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Contribution of Defect Dragging to Dislocation Damping. I. Theory*

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The contribution of dragging of point defects attached to dislocation lines, to dislocation damping, to elastic modulus, and to logarithmic decrement, is developed. It is shown that the dragging leads to an initial increase in decrement in a suitable frequency range, determined by other related parameters: dislocation loop length, line tension, and damping constants. The theory predicts a dependence on frequency of ω^{-1}, in contrast to the Koehler-Granato-Lücke (KGL) frequency dependence of ω, explaining the failure of previous experiments to confirm the KGL theory. In a similar manner, the generally accepted dependences on point-defect density are shown to be incorrect at lower frequencies, below a few kHz in copper. For example, it is shown that the dislocation decrement should be proportional to the two-thirds power of the modulus defect, rather than proportional to the square of the modulus defect as previously expected, at large point-defect densities on dislocation lines.

I. GENERAL INTRODUCTION

Some thirty years have passed since Read suggested that dislocations contribute notably to the internal friction of metals.[1] Since then, several mechanisms for this contribution have been proposed and developed.[2,3] The most successful of these, the one most commonly used for interpretations of experimental observations, was initiated by Koehler in 1950.[4] The model was shortly thereafter developed in more detail by Granato and Lücke.[5] Henceforth we shall refer to this as the KGL theory of internal friction.

The KGL model is a string model for a dislocation. In this model the dislocation is endowed with all the attributes of a string so that the mathematical formalism begins with a string equation

$$A\frac{\partial^2 y}{\partial t^2}+B\frac{\partial y}{\partial t}-C\frac{\partial^2 y}{\partial x^2}=\sigma_0 b e^{-i\omega t} . \quad (1)$$

In Eq. (1), A is the effective mass per unit length of dislocation, B is a viscous damping constant, C is the effective line tension of the dislocation, assumed constant, b is the magnitude of the Burger's vector of the dislocation, σ_0 is the amplitude of the applied harmonic stress of angular frequency ω, y is the displacement of an elemental portion of the string at a distance x from one end of the string, and t is time. $A=\pi\rho b^2$, where ρ is the mass density of the material. Each term in Eq. (1) is a force per unit length of dislocation.

The problem is formulated for a dislocation of length l at zero stress so that the boundary conditions on Eq. (1) are

$$y(0,t)=y(l,t)=0. \quad (2)$$

In all of this article we shall be concerned with frequencies sufficiently low (lower than about 10 kHz), so that the inertial term (the first term in the string equation) may be neglected. This may be verified by inserting generally accepted values of B, greater than 5×10^{-5} dyne sec cm^{-2}, and dislocation velocities, limited by (and well below) the speed of sound, in the first two terms of Eq. (1).

Having disposed of the first term in Eq. (1), one may still question the appropriateness of each of the remaining terms: (i) $B\,\partial y/\partial t$. This term represents the viscous damping of dislocation motion, under stress. Since actual dislocation motion involves the creation and motion of kinks, the mounting of Peierl's barriers, etc., the concept of simple viscous motion is an idealization, but the work of Trott and Birnbaum shows that these complexities are reasonably well averaged over in a string model.[6] Furthermore, Leibfried has calculated B in a model of phonon scattering at dislocations, lending further credence to this term.[7] We shall continue to adopt a viscous-force term but with reservations. As developed later, we believe that impurity effects have been underestimated and, concomitantly, the actual dislocation-line damping, overestimated. Also, the possible validity of other frictional terms should be considered. The classical case of constant sliding friction may be important, for example. (ii) $C\,\partial^2 y/\partial x^2$. This term represents the line tension of a dislocation. Since the existence of a line tension implies a line energy and since such an energy is rather well established, a line-tension term seems justified although other restorative terms can be envisioned. It is also likely that $C = C(x)$ but the disregard of this point seems trivial, particularly at lower stresses.

Mott and Nabarro[8] give

$$C = 2Gb^2/\pi(1-\nu) \simeq \tfrac{1}{2}Gb^2 \quad , \tag{3}$$

where G is the shear modulus of the material and ν is Poisson's ratio. (iii) $\sigma_0 b e^{i\omega t}$. This is the driving-force term. The major concern here, as discussed recently by Trott and Birnbaum,[6] lies in the neglect of orientation factors. In metals, the dislocation motion considered here is restricted to glide planes inclined at various angles to the direction of the applied stress. A simple average over direction generally assumed as part of the KGL driving term, may be the source of considerable difficulty, particularly if the sharp dependences on dislocation loop length l predicted by Granato and Lücke are correct.

We shall adopt the position that Eq. (1)—or an expanded version of this string equation, proposed below—is substantially correct and explore the consequences. We do this, however, with two reservations. First, we will restrict our attention to sufficiently small stresses where the internal friction is amplitude independent. The theory of amplitude-dependent internal friction will not be presented at this time. There is considerably more reason to question the adequacies of Eq. (1) for amplitude-dependent internal friction. Second we would not be surprised if some observations fail to adhere precisely to predictions based on Eq. (1) considering the reservations just cited above. In fact, it will be shown in the accompanying paper[9] that observations coincide with the predictions of an expanded form of Eq. (1) with remarkable accuracy.

With these limitations, we shall present the exact solution to Eq. (1) subsequently. By approximating this solution to the same degree as in the KGL model, we will then arrive at the well-known results presented by Granato and Lücke:

$$\delta = a_\delta (\pi E b^2/C^2)\, \omega B \Lambda l^4 \tag{4}$$

and

$$\Delta E/E = a_E (Eb^2/2C)\, \Lambda l^2 \quad , \tag{5}$$

where a_δ and a_E are numerical factors whose values depend on the type of function adopted to describe the distribution of lengths l. If all dislocations are taken of equal length ("δ-function distribution"), $a_\delta = (5!)^{-1}$ and $a_E = (3!)^{-1}$; if an "exponential distribution" is assumed, $a_\delta = a_E = 1$.

The actual values predicted by Eqs. (4) and (5) are important. To make the comparison with theory, we take representative values of the parameters: $B \sim 4\times 10^{-4}$ dyne cm^{-2} sec, $C \sim 4\times 10^{-4}$ dyne, $l \sim 5\times 10^{-4}$ cm. From experiment, $\Delta E/E \cong 2\times 10^{-2}$. Calculating at 1 kHz ($\omega = 2\pi\times 10^3$ sec^{-1}), we find

$$\delta = (a_\delta/a_E)\, 2\pi(\omega B l^2/C)\, \Delta E/E \sim 2\times 10^{-6} \rightarrow 2\times 10^{-4},$$

depending on which of the (extreme) values of a_δ and a_E are adopted. But experimentally, $10^{-3} < \delta < 10^{-2}$ is common. No sensible readjustment of parameters will produce agreement. The implication is that Eqs. (4) and (5) can be valid only at frequencies well above 1 kHz. Equations (4) and (5) (and related predictions) will subsequently be referred to as the GL predictions.

The salient features of Eqs. (4) and (5) are the dependences of δ on l^4 and of $\Delta E/E$ on l^2 and the dependence of δ on ω: (i) Frequency dependence: $\delta \propto \omega$. Of the two features, this has received the least attention. The situation is best summarized by Heiple and Birnbaum who show that, below about 30–40 kHz, the damping is essentially independent of ω in copper.[10] The observations of Heiple and Birnbaum are probably the most complete ones, but other investigators have noted this discrepancy between experiment and the GL prediction. In fact, Routbort and Sack reported that the decrement increased with decreasing frequency in copper below 40 kHz.[11] By way of contrast, the GL predictions are well borne out in the MHz range. We propose that the damping at low frequencies is determined by another mechanism which results in considerably larger losses.

The mechanism which we shall explore below was, in fact, anticipated by Koehler when he wrote: "If the frequency of the applied stress is in the kilocycle range, the impurity atoms are completely unable to follow the alternating stress, since diffusion is an extremely slow process at room temperature."[4] (Koehler was concerned with damping in copper with small impurity additions.) We propose that "pinning points," to be identified with impurity atoms in Koehler's quotation, do follow the motion of dislocations to varying extents and that this "dragging" is highly dissipative. The most conspicuous evidence for this was recently reported.[12] It consists of an initial *increase* in δ as dislocation lines were loaded with "pinners"; the GL prediction calls for a monatonic decrease, as presented in the paragraphs below and, in more detail, subsequently. (ii) Length dependences: $\delta \propto l^4$ and $\Delta E/E \propto l^2$. These dependencies are certainly the most noted ones in the literature on dislocation damping. In comparing experimental observation with theory, the results of irradiation experiments have played a central role. The unique opportunity provided by irradiation experiments has been based on the following logic: (a) The point defects created by irradiation (vacancies, interstitials) act as new pinning points on arrival at dislocation lines. (b) The concentration of new pinning points can be controlled exceptionally well in irradiation studies. In contrast, periodic doping experiments, where impurity atoms act as pinners, are a good deal less controlled. Furthermore, the concentration of pinners which are most effective in pinning dislocations is very small, compounding the difficulty of impurity-doping experiments. (c) The concentration of point-defect pinners can be simply added to the concentration of pre-existing pinners consisting of original impurity atoms and network nodes (nodal points) in the dislocation network of the material. (d) The sole factor effected by the addition of point defects is the average dislocation loop length l appearing in Eqs. (4) and (5). (e) The manner in which l is effected is given by

$$l = l_0/(1+n) \quad , \tag{6}$$

where n is the number of pinners added to the dislocation of initial length l_0.

With these assumptions, it is clear that a test of the model lies in plotting δ and $\Delta E/E$ against the number of point defects which have been created and subsequently diffused to dislocation lines. In fact, the absolute value of n is not generally precisely known for three reasons. First, the actual number of point defects created in the entire lattice is known only through a displacement cross section which is always more or less open to question. Second, of these displacements, only a fraction reach dislocations since a diffusing point defect may be "captured" by a number of other competing traps. Third, the point defects reaching dislocations may not be stable there but may diffuse along dislocations lines ("pipe diffusion"), thereby possibly arriving at dislocation nodes and ceasing to act as pinners, or "boil off" dislocations entirely.

Much of the details of the potential defect-dislocation interactions are determined by other factors such as impurity content and temperature of the material. For this reason, it is desirable to circumvent the hazards which lie in individual plots of δ vs n and $\Delta E/E$ vs n by making a single cross plot of δ vs $(\Delta E/E)^2$. This plot, according to Eqs. (4) and (5), should be linear. It is in this sense that assumption (b) above is entirely correct. However, full verification of the GL predictions requires still more, since models other than the string model may produce such a cross plot: $\delta \propto (\Delta E/E)^2$.

Most of the reported investigations in which the cross-plot test have been applied have failed to demonstrate a linearity between δ and $(\Delta E/E)^2$. The most prominent successful demonstration of the cross-plot test was made by Thompson and Holmes[13,14] at ~11 kHz. Generally the failures to meet the cross-plot test or to observe the predicted length dependences of δ and $\Delta E/E$ have been explained with the introduction of another degree of complexity—a two-dislocation description.[15,16] Briefly, it has been proposed that both the friction and modulus-defect expressions are due to two families of dislocations, presumably screw and edge types, each with its own set of parameters: average loop length, dislocation density, etc. With further parameters at hand, further agreement with the GL predictions has been effected. Further information concerning the nature of two-dislocation types is provided in the accompanying paper.

During the developments in this paper we shall need to consider the distribution of dislocation loop lengths. We merely remark at this point that the relationship Eq. (6) is generally not applicable, despite its wide-spread use. The expression is due to Koehler who derived it for the case of impurity atoms pinning dislocations. In this case, the number of pinners is relatively large so that it is reasonable to derive a distribution function which treats the position of all pinners, including nodal points, as random on a very long line. The resulting distribution is exponential:

$$f(l) = \Lambda(\bar{l})^{-2} e^{-l/\bar{l}} \quad , \tag{7}$$

where $\bar{l}$ is the average loop length and Λ is the density of dislocations. Only the exponential distribution retains its analytical form as the number of pinners is increased, subsequently making Eq. (6) valid. Any other distribution does not lead to Eq. (6) except in the limit of large number of added pinning points n. Unfortunately, most of the more interesting investigations using irradiation techniques involve depositing a very few pinners on a typical dislocation-line segment.

Regardless of the nature of the dislocation length distribution, the GL predictions as given by Eqs. (4)–(6) never lead to an increase in δ upon irradiation unless one assumes that the defects created during irradiation and arriving at dislocations depin dislocations—i.e., remove some pre-existing pinners from the dislocation line. Indeed, Nielsen, upon observing the decrement initially increasing in copper bombarded with protons at 20 °K, casually speculated that this was the case.[17] The details of the initial decrement increase have now been too deeply explored here to allow this explanation. We shall show in the subsequent paper that a model in which defects are dragged with the dislocations is particularly successful in accounting for this increase and its accompanying characteristics.

The concept of an increase in the decrement is sufficiently novel that a physical argument is in order. Suppose that the dislocation line gives rise to no (or very little) energy dissipation when oscillating under an applied stress. Now add one point defect to the line. Any significant drag of this defect must add to the energy losses observed. This simple, intuitive explanation can be extended. Suppose more point defects are now added. Each defect will contribute to the total energy dissipation; however, the amount of dislocation motion becomes more restricted with additional point defects, tending to minimize the energy dissipation per attached defect. These two effects finally balance so that continuing addition of point defects to a dislocation line must inevitably lead to an internal-friction decrease—the decrement must go through a peak value. It is also clear that the ability of a defect to be dragged depends on the frequency of the stress, dragging being more easily effected at lower frequencies. Thus the dragging peak in the decrement should be important at low frequencies. This is clearly not consistent with the frequency dependence of Eq. (4). Note that the dragging effect should go through a maximum as a function of frequency since, at very low frequencies, the dragged defect will keep up with the bowing dislocation which never attains appreciable velocity. Since the loss term is proportional to velocity, the loss goes to zero.

Models have been proposed previously which could account for an increase in δ due to the addition of defects into a crystal. Kamel[18] suggested that oscillating dislocations drag vacancies through the lattice, but failed to present any quantitative description of such a model. Kessler[19] considered the case of a dislocation oscillating through a dilute cloud of impurity atoms. He did not consider the details of the dislocation displacement; only the average effects were treated. He did, however, show that within his approximations the decrement was proportional to the defect density. Both of these two models and the one to be explored in this paper are concerned with the motion of dragging points perpendicular to the dislocation line. Yamafuji and Bauer[20] considered the case of stress-assisted diffusion of a defect along the core of a dislocation. They showed that such a mechanism can give rise to a relaxation-type peak. However, they assumed that the diffusivity of the defect along the core of the dislocation is much greater than the diffusivity of a defect in the pure lattice. Recent measurements by Thompson *et al.* show that, in copper, the reverse is apparently true below about 400 °K.[14]

A modification of the original zero-point-temperature theory of Granato and Lücke, by Teutonico *et al.*, to include the motion of dislocations in the stress field of point defects could possibly result in an increase in the decrement.[21] However, their analysis is rather intractable and it is difficult to discern the consequences in its present form.

With this background we are prepared to proceed to the analysis of our proposed "dragging model." In this analysis we accept certain concepts *ad hoc*, leaving further discussion to later. For example, what really comprises dragging? One further comment is in order, however. The reader should note that our analysis does not directly ever speak of the shortening of the dislocation loop lengths by the addition of dragging points, such as Eq. (6) would predict. In our analysis, a dislocation loop is endowed with a length determined by the distance between nodal points prior to the addition of dragging points and this length is unaltered in any way.

II. DISLOCATION DRAG OF POINT DEFECTS

A. Introduction

In this section we shall calculate the consequences of our model in the dragging of point defects by oscillating dislocations. Following the general procedures of Nowick,[22] we take the applied stress as

$$\sigma = \sigma_0 e^{-i\omega t} \quad , \tag{8}$$

and the resulting strain ϵ to consist of an elastic strain ϵ_e and an anelastic component ϵ_d due to the dislocation motion

$$\epsilon = (\epsilon_e + \epsilon_d) e^{-i\omega t} \quad . \tag{9}$$

We further anticipate that the dislocation strain will include a component out of phase with the applied stress and take

$$\epsilon_d = \epsilon_d' - i\epsilon_d'' \quad . \tag{10}$$

With these definitions of stress and strain and the additional assumption that $|\epsilon_d| \ll \epsilon_e$ (true even in cases of "high" decrements), the appropriate measure of the internal friction is the logarithmic decrement, δ:

$$\delta = \pi \epsilon_d'' / \epsilon_e \quad . \tag{11}$$

The modulus defect—the apparent fractional decrease in elastic modulus due to dislocation motion—is $\Delta E/E$:

$$\Delta E/E = E\,\epsilon_d'/\sigma \quad . \tag{12}$$

Taking $\bar{y}$ as the average displacement during oscillation of a dislocation segment from its equilibrium position and Λ as the density of dislocations (lines per cm^2)

$$\epsilon_d = \Lambda b \bar{y} \quad . \tag{13}$$

Equations (12) and (13) will be used to calculate the modulus defect. Note that the logarithmic decrement is formally defined through energy considerations

$$\delta = \left(\frac{\sigma_0^2}{E}\right)^{-1} \int_0^\tau \sigma \dot{\epsilon}\, dt \quad , \tag{14}$$

the ratio of energy dissipated per oscillatory cycle to twice the total "stored" energy. Equations (14) and (11) are, however, equivalent if $|\epsilon_d| \ll \epsilon_e$.

We will first consider the case of a dislocation loop containing only a single dragging point. Later we will extend the calculation to consider several dragging points on a loop. To relate these calculations to measured parameters (i.e., δ and $\Delta E/E$), it is necessary to also consider the statistical distribution of the number of point defects on dislocation loops. As a very helpful prelude to these complexities involved in the multidragging-point case, we will consider a limiting form of a continuum of dragging points uniformly distributed along dislocation loops.

B. General Formalism

Neglecting inertial effects, the general equation of motion of an oscillating dislocation dragging defects is

$$B(x)\rho(x)\frac{\partial y(x,t)}{\partial t} - C\frac{\partial^2 y(x,t)}{\partial x^2} = b\sigma_0 e^{-i\omega t} \tag{15}$$

where $\rho(x)$ is the density of dragging point (or the line itself) and $B(x)$ is the viscous drag constant for the defect which is at the point x. In this formulation, B may be associated with several types of defects: impurities, self-interstitial, vacancies, etc. In the usual KGL treatment $\rho(x)$ is unity and $B(x)$ is the constant dislocation line damping term B_l. Again the boundary conditions on Eq. (15) are

$$y(0,t) = y(l,t) = 0. \tag{16}$$

The distance l is now taken to be the distance between absolutely firm pinner points, such as clusters of defect or nodal points.

To solve Eq. (15) we assume the steady-state conditions of

$$y(x,t) = y(x)\, e^{-i\omega t} \quad . \tag{17}$$

We take the case of n dragging points, the ith one located at $x = x_i$. With each dragging point, we associate a viscous drag constant B_d. Retaining a frictional contribution B_l due to the dislocation line itself, we have

$$B(x)\rho(x) \rightarrow B_l + \sum_{i=1}^{n} B_d\, \delta(x - x_i) \tag{18}$$

(note that B_l and B_d are not dimensionally equivalent). We now have the time-independent equation

$$\mathfrak{L} y(x) = \frac{i\sigma_0 b}{\omega B_l} - \frac{B_d}{B_l}\sum_{i=1}^{n}\delta(x - x_i)\, y(x) \quad , \tag{19}$$

with the operator $\mathfrak{L}$ defined by

$$\mathfrak{L} \equiv \alpha^2 \frac{d^2}{dx^2} + 1 \tag{20}$$

and

$$\alpha^2 \equiv iC/\omega B_l \quad . \tag{21}$$

The Green's function for this operator is

$$G(x,x') = [1/\alpha \sinh(l/\alpha)] \times \{\sinh[(l - x')/\alpha]\sinh(x/\alpha)\Theta(x' - x) + \sinh(x'/\alpha)\sinh[(l - x)/\alpha]\Theta(x - x')\} \quad , \tag{22}$$

where Θ is the Heavyside function defined by

$$\Theta(a) = \begin{cases} 0, & a < 0 \\ 1, & a > 0 \end{cases} \quad . \tag{23}$$

The value of the displacement y is obtained as

$$y(x) = \int_0^l G(x,x')\frac{i\sigma_0 b}{\omega B_l}\, dx' - \frac{B_d}{B_l}\int_0^l \sum_{i=1}^{n}\delta(x' - x_i)\, y(x')\, G(x,x')\, dx' \quad , \tag{24}$$

so that

$$y(x) = y_0(x) - \frac{B_d}{B_l}\sum_{i=1}^{n} y(x_i)\, G(x,x_i) \quad , \tag{25}$$

with

$$y_0(x) = \frac{i\sigma_0 b}{\omega B_l \sinh(l/\alpha)} \times \{\sinh(l/\alpha) - \sinh(x/\alpha) - \sinh[(l - x)/\alpha]\}. \tag{26}$$

Note the Eqs. (25) and (26) also give the displacement in the absence of point defects ($n = 0$ or $B_d = 0$) for any segment of length l.

Whereas Eqs. (25) and (26) constitute the full solution to our problem, they are of limited direct value since the forms are not particularly revealing and since this solution applies for a particular set of dragging points—i.e., n dragging points at positions x_i. A suitably averaged effect is noted from experiment. With these complications, further discussion of the general solution is delayed to Sec. III B.

C. One-Dragging-Point Case

With only one dragging point on a dislocation loop ($n = 1$), Eq. (25) reduces to

$$y(x) = y_0(x) - (B_d/B_l)\, y(x_i)\, G(x,x_i) \quad , \tag{27}$$

with

$$y(x_i) = y_0(x_i)$$

$$\times\left(1+\frac{B_d}{B_l}\frac{\sinh(x_i/\alpha)\sinh[(l-x_i)/\alpha]}{\alpha\sinh(l/\alpha)}\right)-1 \ , \quad (28)$$

and $G(x, x_i)$ given by Eq. (22) with $x'=x_i$.

Equation (27) is reasonably simple, but for purposes of calculating the modulus defect, the portion of the line-damping term containing B_l is small and can be ignored. This is not true for calculations of the decrement, however. Nor is it true that the effects due to the dragging points can be neglected, as is clear from the following solution.

If we let $B_l \to 0$, then we obtain for the displacement

$$y(x)=\frac{b\sigma_0}{2C}\left(x(l-x)-\frac{i\omega B_d x_i(l-x_i)}{lC+i\omega B_d x_i(x_i-l)}\times[(x_i-l)\,x\Theta(x_i-x)+(x-l)\,x_i\,\Theta(x-x_i)]\right). \quad (29)$$

In the limit that $\omega B_d \to \infty$, i.e., the point defects behave as hard pinners, Eq. (29) reduces to

$$y(x)=(b\sigma_0/2C)\,x(x_i-x) \text{ for } 0<x<x_i \ , \quad (30)$$

the equation of a string pinned at $x=0$ and $x=x_i$, and

$$y(x)=(b\sigma_0/2C)(x-x_i)(l-x) \text{ for } x_i<x<l \ , \quad (31)$$

the equation of a string pinned at $x=x_i$ and $x=l$. From Eq. (29) and Eq. (12), the modulus defect is

$$\frac{\Delta E}{E}=\frac{\Lambda E b^2}{12C}\left(l^2-\frac{3(\omega B_d)^2[x_i(l-x_i)]^3}{(lC)^2+[\omega B_d x_i(l-x_i)]^2}\right). \quad (32)$$

If $\omega B_d \to \infty$, it is a simple matter to show that $\Delta E/E$ is equivalent to the result for two loops of length x_i and $l-x_i$. For intermediate frequencies Eq. (32) correctly predicts the value of the modulus defect. It is straightforward but tedious to extend the calculation to the several-particle case. This will be examined later.

To calculate the decrement we retain B_l to the lowest nonvanishing order in the solution for $y(x)$. The result for the one-pinner case is

$$\delta=\pi\Lambda E b^2\omega\left(\frac{B_l l^4}{5!\,C^2}+\frac{DB_d}{4lB_l}\frac{[x_i l(x_i-l)]^2}{(l^2C)^2+[\omega B_d x_i l(x_i-l)]^2}\right), \quad (33)$$

with

$$D=l^2B_l+\tfrac{1}{30}B_d(\omega B_l/C)^2 l^5[\gamma^5-1+(1-\gamma)^5] \ , \quad (34)$$

and x_i has been replaced by γl.

Again if $\omega B_d \to \infty$, Eq. (33) correctly predicts the decrement for two loops of length x_i and $l-x_i$. The term in the square brackets of (34) is always negative. Thus D can be positive or negative depending on the magnitude of the other parameters. In particular, note the strong dependence on ω and l. For appropriate loop lengths and frequencies, the decrement *increases* with the addition of point defects to the dislocations.

To facilitate the examination of the dependence of the decrement and modulus defect on the dragging term, we will place the defect at $x_i=\frac{1}{2}l$. It is also convenient to define the dimensionless parameters

$$\mu_l \equiv \omega B_l l^2 C^{-1} \text{ and } \mu_d \equiv \omega B_d l C^{-1} \ . \quad (35)$$

With these simplifications we have for the modulus defect

$$\left.\frac{\Delta E}{E}\right|_{x_i=(1/2)l}=\frac{\Lambda E b^2 l^2}{12C}\left(1-\frac{3}{4}\frac{\mu_d^2}{(16+\mu_d^2)}\right), \quad (36)$$

and for the decrement

$$\delta\,|_{x_i=(1/2)l}=\frac{\pi\Lambda E b^2 l^2}{C}\left(\frac{\mu_l}{5!}+\frac{\mu_d(16-\frac{1}{2}\mu_d\mu_l)}{64(16+\mu_d^2)}\right). \quad (37)$$

Note that in the limit that $\mu_d \to \infty$, both Eqs. (36) and (37) reduce to the usual GL values. Furthermore, the dependence of the dragging terms on ω^2 shows that the dragging mechanism should be of particular importance at low frequencies.

The form of Eq. (37) shows that the decrement can increase due to the addition of a single pinner to the line provided $\mu_d\mu_l \lesssim 32$. Therefore, let us examine the displacement of the dragged defect assuming the above conditions to be held. Using typical values of $C=4\times10^{-4}$ dyn, $B_l=4\times10^{-4}$ dyn cm^{-2}sec, $l=5\times10^{-4}$ cm, as previously, and $\omega=3\times10^3$ sec^{-1}, one obtains $\mu_l \sim 0.75\times10^{-3}$ so that $\mu_d \lesssim 4\times10^4$. From Eq. (29), the displacement is given as

$$y\,|_{x_i=(1/2)l}=(b\sigma_0 l^2/8C)[1+(\tfrac{1}{4}\mu_d)^2]^{-1/2} \ . \quad (38)$$

Since $\mu_d \gg 1$,

$$y\,|_{x_i=(1/2)l} \sim b\sigma_0 l^2/2C\mu_d \ .$$

Assuming a strain amplitude of 10^{-6} and a value of $E=4\times10^{11}$ dyn/cm^2 yields

$$y\,|_{x_i=(1/2)l} \sim 10^{-10} \text{ cm}.$$

A displacement of the defect by only $10^{-2}a_0$(a_0 is the lattice parameter) contributes as much to the decrement as the line damping does. Clearly, this dragging mechanism must be an important source of internal friction, particularly for low frequencies.

The previous calculation—that little displacement of point defects dragged by dislocations is needed to account for appreciable energy loss—suggests that a significant portion of the "background damping" (the decrement in a sample before the addition of point defects) may be due to dragging losses rather than to the line losses as usually attributed. Further evidence for this suggestion comes from the KGL theory or, equivalently, from Eq. (37). In the absence of dragging [letting $B_d \to 0$ in Eq. (37)], theory still predicts a linear frequency dependence of decrement on frequency. This is, as stated in the Introduction, at variance with observation, by a factor of perhaps 10^3 in the range below 1 kHz. It is our contention, supported by experimental evidence presented in the following article, that point-defect dragging is unexpectedly lossy. Since the number of point defects which are involved in much of those observations is typically small, a few per dislocation loop, it seems reasonable to speculate that dragging of intrinsic point defects, jogs, and impurities, may contribute appreciably to background damping. Again, we postpone discussion of the nature of dragging until a later section, but note that, if background damping is indeed due in large part to the dragging of impurities, the treatment of Sec. II D should be generally applicable.

D. High Defect Densities

In some experiments one is interested in measuring changes in the decrement and modulus defect due to the presence of many defects on dislocation lines. Indeed, a salient feature of such experiments frequently is that enough defects can be added to the dislocations so that all dislocation motion is essentially eliminated. Moreover, experimental conditions may be such that an appreciable density of defects exists on the dislocation prior to the experiment, as stated in the previous paragraph. Thus it is important to examine the solution to the vibrating-string equation when the density of defects is large enough so that they can be regarded as distributed uniformly along the dislocation segment. For this case, Eq. (15) reduces to

$$B\frac{\partial y}{\partial t} - C\frac{\partial^2 y}{\partial x^2} = b\sigma_0 e^{-i\omega t} \quad . \tag{39}$$

The solution to Eq. (39) is given by (26) with the substitution $B_l \to B$. (See also Refs. 23 and 24.) The decrement is

$$\delta = \frac{\pi E b^2 l^2 \Lambda}{2C\mu^2}\left[1 - \frac{1}{\mu}\left(\frac{\sin\mu + \sinh\mu}{\cosh\mu + \cos\mu}\right)\right] , \tag{40}$$

and the modulus defect is

$$\frac{\Delta E}{E} = \frac{Eb^2\Lambda l^2}{2C\mu^3}\left(\frac{\sinh\mu - \sin\mu}{\cosh\mu + \cos\mu}\right) , \tag{41}$$

with $\mu^2 = \omega B l^2/2C$. For small μ the decrement and modulus defect are

$$\delta = \pi\Lambda E b^2 \omega B l^4/5!\,C^2 \tag{42}$$

and

$$\Delta E/E = \Lambda E b^2 l^2/12C \quad . \tag{43}$$

The above expressions for δ and $\Delta E/E$ differ slightly in magnitude from those given by GL due to their retention of only the first term in the Fourier-series solution of Eq. (39).

The effect of adding defects to dislocation is, of course, to increase B. Thus B should go as

$$B = B_0 + m B_d \quad , \tag{44}$$

where B_0 is the viscous drag constant for defects which produce the background damping (preirradiation damping), m is the number of defects *added* per unit length of dislocation, and mB_d is the additional drag due to these defects.

A graphical representation of the decrement and modulus defect as a function of the universal parameter $\mu = (\omega B l^2/2C)^{1/2}$, given by Eqs. (40) and (41) is presented in Fig. 1. The effect of adding defects to the dislocation is merely to increase μ. We note that if $\mu_0 \equiv (\omega B_0 l^2/2C)^{1/2}$, the initial value of μ before irradiation, etc., is less than μ_p, then the decrement will increase as μ is increased. Thus, an increase in decrement due to the addition of point defects is seen to be a potentially general effect. However, if $\mu_0 \gtrsim \mu_p$, then both the decrement and modulus defect decrease monotonically. We note that a change in μ_0 may produce a large change in the magnitude of the modulus defect, i.e., if μ_0 is small, the modulus defect is large. Merely increasing μ_0 from a value of one to two reduces the modulus defect by a factor of two.

For high defect densities ($\mu \gtrsim 3$) the decrement and modulus defect change according to

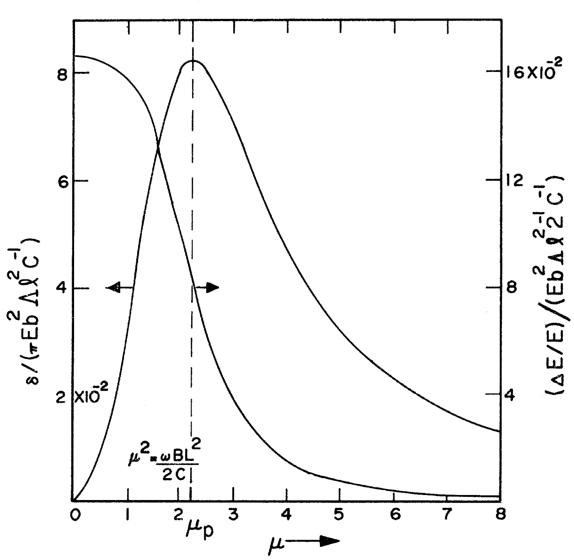

FIG. 1. Decrement and modulus defect plotted as a function of the parameter μ for the continuous-distribution case.

$$\delta = \frac{\pi E b^2 l^2}{2C\mu^2} = \frac{\pi \Lambda E b^2}{\omega(B_0 + mB_d)} \qquad (45)$$

and

$$\frac{\Delta E}{E} = \frac{\Lambda E b^2 l^2}{2C\mu^3} = \frac{\Lambda E b^2 l^2}{2C}\left(\frac{2C}{\omega l^2}\right)^{3/2}\left(\frac{1}{B_0 + mB_d}\right)^{3/2} . \qquad (46)$$

From these expressions we see that the modulus defect should decrease at a *faster* rate than the decrement. Indeed, a ln-ln plot of δ vs $\Delta E/E$ should have an asymptotic slope of $\frac{2}{3}$. This result is entirely contrary to the standard interpretation generally quoted from the GL prediction that the decrement decreases more rapidly.

The solutions to Eq. (39) for δ and $\Delta E/E$, given, respectively, by Eqs. (40) and (41), should be examined closely to note the identity and contrast to the standard GL solutions. The solutions, Eqs. (40) and (41), are exact and hold in the domain usually ascribed to GL predictions, as well as in the dragging model. In the framework of the GL predictions, these solutions are guides to experiments in which the number of defects on dislocation varies by their change in the value of l; for example, Eq. (6) may hold. In the dragging model, variations appear due to change in B. The result of this new orientation, from a variation in l to one in B, is that the low-frequency GL solutions, Eqs. (42) and (43) become high-frequency dragging solutions and $\delta \propto \omega$ is a valid conclusion for relatively high frequencies. In the dragging model, the dependence on frequency is ω^{-1}, corresponding well to the behavior of the δ-vs-μ plot of Fig. 1 for $\mu \gg \mu_p$. This change in frequency dependence explains the reputed failure of experiments to match theory at relatively low frequencies where dragging dominates over line damping. For a fixed concentration of pinning or dragging points, the frequency dependence of decrement should be given by Fig. 1, with ω the variable in the abscissa parameter μ, but with a linear increase becoming dominant for $\mu \gg \mu_p$. For much higher frequencies, inertial effects become important and the theory has been given by Granato and Lücke.

III. STATISTICAL TREATMENT OF DISLOCATION PINNING

A. Firm-Pinner Case

The experimentally measured quantities (δ and $\Delta E/E$) are in effect determined by the average contribution from all dislocation segments. Since the theory is couched in terms of the number of pinners per dislocation segment, it is necessary to (i) determine the number K of pinners per segment when on the average there are n added pinners per segment and (ii) average δ and $\Delta E/E$ for all possible positions of the K pinners along the line. Of course it is assumed that pinners arrive at the dislocation in a completely random manner.

For the hard-pinner case, the effect of adding pinning points is to increase the number of looplets and to shorten the average loop length.

$$\delta = \frac{\pi E b^2}{120C^2}\,\omega B_l\left(\sum_{i=1}^{N} l_i^5\right) , \qquad (47)$$

$$\frac{\Delta E}{E} = \frac{E b^2}{12C}\left(\sum_{i=1}^{N} l_i^3\right) , \qquad (48)$$

where N is the total number of dislocation segments per unit volume and l_i is the length of the ith segment. For dislocation loops of constant length l, $N = \Lambda/l$. Now since $N \sim 10^{10}\ \text{cm}^{-3}$, we can replace the sums by integration, provided we know the distribution function for the l_i's; i.e.,

$$\delta = \frac{\pi E b^2}{120C^2}\,\omega B_l \int_0^\infty \rho(l)\, l^5\, dl , \qquad (49)$$

$$\frac{\Delta E}{E} = \frac{E b^2}{12C}\int_0^\infty \rho(l)\, l^3\, dl . \qquad (50)$$

From the forms of Eqs. (47) and (48) we see that only under very special conditions will

$$\delta \propto \omega B_l (\Delta E/E)^2 . \qquad (51)$$

In fact, the only condition for which this is true when defects are added randomly (i.e., in radiation-damage experiments) is when the initial distribution of l_i's is exponential. This problem was considered some time ago by Thompson and Holmes.[13]

If one knows the loop-length distribution $F(l, n)$ as a function of n, then one merely averages the decrement and modulus defect over all possible values of l. Thompson and Holmes evaluated $F(l, n)$ when the initial distribution $F(l, 0)$ was taken to be exponential in l; see Eq. (7). They showed that for this special case, the distribution remained invariant in form and that l could be replaced by

$$l = l_0/(1+n) . \qquad (52)$$

Since that time, Eq. (52) has been used intact, although Trott and Birnbaum[6] have recently emphasized the perils of this course. It seems more reasonable that for high-purity well-annealed samples, the initial (preirradiated, say) loop-length distribution should be better approximated by a delta function. With this in mind we recalculate, according to the hard-pinner model, the change in δ and $\Delta E/E$ due to the addition of pinners, to an initially δ-function dislocation distribution. This calculation will also serve subsequently as a guide for the pinning-point drag model.

If all segments are the same length and if the pinning points are placed on the segments completely randomly, then the probability that a single segment has K pinners when the average number of pinners is n is given by the Poisson distribution

$$P(K, n) = e^{-n}n^K/K! \quad . \tag{53}$$

Now let φ_K be the average modulus defect when K pinners are on a segment. The total modulus defect will be

$$\varphi(n) = \frac{\Lambda}{l_0} \sum_{K=0}^{\infty} \varphi_K P(K, n) \quad . \tag{54}$$

Thus our problem is now to calculate φ_K. According to Eqs. (47) and (48) the contribution to the modulus defect for a single looplet of length l is proportional to l^3 and to the decrement is proportional to l^5. We will work out the general case for a looplet of length $(x_i - x_{i-1})$ to the pth power. We have $K+1$ looplets per segment and average by letting x_1 range from 0 to x_2, x_2 range from 0 to x_3, $\cdots$, x_K range from 0 to l_0. This averaging can be written as[25]

$$G_{P,K} = \int_0^{l_0} dx_K \int_0^{x_K} dx_{K-1} \cdots \int_0^{x_2} dx_1 \left(\sum_{i=1}^{K+1} (x_i - x_{i-1})^P \right) \Big/ \int_0^{l} dk_K \int_0^{x_K} dx_{K-1} \cdots \int_0^{x_2} dx_1 \ , \tag{55}$$

with $x_0 \equiv 0$, $x_{K+1} = l_0$. Performing the indicated integration yields[25]

$$G_{P,K} = (K+1)!\, p!\, l_0^p/(K+p)! \, . \tag{56}$$

For example,

$$G_{3,K} = 6l_0^3/(K+2)(K+3) \tag{57}$$

and

$$G_{5,K} = 5!\, l_0^5/(K+2)(K+3)(K+4)(K+5) \quad . \tag{58}$$

Averaging these quantities over the Poisson distribution gives

$$\frac{\Delta E}{E} = \frac{\Lambda E b^2 l_0^2}{C} \left(\frac{1}{2n^2}(1+e^{-n}) + \frac{1}{n^3}(e^{-n}-1) \right) \tag{59}$$

and

$$\delta = \frac{\pi \Lambda E b^2 \omega B_l l_0^4}{C^2} \left[e^{-n}\left(\frac{1}{6n^2} + \frac{1}{n^3} + \frac{3}{n^4} + \frac{4}{n^5} \right) - \frac{4}{n^5} + \frac{1}{n^4} \right] \quad . \tag{60}$$

For large n, Eqs. (59) and (60) tend to

$$\frac{\Delta E}{E} = \frac{\Lambda E b^2}{2C} \left(\frac{l_0}{n} \right)^2 \tag{61}$$

and

$$\delta = \frac{\Lambda E b^2 \omega}{C^2} B_l \left(\frac{l_0}{n} \right)^4 \ , \tag{62}$$

which correspond to Eqs. (4) and (5) with

$$l = l_0/n \quad . \tag{63}$$

Equation (63) is, of course, equivalent to Eq. (52) at large n, bringing together conclusions based on initial dislocation length distribution functions of the exponential and δ-function type. The limits for small n of Eqs. (59) and (60) are

$$\frac{\Delta E}{E} = \frac{\Lambda E b^2 l_0^2}{12C} (1 - \tfrac{1}{2}n + \cdots) \tag{64}$$

and

$$\delta = \frac{\pi \Lambda E b^2 \omega B_l l_0^4}{C^2 5!} (1 - \tfrac{2}{3}n + \cdots) \ . \tag{65}$$

For the δ-function initial distribution the initial rate of change of the modulus defect and decrement are $\frac{1}{4}$ and $\frac{1}{6}$ as fast as corresponding changes for the exponential distribution. Also the $l^2 - l^4$ dependence clearly is not observed in the modulus defect and decrement for small n.

B. Dragging Case

The formal procedure for calculating the average modulus defect and decrement are the same for the dragging case as for the firm-pinning-point case. One merely evaluates the modulus defect and decrement contributions for a line segment containing K defects. We have already shown that the line damping can be neglected in calculating the modulus defect and that it probably does not contribute much to the decrement. We therefore consider only the dragging of point defects in this section. Equation (15) reduces in this case to

$$\sum_{i=1}^{K} B_d\, \delta(x - x_i) \frac{\partial y}{\partial t} - C \frac{\partial^2 y}{\partial x^2} = b\sigma_0 e^{-i\omega t} \quad . \tag{66}$$

Analogous to Eq. (25) we have

$$y(x) = y_0(x) - \frac{i\omega B_d}{C} \sum_{i=1}^{K} y(x_i) G'(x, x_i) \ , \tag{67}$$

with

$$G'(x, x_i) = (1/l)[x(x_i - l)\Theta(x_i - x) + x_i(x - l)\Theta(x - x_i)] \tag{68}$$

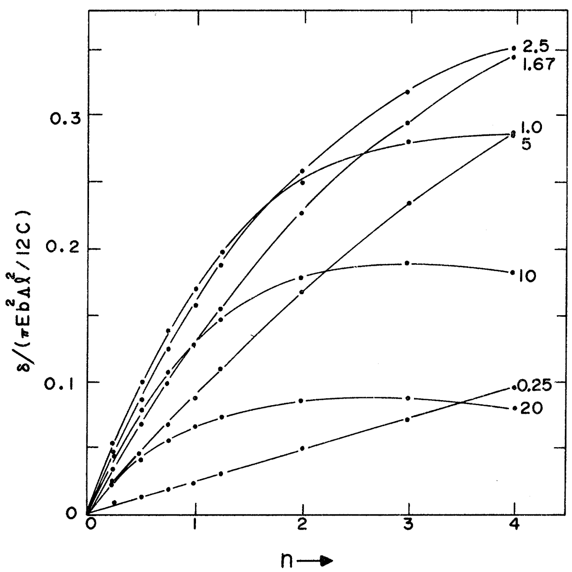

FIG. 2. Decrement plotted as a function of n, the average number of defects per dislocation segment, for the discrete case. Values of $\mu_d = \omega B_d l/C$ are listed beside each curve.

and

$$y_0(x) = b\sigma_0 x(l-x)/2C \quad . \tag{69}$$

We are interested in the average displacement

$$\bar{y} = \frac{1}{l_0}\int_0^{l_0} y(x; x_1, x_2, \cdots, x_K)\,dx$$
$$= \bar{y}(x_1, x_2, \cdots, x_K) \quad , \tag{70}$$

averaged over all possible x_i's subject to the ordering conditions leading to Eq. (55). However, in the case considered here in which defects are dragged, averages of $(x_i - x_{i-1})^P$ no longer suffice. Contributions to the modulus defect and decrement depend on the displacements of the dragged points, as well as on their position along dislocations.

To calculate $\bar{y}$, it is first necessary to calculate y from Eq. (67) and this requires evaluation of $y(x_i)$. For K defects on a line Eq. (67) represents K simultaneous equations in the K values of $y(x_i)$:

$$y(x_j) = y_0(x_j) - \frac{i\omega B_d}{C}\sum_{i=1} y(x_i)\,G(x_j, x_i) \tag{71}$$

or, in parallel operator form,

$$Y_0 = (G+I)Y \quad , \tag{72}$$

with formal solution

$$Y = (G+I)^{-1}Y_0 \quad , \tag{73}$$

in which I is the identity operator. For fuller description of these operators and general methods of solution for $\mu_d = 0$ and $\mu_d = \infty$ and for cases near these extremes, see the matrix developments by Rosenstock.[25]

This averaging was performed numerically by us by using the Monte Carlo technique on a UNIVAC 1108 computer. Random numbers were selected for all x_i's and the following sum and average was evaluated:

$$\bar{y}(\mu_d) = \frac{1}{N}\sum_{s=1}^{N}\bar{y}_s(\mu_d; x_1, x_2, \ldots, x_K) \quad . \tag{74}$$

Usually N was taken as 1000. Plots of the decrement and normalized inverse modulus defect are given in Figs. 2 and 3 as a function of n, the average number of defects per line segment. This averaging was performed by using the Poisson distribution according to Eq. (54) with K ranging from 0 to 10. Cutting the sum off at $K = 10$ introduces a negligible error provided n is no larger than five. We see that the dragging mechanism is significant only over about one order of magnitude range in μ_d. For low frequencies $\mu_d \lesssim 1$ the modulus defect changes slowly with n and the decrement increases. Thus for low-frequency radiation-damage experiments, one would predict an increase in the decrement as point defects were added to the dislocation line.

IV. FURTHER DISCUSSION

In this paper we have examined, in some depth, the behavior of dislocation damping if attached im-

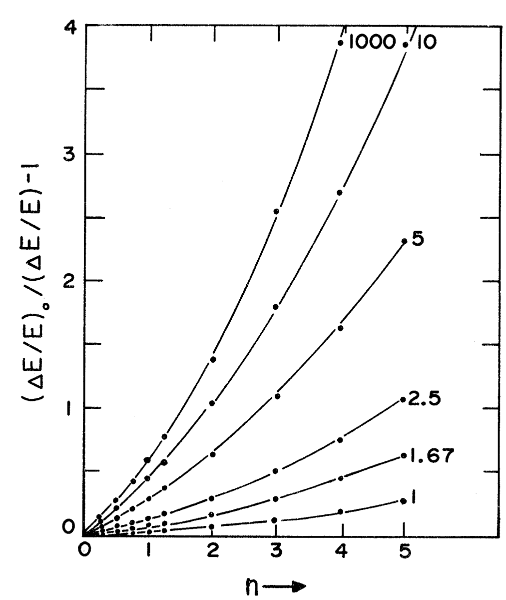

FIG. 3. Normalized inverse modulus defect plotted as a function of n, the average number of defects per dislocation segment for the discrete case. Value of $\mu_d = \omega B_d l/C$ are listed beside each curve.

perfections are allowed to oscillate, albeit in a restricted manner, with the dislocation line in response to an alternating stress. The most distinctive feature of this dragging model is the prediction of an initial increase in damping with the addition of defects at suitably low frequencies and, concomitantly, appropriate values of average dislocation loop length, damping constants, etc. Using typical values we have shown that the amount of actual displacement of the defects may be truly small—a fraction of an interatomic distance on occasion. We have also shown that the dependences of the modulus defect and decrement is altered from the generally accepted characteristics, in the appropriate range of parameters, and that the long-standing discrepancy with the GL predictions on frequency dependence of the decrement are resolved in the dragging model.

Verification of many of these predictions has been reported in a previous note[12] and in the accompanying article.[9]

A number of more speculative matters remain. In particular, it is important to ask: What is the nature of defect drag? Here we distinguish between drag of self-defects (vacancies and self-interstitials) and impurities. The capture of self-defects at dislocations, a well-established effect now, implies that the defect is located in a potential well with high slopes against release from the dislocation. The evidence from the work of Thompson *et al.*[14] and in the accompanying paper points to a wall height in the direction along the dislocation which is considerably lower. Thus motion laterally along dislocations should be relatively simple compared to the drag described in this paper. This motion, discussed by Yamafuji and Bauer,[20] has not been included in our calculations since the ease of motion implies that defects can move in phase with the dislocation in their lateral motion, thereby giving rise to little additional friction above the standard line damping. In contrast, dragging would appear to be only slightly influenced by thermal activation—witness the observations of Nielsen[17] of an initial increase in damping on proton bombardment at 20 °K in copper and our similar observations on electron bombardment, not yet reported elsewhere. Thus we tend to equate drag with an essentially athermal diffusion of self defects. In the case of a self interstitial, this would be an athermal interstitialcy diffusion: the interstitial-atom identity is shifted as the dislocation oscillates.

The large dissipation due to defect drag combined with the apparent inadequacies of line damping to account for background damping (in the absence of defects) raises the question as to the contribution to background from other defects, notably impurities. In the accompanying paper,[9] the importance of thermal history on the nature of the internal friction peak, depicted in Fig. 1, points to the importance of impurities. This is discussed further in that article.

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Contribution of Defect Dragging to Dislocation Damping. II. Experimental*

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Detailed internal-friction and Young's-modulus measurements have been made in electron-irradiated copper. The measurements confirm the defect-dragging-model calculations presented in the previous paper. In particular, an initial increase and subsequent decrease in logarithmic decrement δ is observed. This peaking effect is shown to be highly structurally dependent, vanishing after a suitable high-temperature annealing treatment. The proportionality of δ to the modulus defect $\Delta E/E$ is demonstrated. This dependence, contrary to previous expectations that $\delta \propto (\Delta E/E)^2$, is consistent with the dragging model. Agreement with the dragging model is quite complete if two types of dislocations are assumed, as has been reported by other investigators.

I. INTRODUCTION

In the previous paper,[1] we have developed a model for dislocation damping in metals—actually a low-frequency extension of the Koehler-Granato-Lücke (KGL) model.[2,3] The predictions of this new formulation explain some of the discrepancies noted by many observers but rarely documented in any sufficient detail. Furthermore, the formulation's predictions are notably different from those predictions investigators have generally drawn—incorrectly—from the KGL theory.

This paper presents a more detailed investigation of damping at relatively low frequencies, to test the predictions of our newer model.

II. EXPERIMENTAL PROCEDURE

A. Sample Preparation

The experiments for this work were performed on high-purity (99.999%) copper from the American Smelting and Refining Company. The copper was received in an extruded and swaged rod $\frac{3}{8}$ in. in diameter. From this material a sample was machined into the shape of a cantilevered beam with active sample dimension in the form of a foil of thickness (about 0.006–0.007 in.), 1-cm length, and $\frac{1}{2}$-cm width. After machining, the sample was etched to a final thickness of 0.004–0.005 in. Copper samples of these dimensions usually have a natural resonant frequency of about 500 Hz when resonated in the flexural mode. Following the final etch, samples were mounted in the appropriate irradiation apparatus without any intervening heat treatment. The samples were polycrystalline.

B. Equipment

Our experimental arrangement allows us to continuously (every 2–3 sec) monitor the logarithmic decrement and Young's modulus as a function of time. To do this we employ a capacitive pickup system (the sample forms half of a parallel-plate capacitor) similar to that described by DiCarlo *et al.*,[4] but with the advantage of automatic data logging.

It can be shown[5] that the decrement can be related to the force F required to maintain a damped harmonic oscillator vibrating at constant amplitude through

$$\delta = \delta_0(\omega/\omega_0)^2 F/F_0 , \quad (1)$$

where the subscript 0 refers to preirradiation values. Young's modulus E is obtained from the resonant frequency of the bar by

$$E = kf^2 . \quad (2)$$

The force acting on the sample is proportional to the square of the voltage impressed across the sample and drive plate. This voltage will be referred to as the drive voltage. The decrement is now given by

$$\delta = \delta_0(\omega/\omega_0)^2 V^2/V_0^2 . \quad (3)$$

Usually $\omega/\omega_0 \sim 1$, and so $\delta \propto V^2$. To measure δ_0, the signal from the freely decaying sample amplitude is stored in a storage scope. A photograph is taken and the log decrement measured in the usual manner. It is difficult to estimate uncertainty limits u in the relative changes in the decrement and Young's modulus, but typical values would be