 0.2 sec.

III. RESULTS

A. Regularity of Domain Motion Past a Single Point

1. *Field and Temperature Dependence*

In all the figures of this paper the voltage across the point is presented on the y axis and the time, increasing to the right, along the x. In all cases, limits of the y excursions correspond to the material under the point being completely normal (maximum voltage) and superconducting (minimum voltage). Before commencing to record a trace, the sensitivity was adjusted so that the amplitude of these excursions was some standard value. The actual magnitude of the excursion depended, of course, on the associated change of resistance of the point contact which we found to lie in the range 10^{-3}–10^{-1} Ω, as in Sharvin and Landau's work.[6]

Figures 3–5 exhibit the basic result of this study. They show direct chart-recorder traces obtained for two different samples at different temperatures and for three values of C_N in each case. Both samples exhibit a drastic improvement in regularity at a value of C_N around $C_N\sim0.9$. In all cases, the general degree of regularity for a given C_N and the extent of the range of C_N over which high regularity was achieved, typically $\Delta C_N\sim0.05$, appeared to be practically independent of temperature over the investigated range $1.5<T<3.0$ K. The exact range of C_N was sample dependent, but we have not observed high regularity outside the range $0.85<C_N<0.95$.

Figures 6 and 7 exhibit the extent to which this general pattern is asserted independent of sample condition. Figure 6 shows data for the *worst* sample examined, which displayed severe surface-

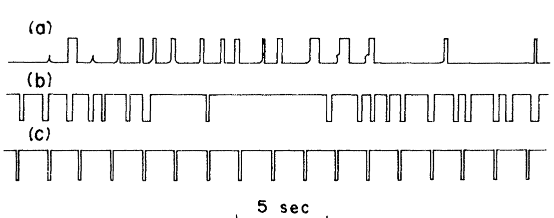

FIG. 4. Sample 1, $T=2.96$ K, $\beta=17°$: (a) $I=4.5$ A, $C_N\sim0.1$; (b) $I=2.0$ A, $C_N=0.80$; (c) $I=3.0$ A, $C_N=0.90$.

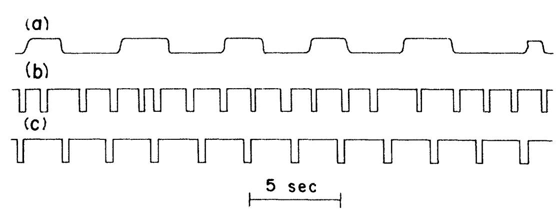

FIG. 5. Sample 2, dimensions $21\times11\times0.5$ mm, $T=2.52$ K, $\beta=8°$; (a) $I=3.5$ A, $C_N=0.36$; (b) $I=2.3$ A, $C_N=0.73$; (c) $I=1.5$ A, $C_N=0.86$.

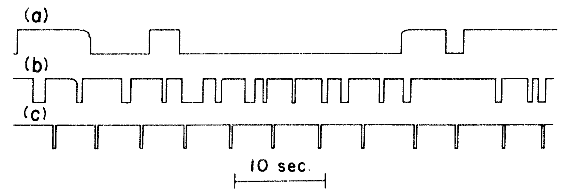

FIG. 6. Sample 3, dimensions $20\times10\times0.5$ mm, $T=2.96$ K, $\beta=10°$: (a) $I=3.0$ A, $C_N=0.4$; (b) $I=1.1$ A, $C_N=0.80$; (c) $I=1.4$ A, $C_N=0.93$.

strain lines. As can be seen, a sharp improvement in regularity still occurs at high C_N. Figure 7 displays data for a 1-mm-thick sample which again exhibits the same pattern. Indeed, both these samples appear little different from the first two, to judge from the data presented so far. The really striking aspect of the achievement of regularity in these cases will become clear when the results in Sec. III B are presented.

2. *Transport Current Dependence*

As found by Sharvin and Landau[6] over a limited range of current, the number of domains passing a point per unit time is a linear function of the transport current, and within that same range we have found that the degree of regularity is almost completely independent of the current. When small deviations occur they tend to do so at *both* low and high currents with the most regular behavior exhibited in the middle of the current range. This point is exhibited in Fig. 8, which also plots frequency as a function of current to demonstrate the linear relationship invariably observed. As can be seen, the value of C_N remains sensibly constant as the transport current is changed for fixed applied field. Note, in addition, that the frequency-current plot is a straight line through the origin. Finally, in the low-C_N region no variation of the transport current was found to effect an improvement in the regularity.

B. Domain Motion Past Two Adjacent Points

As mentioned previously, while obtaining all the single-point data presented in Sec. III A above, we were in fact simultaneously recording the response of another point, the line joining the two points being at an angle of $90° \pm 10°$ to the direction of transport current flow. The results of all our observations can be simply summarized. In the region of high regularity for $C_N \sim 0.9$, the correlation between the two points is essentially perfect, while for all other regions of C_N we were unable to obtain convincing evidence for *long-term* correlations between the two points where the point separation employed varied between 0.1 and 0.5 mm. The behavior at each point was irregular to some degree and the irregularities were not transmitted over distances as short as 0.1 mm.

Data supporting the above remarks are exhibited by Figs. 9–11, which reproduce the data exhibited in Figs. 3–5 together with that obtained from another point the indicated distance away. Note the extreme lack of correlation in Fig. 10, trace (a), where one point exhibits no oscillations at all, even though the other is a mere 0.1 mm away. In the low-C_N region such an observation was by no means

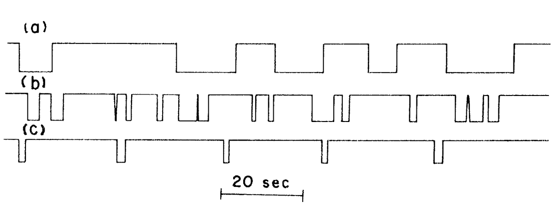

FIG. 7. Sample 4, dimensions $22\times10\times1.0$ mm, $T=2.96$ K, $\beta=10°$: (a) $I=5.8$ A, $C_N=0.42$; (b) $I=3.0$ A, $C_N=0.74$; (c) $I=2.4$ A, $C_N=0.92$.

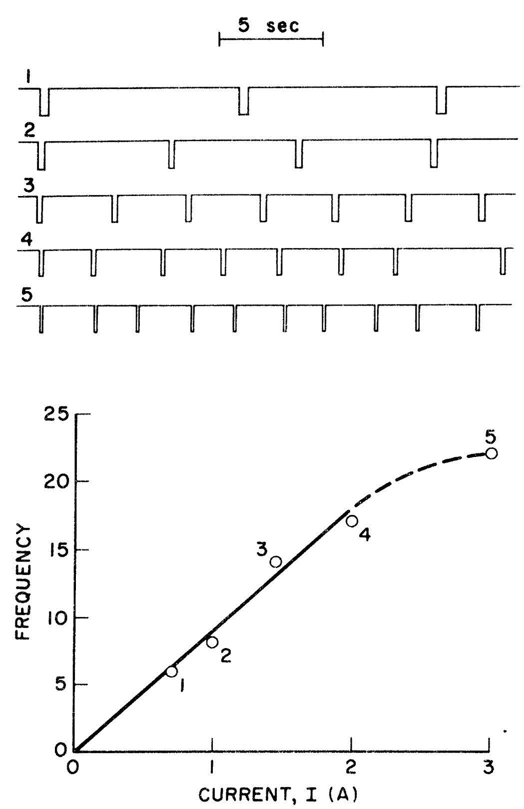

FIG. 8. Sample 1, $T=1.96$ K, $\beta=17°$: Traces 1–5 exhibit oscillations observed for increasing values of the transport current, identified using the lower inset. "Frequency" is the number of oscillations counted during a time period of 50 sec.

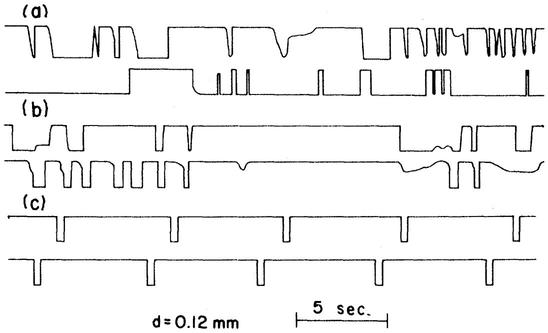

FIG. 9. Upper trace of each pair reproduces that in Fig. 3, while lower trace was recorded simultaneously from a second point the indicated distance d away from the first.

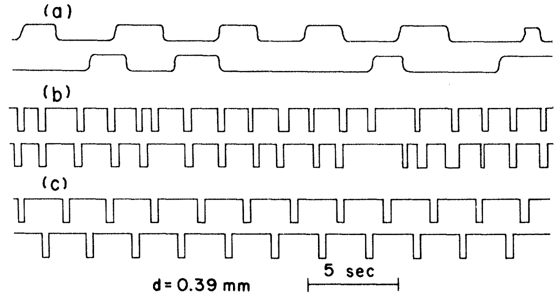

FIG. 11. Upper trace of each pair reproduces that in Fig. 5, while lower trace was recorded simultaneously from a second point the indicated distance d away from the first.

uncommon and was also observed for $C_N > 0.95$, but never in the highly regular region around $C_N \sim 0.90$. From Fig. 12 one can extract the transport current dependence of the time lag between the two points, the current for each trace being identified in the insert of Fig. 8. The domain velocity deduced from these measurements is directly proportional to the transport current over the same range where the frequency itself displays a direct proportionality (Fig. 8). Note that the obviously "missing" domain for one point in trace 4 actually appeared under the other point. We remark, in passing, that in the highly regular region the time separation between the two points was always observed to change sign on reversing the transport current, corresponding to a reversal in direction of the domain motion.

As previously mentioned, the data exhibited for *one* point on sample 3 (highly strained) could scarcely be distinguished from those for samples 1 and 2. However, the data from two points, shown in Fig. 13, show that a rather severe distortion of the domain structure occurred near one point which was nonetheless "healed" by the time it arrived at the other. Despite the severity of the distortion it is clear, and remarkable, that there is still an excellent correlation between the points even though the C_N value obtained from each is quite different. (For samples 1 and 2, the C_N values are identical in the highly regular region.) Finally, we turn to the data on sample 4, the single-point data for which has been shown in Fig. 7. The second point showed no correlation for traces (a) and (b), and since it merely duplicates the behavior in Figs. 9–12, those data will not be reproduced here. Trace (c) was taken with a sweep four times as fast as that in Fig. 7 and the data for both points are shown with correspondingly greater resolution in Fig. 14. The four oscillations shown were taken from a continuous recorded trace of 15 oscillations. As can be seen, the superconducting domain is bro-

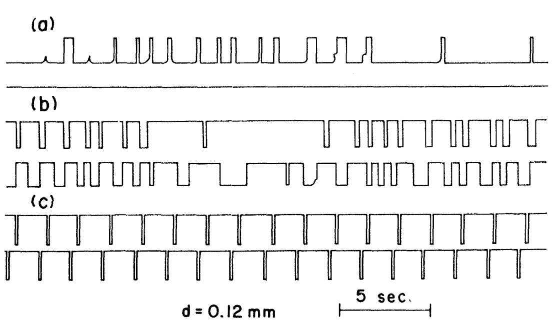

FIG. 10. Upper trace of each pair reproduces that in Fig. 4, while lower trace was recorded simultaneously from a second point the indicated distance d away from the first.

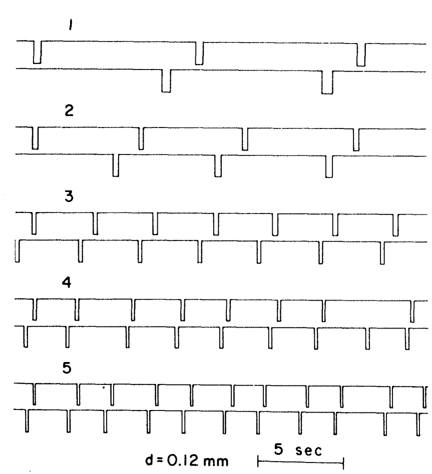

FIG. 12. Upper trace of each pair reproduces that in Fig. 8, while lower trace was recorded simultaneously from a second point the indicated distance d away from the first.

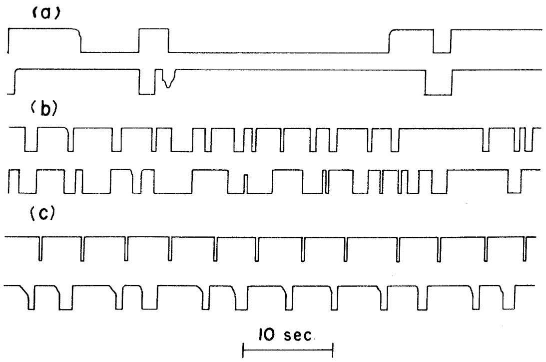

FIG. 13. Upper traces of (a), (b), and (c) reproduce the oscillations shown in Fig. 6, while the lower trace of each pair exhibits oscillations obtained simultaneously from a second point a distance of 0.3 mm away from the first.

ken into many pieces on passing under the first point. The number of resolved pieces varied from three to eight during the entire recorded sequence and the manner of breakage appears quite irregular. In view of this it is quite remarkable that by the time the structure has moved on to the second point the superconducting domain has reassembled itself into a single entity. It should be mentioned that the form of the broken domain depended strongly on temperature and field and we have not observed this extreme type of "domain healing" on any other sample. Clearly, its manifestation depends upon the existence of a chance flaw in the sample near one of the points. It is nonetheless an impressive example of the degree to which the regular behavior for high C_N can assert itself in the presence of flaws in the sample sufficient to completely break up the domain structure at a single point.

IV. DISCUSSION

In terms of the simplest conceivable type of domain motion (uniform translation of the entire domain structure), the drastic improvement in regularity at high-C_N values is very difficult to understand. Any simple argument which invoked a field-dependent effectiveness of the flaws in the sample would give an optimum C_N value that depended on the chance condition of the sample, at variance with the well-marked universal enhancement around $C_N \sim 0.9$.

However, a reasonable physical explanation of the observations may be found if we return to the original observations by Sharvin[4] of the static-domain structure. In that work it was observed that for values of the external field corresponding to low C_N a regular laminar structure was observed, consistent with the original Landau model.[10] However, for field values corresponding to $C_N > 0.86$, this structure was gradually replaced by a system of isolated superconducting domains on which no quantitative observations were made. These appeared to be superconducting domains which, on account of their thinness at the high value C_N, become broken at numerous places along their length. We shall make the hypothesis, then, that the highly regular oscillations we observe near the sample center for $0.85 < C_N < 0.95$ arise from the regular motion of a structure similar to that described by Landau but modified by the existence of gaps in the superconducting domains. With this simple physical picture in mind we can account for the main qualitative features of our observations:

a. Regularity. At the value of C_N when gaps first appear one would expect a drastic enhancement in regularity because the domains have no longer to be dragged through the highly nonuniform fields and currents near the sample ends. They will then experience only the relatively uniform conditions near the sample center. As C_N increases one might anticipate the number and length of the gaps to increase and eventually destroy the regular pattern.

b. Critical Current. The frequency vs current is a straight line through the origin in the highly regular regime (Fig. 8), showing that the usual "critical" current needed to overcome pinning forces is entirely absent or very small. Although our knowledge of pinning in type-I superconductors is rudimentary, it is reasonable to suppose that the ends of a domain would be strongly pinned and their elimination would be partly responsible for the reduction in the critical current.

c. Occasional Flaws in Structure. Although the correlation between the two points is essentially perfect in the highly regular region, it does occasionally happen that a single domain is missing from one of the traces and appears on the other (see Fig. 12, trace 4). This behavior is impossible to understand in terms of motion of a continuous domain structure. However, if the lengths of the isolated domains are somewhat variable, as seems likely, the observation is immediately ex-

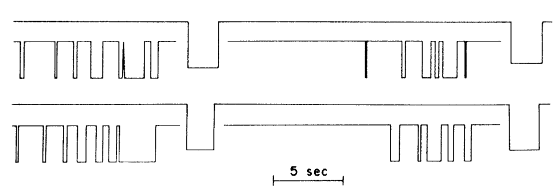

FIG. 14. Upper traces of both pairs show oscillations obtained from the same sequence as that represented by Fig. 7(c), while the lower trace exhibits oscillations obtained simultaneously from a second point 0.5 mm away from the first.

TABLE I. Comparison of observed and calculated domain spacings.

	T (K)	C_N	β	Domain spacing (mm) Obs.	Calc.
Sample 1	1.96	0.94	17°	0.5	1.4
	2.96	0.90	17°	0.5	1.4
Sample 2	2.52	0.86	8°	0.9	2.1

plicable.

Quantitatively, we can compare the spacing of the observed structure with the predictions of the Landau model.[10] Samples 3 and 4 are of course not suitable for this purpose because of the substantial variation in size of the superconducting domains in both cases. However, we can establish domain spacings directly from the traces of Figs. 9–11. The original Landau model referred to fields normal to the sample plane, but a simple extension due to Sharvin allows application to the acute angle case. His expression for the periodicity involves H/H_c, which we have not measured with precision, but assuming the continuous Landau structure it is further easy to show that the relations given by Sharvin[4] lead to the following expression for the periodicity a in terms of C_N, viz.,

$$a=\left(\frac{L\Delta}{\phi(C_N)}\right)^{1/2}\frac{[1+\cos^2\beta(C_N^2-1)]^{1/2}}{\sin\beta}, \qquad (2)$$

where L is the sample thickness, Δ the surface-tension parameter, and ϕ is a dimensionless function calculated by Lifshitz and Sharvin.[11] From Sharvin's static measurements on the lamina structure at lower fields we have that

$$\Delta=\frac{2.5\times10^{-5}}{(1-T/T_c)^{1/2}}\;. \qquad (3)$$

Using Eqs. (2) and (3) [the measured values for T, L, β, C_N, and $\phi(C_N)$ from Ref. 11] one arrives at Table I.

It is interesting to point out that the results in Table I exhibit the same discrepancy between theory and experiment which, to a lesser extent, was noted in the work of Sharvin and Landau on indium.[6] Although that work extended only up to $C_N=0.8$, a clear discrepancy was evident which increased with C_N and was in the same sense as that noted above. However, if the hypothesis we have advanced is correct, then it would be reasonable to expect that modifications would be needed to the Landau theory itself in order to describe the broken structure envisaged. At this stage, our conclusion from the above comparison of theory with experiment is that it is most likely to be that type of structure, rather than some more drastic variant (such as the Landau branching model), which is giving rise to our observed oscillations.

It should be mentioned that Solomon and Harris[3] recently reported the observation of isolated domain motion in tin with the field at a small acute angle to the sample. However, the authors also remarked that the motion occurred between *fixed* superconducting domains. In view of this, it seems unlikely that their observations were made in the highly regular region, since in that region we have never observed one point to give oscillations without the other doing so, although, as pointed out in the text, such behavior did occur outside the regular region.

Finally, none of the results presented here should be interpreted as denying the possibility that for an excellent sample, under ideal conditions, highly regular uniform motion of the whole domain pattern may occur at all fields. Such motion, if it is possible, has apparently yet to be observed.

V. CONCLUSION

We have observed an extremely drastic increase in the regularity of flux motion in superconducting tin at high C_N over a wide temperature range and for a wide range of sample perfection. On the basis of all the evidence discussed above we suggest that this represents the onset of motion of a modified Landau domain structure, freed from the perturbing influence of the ends of the sample.

Note added in proof. One of us (D.E.F.) has now developed[12] a simple method for producing extremely regular motion of the true Landau domain structure over a very wide range of C_N. The method is based directly on the suggestion that concluded this paper. It can therefore be stated that the physical picture hypothesized above now enjoys the additional support of Ref. 12.

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[1]P. R. Solomon, Phys. Rev. Letters 16, 50 (1966).
[2]C. J. van Gurp, Phys. Rev. 178, 650 (1968).
[3]P. R. Solomon and R. E. Harris, Phys. Rev. B 3, 2969 (1971).
[4]Yu. V. Sharvin, Zh. Eksperim. i Teor. Fiz. 33, (1957) [Sov. Phys. JETP 6, 1031 (1958)].
[5]Yu. V. Sharvin, Zh. Eksperim. i. Teor. Fiz. Pis'ma

v Redaktsiyu 2, 287 (1965) [Sov. Phys. JETP Letters 2, 183 (1965)].
[6]Yu. V. Sharvin and I. L. Landau, Zh. Eksperim. i Teor. Fiz. 58, 1943 (1970) [Sov. Phys. JETP 31, 1947 (1970)].
[7]P. Tekel, Zh. Eksperim. i Teor. Fiz. 59, 1943 (1970) [Sov. Phys. JETP 32, 567 (1971)].
[8]B. S. Chandrasekhar, D. E. Farrell, and S. Huang, Phys. Rev. Letters 18, 43 (1967).
[9]J. J. Sabo, Phys. Rev. B 1, 1479 (1970).
[10]L. D. Landau, Zh. Eksperim. i Teor. Fiz. 7, 371 (1937).
[11]E. M. Lifshitz and Yu. V. Sharvin, Dokl. Akad. Nauk SSSR 79, 783 (1951).
[12]D. E. Furrell, Phys. Rev. Letters 28, 154 (1972).

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Joule-Heating Power Dissipation in a Type-II Superconductor Tube

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We have calculated the radial distribution of Joule-heating power dissipation in the wall of a hollow tube of Nb–25-at.% Zr for changing longitudinal applied magnetic field. The superconducting critical current density in the cylinder was assumed to obey the formula $J_c = \alpha_c \times (\mu_0 H_{c2} - B)/(B_0 + B)$, as reported elsewhere. The power-density distribution across the tube wall was calculated as the field increased and decreased from a series of cooled-in background fields representing the idealized physical situation following flux jumps. It was found that at each radial position in the tube the power-density function is far from monotonic. The function has a pronounced discontinuous amplitude increase and may have from one to as many as five changes in slope. For each radial position the power density can be displayed as a three-dimensional surface. Relative temperature changes to be expected in well-cooled and poorly cooled tubes have been estimated from the power-density curves.

INTRODUCTION

Magnetic flux moves in a type-II superconductor by several distinct well-documented processes as the applied magnetic field changes. A viscous *flux flow*[1,2] results, when the Lorentz force on the flux exceeds the local pinning forces provided by any of a number of possible metallurgical structures. Flux flow leads to smooth changes in a well-defined magnetic induction profile in the superconductor and, wherever flux has moved, is accompanied by the spontaneous appearance of a diamagnetic persistent field-dependent critical current density $\vec{J}_c$. During the flux-flow process, power is dissipated in the superconductor by Joule heating due to the $\vec{J}_c \cdot \vec{E}$ process, where $\vec{E}$ is the electric field resulting from the changing induction. We can equally well attribute the heating to resistive losses as local currents are forced to traverse the normal cores of the moving fluxoids.[3]

As the applied field changes monotonically, flux flow is interrupted by catastrophic total instabilities, known as *flux jumps*, which dissipate the body currents; the sample is heated momentarily to the vicinity of its transition temperature T_c, and the induction becomes approximately uniform throughout the material. Jumps are almost universally observed in high-field superconductors and, considering the technological applications of these materials, are their least desirable feature.

Under certain dynamic conditions as the field is changing, the sample is subject to *limited instabilities*, defined by Wipf[4] to be large, but locally and temporally limited, disturbances in the induction. Limited instabilities have been observed through their thermal[5–7] and magnetic[8] effects immediately preceding flux jumps, and, more rarely, under other conditions.[8] Finally, *flux creep* is a small-scale relaxation when the field is not changing and the pinning forces exceed the Lorentz force.[9] Creep has generally been understood to be a thermally activated process, but Wade[10] has recently questioned this activation concept. In the present study, creep will be disregarded.

The qualitative relationship between the thermal and magnetic environment in and around a superconductor, and its flux flow and instability behavior, appear to be generally understood.[1,4,11] However, owing to the complexity of these relationships, only approximate predictions of instabilities in type-II materials with even simple geometries have been made. An accurate knowledge of the rate of heat generation is central to the problem of establishing the conditions under which instabilities occur and propagate. It is our purpose in the following to compute the rate of Joule heating in a supercon-